ADDITAMENTVM
ad Dissertationem
DE VALORIBVS FORMVLAE
INTEGRALIS

$$\int \frac{x^{p-1}\,\partial x}{\sqrt[n]{(1-x^n)^{n-q}}},$$

ab $x=0$ ad $x=1$ extensae.

Auctore
L. EVLERO.

Conuent. exhib. die 17 Octobr. 1776.

§. 1.

Si methodum in praecedente dissertatione traditam ad altiores ordines quam $n=7$ transferre vellemus, ob ingentem aequationum considerandarum numerum labor fieret nimis molestus. Quoniam autem vidimus, non omnes istas aequationes concurrere ad valores singularum formularum determinandos, opus non mediocriter subleuabitur, si quouis casu eas tantum aequationes in computum ducamus, quae immediate ad determinationes formularum perducant, quemadmodum hic pro casu $n=10$ sum ostensurus.

DE-

DETERMINATIO

harum formularum pro casu $n = 10$, vbi formula

$$(p, q) = \int \frac{x^{p-1}\,\partial x}{\sqrt[10]{(1-x^{10})^{10-q}}} = \int \frac{x^{q-1}\,\partial x}{\sqrt[10]{(1-x^{10})^{10-p}}}.$$

§. 2. Hoc casu ergo formulae valorem absolutum recipientes sunt $(10, 1) = 1$, $(10, 2) = \frac{1}{2}$, $(10, 3) = \frac{1}{3}$ et in genere $(10, \alpha) = \frac{1}{\alpha}$. Deinde omnes formulae, in quibus est $p + q = 10$, a circulo pendent ideoque pro cognitis haberi possunt, quas ergo propriis litteris designemus:

$$(1, 9) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{1}{10}\pi} = A, \qquad (6, 4) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{6}{10}\pi} = D,$$

$$(2, 8) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{2}{10}\pi} = B, \qquad (7, 3) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{7}{10}\pi} = C,$$

$$(3, 7) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{3}{10}\pi} = C, \qquad (8, 2) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{8}{10}\pi} = B,$$

$$(4, 6) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{4}{10}\pi} = D, \qquad (9, 1) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{9}{10}\pi} = A,$$

$$(5, 5) = \frac{\pi}{10\,\mathrm{sin.}\,\frac{5}{10}\pi} = E,$$

§. 3. Per has autem formulas circulares reliquas in forma generali contentas neutiquam determinare licet; sed insuper aliquot formulas transcendentes in subsidium vocari oportet, ex quibus cum circularibus illis coniunctis reliquarum omnium valores assignare licebit. Nostro autem casu, quo $n = 10$, sequentes formulas tanquam cognitas spectari conueniet, quae in ordine praecedenti, vbi $n = 9$, erant circulares, nunc

nunc autem in ordinem transcendentium transeunt. Eas igitur sequenti modo designemus:

$$(1, 8) = P, \ (2, 7) = Q, \ (3, 6) = R, \ (4, 5) = S,$$
$$(5, 4) = S, \ (6, 3) = R, \ (7, 2) = Q, \ (8, 1) = P.$$

Scilicet si valores harum litterarum quoque tanquam cognitos spectemus, per eos cum circularibus iunctos reliquas formulas omnes in hoc ordine contentas determinare poterimus. Cum igitur numerus omnium formularum integralium in hoc ordine $n = 10$ contentarum sit 45, ex iis autem nouem vt cognitae spectentur, reliquae 36 per has litteras maiusculas determinari debebunt.

§. 4. Istas autem determinationes ex aequatione generali supra demonstrata peti oportet, quae hac forma continetur:

$$(a, b) \ (a + b, c) = (a, c) \ (a + c, b),$$

vbi assumere licebit semper esse $b > c$, quoniam, si foret $c = b$, aequatio foret identica. Primo igitur vt hinc aequationes, quae immediate determinationes praebeant, nanciscamur, sumamus $a + b = 10$, vt sit $(10, c) = \frac{1}{c}$; tum vero capiatur $c = b - 1$, quo facto pro a ordine scribendo numeros 1, 2, 3, etc. sequentes prodibunt determinationes:

$(1, 9) \ (10, 8) = (1, 8) \ (9, 9)$, siue $\frac{1}{8} A = P \ (9, 9)$, ergo $(9, 9) = \frac{A}{8 P}$.

$(2, 8) \ (10, 7) = (2, 7) \ (9, 8)$, siue $\frac{1}{7} B = Q \ (9, 8)$, ergo $(9, 8) = \frac{B}{7 Q}$.

$(3, 7) \ (10, 6) = (3, 6) \ (9, 7)$, siue $\frac{1}{6} C = R \ (9, 7)$, ergo $(9, 7) = \frac{C}{6 R}$.

(4, 6)

$(4, 6)\,(10, 5) = (4, 5)\,(9, 6)$, fiue $\frac{1}{5}D = S\,(9, 6)$, ergo
$(9, 6) = \frac{D}{5S}$.

$(5, 5)\,(10, 4) = (5, 4)\,(9, 5)$, fiue $\frac{1}{4}E = S\,(9, 5)$, ergo
$(9, 5) = \frac{E}{4S}$.

$(6, 4)\,(10, 3) = (6, 3)\,(9, 4)$, fiue $\frac{1}{3}D = R\,(9, 4)$, ergo
$(9, 4) = \frac{D}{3R}$.

$(7, 3)\,(10, 2) = (7, 2)\,(9, 3)$, fiue $\frac{1}{2}C = Q\,(9, 3)$, ergo
$(9, 3) = \frac{C}{2Q}$.

$(8, 2)\,(10, 1) = (8, 1)\,(9, 2)$, fiue $B = P\,(9, 2)$, ergo
$(9, 2) = \frac{B}{P}$.

§. 5. Ex formulis igitur incognitis illis numero 36 iam octo determinauimus, quae nobis viam fternent ad nouas determinationes, quas primo deriuabimus ex aequatione generali fumendo $a = 1$, $b = 9$, et pro c fcribendo ordine numeros 1, 2, 3 . . . . 8, vnde calculus ita fe habebit:

$$\begin{array}{l|l}
(1, 9)\,(10, 1) = (1, 1)\,(2, 9) & A = (1, 1)\,\frac{B}{P} \text{ ergo } (1, 1) = \frac{AP}{B},\\
(1, 9)\,(10, 2) = (1, 2)\,(3, 9) & \frac{1}{2}A = (1, 2)\,\frac{C}{2Q} \text{ ergo } (1, 2) = \frac{AQ}{C},\\
(1, 9)\,(10, 3) = (1, 3)\,(4, 9) & \frac{1}{3}A = (1, 3)\,\frac{D}{3R} \text{ ergo } (1, 3) = \frac{AR}{D},\\
(1, 9)\,(10, 4) = (1, 4)\,(5, 9) & \frac{1}{4}A = (1, 4)\,\frac{E}{4S} \text{ ergo } (1, 4) = \frac{AS}{E},\\
(1, 9)\,(10, 5) = (1, 5)\,(6, 9) & \frac{1}{5}A = (1, 5)\,\frac{D}{5S} \text{ ergo } (1, 5) = \frac{AS}{D},\\
(1, 9)\,(10, 6) = (1, 6)\,(7, 9) & \frac{1}{6}A = (1, 6)\,\frac{C}{6R} \text{ ergo } (1, 6) = \frac{AR}{C},\\
(1, 9)\,(10, 7) = (1, 7)\,(8, 9) & \frac{1}{7}A = (1, 7)\,\frac{B}{7Q} \text{ ergo } (1, 7) = \frac{AQ}{B},\\
(1, 9)\,(10, 8) = (1, 8)\,(9, 9) & \frac{1}{8}A = (1, 8)\,\frac{A}{8P} \text{ ergo } (1, 8) = \frac{AP}{A},
\end{array}$$

hocque modo feptem nouas determinationes fumus adepti.

§. 6. His autem inuentis consideremus aequationes ex valoribus $a = 1$, $b = 8$, $c = 1, 2, 3 \ldots\ldots 7$ ortas, eritque

$(1, 8)(9, 1) = (1, 1)(2, 8)$	$AP = (1, 1) B$	identica.
$(1, 8)(9, 2) = (1, 2)(3, 8)$	$B = (3, 8) \frac{AQ}{C}$	$(3, 8) = \frac{BC}{AQ}$,
$(1, 8)(9, 3) = (1, 3)(4, 8)$	$\frac{CP}{2Q} = (4, 8) \frac{AR}{D}$	$(4, 8) = \frac{CDP}{2AQR}$,
$(1, 8)(9, 4) = (1, 4)(5, 8)$	$\frac{DP}{3R} = (5, 8) \frac{AS}{E}$	$(5, 8) = \frac{DEP}{3ARS}$,
$(1, 8)(9, 5) = (1, 5)(6, 8)$	$\frac{EP}{4S} = (6, 8) \frac{AS}{D}$	$(6, 8) = \frac{DEP}{4ASS}$,
$(1, 8)(9, 6) = (1, 6)(7, 8)$	$\frac{DP}{5S} = (7, 8) \frac{AR}{C}$	$(7, 8) = \frac{CDP}{5ARS}$,
$(1, 8)(9, 7) = (1, 7)(8, 8)$	$\frac{CP}{6R} = (8, 8) \frac{AQ}{B}$	$(8, 8) = \frac{BCP}{6AQR}$,

§. 7. Nouas determinationes reperiemus ponendo: $a = 1$, $b = 7$, $c = 3, 4, 5, 6$; hinc enim nanciscimur sequentes determinationes:

$(1, 7)(8, 3) = (1, 3)(4, 7)$	$C = (4, 7) \frac{AR}{D}$	$(4, 7) = \frac{CD}{AR}$,
$(1, 7)(8, 4) = (1, 4)(5, 7)$	$\frac{CDP}{2BR} = (5, 7) \frac{AS}{E}$	$(5, 7) = \frac{CDEP}{2ABRS}$,
$(1, 7)(8, 5) = (1, 5)(6, 7)$	$\frac{DEPQ}{3BRS} = (6, 7) \frac{AS}{D}$	$(6, 7) = \frac{DDEPQ}{3ABRSS}$,
$(1, 7)(8, 6) = (1, 6)(7, 7)$	$\frac{DEPQ}{4BSS} = (7, 7) \frac{AR}{C}$	$(7, 7) = \frac{CDEPQ}{4ABRSS}$,

§. 8. Sumamus nunc $a = 1$, $b = 6$, $c = 4, 5$, eritque

$(1, 6)(7, 4) = (1, 4)(5, 6)$	$D = (5, 6) \frac{AS}{E}$	$(5, 6) = \frac{DE}{AS}$,
$(1, 6)(7, 5) = (1, 5)(6, 6)$	$\frac{DEP}{2BS} = (6, 6) \frac{AS}{D}$	$(6, 6) = \frac{DDEP}{2ABSS}$,

Hactenus igitur omnes formulas (p, q) determinauimus, in quibus $p + q > 10$. Ex reliquis autem, vbi $p + q < 9$, iam nacti sumus istas:

(1, 1)

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7),
ita vt adhuc determinandae relinquantur iſtae:

(2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 3), (3, 4), (3, 5),
(4, 4).

§. 9. Pro his inueniendis ſumamus $a = 1$ et $c = 1$, pro b autem ordine capiamus numeros, 2, 3, etc. atque conſequemur has aequationes:

$$\begin{array}{l|l|l}
(1,2)(3,1) = (1,1)(2,2) & \frac{AAQR}{CD} = (2,2)\frac{AP}{B} & (2,2) = \frac{ABQR}{CDP}, \\
(1,3)(4,1) = (1,1)(2,3) & \frac{AARS}{DE} = (2,3)\frac{AP}{B} & (2,3) = \frac{ABRS}{DEP}, \\
(1,4)(5,1) = (1,1)(2,4) & \frac{AASS}{DE} = (2,4)\frac{AP}{B} & (2,4) = \frac{ABSS}{DEP}, \\
(1,5)(6,1) = (1,1)(2,5) & \frac{AARS}{CD} = (2,5)\frac{AP}{B} & (2,5) = \frac{ABRS}{CDP}, \\
(1,6)(7,1) = (1,1)(2,6) & \frac{AAQR}{BC} = (2,6)\frac{AP}{B} & (2,6) = \frac{ABQR}{BCP},
\end{array}$$

ſicque etiamnunc determinandae reſtant formulae (3, 3), (3, 4), (3, 5) et (4, 4).

§. 10. Pro his ſumatur $a = 1$, $c = 2$ et $b = 3, 4, 5$, etc. tum enim prodibunt hae aequationes:

$$\begin{array}{l|l|l}
(1,3)(4,2) = (1,2)(3,3) & \frac{AABRSS}{DDEP} = (3,3)\frac{AQ}{C} & (3,3) = \frac{ABCRSS}{DDEPQ}, \\
(1,4)(5,2) = (1,2)(3,4) & \frac{AABRSS}{CDEP} = (3,4)\frac{AQ}{C} & (3,4) = \frac{ABRSS}{DEPQ}, \\
(1,5)(6,2) = (1,2)(3,5) & \frac{AAQRS}{CDP} = (3,5)\frac{AQ}{C} & (3,5) = \frac{ARS}{DP},
\end{array}$$

Vnica ergo formula reſtat determinanda, ſcilicet (4, 4), quae ex hac aequatione: $(1,4)(5,3) = (1,3)(4,4)$ definietur; erit enim $\frac{AARSS}{DEP} = (4,4)\frac{AR}{D}$, ideoque $(4,4) = \frac{ASS}{EP}$.

 §. 11.

§. 11. Vt nunc omnes has determinationes simul aspectui exponamus, quoniam in hoc ordine $n = 10$ omnino 45 formulae integrales occurrunt, si ex iis vt cognitae spectentur nouem sequentes:

$$(1,9) = A,\ (2,8) = B,\ (3,7) = C,\ (4,6) = D,\ (5,5) = E,$$
$$(1,8) = P,\ (2,7) = Q,\ (3,6) = R,\ (4,5) = S,$$

reliquae triginta sex ex his sequenti modo determinabuntur:

1.	$(9,9) = \frac{A}{8P}$	19.	$(2,6) = \frac{AQR}{CP}$
2.	$(9,8) = \frac{B}{7Q}$	20.	$(3,5) = \frac{ARS}{DP}$
3.	$(9,7) = \frac{C}{6R}$	21.	$(4,4) = \frac{ASS}{EP}$
4.	$(9,6) = \frac{D}{5S}$	22.	$(4,8) = \frac{CDP}{2AQR}$
5.	$(9,5) = \frac{E}{4S}$	23.	$(5,8) = \frac{DEP}{3ARS}$
6.	$(9,4) = \frac{D}{3R}$	24.	$(6,8) = \frac{DEP}{4ASS}$
7.	$(9,3) = \frac{C}{2Q}$	25.	$(7,8) = \frac{CDP}{5ARS}$
8.	$(9,2) = \frac{B}{P}$	26.	$(8,8) = \frac{BCP}{6AQR}$
9.	$(1,1) = \frac{AP}{B}$	27.	$(2,2) = \frac{ABQR}{CDP}$
10.	$(1,2) = \frac{AQ}{C}$	28.	$(2,3) = \frac{ABRS}{DEP}$
11.	$(1,3) = \frac{AR}{D}$	29.	$(2,4) = \frac{ABSS}{DEP}$
12.	$(1,4) = \frac{AS}{E}$	30.	$(2,5) = \frac{ABRS}{CDP}$
13.	$(1,5) = \frac{AS}{D}$	31.	$(5,7) = \frac{CDEP}{2ABRS}$
14.	$(1,6) = \frac{AR}{C}$	32.	$(6,6) = \frac{DDEP}{2ABSS}$
15.	$(1,7) = \frac{AQ}{B}$	33.	$(3,4) = \frac{ABRSS}{DEPQ}$
16.	$(3,8) = \frac{BC}{AQ}$	34.	$(6,7) = \frac{DDEPQ}{3ABRSS}$
17.	$(4,7) = \frac{CD}{AR}$	35.	$(7,7) = \frac{CDEPQ}{4ABRSS}$
18.	$(5,6) = \frac{DE}{AS}$	36.	$(3,3) = \frac{ABCRSS}{DDEPQ}$

§. 12.

§. 12. Eadem methodo, qua hic vſi ſumus pro caſu $n = 10$, haud difficile erit ordines altiores euoluere; neque tamen hinc adhuc elucet, quanam lege omnes determinationes progrediantur, quandoquidem valores certarum formularum continuo magis euadunt complicati. Ceterum valores, quos hic inuenimus, omnibus aequationibus in forma generali

$$(a, b)\,(a+b, c) = (a, c)\,(a+c, b),$$

contentis ſatisfacere deprehenduntur, ita vt perpetuo aequatio identica reſultet, neque idcirco inde ulla noua relatio inter litteras noſtras maiuſculas deduci queat. Tandem probe hic notaſſe iuuabit, quod in omnibus ordinibus, praeter formulas a circulo pendentes, commodiſſime eae formulae, quae in ordine proxime praecedente erant circulares, hic etiam tanquam cognitae accipi queant, quippe quibus determinationes omnes optimo ſucceſſu perfici poſſunt.

METHODVS GENERALIS
determinandi valores formulae

$$(p, q) = \int \frac{x^{p-1}\,\partial x}{\sqrt[n]{(1-x^n)^{n-q}}} = \int \frac{x^{q-1}\,\partial x}{\sqrt[n]{(1-x^n)^{n-p}}},$$

a termino $x = 0$ vsque ad $x = 1$ extenſa.

Vbi praeter formulas circulum inuoluentes, in quibus eſt $p + q = n$, etiam illae pro cognitis accipiuntur, in quibus eſt $p + q = n - 1$.

I. Cum aequatio generalis, vnde omnes hae determinationes ſunt petendae, ſit

$$(a, b)\,(a+b, c) = (a, c)\,(a+c, b),$$

 ſumatur

ſumatur primo $a = n - \alpha$, $b = \alpha$ et $c = \alpha - 1$; eritque aequatio:

$$(n - \alpha, \alpha)(n, \alpha - 1) = (n - \alpha, \alpha - 1)(n - 1, \alpha),$$

vbi eſt $n, \alpha - 1 = \frac{1}{\alpha - 1}$. In primo autem factore, ob $p = n - \alpha$ et $q = \alpha$, eſt $p + q = n$, ideoque datur. In tertio autem autem factore, vbi $p = n - \alpha$ et $q = \alpha - 1$, eſt $p + q = n - 1$, ideoque pariter datur. Hinc ergo colligimus

$$(n - 1, \alpha) = \frac{1}{\alpha - 1} \cdot \frac{(n - \alpha, \alpha)}{(n - \alpha, \alpha - 1)},$$

vbi eſſe debet $\alpha > 1$, ita vt pro α accipi queant omnes numeri a 2 vsque ad $n - 1$; at vero caſu $\alpha = 1$ valor formulae per ſe eſt notus.

II. In aequatione generali iam ſumatur $a = \beta$, $b = n - \beta - 1$ et $c = 1$, eritque noſtra aequatio:

$$(\beta, n - \beta - 1)(n - 1, 1) = (\beta, 1)(\beta + 1, n - \beta - 1),$$

ex qua aequatione colligitur:

$$\beta, 1 = \frac{(\beta, n - \beta - 1)(n - 1, 1)}{(\beta + 1, n - \beta - 1)},$$

vbi eſſe debet $\beta < n - 1$, ita vt hinc omnes formulae $\beta, 1$ definiantur, a valore $\beta = 1$ vsque ad $\beta = n - 1$, quo poſteriore caſu formula $(n - 1, 1)$ per ſe cognoſcitur.

III. Vt hinc etiam alias formas eliciamus, ſumamus $a = 1$, $b = n - 2$, $c = \gamma$, vt oriatur haec aequatio:

$$(1, n - 2)(n - 1, \gamma) = (1, \gamma)(1 + \gamma, n - 2),$$

vbi primus factor ac tertius dantur per N°. II. ſecundus vero per N°. I. vnde quartus deriuatur, ſcilicet:

$$(1 + \gamma, n - 2) = \frac{(1, n - 2)(n - 1, \gamma)}{(1, \gamma)},$$

vbi valores ipſius $1 + \gamma$ a 2 vsque ad $n - 2$ augeri poſſunt.

Cum

Cum igitur per N°. I. fit

$$(n-1, \gamma) = \frac{1}{\gamma - 1} \cdot \frac{(n-\gamma, \gamma)}{(n-\gamma, \gamma-1)},$$

tum vero per N°. II. fit

$$(\gamma, 1) = \frac{(\gamma, n-\gamma-1)(n-1, 1)}{(\gamma+1, n-\gamma-1)},$$

his valoribus fubftitutis fiet

$$(n-2, 1+\gamma) = \frac{1}{\gamma-1} \cdot \frac{(1, n-2)(n-\gamma, \gamma)(\gamma+1, n-\gamma-1)}{(n-\gamma, \gamma-1)(\gamma, n-\gamma-1)(n-1, 1)}.$$

IV. Sumamus nunc $a = 1$, $b = n-3$, $c = \delta$, prodibitque haec aequatio:

$$(1, n-3)(n-2, \delta) = (1, \delta)(1+\delta, n-3),$$

vnde colligitur

$$(n-3, 1+\delta) = \frac{(n-3, 1)(n-2, \delta)}{(\delta, 1)},$$

vbi ergo $1+\delta$ continet numeros $2, 3, 4 \ldots\ldots n-3$, ita vt hinc excludatur $n-3, 1$, quae autem per N°. I. datur. At fi valores ante reperti fubftituantur, fiet

$$(n-3, 1+\delta) = \frac{1}{\delta-2} \cdot \frac{(n-3, 2)(n-2, 1)(n-\delta+1, \delta-1)(\delta, n-\delta)(\delta+1, n-\delta-1)}{(n-2, 2)(n-\delta+1, \delta-2)(\delta-1, n-\delta)(n-1, 1)(\delta, n-\delta-1)},$$

Vnde patet effe debere $\delta > 2$, eodemque modo pro praecedente formula $\gamma > 1$, ita vt hic excludantur cafus $(n-3, 1)$, $(n-3, 2)$, quorum quidem prior per N°. I. datur, alter vero per fe.

V. Statuamus nunc $a = 1$, $b = n-4$ et $c = \varepsilon$, prodibitque haec aequatio:

$$(1, n-4)(n-3, \varepsilon) = (1, \varepsilon)(1+\varepsilon, n-4),$$

vnde concluditur

$$(n-4, 1+\varepsilon) = \frac{(n-4, 1)(n-3, \varepsilon)}{(1, \varepsilon)};$$

vbi fi loco $n-3, \varepsilon$ valor ante inuentus fubftitueretur, factor abfo-

abſolutus ingrederetur $\frac{1}{\varepsilon-3}$, ita vt eſſe debeat $\varepsilon > 3$, ideoque $1+\varepsilon > 4$, vnde hic excluduntur caſus $(n-4,1)$, $(n-4,2)$, $(n-4,3)$, quorum quidem primus ex N°. II. tertius autem per ſe datur, medius vero reuera manet incognitus.

VI. Statuamus porro $a=1$, $b=n-5$, $c=\zeta$, et aequatio erit

$$(1, n-5)(n-\zeta) = (1,\zeta)(1+\zeta, n-5),$$

vnde fit

$$(n-5, 1+\zeta) = \frac{(n-5,1)(n-4,\zeta)}{(1,\zeta)},$$

vbi ob formulam $(n-4,\zeta)$ debet eſſe $\zeta > 4$, ideoque $1+\zeta > 5$, vnde hinc excluduntur caſus $(n-5,1)$, $(n-5,2)$, $(n-5,3)$, $(n-5,4)$, quorum quidem primus ex N°. II. conſtat, quartus vero per ſe datur, ita vt hic occurrant duo caſus etiamnunc incogniti $(n-5,2)$ et $(n-5,3)$.

VII. Simili modo ſi vlterius ſumamus $a=1$, $b=n-6$ et $c=\eta$, prodibit

$$(n-6, 1+\eta) = \frac{(n-6,1)(n-5,\eta)}{(1,\eta)},$$

vbi reuera occurrunt tres ſequentes caſus: $(n-6,2)$, $(n-6,3)$, $(n-6,4)$, qui adhuc manent incogniti, atque hoc modo progredi licebit, quousque neceſſe fuerit; vnde patet numerum caſuum incognitorum continuo augeri, ita vt terminorum p et q alter futurus ſit vel 2, vel 3, vel 4, etc. qui igitur caſus adhuc definiendi reſtant.

VIII. Sumamus nunc primo $a=1$, $b=\theta$, $c=1$, vt aequatio noſtra fiat

$$(1,\theta)(1+\theta,1) = (1,1)(2,\theta),$$

vnde

vnde concludimus

$$(2, \theta) = \frac{(1, \theta)(1 + \theta, 1)}{(1, 1)},$$

quae formula iam omnes casus exclusos suppeditat, in quibus alter terminus erat 2.

IX. Deinde sumamus $a = 2$, $b = \varkappa$ et $c = 1$, vt aequatio prodeat $(2, \varkappa)(2 + \varkappa, 1) = (2, 1)(3, \varkappa)$, vnde fit

$$(3, \varkappa) = \frac{(2, \varkappa)(2 + \varkappa, 1)}{(2, 1)},$$

vbi cum $(2, \varkappa)$ per praecedentem N^{rum} detur, nunc etiam ii casus innotescunt, vbi alter terminus erat 3.

X. Sumatur porro $a = 3$, $b = \varkappa$, $c = 1$, eritque $(3, \varkappa)(3 + \varkappa, 1) = (3, 1)(4, \varkappa)$, vnde fit

$$(4, \varkappa) = \frac{(3, \varkappa)(3 + \varkappa, 1)}{(3, 1)},$$

vnde igitur ii casus eliciuntur, vbi alter terminus erat 4. Eodem modo pro reliquis proceditur; sicque omnes plane casus in formula proposita contenti plene sunt determinati.