$$p = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} - \frac{3a}{4}\sin\zeta - \frac{2a}{4}\sin\eta - \frac{a}{4}\sin\theta$$

$$p = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta - \tfrac{2}{4}a\sin\eta - \tfrac{1}{4}a\sin\theta$$

$$r = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta + \tfrac{2}{4}a\sin\eta - \tfrac{1}{4}a\sin\theta$$

$$s = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta + \tfrac{2}{4}a\sin\eta + \tfrac{3}{4}a\sin\theta$$

atque

$$x = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} - \tfrac{3}{4}a\cos\zeta - \tfrac{2}{4}a\cos\eta - \tfrac{1}{4}a\cos\theta$$

$$y = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta - \tfrac{2}{4}a\cos\eta - a\cos\theta$$

$$z = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta + \tfrac{2}{4}a\cos\eta - \tfrac{1}{4}a\cos\theta$$

$$v = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta + \tfrac{2}{4}a\cos\eta + \tfrac{3}{4}a\cos\theta$$

Problema. V.

Fig. 8. 32. *Augeatur nunc numerus corpusculorum in infinitum, filorum autem longitudines evanescant, ita ut hoc modo funis perfecte flexibilis formetur, cujus, si super plano horizontali utcunque projiciatur, motus & situs ad quodvis tempus assignari debet.*

Solutio.

Pervenerit iste funis elapso tempore t in situm AMG, ex cujus singulis punctis M perpendicula ad axem Oo demissa con-

concipiantur. Vocetur abscissa quæcunque $OP = X$, applicata $PM = Y$, & longitudo portionis funis $AM = S$, ejus vero massa exprimatur per functionem quamcunque Σ ipsius S, sitque præterea angulus $AMP = \Phi$, ita ut sit $dX = dS \sin \Phi$ & $dY = dS \cos \Phi$. Ponatur ut ante $Oa = p$ & $Aa = x$, quia invenimus

$$p = \frac{\mathfrak{A}t + \mathfrak{a} + Aa \sin \zeta + (A+B) b \sin \eta + (A+B+C) c \sin \theta + \&c.}{A + B + C + E + \&c.}$$

$$x = \frac{\mathfrak{B}t + \mathfrak{b} + Aa \cos \zeta + (A+B) b \cos \eta + (A+B+C) c \cos \theta + \&c.}{A + B + C + D + \&c.}$$

si massam totius funis ponamus $= H$, his formulis ad casum præsentem translatis habebimus

$$Oa = \frac{\mathfrak{A}t + \mathfrak{a} + \int \Sigma\, dS \sin \Phi}{H} - \int dS \sin \Phi$$

$$Aa = \frac{\mathfrak{B}t + \mathfrak{b} + \int \Sigma\, dS \cos \Phi}{H} - \int dS \cos \Phi$$

his integralibus per totam funis longitudinem AMC extensis. Erit autem $\int dS \sin \Phi = ag$ & $\int dS \cos \Phi = Gg - Aa$. Deinde cum supra invenerimus: $Ap + Bq + Cr + Ds + \&c. = (A + B + C + D + \&c.)(p + a \sin \zeta + b \sin \eta + c \sin \theta + \&c.) - Aa \sin \zeta - (A+B) b \sin \eta - (A+B+C) c \sin \theta - \&c.$ erit hæc expressio ad præsentem casum translata pro arcu $AM = \Sigma (Oa + \int dS \sin \Phi) - \int \Sigma\, dS \sin \Phi = Oa . \Sigma + \int d\Sigma \int dS \sin \Phi$. Simili modo & altera expressio $Ax + By + Cz + Dv + \&c.$ nostro casu transit in hanc $Aa . \Sigma + \int d\Sigma \int dS \cos \Phi$; unde erit $\frac{\sin \Phi}{\cos \Phi} = \frac{dd.(Oa . \Sigma + \int d\Sigma \int dS \sin \Phi)}{dd.(Aa . \Sigma + \int d\Sigma \int dS \cos \Phi)}$, quæ differentialia

secunfecundi gradus ex variabilitate temporis fola funt defumenda, ita ut S & Σ tanquam conftantia tractentur.

Quoniam ergo angulus Φ cum tempore t variatur, etiamfi arcus S idem maneat, quantitas Φ erit functio duarum variabilium S & t. Ponatur propterea $d\Phi = M\,dS + N\,dt$ eritque pofito S conftante & folo t variabilis.

$$d.\,Oa = \frac{\mathfrak{A}\,dt + dt\int \Sigma N\,dS \cos\Phi}{H} - dt\int N\,dS \cos\phi$$

$$d.\,Aa = \frac{\mathfrak{B}\,dt - dt\int \Sigma N\,dS \sin\phi}{H} + dt\int N\,dS \sin\phi$$

Quia vero N eft porro functio ipfarum S & t ponatur $dN = P\,dS + Q\,dt$, eritque pofito folo t variabili, & dt conftante

$$dd.\,Oa = -\frac{dt^2\int \Sigma N^2\,dS \sin\phi + dt^2\int \Sigma Q\,dS \cos\phi}{H} + dt^2\int N^2\,dS \sin\phi - dt^2\int Q\,dS \cos\phi$$

$$dd.\,Aa = -\frac{dt^2\int \Sigma N^2\,dS \cos\phi - dt^2\int \Sigma Q\,dS \sin\phi}{H} + dt^2\int N^2\,dS \cos\phi + dt^2\int Q\,dS \sin\phi$$

quæ fingula integralia ad totam curvam erunt extendenda; ita ut ea abeant in functiones ipfius t tantum; hancobrem fit $dd.\,Oa = E\,dt^2$ & $dd.\,Aa = F\,dt^2$, erunt E & F functiones ipfius t tantum. Deinde quæ pertinent ad folum arcum indefinitum AM, erunt differentialia, quæ ex fola variabilitate ipfius t oriuntur:

$$d.\int d\Sigma \int dS \sin\phi = dt \int d\Sigma \int N\, dS \cos\phi$$

$$dd.\int d\Sigma \int dS \sin\phi = -dt^2 \int d\Sigma \int N^2\, dS \sin\phi + dt^2 \int dz \int Q\, dS \cos\phi$$

$$d.\int dz \int dS \cos\phi = -dt \int dz \int N\, dS \sin\phi$$

$$dd.\int dz \int dS \cos\phi = -dt^2 \int dz \int N^2\, dS \cos\phi - dt^2 \int dz \int Q\, dS \sin\phi$$

Ex his ergo obtinebitur sequens aequatio;

$$\frac{\sin\phi}{\cos\phi} = \frac{Ez - \int dz \int N^2\, dS \sin\phi + \int dz \int Q\, dS \cos\phi}{Fz - \int dz \int N^2\, dS \cos\phi - \int dz \int Q\, dS \sin\phi}$$

Quo autem naturam hujus curvae ejusque motus commodius exprimamus, ponamus:

$$Oa.z + \int dz \int dS \sin\phi = T$$

$$Aa.z + \int dz \int dS \cos\phi = V$$

sitque nobis δ character differentialium, quae ex variabilitate ipsius t nascuntur, manente d charactere differentialium tantum, quae ex sola variabilitate ipsius S seu z ortum trahunt. Erit ergo $\frac{\sin\phi}{\cos\phi} = \frac{\delta\delta T}{\delta\delta V}$: tum vero differentiando formulas antecedentes ponendo tantum z vel S variabili, ob Oa & Aa constantes erit

$$Oa.dz + dz \int dS \sin\phi = dT$$

$$Aa.dz + dz \int dS \cos\phi = dV$$

& statuendo dz constante, ita ut S tanquam functio ipsius z spectetur, si denuo differentialia sumantur, erit

$$dz\, dS \sin\phi = ddT$$

$$dz\, dS \cos\phi = ddV$$

ideoque $\frac{\sin\phi}{\cos\phi} = \frac{ddT}{ddV}$, unde obtinetur ista æquatio

$$\frac{\delta\delta T}{\delta\delta V} = \frac{ddT}{ddV}$$

Præterea vero cum sit $\sin\phi = \frac{dd\,T}{d\Sigma dS}$, & $\cos\phi = \frac{dd\,V}{d\Sigma dS}$ debebit esse $ddT^2 + ddV^2 = d\Sigma^2 dS^2$. Quæstio ergo huc redit ut investigentur duæ hujusmodi functiones ipsarum Σ & s quæ sint T & V, ita ut earum differentialia secunda posito solo Σ variabili eandem inter se teneant rationem, quam earundem functionum differentialia secunda, si solum s ponatur variabile: præterea vero debet esse $ddT^2 + ddV^2 = dS^2 d\Sigma^2$: quibus inventis erit

$$\sin\phi = \frac{dd\,T}{dS\,d\Sigma} \quad \& \cos\phi = \frac{dd\,V}{dS\,d\Sigma}.$$

Hincque porro coordinatæ X & Y reperientur. Vel cum sit $dX = dS\sin\phi$ & $dY = dS\cos\phi$, erit $ddT = d\Sigma dX$ & $ddV = d\Sigma dY$, unde fit $T = \int X d\Sigma$ & $V = \int Y d\Sigma$. Quare X & Y ita comparatæ esse debent, ut sit $\frac{\delta\delta T}{\delta\delta V} = \frac{d\,X}{d\,Y}$ vel $\frac{\int d\Sigma\delta\delta X}{\int d\Sigma\delta\delta Y} = \frac{dX}{dY}$. Reducta ergo est solutio hujus problematis mechanici ad problema analyticum; in quo acquiescere oportet. Q. E. J.

Scholion. I.

33. Quo hæc clarius explicentur, ponamus funem ubique esse æque crassum, ita ut sit massa Σ proportionalis longi-

longitudini S. Sit portio AM $= S$, angulus AMP $= \phi$, & longitudo tota AMG $= h$, eritque elapso tempore t

$$O\alpha = \mathfrak{A}t + \mathfrak{a} + \frac{\int s\,ds\,\sin\phi - h\int ds\,\sin\phi}{h}$$

ubi integralia $\int ds \sin\phi$ ita debent capi, ut evanescant posito $s = 0$, tum vero statui oportet $s = h$, sicque $O\alpha$ exprimetur per functionem ipsius t. Simili modo erit

$$Aa = \mathfrak{B}t + \mathfrak{b} + \frac{\int s\,ds\,\cos\phi - h\int ds\,\cos\phi}{h}$$

ubi integralia $\int ds \cos\phi$ & $\int s ds \cos\phi$ pariter ita accipi debent ut evanescant posito $s = 0$, quo facto ubique faciendum est $s = h$.

Statuatur brevitatis ergo:	fiatque casu $s = h$
$\int ds \sin\phi = P$	$P = A$
$\int ds \cos\phi = Q$	$Q = B$
$\int s ds \sin\phi = R$	$R = C$
$\int s ds \cos\phi = S$	$S = D$

erit $$O\alpha = \mathfrak{A}t + \mathfrak{a} + \frac{C - Ah}{h};$$

$$Aa = \mathfrak{B}t + \mathfrak{b} + \frac{D - Bh}{h}$$

Deinde superior expressio generalis $\int dz \int dS \cdot \sin\phi$ abit hic in Ps —, & $\int dz \int dS \cos\phi$ in $Qs - S$; ideoque si δ sumatur pro charactere differentiationis, si t tantum variabile spectetur, erit

$$\frac{\sin\phi}{\cos\phi} = \frac{\delta\delta(Cs - Ahs + Phs - Rh)}{\delta\delta(Ds - Bhs + Qhs - Sh)}$$

ubi omisimus $\mathfrak{A}t + a$ & $\mathfrak{B}t + b$, quia horum differentialia secunda evanescunt. Quanquam autem hinc idoneae functiones ipsarum t & s pro angulo ϕ adhibendae inveniri vix possunt, tamen ope harum valores, quos quis forte pro ϕ exhibuerit, facile explorari possunt, utrum problemati satisfaciant necne.

Scholion. 2.

34. Quamvis autem hoc problema sit difficillimum, si in genere consideretur, tamen unus extat casus specialis, quo solutu sit facillimum. Hic locum habet, si angulus ϕ exprimatur per functionem ipsius s tantum, ita ut in eam tempus t non ingrediatur. Quia enim tum formulae $Cs - Ahs + Phs - Rh$ & $Ds - Bhs + Qhs - Sh$ a sola variabili s pendent, earum differentialia, quae prodeunt, si solum t variabile ponatur, evanescent, sicque aequationi ultimae satisfit. Hoc igitur casu funis instar corporis rigidi motu sibi parallelo feretur, ita ut singulae ejus partes perpetuo ad axem Oo eandem inclinationem conservent, & singulorum punctorum M celeritates, tam secundum directionem axis Oo, quam axis ad eum normalis $O\omega$ inter se erunt aequales. Quare si funi initio hujusmodi motus fuerit impressus, ut primo saltem momento singula elementa ad axem Oo eandem inclinationem retineant, tum eodem motu perpetuo promoveri perget.

Coroll. 1.

35. Si ponatur $Cs - Ahs + Phs - Rh = T$ & $Ds - Bhs + Qhs - Sh = \Theta$, erunt T & Θ ejusmodi functiones, quae evanescunt tam si ponatur $s = 0$, quam si fiat

fiat $s = h$. Ita autem præterea istæ functiones T & Θ debent esse comparatæ, ut sit

$$ddT = h\,ds^2 \sin\varphi \quad dd\Theta = h\,ds^2 \cos\varphi.$$

in quibus differentiationibus solum s positum est variabile & ds constans. Sin autem solum t variabile statuatur, debet esse

$$\delta\delta T = V\,dt^2 \sin\varphi \quad \& \quad \delta\delta\Theta = V\,dt^2 \cos\varphi$$

existente V functione quacunque.

Coroll. 2.

36. Si igitur hujusmodi functiones T & Θ inveniri possent, haberetur solutio particularis problematis; sin autem omnes omnino functiones his proprietatibus gaudentes in formulis generalibus comprehendi possent, tum haberetur solutio problematis generalis absoluta, qualis desideratur.

Coroll. 3.

37. Sint M & N & φ functiones ipsarum s & t, & ponatur:

$$T = M \sin\varphi + N \cos\varphi; \quad \Theta = M \cos\varphi - N \sin\varphi$$

atque problemati satisfiet, si primum T & Θ evanescant tam casu $s = 0$ quam casu $s = h$; quod fit, si his casibus & M & N fiant $= 0$. Deinde vero requiritur ut sit

$$h\,ds^2 = ddM - M\,d\varphi^2 - 2dN\,d\varphi - N\,dd\varphi$$

Præterea vero debet esse

$$ddN - N\,d\varphi^2 + M\,dd\varphi + 2dM\,d\varphi = 0$$
$$\delta\delta N - N\,\delta\varphi^2 + M\,\delta\delta\varphi + 2\delta M\,\delta\varphi = 0$$

in quarum æquationum prima & secunda t & dt sunt constantia, in tertia vero s & ds constantia sunt posita.

Coroll. 4.

38. Ex æquationibus prima & secunda eliminando terminos continentes $d\phi^2$ obtinebitur hæc:

$$(M^2+N^2)dd\phi + 2d\phi(MdM+NdN) = NddM - MddN - Nhds^2$$

ex cujus integratione eruitur

$$(M^2+N^2)d\phi = NdM - MdM - hds\int Nds$$

hincque porro $\phi = A \,\mathrm{tag}\, \frac{M}{N} - h\int\frac{ds\int Nds}{M^2+N^2}$. Qui valor si in alterutra æquatione substituatur ponendo $\int Nds = K$ prodibit

$$MddM + NddN = hMds^2 + \frac{hhK^2 ds}{M^2+N^2} - \frac{(NdM-MdN)^2}{M^2+N^2}$$

Addatur utrinque $dM^2 + dN^2$ & ponatur $M^2+N^2 = v^2$ erit

$$vddv = \frac{hhK^2ds^2}{v^2} + hMds^2 = \frac{hhK^2ds^2}{v^2} + hds^2\sqrt{(v^2-N^2)}$$

ex qua si definiatur v, ob K datum, per N, invenietur valor idoneus pro M substituendus. Tum vero N ex tertia æquatione determinari debet.

Co-

Coroll. 5.

30. Si sit $N = 0$, aequatio secunda statim dat $M^2 d\phi = E ds$ & $d\phi = \frac{E ds}{M^2}$, qui valor in prima $h ds^2 = ddM - M d\phi^2$ substitutus dabit, $h ds^2 = ddM - \frac{E^2 ds^2}{M^3}$. Statuatur $ds = v dM$, ut, ob ds constans sit $ddM = \frac{-dv dM}{v}$, erit substitutione facta;

$$h v^2 dM = \frac{-dv}{v} - \frac{E^2 v^2 dM}{M^3} \quad \text{seu}$$

$$h dM = \frac{-dv}{v^3} - \frac{E^2 dM}{M^3}$$

cujus integrale est:

$$hM = \frac{1}{2v^2} + \frac{E^2}{2M^2} - F \quad \text{seu}$$

$$2hv^2M^3 = M^2 + E^2v^2 - 2Fv^2M^2 \quad \text{unde}$$

$$v = \frac{\pm M}{\sqrt{(2hM^3 + 2FM^2 - E^2)}}$$

$$\& \quad s = \int \frac{\pm M dM}{\sqrt{(2hM^3 + 2FM^2 - E^2)}}$$

Quoniam vero M debet evanescere posito tam $s = 0$ quam $s = h$, fiat $\int \frac{\pm M dM}{\sqrt{(2hM^3 + 2FM^2 - E^2)}} = \pm G$ posito $M = 0$, atque G determinabitur per E, F & constantes.

stantes. Fiat ergo $G = \frac{1}{2}h$, & conditiones requisitae implebuntur, si ponatur

$$s = \tfrac{1}{2}h \pm \int \frac{M\,dM}{\sqrt{(2hM^3 + 2FM^2 - E^2)}}$$

unde erit $d\phi = \dfrac{\pm E\,dM}{M\sqrt{(2hM^3 + 2FM^2 - E^2)}}$

ubi E & F erunt quantitates tum ex constantibus tum ex t compositae. Praeterea vero si fieri potest, ita debent esse comparatae, ut etiam tertiae aequationi $M\partial\partial\phi + 2\partial M\,d\phi = 0$ seu $MM\partial\phi = H\,dt$ satisfiat, quod fiet si $d\phi + \partial\phi = \dfrac{E\,ds + H\,dt}{MM}$ fuerit integrabile: ejus enim integrale verum dabit angulum ϕ. Ubi notandum est, H designare functionem quamcunque ipsius s non involventem t, uti E est functio ipsius t non continens s. Hancobrem $\dfrac{E\,ds + H\,dt}{MM}$ erit integrabile si fuerit $MM = EH$ in functionem quampiam quantitatis $\int\frac{ds}{H} + \int\frac{dt}{E}$, seu si sit $\dfrac{MM}{EH}$ functio hujus quantitatis $\int\frac{ds}{H} + \int\frac{dt}{E}$.

Problema. VI.

Fig. 9. 40. *Constet corpus ABC duobus articulis AB & BC in B flexura invicem conjunctis, ita ut ambo circa B liberrime circumagi queant; quaeriturque motus, quo hoc corpus super plano, horizontali politissimo sit progressurum, postquam ipsi semel motus quicunque fuerit impressus.*

So-

Solutio.

Sumtis pro lubitu in plano horizontali duobus axibus orthogonalibus $O\upsilon$ & $O\omega$ sese decussantibus, ad quos quovis momento positio corporis refertur, pervenerit elapso tempore quocunque t, corpus in situm ABC, ex cujus punctis A, B, C ad axem $O\upsilon$ demittantur perpendicula Aa, Bb, Cc. Sit porro K centrum gravitatis articuli AB, & L centrum gravitatis articuli BC, ex quibus pariter ad axem $O\upsilon$ normales ducantur KP & LQ, vocenturque:

$$AK = a;\ BK = b;\ BL = \beta;\ LC = \epsilon:$$
$$OP = p;\ PK = x;\ OQ = q;\ QL = y:$$
$$\text{Ang. } AKP = \zeta;\quad \& \text{ ang. } BLQ = \eta.$$

ex quibus erit

$$q - p = b \sin\zeta + \beta \sin\eta \quad \& \quad y - x = b \cos\zeta + \beta \cos\eta.$$

Motus autem utriusque articuli, constat ex motu centri gravitatis, quem resolvamus secundum directiones amborum axium $O\upsilon$ & $O\omega$, & ex motu rotatorio, quo uterque articulus circa suum gravitatis centrum gyrabitur. Erit ergo

Celeritas puncti	secundum directionem $O\upsilon$	secundum directionem $O\omega$
K	$= \frac{dp}{dt}$	$= \frac{dx}{dt}$
L	$= \frac{dq}{dt}$	$= \frac{dy}{dt}$

Celeritas rotatoria articuli AB circa centrum gravitatis K erit $= \frac{d\zeta}{dt}$ in distantia $= 1$.

Cele-

Celeritas rotatoria articuli BC circa centrum gravitatis L erit $= \frac{d\eta}{dt}$ in distantia $= 1$.

Nunc ad motum determinandum sit:

massa articuli $AB = K$: articuli $BC = L$

momentum inertiæ articuli $AB = Kk^2$; articuli $BC = Ll^2$

Momenta hæc inertiæ respectu utriusque articuli centri gravitatis sumi ponuntur, estque momentum inertiæ aggregatum omnium corporis particularum per quadrata distantiarum suarum ab axe, circa quem corpus mobile concipitur, multiplicatarum. His positis si ambo articuli a se invicem essent dissoluti, uterque eundem motum tam progressivum centri gravitatis quam rotatorium circa axem verticalem per centrum gravitatis transeuntem perpetuo conservaret. Quoniam autem junctura in B sunt colligati, ambo isti motus se mutuo continuo perturbabunt, hæque perturbationes provenient a vi, quam junctura in B sustinet. Quæ vis cum sit incognita, ponamus ab ea articulum AB urgeri duabus viribus, altera in directione Bk quæ sit $= B$, altera in directione $B\varkappa$ quæ sit $= \mathfrak{B}$; atque alter articulus BC iisdem viribus, at in directionibus oppositis urgebitur, scilicet in directione Bl vi $= B$ & in directione $B\lambda$ vi $= \mathfrak{B}$. His ergo viribus primum motus centri gravitatis afficietur, ac primo quidem vis $B\varkappa = \mathfrak{B}$ accelerabit motum centri gravitatis K secundum $O\sigma$, & vis $Bk = B$ motum in directione $O\omega$; contra vero vis $B\lambda = \mathfrak{B}$ retardabit motum centri gravitatis L secundum $O\sigma$, & vis $Bl = B$ motum in directione $O\omega$. Hinc ex legibus sollicitationum erit:

$$2K\,ddp = \mathfrak{B}\,dt^2; \qquad 2K\,ddx = B\,dt^2$$

2 L

$$2L\,ddq = -\mathfrak{B}dt^2;\quad 2L\,ddy = -B\,dt^2$$

Porro momentum vis $B\kappa = \mathfrak{B}$ respectu centri gravitatis K est $= \mathfrak{B}b\cos\zeta$, eoque motus rotatorius articuli AB accelerabitur, quoniam id tendit ad augendum angulum AKP $= \zeta$: momentum autem vis $Bk = B$ erit $= Bb\sin\zeta$, eoque motus rotatorius articuli AB retardabitur, quoniam id tendit ad angulum AKP $= \zeta$ minuendum. Unde ex legibus sollicitationum erit:

$$2Kkk\,dd\zeta = \mathfrak{B}b\,dt^2\cos\zeta - Bb\,dt^2\sin\zeta$$

Deinde momentum vis $B\lambda = \mathfrak{B}$ respectu centri gravitatis L est $= \mathfrak{B}\beta\cos\eta$, tenditque ad motum rotatorium articuli BC accelerandum, momentum autem vis $Bl = B$, quod est $= B\beta\sin\eta$, retardabit eundem motum rotatorium, unde erit;

$$2Lll\,dd\eta = \mathfrak{B}\beta\,dt^2\cos\eta - B\beta\,dt^2\sin\eta.$$

His sollicitationibus ad calculum revocatis, priores aequationes pro motu progressivo utriusque centri gravitatis inventae dabunt,

$$K\,ddp + L\,ddq = 0 \quad \& \quad K\,ddx + L\,ddy = 0$$

unde integrando elicitur

$$Kp + Lq = \mathfrak{F}t + f$$

$$Kx + Ly = \mathfrak{G}t + g$$

Cum igitur sit $q = p + b\sin\zeta + \beta\sin\eta$ & $y = x + b\cos\zeta + \beta\cos\eta$ erit:

$$(K+L)p = \mathfrak{F}t + f - Lb\sin\zeta - L\beta\sin\eta$$

$$(K+L)x = \mathfrak{G}t + g - Lb\cos\zeta - L\beta\cos\eta$$

seu $p = \frac{\mathfrak{F}t + f - Lb\sin\zeta - L\beta\sin\eta}{K+L}$

$x = \frac{\mathfrak{G}t + g - Lb\cos\zeta - L\beta\cos\eta}{K+L}$

ideoque $ddp = -\frac{Lbdd.\sin\zeta - L\beta dd.\sin\eta}{K+L}$

$ddq = \frac{Kbdd.\sin\zeta + K\beta dd.\sin\eta}{K+L}$

$ddx = -\frac{Lbdd.\cos\zeta - L\beta dd.\cos\eta}{K+L}$

$ddy = \frac{Kbdd.\cos\zeta + K\beta dd.\cos\eta}{K+L}$

Ergo hinc vires $\mathfrak{B}$ & B ita definientur ut sit:

$$\mathfrak{B}dt^2 = -\frac{2KL}{K+L}(bdd.\sin\zeta + \beta dd.\sin\eta)$$

$$Bdt^2 = -\frac{2KL}{K+L}(bdd.\cos\zeta + \beta dd.\cos\eta)$$

Qui valores si in aequationibus ex sollicitationibus motus rotatorii utriusque articuli ortis substituantur, prodibit:

$$Kkkdd\zeta = -\frac{KL}{K+L}\left(\begin{array}{l} +bb\cos\zeta.dd\sin\zeta + b\beta\cos\zeta.dd.\sin\eta \\ -bb\sin\zeta.dd\cos\zeta - b\beta\sin\zeta.dd\cos\eta \end{array}\right)$$

$$Llldd\eta = -\frac{KL}{K+L}\left(\begin{array}{l} +b\beta\cos\eta dd.\sin\zeta + \beta\beta\cos\eta.dd.\sin\eta \\ -b\beta\sin\eta dd.\cos\zeta - \beta\beta\sin\eta.dd.\cos\eta \end{array}\right)$$

Cum ergo sit $\cos m\, dd.\sin n - \sin m\, dd.\cos n = ddn\cos(m-n) + dn^2\sin(m-n)$ habebuntur istae aequationes.

—(K

$$-\frac{(K+L)kkdd\zeta}{Lb} = bdd\zeta + \beta dd\eta \operatorname{cof}(\zeta-\eta) + \beta d\eta^2 \operatorname{fin}(\zeta-\eta)$$

$$-\frac{(K+L)lldd\eta}{K\beta} = \beta dd\eta + bdd\zeta \operatorname{cof}(\zeta-\eta) - bd\zeta^2 \operatorname{fin}(\zeta-\eta)$$

ex quibus per integrationem elicitur:

$$-\frac{(K+L)kkd\zeta}{Lb} = bd\zeta + \beta d\eta \operatorname{cof}(\zeta-\eta) + \beta\int d\zeta d\eta \operatorname{fin}(\zeta-\eta)$$

$$-\frac{(K+L)lld\eta}{K\beta} = \beta d\eta + bd\zeta \operatorname{cof}(\zeta-\eta) - b\int d\zeta d\eta \operatorname{fin}(\zeta-\eta)$$

Multiplicetur prior per b posterior per β & addantur, atque prodibit:

$$-\frac{(K+L)kkd\zeta}{L} - \frac{(K+L)lld\eta}{K} = bbd\zeta + \beta\beta d\eta + b\beta(d\zeta+d\eta)\operatorname{cof}(\zeta-\eta) \pm fdt\sqrt{f}$$

$$\text{seu} \pm fdt\sqrt{f} = \frac{(K+L)(Kkkd\zeta+Llld\eta)}{KL} + bbd\zeta + \beta\beta d\eta + b\beta(d\zeta+d\eta)\operatorname{cof}(\zeta-\eta)$$

Sit $\zeta+\eta = v$ & $\zeta-\eta = u$, ut fit $\zeta = \frac{v+u}{2}$ & $\eta = \frac{v-u}{2}$

erit:

$$\pm fdt\sqrt{f} = \frac{(K+L)kkdv}{2L} + \frac{(K+L)kkdu}{2L} + b\beta dv \operatorname{cof} u$$
$$+ \frac{(K+L)lldv}{2K} - \frac{(K+L)lldu}{2K}$$

$$+\frac{bbdv}{2} + \frac{bbdu}{2}$$

$$+\frac{\rho\rho dv}{2} - \frac{\rho\rho du}{2}$$

Ponatur porro $\frac{f dt \sqrt{f}}{b\rho} = \frac{dt}{\sqrt{f}}$ atque

$$\frac{(K+L)kk}{2Lb\rho} + \frac{(K+L)ll}{2Kb\rho} + \frac{b}{2\rho} + \frac{\rho}{2b} = m$$

$$\frac{(K+L)kk}{2Lbc} - \frac{(K+L)ll}{2Kbc} + \frac{b}{2c} - \frac{c}{2b} = n \quad \text{erit.}$$

$$\frac{dt}{\sqrt{f}} = m\,dv + n\,du + dv \cos u$$

superiores vero æquationes differentio-differentiales in has formas transmutentur.

$$0 = \frac{(K+L)kk\,dd\zeta}{Lbc} + \frac{b}{c}\,dd\zeta + dd\eta \cos(\zeta-\eta) + d\eta^2 \sin(\zeta-\eta)$$

$$0 = \frac{(K+L)ll\,dd\eta}{Kbc} + \frac{c}{b}\,dd\eta + dd\zeta \cos(\zeta-\eta) - d\zeta^2 \sin(\zeta-\eta)$$

quæ invicem subtractæ dabunt:

$$0 = n\,ddv + m\,ddu - ddu \cos u + \tfrac{1}{2}(dv^2 + du^2)\sin u$$

Prior vero $\frac{dt}{\sqrt{f}} = m\,dv + n\,du + dv \cos u$ suppeditat

$$dv = \frac{dt:\sqrt{f} - n\,dv}{m + \cos u} \;\&\; ddv = \frac{-n\,ddu}{m+\cos u} + \frac{dt\,du \sin u : \sqrt{f} - n\,du^2 \sin u}{(m+\cos u)^2}$$

atque

atque $\frac{1}{2}dv^2 \sin u = \frac{\frac{1}{2}dt^2 \sin u : f - ndtdu \sin u : \sqrt{f} + \frac{1}{2}n^2 du^2 \sin u}{(m+\cos u)^2}$

quibus valoribus substitutis obtinebitur:

$$o = \frac{-mnddu}{m+\cos u} + \frac{ndtdu \sin u : \sqrt{f} - nndu^2 \sin u}{(m+\cos u)^2}$$
$$+ mddu \quad + \frac{\frac{1}{2}dt^2 \sin u : f - ndtdu \sin u : \sqrt{f} + \frac{1}{2}n^2 du^2 \sin u}{(m+\cos u)^2}$$
$$- ddu \cos u$$
$$+ \frac{1}{2}du^2 \sin u$$

quae reducitur ad hanc:

$$o = (mm - mn)ddu - ddu \cos u^2 + \frac{\frac{1}{2}dt^2 \sin u : f - \frac{1}{2}n^2 du^2 \sin u}{m+\cos u}$$
$$+ \frac{1}{2}mdu^2 \sin u + \frac{1}{2}du^2 \sin u \cos u$$

seu $o = 2(mm - nu)ddu - 2ddu \cos u^2 + du^2 \sin u \cos u$

$$+ \frac{dt^2 \sin u : f + (m^2 - n^2)du^2 \sin u + mdu^2 \sin u \cos u}{m+\cos u}$$

Sit $dt = \frac{du}{\sqrt{\omega}}$ erit $ddt = o = \frac{ddu}{\sqrt{\omega}} - \frac{dud\omega}{2\omega\sqrt{\omega}}$, ideoque

$ddu = \frac{dud\omega}{2\omega}$, quibus valoribus pro ddu & dt substitutis fiet

$$o = (mm - nn)\frac{d\omega}{\omega} - \frac{d\omega \cos u}{\omega} + du \sin u \cos u +$$
$$\frac{du \sin u : f\omega + (m^2 - n^2)du \sin u + mdu \sin u \cos u}{m+\cos u}$$

sive

five

$$\frac{(mm-nn)\,d\omega - d\omega \cos u^2 + \omega du \sin u \cos u}{m+\cos u} + \frac{du \sin u : f + (m^2-n^2)\,\omega du \sin u + m\omega du \sin u \cos u}{(m+\cos u)^2} = 0$$

cujus integrale eft

$$\frac{(mm-nn)\,\omega}{m+\cos u} - \frac{\omega \cos u^2}{m+\cos u} + \frac{1}{f(m+\cos u)} = \frac{1}{g}.$$

hinc fit $\omega = \frac{f(m+\cos u) - g}{fg\,(mm-nn-\cos u^2)}$ atque

$dt = \frac{du\sqrt{fg(mm-nn-\cos u^2)}}{\sqrt{(mf-g+f\cos u)}}$ quo invento habebitur

$dv = \frac{dt:\sqrt{f} - n\,du}{m+\cos u}$ ficque per unicam variabilem u determinabuntur t, v, porroque anguli ζ & η, quibus inventis reliquæ quantitates p, q, x, & y innotefcent, ex quibus non folum pofitio corporis, fed etiam ejus motus definitur. Q. E. J.

Coroll. 1.

41. Ex æquationibus $K\,ddp + L\,ddq = 0$ & $K\,ddx + L\,ddy = 0$ intelligitur corporis ABC centrum gravitatis uniformiter in directum progredi.

Coroll. 2.

42. Eandem vero quoque vivarum quantitatem confervari, hoc modo patebit:

$$\frac{2K\,dp\,ddp}{dt^2} = \mathfrak{B}\,dp;\quad \frac{2K\,dx\,ddx}{dt^2} = B\,dx$$

$$\frac{2L\,dq\,ddq}{dt^2} = -\mathfrak{B}\,dq;\quad \frac{2K\,dy\,ddy}{dt^2} = -B\,dy$$

Est vero $dp - dq = -bd.\sin\zeta - {}^{c}d.\sin\eta$

$dx - dy = -bd.\cos\zeta - {}^{c}d\cos\eta$

Ergo $\frac{K(dp^2 + dx^2) + L(dq^2 + dy^2)}{dt^2} = -\int\mathfrak{B}(bd.\sin\zeta + bd.\sin\eta) - \int B(bd.\cos\zeta + {}^{c}d.\cos\eta)$

Porro vero est $\frac{Kkkd\zeta^2}{dt^2} = \int\mathfrak{B}\,bd.\sin\zeta + \int B\,bd.\cos\zeta$ atque

$\frac{Llld\eta^2}{dt^2} = \int\mathfrak{B}\,{}^{c}d.\sin\eta + \int B\,{}^{c}d.\cos\eta$; quibus in unam summam collectis erit.

$$\frac{K(dp^2 + dx^2 + kkd\zeta^2) + L(dq^2 + dy^2 + lld\eta^2)}{dt^2} = \text{Constanti}$$

At vero ista expressio exhibet vim vivam totius corporis,

nam $\frac{K(dp^2 + dx^2 + kkd\zeta^2)}{dt^2}$ est vis viva articuli A B; atque

$\frac{L(dq^2 + dy^2 + lld\eta^2)}{dt^2}$ est vis viva articuli BC.

Scholion.

43. Cum generaliter æquatio ultimo inventa $dt = \frac{du\sqrt{fg(mm - nn - \cos u^2)}}{\sqrt{(mf - g + f\cos u)}}$ integrationem non admittat,