$$\frac{2K\,dp\,ddp}{dt^2} = \mathfrak{B}\,dp;\quad \frac{2K\,dx\,ddx}{dt^2} = B\,dx$$

$$\frac{2L\,dq\,ddq}{dt^2} = -\mathfrak{B}\,dq;\quad \frac{2K\,dy\,ddy}{dt^2} = -B\,dy$$

Est vero $dp - dq = -bd.\sin\zeta - cd.\sin\eta$
$dx - dy = -bd.\cos\zeta - cd\cos\eta$

Ergo $\dfrac{K(dp^2+dx^2)+L(dq^2+dy^2)}{dt^2} = -\int\mathfrak{B}(bd.\sin\zeta + bd.\sin\eta) - \int B(bd.\cos\zeta + cd.\cos\eta)$

Porro vero est $\dfrac{Kkkd\zeta^2}{dt^2} = \int\mathfrak{B}\,bd.\sin\zeta + \int B\,bd.\cos\zeta$ atque $\dfrac{Llld\eta^2}{dt^2} = \int\mathfrak{B}\,cd.\sin\eta + \int B\,cd.\cos\eta$; quibus in unam summam collectis erit.

$$\frac{K(dp^2+dx^2+kkd\zeta^2)+L(dq^2+dy^2+lld\eta^2)}{dt^2} = \text{Constanti}$$

At vero ista expressio exhibet vim vivam totius corporis, nam $\dfrac{K(dp^2+dx^2+kkd\zeta^2)}{dt^2}$ est vis viva articuli A B; atque $\dfrac{L(dq^2+dy^2+lld\eta^2)}{dt^2}$ est vis viva articuli BC.

Scholion.

43. Cum generaliter æquatio ultimo inventa $dt = \dfrac{du\sqrt{fg(mm-nn-\cos u^2)}}{\sqrt{(mf-g+f\cos u)}}$ integrationem non admittat, casus

casus sunt perpendendi, quibus integratio succedit, atque ideo motus commodius definiri queat. Sic enim obtinebimus, ut quoties exemplum proponitur ad unum horum casuum pertinens, solutionem facilius exhibere queamus. Occurrunt autem potissimum tres casus sequentes.

Casus. I.

44. Æquationi scilicet $dt = \frac{du\sqrt{fg(mm - nn - \cos u^2)}}{\sqrt{(mf - g + f\cos u)}}$ primum satisfit, si $mf - g + f\cos u = 0$, unde erit $\cos u = \frac{g - mf}{f}$. Hic ergo casus locum habet, si angulus $u = \zeta - \eta$ maneat constans, seu si corpus ABC initio ita fuerit projectum, ut angulus ABC non varietur. Tum igitur, quasi nullam in B haberet juncturam, instar corporis rigidi uniformiter in directum feretur. Erit autem $dv = \frac{dt\sqrt{f}}{m + \cos u} = \frac{dt\sqrt{f}}{g}$, & $v = \frac{t\sqrt{f}}{g}$; unde si ponatur $u = 2i$ fiet $\zeta = at + i$ & $\eta = at - i$ posito $\frac{\sqrt{f}}{g} = 2a$. Ex his elicitur:

$$p = \frac{\mathfrak{F}t + \mathfrak{f} - Lb\sin(at + i) - Lc\sin(at - i)}{K + L}$$

$$q = \frac{\mathfrak{F}t + \mathfrak{f} + Kb\sin(at + i) + Kc\sin(at - i)}{K + L}$$

$$x = \frac{\mathfrak{G}t + \mathfrak{g} - Lb\cos(at + i) - Lc\cos(at - i)}{K + L}$$

$y =$

$$y = \frac{\mathfrak{G}t + \mathfrak{g} + Kb\cos(\alpha t + i) + Kc\cos(\alpha t - i)}{K + L}$$

Hinc ergo erunt celeritates punctorum K & L secundum directiones axium $O o$ & $O \omega$.

$$\frac{dp}{dt} = \frac{\mathfrak{F} - L\alpha b\cos(\alpha t + i) - L\alpha c\cos(\alpha t - i)}{K + L}$$

$$\frac{dq}{dt} = \frac{\mathfrak{F} + K\alpha b\cos(\alpha t + i) + K\alpha c\cos(\alpha t - i)}{K + L}$$

$$\frac{dx}{dt} = \frac{\mathfrak{G} + L\alpha b\sin(\alpha t + i) + L\alpha c\sin(\alpha t - i)}{K + L}$$

$$\frac{dy}{dt} = \frac{\mathfrak{G} - K\alpha b\sin(\alpha t + i) - K\alpha c\sin(\alpha t - i)}{K + L}$$

At vero celeritas rotatoria utriusque articuli erit

articuli AB $= \alpha$

articuli BC $= \alpha$.

Si ergo ambobus articulis initio aequales motus rotatorii imprimantur, insuperque celeritates progressivae punctorum K & L ejusmodi fuerint, ut in expressionibus inventis contineantur, tum corpus quasi nullam haberet flexuram promovebitur. Si centrum gravitatis totius corporis quiescere assumatur, erunt $\mathfrak{F} = 0$ & $\mathfrak{G} = 0$, atque si id in puncto O existat, fiet simul $\mathfrak{f} = 0$ & $\mathfrak{g} = 0$. Quod si ergo ponamus corpus ABC initio super linea $O o$ in directum jacuisse, quia tum erat $t = 0$, debebit esse $i = 90°$, & celeritates secundum directionem $O o$ evanescent, reliquae vero in directione $O \omega$ erunt

$$\frac{dx}{dt} = \frac{L\alpha b - L\alpha c}{K + L} \quad \& \quad \frac{dy}{dt} = -\frac{K\alpha b + K\alpha c}{K + L}$$

Casus II.

45. Hic casus secundus locum habet si fuerit $f = o$ & $n = 0$, hoc est si fuerit

$$(K+L)(Kkk - Lll) = KL(ee - bb)$$

tum autem fiet $dv = 0$, ideoque $v =$ constanti. Sit ergo $v = 2i$, fietque $dt = \frac{du\sqrt{g(mm - \cos u^2)}}{\sqrt{(m+\cos u)}} = du\sqrt{g}$

Fig. 10. $(m - \cos u)$. Quoniam v est constans, recta angulum KBL bisecans, quae sit MBN, cum axe Oo angulum constantem OND perpetuo constituet: cum enim sit KBL $= 180° - \zeta + \eta$, erit KBM $= 90° - \frac{1}{2}\zeta + \frac{1}{2}\eta$ & MB$b = 90° + \frac{1}{2}v$, ergo OMB $= \frac{1}{2}v = i$.

Quoties ergo hujusmodi corpus, in quo est

$$K(Kkk + Lkk + Lbb) = L(Kll + Lll + Kee)$$

initio ita projiciatur, ut positio rectae angulum KBL bisecantis non varietur, tum motus ad hunc casum secundum pertinebit, atque linea MN perpetuo eandem inclinationem ad axem conservabit. Cum deinde sit angulus KBL $= 180 - \zeta + \eta = 180 - u$, ex quovis angulo KBL, quem inter motum induit, definiri poterit tempus ab initio elapsum $t = \int du\sqrt{g(m - \cos u)}$. Quia ergo est $\zeta = \frac{v+u}{2}$ & $\eta = \frac{v-u}{2}$ erit $\frac{d\zeta}{dt} = \frac{du}{2dt} = \frac{1}{2\sqrt{g(m-\cos u)}}$ & $\frac{d\eta}{dt} = -\frac{1}{2\sqrt{g(m-\cos u)}}$, quarum illa exprimit celeritatem rotatoriam articuli AB, haec vero articuli BC. Praeterea

terea

terea vero si ponamus centrum gravitatis in puncto O quiescens, erit

$$p = -\frac{L(b\sin\zeta + c\sin\eta)}{K+L};\quad x = -\frac{L(b\cos\zeta + c\cos\eta)}{K+L}.$$

$$q = \frac{K(b\sin\zeta - c\sin\eta)}{K+L};\quad y = \frac{K(b\cos\zeta - c\cos\eta)}{K+L}$$

Celeritates vero punctorum K & L erunt

$$\frac{dp}{dt} = -\frac{L(b\cos\zeta - c\cos\eta)}{2(K+L)\sqrt{g(m-\cos u)}};\quad \frac{dx}{dt} = \frac{L(b\sin\zeta - c\sin\eta)}{2(K+L)\sqrt{g(m-\cos u)}}$$

$$\frac{dq}{dt} = \frac{K(b\cos\zeta - c\cos\eta)}{2(K+L)\sqrt{g(m-\cos u)}};\quad \frac{dy}{dt} = -\frac{K(b\sin\zeta - c\sin\eta)}{2(K+L)\sqrt{g(m-\cos u)}}$$

Si initio corpus ABC super axe $O\vartheta$ in directum jacuerit, ita ita ut fuerit $\zeta = 90^\circ$ & $\eta = 90^\circ$, ideoque $v = 180^\circ$, & $u = 0$ celeritates secundum axem $O\vartheta$ evanescent; at secundum axem $O\omega$ erit

$$\text{celeritas puncti K} = \frac{L(b-c)}{2(K+L)\sqrt{gm}}$$

$$\text{celeritas puncti L} = -\frac{K(b-c)}{2(K+L)\sqrt{gm}}$$

& celeritas rotatoria articuli AB $= \frac{1}{2\sqrt{gm}}$

celeritas rotatoria articuli BC $= \frac{-1}{2\sqrt{gm}}$

Casus. III.

46. Sit utriusque articuli universa materia in ejus

centro gravitatis concentrata, erit $k=0$ & $l=0$; unde fit $m=\frac{b}{2c}+\frac{c}{2b}$; $n=\frac{b}{2c}-\frac{c}{2b}$ & $m^2-n^2=1$.

Habebimus ergo $dt=\frac{du\sin u.\ \sqrt{fg}}{\sqrt{(mf-g+f\cos u)}}$; cujus integrale est

$$t=\sqrt{h}-\frac{2\sqrt{g}}{\sqrt{f}}\sqrt{(mf-g+f\cos u)}$$

unde fit $2\sqrt{g(mf-g+f\cos u)}=\sqrt{fh}-t\sqrt{f}$ &

$4g(mf-g+f\cos u)=fh-2ft\sqrt{h}+ftt$ ideoque

$$\cos u=\frac{4gg-mfg+fh-ft\sqrt{h}+ftt}{4fg}.$$ Deinde vero erit

$$dv=\frac{4gdt\sqrt{f}}{4gg+(\sqrt{fh}-t\sqrt{f})^2}-\frac{ndu}{m+\cos u}.$$

Partis prioris integrale est $=-2\,\mathrm{A\,tang.}\frac{\sqrt{fg}-t\sqrt{f}}{2g}$, ad posterius integrale inveniendum ponatur $\cos u=s$, erit $du=\frac{-ds}{\sqrt{(1-ss)}}$ & $\frac{-ndu}{m+\cos u}=\frac{nds}{(m+s)\sqrt{(1-ss)}}$; fit $s=\frac{2r}{1+rr}$, erit $\sqrt{(1-ss)}=\frac{1-rr}{1+rr}$ & $ds=\frac{2dr(1-rr)}{(1+rr)^2}$ atque $\frac{ds}{\sqrt{(1-ss)}}=\frac{2dr}{1+rr}$; unde fit $\frac{nds}{(m+s)\sqrt{(1-ss)}}=\frac{2ndr}{m+2r+mrr}$; cujus integrale est $2\,\mathrm{A\,tag}\frac{nr}{m+r}$ ob $n=\sqrt{(mm-1)}$. Ergo ob $rr=\frac{1-\sqrt{(1-ss)}}{1+\sqrt{(1-ss)}}=\frac{(1-\sqrt{(1-ss)})^2}{ss}$,

erit

erit $r = \frac{1-\sqrt{(1-ss)}}{s} = \frac{1-\sin u}{\cos u}$, atque $\int \frac{-ndu}{m+\cos u}$

$$= 2A\,\mathrm{tag}\,\frac{n-n\sin u}{m\cos u+1-\sin u} = 2A\,\mathrm{tag}.\,\frac{n}{1+m\,\mathrm{tag}(45^\circ+\frac{1}{2}u)}.$$

Quapropter erit

$$v = -2A\,\mathrm{tág}\sqrt{\frac{mf-g+f\cos u}{g}} + A\,\mathrm{tág}.\frac{n(1-\sin u)}{m\cos u+1-\sin u} \text{ vel}$$

$$v = -2A\,\mathrm{tág}\sqrt{\frac{mf-g+f\cos u}{g}} + A\,\mathrm{tág}\,\frac{n(1-\sin u+m\cos u)}{1+nn\sin u+m\cos u}$$

Ceterum cum sit $\cos u = \frac{4gg-4mfg+f(\sqrt{h}-t)^2}{4fg}$; patet t non ultra certum limitem augeri posse: Cumprimum enim ista fractio unitatem superat, angulus u fit imaginarius sicque motus continuatio cessare debebit, qui casus maxime est notabilis.

Ponamus initio quo $t=0$, ambos articulos AB & BC in directum jacuisse, seu fuisse $\eta = \zeta$, ideoque $u = 0$, & $\cos u = 1$: erit ergo $\sqrt{h} = \frac{2\sqrt{g}}{\sqrt{f}}\sqrt{(mf-g+f)}$ ideoque

$$t = \frac{2\sqrt{g}}{\sqrt{f}}\sqrt{(mf-g+f)} - \sqrt{(mf-g+f\cos u)})$$

crescente ergo tempore crescet angulus u, neque vero ultra duos rectos augeri potest, quia tum t iterum decresceret: motus ergo eousque tantum continuabitur, quoad fiat $u = 180$ & $\cos u = -1$, tantumque tempus, quo motus durabit, erit

$$t = \frac{2\sqrt{g}}{\sqrt{f}}(\sqrt{(mf-g+f)} - \sqrt{(mf-g-f)})$$

Quod

Quod paradoxon refolvetur, fi ad celeritatem rotatoriam attendatur: cum enim fit celeritas rotatoria articuli $AB = \frac{d\zeta}{dt} = \frac{dv + du}{2dt}$ ob $\frac{du}{dt} = \frac{\sqrt{(mf - g + f\cos u)}}{\sin u . \sqrt{fg}}$ fiet ea, fi $\sin u = 0$, infinita, ex quo intelligitur hoc cafu in expreffione $mm - nn$ terminos continentes kk & ll, etiamfi minimi ftatuantur, negligi non poffe. Quocirca motus tantum definiri poterit, quamdiu $\sin u$ non eft $= 0$. Si igitur initio motus ftatuatur $u = 0$, tum neque ipfum motus initium recte definitur, neque per hunc calculum motum eousque profequi licet, quoad fiat $u = 180°$. Quamdiu autem u non fit $= 180$, motus recte affignatur eritque

$$\sqrt{(mf - g + f\cos u)} = \sqrt{(mf - g + f)} - \frac{t\sqrt{f}}{2\sqrt{g}} \text{ feu}$$

$$f\cos u = f - \frac{t\sqrt{f(mf - g + f)}}{\sqrt{g}} + \frac{ftt}{4g} \text{ ideoque}$$

$$\cos u = 1 - \frac{t\sqrt{(mf - g + f)}}{\sqrt{fg}} + \frac{tt}{4g}.$$

Cum deinde fit $\frac{du}{dt} = \frac{\sqrt{(mf - g + f\cos u)}}{\sin u . \sqrt{fg}}$ & $\frac{dv}{dt} =$

$$\frac{1}{(m + \cos u)\sqrt{f}} - \frac{ndu}{(m + \cos u)dt} = \frac{t_1}{(m + \cos u)\sqrt{f}} - \frac{n\sqrt{(mf - g + f\cos u)}}{\sin u(m + \cos u)\sqrt{fg}},$$

hinc ad quodvis tempus motus rotatorius utriusque articuli affignabitur. Hincque porro per folutionem generalem celeritates veras utriusque centri gravitatis K & L definire licet.

Scho-

Scholion.

47. Generaliter autem ex statu ac motu initiali constantes f & g, quae in solutione insunt, definiri sicque solutio ad quemvis casum particularem accommodari potest. Ponamus, quo omnes casus reduci possunt, commune centrum gravitatis in puncto O quiescere, ita ut sit $\mathfrak{z} = 0$, $\mathfrak{f} = 0$ $\mathfrak{B} = 0$, & $\mathfrak{q} = 0$. Deinde assumamus initio ambos articulos indirectum fuisse sitos, ita ut fuerit $\zeta = \eta$ & $u = 0$, & quoniam positio axis Ov est arbitraria, sumamus initio lineam rectam ABC in axem Ov incidisse, ita ut fuerit $\zeta = 0$, & $\eta = 0$, ideoque & $v = 0$. Fuit ergo initio Fig. 11

$$OK = \frac{Lb + L\beta}{K + L} \text{ \& } OL = \frac{Kb + K\beta}{K + L}.$$

Deinde cum sit $\frac{du}{dt} = \frac{\sqrt{(mf - g + f\cos u)}}{\sqrt{fg(mm - nn - \cos u^2)}}$ & $\frac{dv}{dt} =$

$$\frac{1}{(m + \cos u)\sqrt{f}} - \frac{n\,du}{(m + \cos u)\,dt} = \frac{1}{(m + \cos u)\sqrt{f}} - \frac{n\sqrt{(mf - g + f\cos u)}}{(m + \cos u)\sqrt{fg(mm - nn - \cos u^2)}}$$

ob $\zeta = \frac{v + u}{2}$ & $\eta = \frac{v - u}{2}$ erit

$$\frac{d\zeta}{dt} = \frac{1}{2(m + \cos u)\sqrt{f}} + \frac{(m - n + \cos u)\sqrt{(mf - g + f\cos u)}}{2(m + \cos u)\sqrt{fg(mm - nn - \cos u^2)}}$$

$$\frac{d\eta}{dt} = \frac{1}{2(m + \cos u)\sqrt{f}} - \frac{(m + n + \cos u)\sqrt{(mf - g + f\cos u)}}{2(m + \cos u)\sqrt{fg(mm - nn - \cos u^2)}}$$

Initio igitur quo erat $u = 0$, fuit

$$\frac{d\zeta}{dt} = \frac{1}{2(m+1)\sqrt{f}} + \frac{(m-n+1)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(m^2-n^2-1)}}$$

$$\frac{d\eta}{dt} = \frac{1}{2(m+1)\sqrt{f}} + \frac{(m+n+1)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(m^2-n^2-1)}}.$$

Cum igitur punctorum K & L celeritates sint:

in directione Oo	in directione $O\omega$
K; $\frac{dp}{dt} = \frac{-Lb\,d\zeta\cos\zeta - L\beta\,d\eta\cos\eta}{(K+L)dt}$	$\frac{dx}{dt} = \frac{Lb\,d\zeta\sin\zeta + L\beta\,d\eta\sin\eta}{(K+L)dt}$
L; $\frac{dq}{dt} = \frac{Kb\,d\zeta\cos\zeta + K\beta\,d\eta\cos\eta}{(K+L)dt}$	$\frac{dy}{dt} = -\frac{Kb\,d\zeta\sin\zeta - K\beta\,d\eta\sin\eta}{(K+L)dt}$

initio celeritates punctorum secundum directionem $O\omega$ evanuerunt. In directione autem Oo erant celeritates inter se ut L ad K & altera alterius erat negativa. Quod si ergo ponamus celeritatem puncti K in directione Kk fuisse $= \frac{L\sqrt{h}}{K+L}$ & celeritatem puncti L in directione Ll fuisse $= \frac{K\sqrt{h}}{K+L}$ erit $\sqrt{h} = \frac{b\,d\zeta}{dt} + \frac{\beta\,d\eta}{dt}$ seu

$$\sqrt{h} = \frac{b+\beta}{2(m+1)\sqrt{f}} + \frac{(b-\beta)\sqrt{(mf-g+f)}}{2\sqrt{fg(m^2-n^2-1)}} - \frac{n(b+\beta)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(m^2-n^2-1)}}$$

Pona-

Ponamus:
$\frac{1}{2(m+1)\sqrt{}}$
& $\frac{\sqrt{}}{2\sqrt{}}$
eratque ini
Articuli
articuli
& celeritas
& celeritas
Erit autem
$2(m+1)$
seu $\mu =$
Quia po
erit $(m+$
1

Ponamus:

$$\frac{1}{2(m+1)\sqrt{f}} - \frac{n\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(mm-nn-1)}} = \frac{\mu}{\sqrt{f}}$$

& $\frac{\sqrt{(mf-g+f)}}{2\sqrt{fg(mm-nn-1)}} = \frac{\nu}{\sqrt{f}}$.

eratque initio celeritas rotatoria circa centrum gravitatis

Articuli AB $= \frac{\mu+\nu}{\sqrt{f}}$

articuli BC $= \frac{\mu-\nu}{\sqrt{f}}$

& celeritas puncti K secundum K$k = \frac{L((b+\beta)\mu+(b-\beta)\nu)}{(K+L)\sqrt{f}}$

& celeritas puncti L secundum L$l = \frac{K((b+\beta)\mu+(b-\beta)\nu)}{(K+L)\sqrt{f}}$

Erit autem $\frac{\mu}{\sqrt{f}} + \frac{n\nu}{(m+1)\sqrt{f}} = \frac{1}{2(m+1)\sqrt{f}}$, ideoque $2(m+1)\mu + 2n\nu = 1$; ergo $\nu = \frac{1}{2n} - \frac{(m+1)\mu}{n}$

seu $\mu = \frac{1-2n\nu}{2(m+1)}$.

Quia porro est $\sqrt{(mf-g+f)} = 2\nu\sqrt{g(mm-nn-1)}$ erit $(m+1)f-g = 4\nu^2 g(mm-nn-1)$; ideoque $g = \frac{(m+1)f}{1+4\nu^2(m^2-n^2-1)}$.

Cum deinde fit $\sqrt{h} = \frac{(b+\beta)\mu + (b-\beta)\nu}{\sqrt{f}}$; fiet

$$\sqrt{f} = \frac{(b+\beta)\mu + (b-\beta)\nu}{\sqrt{h}} = \frac{b+\beta}{2(m+1)\sqrt{f}} + \frac{\nu b(m-n+1) - \nu\beta(m+n+1)}{(m+1)\sqrt{h}}$$

seu $\sqrt{f} = \frac{b+\beta}{2(m+1)\sqrt{h}} = \frac{\nu(K+L)(Lbll - K\beta kk)}{KLb\beta(m+1)\sqrt{h}}$

estque $m^2 - n^2 - 1 = \frac{K+L}{KLb^2\beta}((K+L)k^2 ll + K\beta^2 k^2 + Lbb\,ll)$

Hinc ergo ex motu initiali definiuntur litteræ f & g, quibus cognitis deinceps ad quodvis tempus motus & situs corporis determinabitur.

Problema. VII.

Fig. 12. 48. *Constet corpus ex quotcunque articulis AB, BC, CD, DE &c. flexuris in B, C, D, &c. invicem conjunctis, quæritur, quæ motus, quo hoc corpus super plano politissimo horizontali sit progressurum, postquam ipsi semel motus quicunque fuerit impressus.*

Solutio.

Sumtis pro lubitu in plano horizontali duobus axibus orthogonalibus $O o$, $O \omega$, ad quos quovis momento positio corporis referatur, perveneritque elapso tempore t in situm, quem figura repræsentat. Sint singulorum articulorum centra gravitatis in K, L, M, N &c. unde ad

axem

axem Oo demittantur perpendicula KP, LQ, MR, NS, &c. & vocentur:

$$AK = \alpha;\ BK = b;\ BL = \beta;\ CL = c;\ CM = \gamma;\ DM = d;\ DN = \delta\ \&c.$$

$$OP = p;\quad OQ = q;\quad OR = r;\quad OS = s\ \&c.$$

$$PK = x;\quad QL = y;\quad RM = z;\quad SN = v\ \&c.$$

Anguli: $AKP = \zeta$; $BLQ = \eta$; $CMR = \theta$; $DNS = \iota$; &c.

ex quibus oriuntur ſequentes æquationes:

$q - p = b \sin\zeta + \beta \sin\eta$	$y - x = b \cos\zeta + \beta \cos\eta$
$r - q = c \sin\eta + \gamma \sin\theta$	$z - y = c \cos\eta + \gamma \cos\theta$
$s - r = d \sin\theta + \delta \sin\iota$	$v - z = d \cos\theta + \delta \cos\iota$
&c.	&c.

Deinde ſi punctorum K, L, M, N &c. motus reſolvantur in laterales ſecundum directiones axium Oo, $O\omega$, erit

Celeritas puncti	in directione Oo	in directione $O\omega$
K	$= \frac{dp}{dt}$	$= \frac{dx}{dt}$
L	$= \frac{dq}{dt}$	$= \frac{dy}{dt}$
M	$= \frac{dr}{dt}$	$= \frac{dz}{dt}$
N	$= \frac{ds}{dt}$	$= \frac{dv}{dt}$
	&c.	&c.

Celeritates vero angulares cujusque articuli circa ſuum gravitatis centrum ex incrementis angulorum ζ, η, θ, &c.

defi-

definientur, ita ut in distantia a quovis centro gravitatis $= 1$ sit;

Celeritas rotatoria

Articuli AB circa K $= \frac{d\zeta}{dt}$

Articuli BC circa L $= \frac{d\eta}{dt}$

Articuli CD circa M $= \frac{d\vartheta}{dt}$

Articuli DE circa N $= \frac{d\iota}{dt}$

&c.

Si denique:	Massa	Momentum inertiæ
Articuli AB	$=$ K	$=$ Kkk
Articuli BC	$=$ L	$=$ Lll
Articuli CD	$=$ M	$=$ Mmm
Articuli DE	$=$ N	$=$ Nnn
&c.	&c.	&c.

Vires, quas juncturæ sustinent, & quibus motus articulorum alterantur, pariter secundum directiones Oo & Oω resolvantur vocenturque

Vis $Bk = Bl = B$	Vis $B\kappa = B\lambda = \mathfrak{B}$
Vis $Cl = Cm = C$	Vis $C\lambda = C\mu = \mathfrak{C}$
Vis $Dm = Dn = D$	Vis $D\mu = D\nu = \mathfrak{D}$
&c.	&c.

His

afficient…

$2K\,ddp$ …

$2L\,ddq$ …

$2M\,dd$ …

$2N\,dd$ …

T…

a momen…

$2K\,kk\,dd\zeta$ …

$2L\,ll\,dd\eta$ …

$2M\,mm\,dd\vartheta$ …

$2N\,nn\,dd\iota$ …

Ex pr…

tiones

$K\,ddp$ …

$K\,ddx$ …

quæ bis in…

$Kp +$ …

$Lx +$ …

His igitur motus punctorum K, L, M, N, &c. ita afficientur, ut sit:

$$2K\,ddp = \mathfrak{B}\,dt^2 \qquad 2K\,ddx = B\,dt^2$$
$$2L\,ddq = \mathfrak{C}\,dt^2 - \mathfrak{B}\,dt^2 \qquad 2L\,ddy = C\,dt^2 - B\,dt^2$$
$$2M\,ddr = \mathfrak{D}\,dt^2 - \mathfrak{C}\,dt^2 \qquad 2M\,ddz = D\,dt^2 - C\,dt^2$$
$$2N\,dds = \cdots - \mathfrak{D}\,dt^2. \qquad 2N\,ddv = \cdots - D\,dt^2$$

&c. &c.

Tum vero singulorum articulorum motus rotatorii a momentis harum virium ita accelerabuntur; ut fiat:

$$2Kkk\,dd\zeta = (\mathfrak{B}b\cos\zeta - Bb\sin\zeta)\,dt^2$$
$$2Lll\,dd\eta = (\mathfrak{C}c\cos\eta - Cc\sin\eta)\,dt^2 + (\mathfrak{B}\beta\cos\eta - B\beta\sin\eta)\,dt^2$$
$$2Mmm\,dd\theta = (\mathfrak{D}d\cos\theta - Dd\sin\theta)\,dt^2 + (\mathfrak{C}\gamma\cos\theta - C\gamma\sin\theta)\,dt^2$$
$$2Nnn\,dd\iota = \cdots\cdots + (\mathfrak{D}\delta\cos\iota - D\delta\sin\iota)\,dt^2$$

&c.

Ex prioribus sollicitationibus oriuntur istae binae aequationes

$$K\,ddp + L\,ddq + M\,ddr + N\,dds + \&c. = 0$$
$$K\,ddx + L\,ddy + M\,ddz + N\,ddv + \&c. = 0$$

quae bis integratae dabunt:

$$Kp + Lq + Mr + Ns + \&c. = \mathfrak{F}t + f$$
$$Lx + Ly + Mz + Nv + \&c. = \mathfrak{G}t + \mathfrak{F}$$

quibus

quibus communi centri gravitatis totius corporis motus uniformis secundum lineam rectam indicatur. Cum ergo sit

$$q = p + b\sin\zeta + \epsilon\sin\eta$$

$$r = p + b\sin\zeta + (c + \epsilon)\sin\eta + \gamma\sin\theta$$

$$s = p + b\sin\zeta + (c + \epsilon)\sin\eta + (d + \gamma)\sin\theta + \delta\sin\iota$$

&c.

$$y = x + b\cos\zeta + \epsilon\cos\eta$$

$$z = x + b\cos\zeta + (c + \epsilon)\cos\eta + \gamma\cos\theta$$

$$v = x + b\cos\zeta + (c + \epsilon)\cos\eta + (d + \gamma)\cos\theta + \delta\cos\iota$$

&c.

Ponatur massa totius corporis K + L + M + N + &c. = S fietque:

$$p = \mathfrak{z}t + f \frac{\begin{matrix} -(S-L)b\sin\zeta - (S-K-L)c\sin\eta - (S-K-L-M)d\sin\theta \\ -(S-L)b\sin\eta - (S-K-L)\gamma\sin\theta - (S-K-L-M)\delta\sin\iota \end{matrix} \ \&c.}{S}$$

$$q = \mathfrak{z}t + f \frac{\begin{matrix} +Kb\sin\zeta - (S-K-L)c\sin\eta - (S-K-L-M)d\sin\theta \\ +K\epsilon\sin\eta - (S-K-L)\gamma\sin\theta - (S-K-L-M)\delta\sin\iota \end{matrix} \ \&c.}{S}$$

$$r = \mathfrak{z}t + f \frac{\begin{matrix} +Kb\sin\eta + (K+L)c\sin\eta - (S-K-L-M)d\sin\theta \\ +K\epsilon\sin\eta + (K+L)\gamma\sin\theta - (S-K-L-M)\delta\sin\iota \end{matrix} \ \&c.}{S}$$

$s =$

$$s = \mathfrak{F}t + \mathfrak{f}\,\frac{\begin{array}{c} +Kb\sin\zeta + (K+L)c\sin\eta + (K+L+M)d\sin\theta \\ +K\epsilon\sin\eta + (K+L)\gamma\sin\theta + (K+L+M)\delta\sin\iota \end{array} \quad \&c.}{S}$$

&c.

Similique modo applicatæ x, y, z, u, &c. definientur prodibitque:

$$x = \mathfrak{G}t + \mathfrak{g}\,\frac{\begin{array}{c} -(S-K)b\cos\zeta - (S-K-L)c\cos\eta \\ -(S-K-L-M)d\cos\theta \\ -(S-K)\epsilon\cos\eta - (S-K-L)\gamma\cos\theta \\ -(S-K-L-M)\delta\cos\iota \end{array} \quad \&c.}{S}$$

$$y = \mathfrak{G}t + \mathfrak{g}\,\frac{\begin{array}{c} +Kb\cos\zeta - (S-K-L)c\cos\eta \\ -(S-K-L-M)d\cos\theta \\ +K\epsilon\cos\eta - (S-K-L)\gamma\cos\theta \\ -(S-K-L-M)\delta\cos\iota \end{array} \quad \&c.}{S}$$

$$z = \mathfrak{G}t + \mathfrak{g}\,\frac{\begin{array}{c} +Kb\cos\zeta + (K+L)c\cos\eta \\ -(S-K-L-M)d\cos\theta \\ +K\epsilon\cos\eta + (K+L)\gamma\cos\theta \\ -(S-K-L-M)\delta\cos\iota \end{array} \quad \&c.}{S}$$

$$v = \mathfrak{G}t + g\,\frac{\begin{array}{l} +Kb\cos\zeta + (K+L)\,\epsilon\cos\eta + (K+L+M)\,\delta\cos\theta \\ +K\epsilon\cos\eta + (K+L)\,\gamma\cos\theta + (K+L+M)\,\delta\cos\iota \ \&c. \end{array}}{S}$$

&c.

Deinde ex æquationibus, quæ sollicitationes punctorum K, L, M, N &c. suppeditaverunt, colligitur fore.

$$\frac{2K(dpddp + dxddx) + 2L(dqddq + dyddy) + 2M(drddr + dzddz)\ \&c.}{dt^2}$$

$$= \mathfrak{B}(dp - dq) + \mathfrak{C}(dq - dr) + \mathfrak{D}(dr - ds) + \&c.$$

$$+ B(dx - dy) + C(dy - dz) + D(dz - dv) + \&c. =$$

$$-\mathfrak{B}b\,d\zeta\cos\zeta - \mathfrak{B}\epsilon\,d\eta\cos\eta - \mathfrak{C}\gamma\,d\theta\cos\theta - \mathfrak{D}\delta\,d\iota\cos\iota - \&c.$$
$$-\mathfrak{C}\epsilon\,d\eta\cos\eta - \mathfrak{D}\delta\,d\theta\cos\theta$$

$$+ Bb\,d\zeta\sin\zeta + B\epsilon\,d\eta\sin\eta + C\gamma\,d\theta\sin\theta + D\delta\,d\iota\sin\iota + \&c.$$
$$+ C\epsilon\,d\eta\sin\eta + D\delta\,d\theta\sin\theta$$

At sollicitationes motus rotatorii dabunt

$$\frac{2Kk^2\,d\zeta dd\zeta + 2Ll^2\,d\eta dd\eta + 2Mm^2\,d\theta dd\theta + \&c.}{dt^2} =$$

𝔅

$$\mathfrak{B}\,b\,d\zeta\cos\zeta + \mathfrak{B}\,\epsilon\,d\eta\cos\eta + \mathfrak{C}\,\gamma\,d\theta\cos\theta + \mathfrak{D}\,\delta\,d\iota\cos\iota$$
$$+ \mathfrak{C}\,\epsilon\,d\eta\cos\eta + \mathfrak{D}\,d\,d\theta\cos\theta \qquad \&c.$$
$$- B\,b\,d\zeta\sin\zeta - B\,\epsilon\,d\eta\sin\eta - C\,\gamma\,d\theta\sin\theta - D\,\delta\,d\iota\sin\iota$$
$$- C\,\epsilon\,d\eta\sin\eta - D\,d\,d\theta\sin\theta$$

quæ expreſſio cum præcedenti ſit æqualis & contrariis ſignis affecta ſequitur fore:

$$\frac{K(dp^2 + dx^2 + k^2 d\zeta^2) + L(dq^2 + dy^2 + l^2 d\eta^2) + M(dr^2 + dz^2 + m^2 d\theta^2) + \&c.}{dt^2}$$

= Conſt. ſicque patet pepetuo eandem virium vivarum quantitatem conſervari.

Quærantur denique valores virium $\mathfrak{B}$, B, $\mathfrak{C}$, C, $\mathfrak{D}$, D, &c. quæ erunt:

$$\tfrac{1}{2}\mathfrak{B}\,dt^2 = K\,ddp$$

$$\tfrac{1}{2}\mathfrak{C}\,dt^2 = K\,ddp + L\,ddq$$

$$\tfrac{1}{2}\mathfrak{D}\,dt^2 = K\,ddp + L\,ddq + M\,ddr$$

&c.

$$\tfrac{1}{2}\,B\,dt^2 = K\,ddx$$

$$\tfrac{1}{2}\,C\,dt^2 = K\,ddx + L\,ddy$$

$$\tfrac{1}{2}\,D\,dt^2 = K\,ddx + L\,ddy + M\,ddz$$

&c.

Si igitur valores pro 𝔅, ℭ, 𝔇, &c. & B, C, D, &c. hinc oriundi in æquationibus ex motibus rotatoriis ortis substituantur; prodibit

$$K\,kk\,dd\zeta = b(K\,ddp\cos\zeta - L\,ddx\sin\zeta)$$

$$L\,ll\,dd\eta = \begin{cases} -K\,c\,ddp\cos\eta + L\,c\,ddq\cos\eta + K\,c\,ddp\cos\eta \\ -K\,c\,ddx\sin\eta - L\,c\,ddy\sin\eta - K\,c\,ddx\sin\eta \end{cases}$$

$$M\,mm\,dd\theta = \begin{cases} -\cos\theta\,(K(\gamma+d)\,ddp + L(\gamma+d)\,ddq + M\,ddr) \\ -\sin\theta\,(K(\gamma+d)\,ddx + L(\gamma+d)\,ddy + M\,ddz) \end{cases}$$

$$N\,mm\,dd\iota = \begin{cases} +\cos\iota\,(K(\delta+\epsilon)\,ddp + L(\delta+\epsilon)\,ddq + M(\delta+\epsilon) \\ \qquad ddr + N\,\epsilon\,dds) \\ -\sin\iota\,(K(\delta+\epsilon)\,ddx + L(\delta+\epsilon)\,ddy + M(\delta+\epsilon) \\ \qquad ddz + N\,\epsilon\,ddv) \end{cases}$$

&c.

Quod

Quod si nunc hic loco p, q, r, &c. & x, y, z, &c. valores supra inventi substituantur, habebuntur tot æquationes differentio-differentiales, quot sunt anguli ζ, η, θ, &c. per easque ergo isti anguli determinabuntur ex tempore jam elapso $= t$, quibus inventis simul valores coordinatarum p, x, q, y, r, z, &c. innotescent, sicque ad datum tempus positio totius corporis una cum motu cujusque articuli definiri poterit.

Q. E. J.