$$\frac{dx}{dt} = \frac{(B+C)\,ad\zeta + C\,bd\eta}{(A+B+C)\,dt};\quad \frac{dy}{dt} = \frac{-A\,ad\zeta + C\,bd\eta}{(A+B+C)\,dt};$$

atque $$\frac{dz}{dt} = \frac{-A\,ad\zeta - (A+B)\,bd\eta}{(A+B+C)\,dt}.$$

Scholion.

21. Solutio ergo hujus problematis ab integratione formulae $t = \int \frac{du\sqrt{fg(mm - nn - 4\cos u^2)}}{2\sqrt{(mf - g + 2f\cos u)}}$, & propterea per quadraturam curvae cujuspiam construi potest, ita ut ad quemvis valorem ipsius t valor anguli u assignetur: quo autem invento nova opus est quadratura ad angulum v determinandum; quamobrem solutio practica hujus problematis maxime est operosa. Dantur tamen nonnulli casus, quibus solutio multo fit simplicior ac tractabilior, quos hic seorsim evolvamus.

Casus. I.

22. Quoniam invenimus inter t & u hanc aequationem

$$2\,dt\sqrt{(mf - g + 2f\cos u)} = du\sqrt{fg}(m^2 - n^2 - 4\cos u^2)$$

manifestum est huic aequationi satisfieri, si fuerit:

$mf - g + 2f\cos u = 0$ seu $\cos u = \frac{g - mf}{2f}$; fiet enim u constans, & $du = 0$, unde utrumque membrum evanescit: Sit igitur $u = 2\alpha$, ut sit $\cos 2\alpha = \frac{g - mf}{2f}$, eritque $m + 2\cos u = \frac{g}{f}$; ideoque $v = 2\int \frac{dt\sqrt{f}}{g}$, ac propterea $v =$

$\frac{2t\sqrt{f}}{g}+2\beta$. Hinc fit $\zeta=\frac{t\sqrt{f}}{g}+\beta+\alpha$ & $= \frac{t\sqrt{f}}{g}+\beta-\alpha$. Cum igitur differentia angulorum ζ & η sit constans, angulus ABC perpetuo idem manebit, corpusculaque A, B, C perinde movebuntur, ac si corpus inflexile constituerent. Si igitur ponamus centrum gravitatis perpetuo in puncto O quiescere, casus iste locum habebit, si initio quo $t=0$, corpuscula ita fuerint collocata, ut esset:

$$p=\frac{-(B+C)a\sin(\beta+\alpha)-Cb\sin(\beta-\alpha)}{A+B+C}$$

$$q=\frac{+Aa\sin(\beta+\alpha)-Cb\sin(\beta-\alpha)}{A+B+C}$$

$$r=\frac{+Aa\sin(\beta+\alpha)+(A+B)b\sin(\beta-\alpha)}{A+B+C}$$

Atque $x=\frac{-(B+C)a\cos(\beta+\alpha)-Cb\cos(\beta-\alpha)}{A+B+C}$

$$y=\frac{Aa\cos(\beta+\alpha)-Cb\cos(\beta-\alpha)}{A+B+C}$$

$$z=\frac{Aa\cos(\beta+\alpha)+(A+b)\cos(\beta-\alpha)}{A+B+C}$$

Perpetuo vero celeritates corpusculorum ob $d\zeta=d\eta=\frac{dt\sqrt{f}}{g}$ ita se habebunt, ut sit

$$\frac{dp}{dt}=\frac{x\sqrt{f}}{g};\ \frac{dq}{dt}=\frac{y\sqrt{f}}{g};\ \frac{dr}{dt}=\frac{z\sqrt{f}}{g}$$

yd

$$\frac{dx}{dt} = -\frac{p\sqrt{f}}{g};\quad \frac{dy}{dt} = -\frac{q\sqrt{f}}{g};\quad \frac{dz}{dt} = -\frac{r\sqrt{f}}{g}$$

unde colliguntur distantiæ singulorum corpusculorum tam inter se quam a puncto O constantes. Atque cum anguli ζ & η æqualiter & uniformiter crescant, singula corpora circa centrum gravitatis O æquali motu rotatorio uniformiter gyrabuntur.

Ponamus initio motus omnia corpuscula in linea recta $O\varrho$ posita fuisse, atque ob $x = 0$, $y = 0$, $z = 0$, erit $\beta + \alpha = 90^\circ$ & $\beta - \alpha = 90^\circ$, unde $\alpha = 0$, & $\beta = 90^\circ$. Iste ergo casus locum habebit si fuerit: Fig. 4.

$$OA = \frac{(B+C)a + Cb}{A+B+C} = -p$$

$$OB = \frac{-Aa + Cb}{A+B+C} = -$$

$$OC = \frac{Aa + (A+B)b}{A+B+C} = r$$

atque si singulis corpusculis secundum directionem $O\omega$ impressæ fuerint celeritates, quæ sint inter se uti OA, OB, & OC, tum filum ABC circa punctum O instar virgæ rigidæ rotabitur motu uniformi.

Casus II.

23. Ponatur $f = \infty$ & $n = 0$ seu $\frac{(B+C)a}{Cb} = \frac{(A+B)b}{Aa}$, eritque η quantitas constans, sit ea $\eta = 2\alpha$, tum

vero erit $t = \int \frac{du\sqrt{g(m - 2\cos u)}}{2}$, ex qua aequatione facilius ad datum tempus angulus u definiri potest. Cum igitur sit $\zeta = \alpha + \frac{1}{2}u$ & $\eta = \alpha - \frac{1}{2}u$, quantum alter augetur, tantum alter diminuitur: hincque anguli ABb & CBb aequaliter perpetuo vel crescent decrescent. Si recta AB producatur in γ, erit angulus $CB\gamma = \zeta - \eta = u$, hicque perpetuo a tempore jam elapso t ita pendebit, ut sit $t = \frac{1}{2}\int du\sqrt{g(m - 2\cos u)}$. Facilius autem hinc ex angulo $CB\gamma$ tempus jam elapsum t determinari poterit. Ut vero ex angulo u positio omnium corpusculorum definiri queat, notandum est, quia $\frac{(B+C)a}{Ca} = \frac{(A+B)b}{Aa}$ fore

$$(B+C)a = \tfrac{1}{2}mCb \ \& \ (A+B)b = \tfrac{1}{2}mAa,$$

& $m = 2\sqrt{\frac{(A+B)(B+C)}{AC}}$. Hincque obtinebitur

$\frac{a}{b} = \sqrt{\frac{(A+B)C}{(B+C)A}}$; quod est requisitum, ut praesens casus locum habere possit. Erit ergo elapso tempore t, quo angulus $CB\gamma = u$ est ortus:

$$p = \frac{-(A+C)a\sin(\alpha + \frac{1}{2}u) - Cb\sin(\alpha - \frac{1}{2}u)}{A+B+C}$$

$$q = \frac{Aa\sin(\alpha + \frac{1}{2}u) - Cb\sin(\alpha - \frac{1}{2}u)}{A+B+C}$$

$$r = \frac{Aa\sin(\alpha + \frac{1}{2}u) + (A+B)b\sin(\alpha - \frac{1}{2}u)}{A+B+C}$$

$x = -($

$= Aac$

$z = Aac$

Praeterea

Cel. corp. A

Cel. corp. B

Cel. corp. C

At

Cel. Corp. A

Cel. Corp. B:

Cel. Corp. C:

siquidem pot

24.

ter se esse ae

$x =$

$$x = \frac{-(B+C)a\cos(\alpha+\tfrac{1}{2}u) - Cb\cos(\alpha-\tfrac{1}{2}u)}{A+B+C}$$

$$= \frac{Aa\cos(\alpha+\tfrac{1}{2}u) - Cb\cos(\alpha-\tfrac{1}{2}u)}{A+B+C}$$

$$z = \frac{Aa\cos(\alpha+\tfrac{1}{2}u) + (A+B)b\cos(\alpha-\tfrac{1}{2}u)}{A+B+C}$$

Præterea vero corpusculorum celeritates ita se habebunt; Secundum directionem Oυ

$$\text{Cel. corp. A} = -\frac{(B+C)a\cos(\alpha+\tfrac{1}{2}u) + Cb\cos(\alpha-\tfrac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

$$\text{Cel. corp. B} = \frac{Aa\cos(\alpha+\tfrac{1}{2}u) + Cb\cos(\alpha-\tfrac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

$$\text{Cel. corp. C} = \frac{Aa\cos(\alpha+\tfrac{1}{2}u) - (A+C)b\cos.(\alpha-\tfrac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

At vero secundum directionem Oω erit

$$\text{Cel. Corp. A} = \frac{(B+C)a\sin(\alpha+\tfrac{1}{2}u) - Cb\sin(\alpha-\tfrac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

$$\text{Cel. Corp. B} = -\frac{Aa\sin(\alpha+\tfrac{1}{2}u) - Cb\sin(\alpha-\tfrac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

$$\text{Cel. Corp. C} = -\frac{Aa\sin(\alpha+\tfrac{1}{2}u) + (A+B)b\sin(\alpha-\tfrac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

siquidem ponamus centrum gravitatis in puncto O quiescere.

Exemplum.

24. Ponamus corpora extrema A & C inter se esse æqualia, & æqualiter a medio B remota, ita ut Fig. 5.

ds

sit $C = A$ & $b = a$, eritque $m = \frac{2(A+B)}{A}$. Ponamus insuper hæc tria corpora initio in directum fuisse posita, ita ut sumto $t = 0$ fiat quoque $u = 0$, atque ob x, y & $z = 0$ oportebit esse $\alpha = 90^\circ$. Ut igitur motus ad casum II. componatur, celeritates corporum secundum directionem Oo evanescent, celeritates vero in directionibus ad axem normalibus ita erunt.

Corpus A habebit celeritatem $= \frac{a\sqrt{AB}}{(2A+B)\sqrt{2g}}$ in directione $A\alpha$.

Corpus B habebit celeritatem $= \frac{2Aa\sqrt{A:B}}{(2A+B)\sqrt{2g}}$ in directione $B\beta$.

Corpus C habebit celeritatem $= \frac{a\sqrt{AB}}{(2A+B)\sqrt{2g}}$ in directione $C\gamma$.

Cum igitur celeritates extremorum A & C sint æquales, ponantur debitæ altitudini $= k$; erit $\frac{a\sqrt{AB}}{(2A+B)\sqrt{2g}} = \sqrt{k}$ & $\sqrt{2g} = \frac{a\sqrt{AB}}{(2A+B)\sqrt{k}}$; & celeritas medii in directione $B\beta$ erit $= \frac{2A\sqrt{k}}{B}$, & altitudo huic celeritati debita $= \frac{4A^2k}{BB}$.

Nunc quæramus statum horum corporum elapso tempore t, quo

quo generatur angulus u, ut fit $t = \int \frac{du\sqrt{g(m - 2\cos u)}}{2}$

$$= \int \frac{dt\sqrt{2g\left(\frac{A+B}{A} - \cos u\right)}}{2} = \frac{a\sqrt{B}}{2(2A+B)\sqrt{k}} \int du \sqrt{(B + A - A\cos u)}$$

Fig. 6.

Inventoque hoc angulo $CB\gamma = u$, habebitur:

$p = -a\cos\frac{1}{2}u;\ q = 0;\ r = a\cos\frac{1}{2}u$

$x = \frac{Ba\sin\frac{1}{2}u}{2A+B};\ y = -\frac{2Aa\sin\frac{1}{2}u}{2A+B};\ z = \frac{Ba\sin\frac{1}{2}u}{2A+B}$

Celeritates vero corpusculorum ita se habebunt, ob

$$(A+B+C)\sqrt{g(m-2\cos u)} = \frac{a\sqrt{B(B+A-A\cos u)}}{\sqrt{k}}$$

Secundum directionem Oo

Corp. A $= \frac{(2A+B)\sin\frac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$

Corp. B $= 0$

Corp. C $= \frac{-(2A+B)\sin\frac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$

Secundum directionem $O\omega$

Corp. A $= \frac{B\cos\frac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$

Corp. B $= \frac{-2A\cos\frac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$

Corp. C $= \frac{B\cos\frac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$

 Oc-

Occupabunt ergo corpuscula ABC elapso tempore t, quo fila AB & CB ad angulum $CB\gamma = u$ inflectuntur, ejusmodi situm, quem figura indicat; eruntque ipsi anguli ad e & f, quibus fila ad axem $O\omega$ inclinantur, $BeO = BfO = \frac{1}{2}u$.

Si ergo ponatur angulus $\frac{1}{2}u = \phi$, ob $1 - \cos 2\phi = 2 \sin \phi^2$

erit $t = \frac{a\sqrt{B}}{(2A+B)\sqrt{k}} \int d\phi \sqrt{(B + 2A \sin \phi^2)}$, hincque simul celeritates ob $\sqrt{B(B+A-A\cos u)} = \sqrt{B(B+2A\sin\phi^2)}$ simplicius exprimentur.

Scholion.

25. Potest hoc exemplum, quod Celeb. Daniel Bernoulli ad methodi meae bonitatem explorandam mihi evolvendum proposuit, etiam sine subsidio solutionis generalis hic traditae ex cognitis mechanicae principiis resolvi. Fig. 6. Cum enim utrinque omnia sint aequalia, perspicuum est corpusculum medium B alium motum habere non posse nisi secundum rectam $O\omega$, corpuscula vero extrema A & C aequaliter ad rectam $O\omega$ vel accedere vel ab ea recedere oportere. Ex qua circumstantia per principium conservationis virium vivarum singulorum corpusculorum motus sequenti modo determinari poterunt. Positis ut ante corpusculorum A & C massis $= A$, corpusculi B massa $= B$, & longitudine fili $AB = BC = a$, atque angulo $BeO = BfO = \phi$, sit celeritas corpusculi B in directione $B\beta$ debita altitudini v; utriusque corpusculi A & C sit celeritas rotatoria circa B debita altitudini u; erit utriusque celeritas secundum directionem $O\omega$ debita altitudine $= u \cos\phi^2$, & celeritas, qua utrumque directe ad $O\omega$ accedit debita altitudini

dixi $u \sin\phi^2$. Hinc erit celeritas, qua utrumque ab axe $O\omega$ recedit $= \cos\phi\sqrt{u} - \sqrt{v}$, et quia centrum gravitatis in quiete manere ponitur, erit $B\sqrt{v} = 2A(\cos\phi\sqrt{u} - \sqrt{v})$, ideoque $\sqrt{u} = \frac{(2A+B)\sqrt{v}}{2A\cos\phi}$ seu $\sqrt{v} = \frac{2A\cos\phi.\sqrt{u}}{2A+B}$. Fig. 5.

Deinde cum initio utriusque corpusculi A & C celeritas secundum directionem $O\omega$ posita sit $= \sqrt{k}$, celeritas corpusculi B in directione $B\beta = \frac{2A\sqrt{k}}{B}$, summa virium vivarum erat $= 2Ak + \frac{4AAk}{B} = \frac{2Ak}{B}(2A+B)$, quae perpetuo eadem manere debebit. Praesenti autem casu est corporis B vis viva $= Bv$, & corpus A, quia habet duplicem celeritatem alteram $= \cos\phi\sqrt{u} - \sqrt{v}$, alteram vero $= \sin\phi\sqrt{u}$, erit ejus vis viva $= A(\cos\phi\sqrt{u} - \sqrt{v})^2 + Au\sin\phi^2$ Fig. 6

$$= A\left(\frac{B\cos\phi\sqrt{u}}{2A+B}\right)^2 + Au\sin\phi^2 = Au\left(\sin\phi^2 + \frac{BB\cos\phi^2}{(2A+B)^2}\right),$$

cui cum vis viva corporis C sit aequalis, erit summa virum vivarum $= Bv + \frac{2Au}{(2A+B)^2}((2A+B)^2\sin\phi^2 + B^2\cos\phi^2)$

At est $Bv = \frac{4AAB\cos\phi^2}{(2A+B)^2} = \frac{2Au}{(2A+B)^2}.\, 2AB\cos\phi^2$;

unde tota vis viva erit $= \frac{2Au}{(2A+B)^2}((2A+B)^2\sin\phi^2 + (2A+B)B\cos\phi^2) = \frac{2Au}{2A+B}(2A\sin\phi^2 + B)$, quae cum aequalis esse debeat summae virium vivarum initiali, erit

$$\frac{2Au}{2A+B}(2A\sin\phi^2+B) = \frac{2Ak}{B}(2A+B), \text{ atque } u(2A\sin\phi^2+B) = \frac{k}{B}(2A+B)^2, \text{ hincque } \sqrt{u} = \frac{(2A+B)\sqrt{k}}{\sqrt{B(B+2A\sin\phi^2)}}$$

Quoniam nunc celeritas rotatoria corpusculi A est $\sqrt{u}$, hac tempusculo infinite parvo dt arculum radio AB $= a$ describet $= ad\phi$, eritque idcirco $\frac{ad\phi}{\sqrt{u}} = dt$, unde habebitur $dt = \frac{ad\phi\sqrt{B(B+2A\sin\phi^2)}}{(2A+B)\sqrt{k}}$ & $t = \frac{a\sqrt{B}}{(2A+B)\sqrt{k}}\int d\phi\sqrt{(B+2A\sin\phi^2)}$ quæ est ea ipsa æquatio, quam ante invenimus; hocque adeo consensu methodi bonitas atque solutionis generalis veritas comprobatur.

Problema. IV.

Fig. 7.

26. *Sint nunc corpuscula quotcunque* A, B, C, D, E, *&c. filo colligata, quæ si super plano horizontali utcunque projiciantur, eorum motum investigare.*

Solutio.

Ductis ex singulis corpusculis ad axem fixum Oo perpendiculis vocentur:

	$Oa = p$;	$Ob = q$;	$Oc = r$;	$Od = s$	&c.
	$Aa = x$;	$Bb = y$;	$Cc = z$;	$Dd = v$	&c.
Filum	$AB = a$;	$BC = b$;	$CD = c$;	$DE = d$;	&c.
Ang.	$ABb = \zeta$;	$BCc = \eta$;	$CDd = \theta$;	$DEe = \iota$;	&c.

Ex

Ex quibus denominationibus deducuntur sequentes aequationes.

$$q-p=a\sin\zeta;\quad r-q=b\sin\eta;\quad s-r=c\sin\theta;\quad \&c.$$
$$y-x=a\cos\zeta;\quad z-y=b\cos\eta;\quad v-z=c\cos\theta;\quad \&c.$$

Cum nunc spatiola hinc exprimi queant, quae a singulis corpusculis tempusculo infinite parvo, quod sit $=dt$, describuntur, celeritates eorum tam secundum directionem axi Oo parallelam, quam ad Oo normalem sequenti modo definientur.

Corpusculi	celeritas in directione Oo erit	celeritas in directione $O\omega$ erit
A	$\frac{dp}{dt}$	$\frac{dx}{dt}$
B	$\frac{dq}{dt}$	$\frac{dy}{dt}$
C	$\frac{dr}{dt}$	$\frac{dz}{dt}$
D	$\frac{ds}{dt}$	$\frac{dv}{dt}$
	&c.	&c.

Denotent jam litterae A, B, C, D, &c. respective massas corpusculorum, & quia eorum motus a tensione filorum alteratur, sit tensio fili $AB=P$; tensio fili $BC=Q$; tensio fili $CD=R$, &c. quibus positis.

Corpusculum	sollicitabitur in directione Oo vi	sollicitabitur in directione $O\omega$ vi
A	$P\sin\zeta$	$P\cos\zeta$
B	$Q\sin\eta - P\sin\zeta$	$Q\cos\eta - P\cos\zeta$
C	$R\sin\theta - Q\sin\eta$	$R\cos\theta - Q\cos\eta$
D	$S\sin\iota - R\sin\theta$	$S\cos\iota - R\cos\theta$
	&c.	&c.

Ex his ſollicitationibus ſequentes orientur accelerationes

$$\frac{2A\,ddp}{dt^2} = P \sin\zeta \qquad \frac{2A\,ddx}{dt^2} = P \cos\zeta$$

$$\frac{2B\,ddq}{dt^2} = Q \sin\eta - P \sin\zeta \qquad \frac{2B\,ddy}{dt^2} = Q \cos\eta - P \cos\zeta$$

$$\frac{2C\,ddr}{dt^2} = R \sin\theta - Q \sin\eta \qquad \frac{2C\,ddz}{dt^2} = R \cos\theta - Q \cos\eta$$

$$\frac{2D\,dds}{dt^2} = S \sin\iota - R \sin\theta \qquad \frac{2D\,ddv}{dt^2} = S \cos\iota - R \cos\theta$$

&c. &c.

Ex his æquationibus additis orientur iſtæ duæ æquationes

$$2A\,ddp + 2B\,ddq + 2C\,ddr + 2D\,dds + \&c. = 0$$

$$2A\,ddx + 2B\,ddy + 2C\,ddz + 2D\,ddv + \&c. = 0$$

quæ integratæ dant, diviſione per 2 inſtituta:

$$A\,dp + B\,dq + C\,dr + D\,ds + \&c. = \mathfrak{A}\,dt$$

$$A\,dx + B\,dy + C\,dz + D\,dv + \&c. = \mathfrak{B}\,dt$$

& integralibus denuo ſumtis;

$$Ap + Bq + Cr + Ds + \&c. = \mathfrak{A}t + \mathfrak{a}$$

$$Ax + By + Cz + Dv + \&c. = \mathfrak{B}t + \mathfrak{b}$$

quibus motus uniformis in directum centri gravitatis indicatur. Cum igitur ſit:

$$q = p + a \sin\zeta$$

$$r = p + a \sin\zeta + b \sin\eta$$

$$s = p + a \sin\zeta + b \sin\eta + c \sin\theta$$

&c.

$y =$

$$y = x + a\cos\zeta$$
$$z = x + a\cos\zeta + b\cos\eta$$
$$v = x + a\cos\zeta + b\cos\eta + c\cos\theta \quad \&c.$$

Obtinebitur:

$$p = \frac{\mathfrak{A}t + a - (B+C+D+\&c.)a\sin\zeta - (C+D+E+\&c.)b\sin\eta}{A+B+C+D+\&c}$$
$$\frac{-(D+E+\&c.)c\sin\theta - \&c.}{A+B+C+D+\&c.}$$

$$x = \frac{\mathfrak{B}t + b - (B+C+D+\&c.)a\cos\zeta - (C+D+E+\&c.)b\cos\eta}{A+B+C+D+\&c.}$$
$$\frac{-(D+E+\&c.)c\cos\theta - \&c.}{A+B+C+D+\&c.}$$

quæ formulæ in ſequentes transmutabuntur:

$$p = \frac{\mathfrak{A}t + a + Aa\sin\zeta + (A+B)b\sin\eta + (A+B+C)c\sin\theta + \&c.}{A+B+C+E+\&c.}$$
$$-a\sin\zeta - b\sin\eta - c\sin\theta - \&c.$$
$$x = \frac{\mathfrak{B}t + b + Aa\cos\zeta + (A+B)b\cos\eta + (A+B+C)c\cos\iota + \&c.}{A+B+C+D+E+\&c.}$$
$$-a\cos\zeta - b\cos\eta - c\cos\iota - \&c.$$

quibus inventis ſimul litterarum q, r, s, &c. & y, z, v, &c. valores innoteſcent. Perducta eſt ergo quæſtio ad determinationem angulorum $\zeta, \eta, \theta, \iota$, &c. qui ex duplicibus expreſſionibus tenſionum P, Q, R; &c. elicientur:

$$2Pdt^2 = \frac{A\,ddp}{\sin\zeta} = \frac{A\,ddx}{\cos\zeta}$$
$$2Q\,dt^2 = \frac{A\,ddp + B\,ddq}{\sin\eta} = \frac{B\,ddx + B\,ddy}{\cos}$$

$$2R\,dt^2 = \frac{Addp + Bddq + Cddr}{\sin\iota} = \frac{Addx + Bddy + Cddz}{\cos\iota}$$
&c.

ex his enim erit:

$$\frac{\sin\zeta}{\cos\zeta} = \frac{Addp}{Addx}$$

$$\frac{\sin\eta}{\cos\eta} = \frac{Addp + Bddq}{Addx + Bddy}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{Addp + Bddq + Cddr}{Addx + Bddy + Cddz}$$
&c.

Ponatur summa omnium corpusculorum $A + B + D + \&c. = H$, & cum sit:

$$p = \frac{\mathfrak{A}t + \mathfrak{a} - (H-A)a\sin\zeta - (H-A-B)b\sin\eta - (H-A-B-C)c\sin\theta - \&c.}{H}$$

$$x = \frac{\mathfrak{B}t + \mathfrak{b} - (H-A)a\cos\zeta - (H-A-B)b\cos\eta - (H-A-B-C)c\cos\theta - \&c.}{H}$$

ob:

$$Ap + Bq = (A+B)p + Ba\sin\zeta$$
$$Ap + Bq + Cr = (A+B+C)p + (B+C)a\sin\zeta + Cb\sin\eta$$
$$Ax + Bq + Cr + Ds = (A+B+C+D)p + (B+C+D)a\sin\zeta + (C+D)b\sin\theta \quad \&c.$$
$$Ax + By = (A+B)x + Ba\cos\zeta$$
$$Ax + By + Cz = (A+B+C)x + (B+C)a\cos\zeta + Cb\cos\eta$$
$$Ax + By + Cz + Dv = (A+B+C+D)x + (B+C+D)a\cos\zeta + (C+D)b\cos\eta + Dc\cos\theta \quad \&c.$$

His

His igitur valoribus fubſtitutis habebitur:

$$\frac{\sin\zeta}{\cos\zeta} = \frac{A(H-A)\,add\sin\zeta + A(H-A-B)\,bdd\sin\eta + A(H-A-B-C)\,cdd.\sin\theta + \&c.}{A(H-A)\,add.\cos\zeta + A(H-A-B)\,bdd\cos\eta + A(H-A-B-C)\,cdd\cos\theta + \&c.}$$

$$\frac{\sin\eta}{\cos\eta} = \frac{A(H-A-B)\,add.\sin\zeta + (A+B)(H-A-B)\,bdd.\sin\eta + (A+B)(H-A-B-C)\,cdd.\sin\theta + \&c.}{A(H-A-B)\,add.\cos\zeta + (A+B)(H-A-B)\,bdd.\cos\eta + (A+B)(H-A-B-C)\,cdd.\cos\theta + \&c.}$$

$$\frac{\sin\iota}{\cos\iota} = \frac{A(H-A-B-C)\,add.\sin\zeta + (A+B)(H-A-B-C)\,bdd.\sin\eta + (A+B+C)(H-A-B-C)\,cdd.\sin\theta + \&c.}{A(H-A-B-C)\,cdd.\cos\zeta + (A+B)(H-A-B-C)\,bdd.\cos\eta + (A+B+C)(H-A-B-C)\,cdd.\cos\theta + \&c.}$$

&c.

Sicque prodibunt tot æquationes, quot habentur anguli ζ, η, θ, &c. ex quibus adeo ſinguli determinabuntur. Præterea vero ex formulis pro ſollicitationibus momentaneis inventis eruentur ſimili modo, quo ſupra fuimus uſi:

$$2A(dpddp + dxddx) + 2B(dqddq + dyddy) + 2C(drddr + dzddz) + \&c. = 0$$

quæ integrata dabit:

$$A(dp^2 + dx^2) + B(dq^2 + dy^2) + C(dr^2 + dz^2) + \&c. = \mathfrak{S}\,dt^2$$

qua conſervatio virium vivarum continetur. Solutionem ergo illæ æquationes differentio-differentiales complectentur, quæ ſi integrationem admitterent, uti caſu trium corpuſculorum uſu venit, problema perfecte eſſet ſolutum. Suffecit ergo hujus problematis solutionem ad resolutionem aequationum analyticarum perduxisse. Q.E.J.

Coroll. I.

27. Ex aequationibus, quibus anguli ζ, η, θ, &c. definiuntur, si fractiones tollantur, atque lemma hoc in subsidium vocetur $\cos m\, dd.\sin n - \sin m\, dd.\cos n = ddn \cos(m-n) + dn^2 \sin(m-n)$, orientur sequentes:

$$0 = A(H-A)a\,dd\zeta + A(H-A-B)b(dd\eta\cos(\zeta-\eta) + d\eta^2\sin(\zeta-\eta)) + A(H-A-B-C)c(dd\theta\cos(\zeta-\theta) + d\theta^2\sin(\zeta-\theta)) + A(H-A-B-C-D)d(dd\iota\cos(\zeta-\iota) + d\iota^2\sin(\zeta-\iota)) + \&c.$$

$$0 = A(H-A-B)a(dd\zeta\cos(\zeta-\eta) - d\zeta^2\sin(\zeta-\eta)) + (A+B)(H-A-B)b\,dd\eta + (A+B)(H-A-B-C)c(dd\theta\cos(\eta-\theta) + d\theta^2\sin(\eta-\theta)) + (A+B)(H-A-B-C-D)d(dd\iota\cos(\eta-\iota) + d\iota^2\sin(\eta-\iota)) + \&c.$$

$$0 = A(H-A-B-C)a(dd\zeta\cos(\zeta-\theta) - d\zeta^2\sin(\zeta-\theta)) + (A+B)(H-A-B-C)b(dd\eta\cos(\eta-\theta) - d\eta^2\sin(\eta-\theta)) + (A+B+C)(H-A-B-C)c\,dd\theta + (A+B+C)(H-A-B-C-D)d(dd\iota\cos(\theta-\iota) + d\iota^2\sin(\theta-\iota)) + \&c.$$

$0 =$

$$0 = A(H-A-C-D)\,a\,(dd\zeta \cos(\zeta-\iota) - d\zeta^2 \sin(\zeta-\iota)) + (A+B)$$
$$(H-A-B-C-D)\,b\,(dd\eta \cos(\eta-\iota) - d\eta^2 \sin(\eta-\iota))$$
$$+ (A+B+C)(H-A-B-C-D)\,c\,(dd\theta \cos(\theta-\iota) - d\theta^2 \sin$$
$$(\theta-\iota))) + (A+B+C+D)(H-A-B-C-D)\,dd d\iota + \&c.$$
&c.

Coroll. 2.

28. Cum deinde sit $\int(ddn \cos(m-n) + dn^2 \sin(m-n)) = dn \cos(m-n) + \int dm\,dn \sin(m-n)$, erit aequationibus illis hoc modo integratis:

$$0 = A(H-A)\,a\,d\zeta$$
$$+ A(H-A-B)\,b\,d\eta \cos(\zeta-\eta) + A(H-A-B)\,b\int d\zeta\,d\eta \sin(\zeta-\eta)$$
$$+ A(H-A-B-C)\,c\,d\theta \cos(\zeta-\theta) + A(H-A-B-C)\,c\int d\zeta\,d\theta \sin(\zeta-\theta)$$
$$+ A(H-A-B-C-D)\,d\,d\iota \cos(\zeta-\iota) + A(H-A-B-C-D)\,d\int d\zeta\,d\iota \sin$$
$$(\zeta-\iota) \qquad \&c.$$

$$0 = A(H-A-B)\,a\,d\zeta \cos(\zeta-\eta) - A(H-A-B)\,a\int d\zeta\,d\eta \sin(\zeta-\eta)$$
$$+ (A+B)(H-A-B)\,b\,d\eta$$
$$+ (A+B)(H-A-B-C)\,c\,d\theta \cos(\eta-\theta) + (A+B)(H-A-B-C)$$
$$c\int d\eta\,d\theta \sin(\eta-\theta)$$
$$+ (A+B)(H-A-B-C-D)\,d\,d\iota \cos(\eta-\iota) + (A+B)$$
$$(H-A-B-C-D)\,d\int d\eta\,d\iota \sin(\eta-\iota) \qquad \&c.$$

$$0 = A(H-A-B-C)\,a\,d\zeta \cos(\zeta-\theta) - A(H-A-B-C)\,a\int d\zeta\,d\theta \sin(\zeta-\theta)$$
$$+ (A+B)(H-A-B-C)\,b\,d\eta \cos(\eta-\theta) - (A+B)(H-A-B-C)$$
$$b\int d\eta\,d\theta \sin(\zeta-\theta)$$
$$+ (A+B+C)(H-A-B-C)\,c\,d\theta$$
$$+ (A+B+C)(H-A-B-C-D)\,d\,d\iota \cos(\eta-\iota) + (A+B+C)$$
$$(H-A-B-C-D)\,d\int d\theta\,d\iota \sin(\theta-\iota) \qquad \&c.$$

$$o = A(H-A-B-C-D)\,a\,d\zeta \cos(\zeta-\iota) - A(H-A-B-C-D)\,af\,d\zeta\,d\iota \sin(\zeta-\iota)$$
$$+ (A+B)(H-A-B-C-D)\,b\,d\zeta \cos(\eta-\iota) - (A+B)(H-A-B-C-D)\,bf\,d\eta\,d\iota \sin(\eta-\iota)$$
$$+ (A+B+C)(H-A-B-D)\,c\,d\theta \cos(\theta-\iota) - (A+B+C)(H-A-B-C-D\,cf\,d\theta\,d\iota \sin(\theta-\iota)$$
$$+ (A+B+C+D)(H-A-B-D)\,dd\iota$$
&c.

Coroll. 3.

29. Si harum æquationum prima mutiplicetur per a secundum per b, tertia per c, quarta per d, &c. omnesque invicem addantur termini integrales destruentur, prodibitque sequens æquatio integralis.

$$\mathfrak{Z}dt = A(H-A)\,aa\,d\zeta + (A+B)(H-A-B)\,bb\,d\eta + (A+B+C)(H-A-B-C)\,cc\,d\theta + \&c.$$
$$+ A(H-A-B)\,ab(d\zeta+d\eta)\cos(\zeta-\eta)$$
$$+ A(H-A-B-C)\,ac(d\zeta+d\theta)\cos(\zeta-\theta)$$
$$+ A(H-A-B-C-D)\,ad(d\zeta+d\iota)\cos(\zeta-\iota) \quad \&c.$$
$$+ (A+B)(H-A-B-C)\,bc(d\eta+d\theta)\cos(\eta-\theta)$$
$$+ (A+B)(H-A-B-C-D)\,bd(d\eta+d\iota)\cos(\eta-\iota) \quad \&c.$$
$$+ (A+B)(H-A-B-C-D)\,bd(d\theta+d\iota)\cos(\theta-\iota) \quad \&c.$$

Eadem vero æquatio jam continetur in ea, qua conservationem virium vivarum sumus complexi.

Scholion.

30. Quo igitur positio corpusculorum horum A, B, C, D, &c. ad quodvis tempus definiri, atque adeo eorum motus

motus per
differential
admodum
hic multitu
que quema
sit, perspic
ponatur,

31.
sintque tan
inter se æ
bus sequent

$o = 3\,d d\zeta$ …
cos(
$o = 2\,d d\zeta$ c
cos(
$o = d d\zeta$ co
— 2
ex quibus
$\mathfrak{Z}dt = 3\,d\zeta$ …

Unde
possent, fo

motus perfecte cognosci possit, aequationes has differentio-differentiales inventas resolvi atque integrari oportet, quemadmodum in casu trium corporum fieri licuit. At vero hic multitudo variabilium similem tractationem impedit, neque quemadmodum hinc commoda constructio obtineri possit, perspicitur. Quae difficultas, quo clarius ob oculos ponatur, quatuor tantum corpora contemplemur.

Exemplum.

31. Sint quatuor corpuscula filis inter se connexa, sintque tam ipsa corpuscula A, B, C, D, quam fila a, b, c, inter se aequalia, erit $H = 4A$, atque anguli ζ, η, θ ex tribus sequentibus aequationibus investigari debebunt.

$$0 = 3dd\zeta + 2dd\eta \cos(\zeta - \eta) + 2d\eta^2 \sin(\zeta - \eta) + dd\theta \cos(\zeta - \theta) + d\theta^2 \sin(\zeta - \theta)$$

$$0 = 2dd\zeta \cos(\zeta - \eta) - 2d\zeta^2 \sin(\zeta - \eta) + 4dd\eta + 2dd\theta \cos(\eta - \theta) + 2d\theta^2 \sin(\eta - \theta)$$

$$0 = dd\zeta \cos(\zeta - \theta) - d\zeta^2 \sin(\zeta - \theta) + 2dd\eta \cos(\eta - \theta) - 2d\eta^2 \sin(\eta - \theta) + 3dd\theta$$

ex quibus sequens aequatio integralis (29) elicitur

$$\mathfrak{S}dt = 3d\zeta + 4d\eta + 3d\theta + 2(d\zeta + d\eta)\cos(\zeta - \eta) + (d\zeta + d\theta)\cos(\zeta - \theta) + 2(d\eta + d\theta)\cos(\eta - \theta)$$

Unde si valores angulorum ζ, η, & θ per t exprimi possent, foret:

$$p = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} - \frac{3a}{4}\sin\zeta - \frac{2a}{4}\sin\eta - \frac{a}{4}\sin\theta$$

$$q = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta - \tfrac{2}{4}a\sin\eta - \tfrac{1}{4}a\sin\theta$$

$$r = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta + \tfrac{2}{4}a\sin\eta - \tfrac{1}{4}a\sin\theta$$

$$s = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta + \tfrac{2}{4}a\sin\eta + \tfrac{3}{4}a\sin\theta$$

atque

$$x = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} - \tfrac{3}{4}a\cos\zeta - \tfrac{2}{4}a\cos\eta - \tfrac{1}{4}a\cos\theta$$

$$y = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta - \tfrac{2}{4}a\cos\eta - a\cos\theta$$

$$z = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta + \tfrac{2}{4}a\cos\eta - \tfrac{1}{4}a\cos\theta$$

$$v = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta + \tfrac{2}{4}a\cos\eta + \tfrac{3}{4}a\cos\theta$$

Problema. V.

Fig. 8. 32. *Augeatur nunc numerus corpusculorum in infinitum, filorum autem longitudines evanescant, ita ut hoc modo funis perfecte flexibilis formetur, cujus, si super plano horizontali utcunque projiciatur, motus & situs ad quodvis tempus assignari debet.*

Solutio.

Pervenerit iste funis elapso tempore t in situm A M G, ex cujus singulis punctis M perpendicula ad axem Oo demissa con-

concipia[illegible]
cata PM
vero ma
ipsius S,
$=dS$ sin
& $Aa =$

$p = \frac{\mathfrak{A}t \ldots}{}$

$x = \frac{\mathfrak{B}t \ldots}{}$

si massam
sum præs

$Oa = \frac{\mathfrak{A}t \ldots}{}$

$Aa = \frac{\mathfrak{B} \ldots}{}$

ßis integra
fs. Erit
Deinde cu
(A+B+C
$-Aa\sin\zeta$
hæc expre
$\Sigma(Oa + f$
mili modo
stro casu tr
$\frac{\sin\Phi}{\cos\Phi} =$