Leonhard Euler, “On the Solar Astronomical Year of the Indians”

tr. Kim Plofker, July 2002; revised March 2006

[This work is an appendix to two appendices to a book by Euler's friend and Imperial Academy colleague T. S. Bayer, *Historia regni Graecorum Bactriani* (History of the Bactrian kingdom of the Greeks), St. Petersburg, 1738. The original appendices were written by a Danish missionary in Tranquebar, C. T. Walther (“The Indian Doctrine of Time,” pp. 145–190), and by Bayer himself based on his correspondence with Walther and other Tranquebar missionaries (“Supplement to the Indian Doctrine of Time,” pp. 191–200). Euler's contribution (Eneström 18, henceforth “original text”) appears on pp. 201–213; the edition in Euler's *Opera omnia* (henceforth “edition”) is pp. 5–12 of Series II, vol. 30. My editorial additions are in square brackets, and I have used the standard English names of weekdays, planets, and zodiacal signs in place of the astrological symbols in the Latin.]

1. The Indians do not locate the start of any year, as is the custom with us, at the beginning of some day, but at that moment of time in which they consider the sun to arrive at a certain fixed point.[1] But whether this point is the beginning of the sign Aries or instead the beginning of the constellation Aries is not sufficiently clear from the description, in which they say the year begins at that moment in which the sun enters Aries. But this endpoint may be defined not only from the amount of the year, but also from the beginning of some assigned year.

2. As for the length of this astronomical year of the Indians, it can be deduced from the beginnings of the years 1728, '29, '30, '31, '32 included on page 168.[2] Certainly, by the calculation accepted among the Indians, in which they divide the day into 60 hours, the hour into 60 minutes, the minute into 60 seconds, it is clear enough that their year contains 365 days, 15 hours, 31 minutes and 15 seconds;[3] which quantity, according to our manner of dividing time, produces 365 days, 6 hours, 12 minutes and 30 seconds.

3. If this quantity is reduced to days and parts of a day, the year of the Indians will be found to contain $365 + 1/4 + 5/576$ days, which quantity is thoroughly understood to have been accepted in their astronomical tables, from the method described on page 194[4] for computing the beginning of any year. The stated divisions by the number 576, which are often repeated, clearly proclaim it. This will appear more clearly hereafter, when I have derived from this length of the year the same rule of stated calculations that the Indians make use of.

4. Therefore the year of the Indians exceeds our tropical year comprising 365 days, 5 hours, $48'$, $57''$, and the excess is $23'$, $33''$.[5] For that reason the start of the Indian year constantly occurs later with respect to ours, and indeed after every 61 years the start of the Indian year will have advanced almost an entire day; so that, if in the year 1730 the [Indian] year began on the first day of April (old style), in the year 1791 according to us the start of the [Indian] year would fall on the second day of April, and in the year 1852 on the third day of April. And so among the Indians, they will

[1] Original text inserts “(p. 164)” here, referring to the start of the section “On the Year” in Walther's “Doctrine.”

[2] Original text has 199 for 168; the latter is correct. The table in question is as follows (reproduced in edition in a footnote, in which the blank in the first row is filled in with the equivalent of “1 April, morning”):

42 .	Kīlaka	1728		$1.15'$	Tuesday
43 .	Saumya	1729	1 April, midday	$16.46'$	Saturday
44 .	Sadhārana	1730	31 March, night	$32.17'$	Sunday
45 .	Virodhakṛt	1731	31 March, night	$47.48'$	Monday
46 .	Paridhāvī	1732	1 April, morning	$3.20'$	Wednesday

[3] This year-length is a standard parameter of the school of Āryabhaṭa, dominant in South India.

[4] Refers to the discussion in the section “Calculation of the Indian Year” of Bayer's “Supplement.”

[5] Edition has 32 for 33.

reckon an interval of 22333[6] of our years less by an entire year, and it will be equal to 22332[7] of the years according to them.

5. Therefore the beginnings of years for the Indians do not always fall in the same season of the year, as customarily happens for us, but successively in one season or another, so that in an interval of 22332 years the beginning of the year progresses through all the seasons. From which it is apparent that the beginning of their year implies neither the entrance of the sun into the equinoctial point nor [its entrance] into the solstice, but rather refers to moving and changeable places in the [tropical] ecliptic, the nature of which will be not at all difficult to define, in the following way.

6. The year of the Indians agrees accurately enough with our sidereal year, in which the sun returns to the same point in the heavens with respect to the fixed stars, the length of which year is put by the astronomers at 365 days, 6 hours, 10 minutes, so that the year of the Indians differs from this year only by 2 minutes.[8] This small error, owing to the inadequacy of observations which were retained [though] very ancient, is easy to forgive. Therefore there is no doubt [that] in fact the Indians were accustomed to measure their years from the periodic motion of the sun with respect to the fixed [stars], especially since at the time when they composed their tables, they could hardly be supposed to have observed the difference between the tropical and sidereal year.

7. Moreover, the Indians have established the start of the year 1734 according to our calendar (new style) on the tenth day of April, at which time the sun was at Aries 20°. Therefore, the fixed star that was observed by them at Aries 20° at that time is the fixed point from which, when the sun reaches it, they establish the beginning of their year. Therefore this fixed point is not the first star of Aries, which naturally at this time was found at Aries 29°, but another fixed star [that] should be 9° before the first star[9] of Aries. It may perhaps be possible to conjecture what this may be from the catalogue of fixed [stars]; but it will be a fairly noticeable star in the constellation Pisces.

8. Since now the length of the year which the Indians accepted is known, it will be easy to determine from the given beginning of some year the beginning of the following one, by adding to the beginning of the elapsed year one weekday, 15 [sexagesimal] hours, 31′, 15″. So if the beginning of the year 1731 by the defined calculation is weekday 2, hour 47, 48′, 45″, the beginning of the following year must fall on weekday 4, hour 3, 20′, 0″.

9. Therefore by this rule those same beginnings of all years are easily found, which would be extracted by laborious calculation done in the prescribed manner.

[6] Original text has instead 22311, here and in the following section.
[7] Original text has 22310.
[8] Edition alters 2 to 3; in fact, the precise value is $2\frac{1}{2}$.
[9] Original text and edition abbreviate "the first star" as "*1".

Beginnings of years

		Weekday	Hours	Minutes	Seconds
Year	1718	7	26	2	30
	1719	1	41	33	45
	1720	2	57	5	0
	1721	4	12	36	15
	1722	5	28	7	30
	1723	6	43	38	45
	1724	7	59	10	0
	1725	2	14	41	15
	1726	3	30	12	30
	1727	4	45	43	45
	1728	6	1	15	0
	1729	7	16	46	15
	1730	1	32	17	30
	1731	2	47	48	45
	1732	4	3	20	0
	1733	5	18	51	15
	1734	6	34	22	30
	1735	7	49	53	45
	1736	2	5	25	0
	1737	3	20	56	15
	1738	4	36	27	30
	1739	5	51	58	45
	1740	7	7	30	0

10. The first day of the year according to the Indians is always the first day of the month April, but this [does] not always coincide with the weekday in which the beginning of the year occurs; but if the beginning of the year is celebrated in the daytime, they consider that same day (but if the beginning of the year happens [to fall] in the night, then the following day) as the first day of April. Therefore from the table shown it at once appears in which weekday the first day of which year must occur. Certainly, the first day of any year will be the same weekday which is displayed in the table, or in which the beginning of the year occurs, if the number of [sexagesimal] hours is less than 30. And if the number of hours exceeds 30, then the first day of the year or the first day of April is transferred to the following weekday. But with the weekdays known in which the first days of the individual years occur, it immediately appears which years must be leap; for when there is a jump of two weekdays, then the past year will be leap and will contain 366 days.

11. Therefore from the known beginnings of the years from 1718 up to 1740 follows the following table, in which is shown what weekday the first day of any year occurs in, and which years are leap.

The first day is

		Weekday	
Year	1718	7	leap
	1719	2	
	1720	3	
	1721	4	
	1722	5	leap
	1723	7	
	1724	1	
	1725	2	
	1726	3	leap

1727	5	
1728	6	
1729	7	leap
1730	2	
1731	3	
1732	4	
1733	5	leap
1734	7	
1735	1	
1736	2	
1737	3	leap
1738	5	
1739	6	
1740	7	

12. These correspond excellently with those on p. 160,[10] where the years 1718, 1722, 1726 and 1729 are presented as leap. However, what the Indians say [about] the calculation [being] reconciled after 60 years, or the first day of the year returning to the same weekday, is in fact an approximation to the truth, since in 61 years[11] the first day of the year moves forward 6 weekdays and 46 hours,[12] which is nearly 7 weekdays or an entire week. However, [this] is still not absolutely true: for it can happen that, while the first day of this year occurs on the first weekday, [the first day] of the sixtieth after that occurs not in the first but only in the seventh. Furthermore, observe the agreement of our table with p. 168, where it is mentioned that in the year 1729 the first day of April occurred on Saturday or the seventh weekday, in the year 1730 on Monday, in the year 1731 on Tuesday, which agrees entirely with the table.[13] In fact, while on p. 197 it is left in doubt whether the beginning of the year 1734 occurs in the 4th or 22nd hour of the night,[14] from this it is clear [that] the former opinion is preferable, for the beginning of the year occurs in hour 34, which is the 4th hour of the night. Perhaps this 22 derived its origin from the number of minutes.

13. Although the beginning of any year can be found easily by the method used above, yet the rule described on p. 194[15] not only is not to be despised, but is very useful for [finding] the given beginning of any elapsed year. Also, just as a calculation of this sort must be undertaken with a specified year whose beginning is known, so in that rule the first year of the Kaliyuga is used, whose beginning should have occurred in the third weekday, hour 51, minute 8, second 45 by the Indian measure. Whence originates the number 1237, which is employed in the calculation.

14. Therefore for any proposed year, of which the beginning is to be determined, first of all the time elapsed from the beginning of the first year of the Kaliyuga up to the proposed year is to be investigated, which is made by adding to the past year of the sexagesimal era the number of years elapsed from [the beginning of] the Śaka era up to the beginning of the sexagesimal era in which is the proposed year; if 3179 years are added to which, [there] results the interval of elapsed time from the first year of the Kaliyuga to the proposed year.

15. If the number of years so determined is multiplied by the amount of the year in days, namely

[10] In the section "On the Months" of Walther's "Doctrine."

[11] Original text has 60 for 61.

[12] Edition adds "46′".

[13] In the referenced table (see footnote 2), 1730 is said to begin in the night—that is, after the lapse of 30 sexagesimal hours—of Sunday and 1731 in the night of Monday. Euler is here transferring their beginnings to the following weekday, as he described in section 10. (It is not clear why he did not do the same with the beginning of 1726, which in his first table is stated to fall 12′30″ into the night of Tuesday, which should shift it to Wednesday.) The edition, inexplicably, has "Saturday" instead of "Monday" in this sentence and "Wednesday" for "Tuesday".

[14] This part of Bayer's "Supplement" reports some differences of opinion among Indian calendars for recent years. (Edition has 198 for 197.)

[15] Euler explains and illustrates this procedure in the following sections; the edition has 193 (the page on which the verbal description of the rule begins) for 194 (which shows the worked calculation).

$365 + 1/4 + 5/576$, there comes out the number of days elapsed from the first year of the Kaliyuga, from which number—because the beginning of the first year of the Kaliyuga occurs not in the beginning of the first day, but in hour 51, 8′, 45″ of it, and because the required weekday is sought not from the first, but from the sixth—the number $2\frac{85}{576}$ or $\frac{1237}{576}$ should be subtracted. Therefore if this sum is divided by 7 (or into sevenths), [and] however many can be made are subtracted, the integer part of the remainder will give the weekday in which the beginning of the proposed year occurs, counted from the sixth weekday. But the fractional part, converted to sexagesimal parts, will give first the hour, then the minute (first as well as second [part]) in which the beginning of the year occurs. So this calculation in accordance with the arithmetic rules most accurately agrees with the method of the Indians [previously] described, and hence the reasoning of the whole stated procedure is understood. But since from the rule as it is described it may hardly be clear in what way the fractions should be handled, we will demonstrate the matter by an example.

16. Therefore, let it be proposed to investigate the beginning of the present year 1736 after the manner of the Indians. Therefore the [position in the] cycle of the past year will be 49. And the calculation will be as follows:

	60	
	20	
	1200	
	49	
	1249	past year of the Cycle
	409	
	1658	Śaka era
	3179	
	4837	Kaliyuga by $365 + \frac{1}{4} + \frac{5}{576}$
	365	
	1765505	
	$1209\ \frac{1}{4}$	$\frac{1}{4}$ Kaliyuga
	$1766714\ \frac{1}{4}$	
	4837	
	5	
	24185	
	1237	by the rule
	22948	
576 }	$39\ \frac{484}{576}$	

Therefore $1766754\frac{52}{576}$ is the number of elapsed days; [when it is] divided by 7, the remainder will be $3\frac{52}{576} = 3\frac{13}{144}$ weekdays. So the beginning of the year occurs in weekday 3 counted from the sixth, that is, in weekday 2. And the fraction gives 5 hours and 25 minutes of that weekday, just as we stated above.

17. And this calculation can be rendered more easy and more brief in several ways; namely, where

it should be multiplied by 365, multiplication by unity can be substituted in place of it, since 364 can be divided by 7. Then if the numbers to be divided by 576 have a common divisor with 576, the calculation can also be made easier by reducing fractions. But these are of no great moment. Moreover, if 1813 is subtracted in place of 1237, then the weekday in which the beginning of the year occurs will be immediately obtained.

18. As to [what] pertains to the months of this Indian solar year, the number of days which is allotted to individual months does not seem to me at all to be assigned at whim. For the Indians have as a month the space of time in which the sun traverses a twelfth part of the ecliptic, so that the length of a month depends on the speed of the sun. Therefore, since the sun progresses more slowly in the summer than in the winter, it is no wonder that the Indians make their summer months longer than the winter [ones].

19. Hence it is, that among the Indians the same month in different years does not consist of the same number of days. For just as they are accustomed to do in the case of the years, so also in the case of the months, without doubt, they consider the first day of the month to be that in which—either in itself or in the past night—the sun enters a new part of the ecliptic. From which, it turns out, it is necessary that the same month sometimes exceeds by one day, just as happens in the case of the year. Again, it is clear [that] the Indians indeed did not have an intercalary day in leap years, since the motion of the sun determines the quantity of any month.

20. Therefore, since from the inequality of the months it is established [that] the inequality of the motion of the sun is not unknown to the Indians, it would be worthwhile to know what sort of table of solar equation they use, which, however it may be, will not be much different from our tables.

21. What the Yoga or 27 constellations of the zodiac are to the Indians also does not seem obscure to me. For from these constellations they form a month of the fourth type: this is clearly enough to be understood [as] periodic lunar months, which are completed in about 27 days. Wherefore, since the moon takes 27 days to go around the zodiac, one Yoga is seen to be a twenty-seventh part of the zodiac and an assemblage of stars existing in a space of this sort is without doubt one such constellation of which 27 are numbered in the zodiac. Consequently, when they allot these Yogas in the calendars, without doubt they wish to indicate by them the part of the zodiac where the moon is on some day; and since the moon sometimes in one day can enter into two constellations of this sort, it is no wonder if sometimes two such constellations are found written in one day.

Rule for computing the beginning of any year

First, the number of the sixty[-year cycle] in which the given year occurs, is multiplied by $5\frac{25}{48}$ and to the product is added $3\frac{71}{96}$. Then the cycle of the sought year is multiplied by $1 + 1/4 + 5/576$ and the product is added to the former; when this is done, the sum is divided by 7, and the remainder will indicate the weekday together with the hours and minutes in which time the beginning of the year occurs.

As an example, if the beginning of the year 1736 is sought, whose cycle is 50 and index of the sixty[-year] cycle 20, the calculation will be in the following manner:

	20	
	$5\ \frac{25}{48}$	
	100	
	$10\ \frac{5}{12}$	
	$3\ \frac{71}{96}$	add
	$114\ \frac{15}{96}$	
	50	
	$1 + \frac{1}{4} + \frac{5}{576}$	
	50	
	$12\ \frac{1}{2}$	
	$\frac{250}{576}$	
	$62\ \frac{538}{576}$	
Add	$114\ \frac{90}{576}$	
divide by 7	$177\ \frac{13}{144}$	
there will remain	$2\ \frac{13}{144}$	This fraction is
	13	
	60	
	780	
144 }	720	} 5 hours
	60	
	60	
	3600	
144 }	2880	} 25 minutes
	720	

Therefore the beginning of the year falls in weekday 2, hour 5, minute 25, as above.