DE MOTU
Corporum Flexibilium.

1.

Tabb. III. & IV. Si duo copora rigida ita inter se conjungantur, ut utrumque circa juncturam libere moveri possit, ea invicem flexura connecti dicuntur. Axis autem flexuræ vocatur linea recta, circa quam ambo corpora libere gyrari queant. Si in extremitate alterius corporis tertium simili flexura coaptetur, tria habebuntur corpora duabus flexuris inter se connexa, quatuor autem corpora tribus flexuris connectentur, & ita porro. Hujusmodi corpus flexibile pluribus flexuris instructum catena repræsentat, cujus singuli articuli flexuris inter se sunt connexi, numerusque flexurarum unitate deficiet a numero articulorum catenam constituentium. Funis autem & filum, si sint perfecte flexibilia, considerari possunt, tanquam constarent ex pluribus minimis articulis flexuris inter se connexis. Hinc ope fili plurima corpora rigida ita invicem colligari possunt, ut corpus flexibile constituant. Hocque casu cum partes quaquaversus inter se flecti queant, quævis linea recta per flexuram ad fili connectentis directionem normalis locum axis flexuræ tenere poterit: ejus vero tantum ratio erit habenda, circa quem motus actu absolvitur.

2. Ad motum ergo hujusmodi corporum flexibilium definiendum, singulorum primo articulorum motus investigari debet. Deinde cum flexuræ impediant, quo minus partes a se invicem disjungantur, manifestum est hos singu-

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3.
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Euleri (

singularum partium motus quodammodo a se invicem pendere. Binorum enim quorumque articulorum extremitates, quae flexuris inter se sunt connexae, perpetuo motum communem habere debent; ipsi vero articuli circa hanc flexuram motu angulari movebuntur. Hic igitur primum ipsarum singularum flexurarum motus sunt considerandi, qui etsi in infinitum variare possunt, tamen hac lege inter se constringuntur, ut binae contiguae perpetuo aequali intervallo a se invicem distent. Haec igitur motuum multiplicitas problema solutu difficillimum reddere videtur; interim tamen operam dabo, ut quantum fieri licet, hujus problematis tantopere complicati solutionem planam ac facilem exhibeam; quod commodissime fieri poterit, si a casibus simplicissimis investigationem ordiamur. Primum ergo unius articuli solitarii, qui cum aliis omnino non sit connexus, motum determinabo.

Problema. I.

3. *Determinare motum virgae rigidae* AB *super plano horizontali utcunque projectae.* Fig. 1.

Solutio.

Pervenerit haec virga tempore quocunque t elapso in situm AB, ad quem definiendum pro lubitu assumatur in eodem plano horizontali linea recta fixa OP pro axe, ad quem ex virgae terminis A & B demittantur perpendicula Aa, Bb. Sit G centrum gravitatis virgae AB, unde pariter ad axem normalis ducatur GP, ponaturque $OP = p$; $PG = x$, & angulus $AGP = \zeta$. Quod si ergo ad quodvis tempus determinare noverimus valores litterarum p, x, & ζ, habebimus non solum locum & positionem virgae AB,

sed

sed etiam ejus motum verum. Quodsi enim motus puncti G resolvatur in binos laterales, quorum alterius directio sit parallela lineæ OP, alterius vero incidat in ipsam rectam PG, erit celeritas illius $= \frac{dp}{dt}$; hujus vero $= \frac{dx}{dt}$, denotante dt tempusculum, quo variabiles p & x augmenta accipiunt dp & dx. Cognito autem motu centri gravitatis G, motus virgæ angularis circa punctum G celeritas erit $= \frac{d\zeta}{dt}$; si quidem angulum AGP $= \zeta$ motu angulari augeri ponamus; scilicet $\frac{d\zeta}{dt}$ definiet celeritatem, qua virgæ punctum, quod a centro gravitatis G intervallo $= 1$ circa G gyratur. Quoniam hanc virgam a nullis viribus sollicitari ponimus, atque planum horizontale, super quo fit motus, omni asperitate destitutam assumimus, tam motus centri gravitatis, quam motus angularis virgæ circa centrum gravitatis erit uniformis, uti ex mechanicis constat. Erit ergo $\frac{dp}{dt} = \mathfrak{A}$, $\frac{dx}{dt} = \mathfrak{B}$ & $\frac{d\zeta}{dt} = \mathfrak{G}$; unde fit $p = \mathfrak{A}t + a$, $x = \mathfrak{B}t + b$ & $\zeta = \mathfrak{G}t + g$. Ex quibus æquationibus locus & positio virgæ ad quodvis tempus definitur, hincque simul ejus motus innotescit. Q. E. I.

Coroll. 1.

4. Ponamus motus initio, cum esset $t = 0$, centrum gravitatis G in O esse versatum, directionemque virgæ AB ad rectam OP fuisse normalem, ita ut tum angulus ζ fuerit

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erit $p =$

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7.
AB inter
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fuerit $=0$, erit $a=0$, $b=0$, $g=0$, ideoque hoc casu erit $p=\mathfrak{A}t$; $x=\mathfrak{B}t$; & $\zeta=\mathfrak{G}t$.

Coroll. 2.

7. Si praeterea assumamus initio motus celeritatem centri gravitatis fuisse debitam altitudini $=a$, ejusque directionem incidisse in axem OP, erit $\mathfrak{A}=\sqrt{a}$ & $\mathfrak{B}=0$. fiet ergo $p=t\sqrt{a}$ & $x=0$, unde constat centrum gravitatis G perpetuo in axe OP motu uniformi esse progressurum, & celeritatem rotatoriam fore constantem.

Scholion.

6. Planissima haec sunt ex principiis mechanicae, quibus constat, omne corpus rigidum, quod a nullis viribus sollicitatur constanter ita moveri, ut ejus centrum gravitatis motu aequabili lineam rectam describat, singulaeque ejus partes circa centrum gravitatis motu uniformi rotentur. Interim tamen hoc problema praemittere visum est, ut ex modo solutionis via ad sequentia resolvenda planior redderetur. Ceterum hinc jam motus definiri potest duorum corpusculorum minimorum filo connexorum; si enim corpuscula sint minima, eorum motus, quo forteutrumque circa suum centrum gravitatis rotatur, negligi potest: movebunturque ambo, quamdiu filum tensum manet, instar virgae rigidae, quemadmodum ex solutione sequentis problematis perspicietur.

Problema. II.

7. *Duorum corpusculorum A et B filo inertiae experte AB inter se colligatorum motum super plano horizontali determinare, postquam utcunque fuerint projecta.* Fig. 2.

Solutio.

Sumta recta Oab pro axe, pervenerint corpuſcula hæc elapſo tempore t in ſitum AB, ex quibus ad axem perpendicula ducantur Aa & Bb. Sit longitudo fili $AB = a$, quæ perpetuo eadem manet; & vocetur corpuſculi A maſſa $= A$, corpuſculi B maſſa $= B$; ſintque $Oa = p$; $Aa = x$; $Ob = q$ & $Bb = y$; angulus vero ABb ponatur $= \zeta$; eritque

$$ab = q - p = a \sin \zeta \ \& \ y - x = a \cos \zeta$$

Tum vero erit celeritas corpuſculi A ſecundum directionem $Oa = \frac{dp}{dt}$, & ſecundum directionem $aA = \frac{dx}{dt}$: ſimilique modo corpuſculi B celeritas ſecundum directionem Ob erit $= \frac{dq}{dt}$ & ſecundum directionem $bB = \frac{dy}{dt}$. Exprimat jam P tenſionem fili AB, qua vi corpus A verſus B, at corpus B verſus A trahitur. Corporis ergo A

motus $\frac{dp}{dt}$ accelerabitur vi $= P \sin \zeta$;

motus $\frac{dx}{dt}$ accelerabitur vi $= P \cos \zeta$; Corporis vero B

motus $\frac{dq}{dt}$ retardabitur vi $= P \sin \zeta$;

motus $\frac{dy}{dt}$ retardabitur vi $= P \cos \zeta$. Hinc erit ex natura ſollicitationum, poſito elemento dt conſtante:

$$2A\,ddp = P\,dt^2 \sin \zeta; \quad 2A\,ddx = P\,dt^2 \cos \zeta$$

$$2B\,ddq = -P\,dt^2 \sin \zeta; \quad 2B\,ddy = -P\,dt^2 \cos \zeta.$$

Ex

Ex his erit $A\,ddp + B\,ddq = 0$; & bis integrando $Ap + Bq = \mathfrak{A}t + \mathfrak{a}$, similique modo erit $A\,ddx + B\,ddy = 0$ unde $Ax + By = \mathfrak{B}t + \mathfrak{b}$. Cum ergo $q - p = a \sin\zeta$ erit $p = \frac{\mathfrak{A}t + \mathfrak{a} - Ba\sin\zeta}{A+B}$ & $q = \frac{\mathfrak{A}t + \mathfrak{a} + Aa\sin\zeta}{A+B}$. Atque ob $y - x = a\cos\zeta$, erit $x = \frac{\mathfrak{B}t + \mathfrak{b} - Ba\cos\zeta}{A+B}$ & $y = \frac{\mathfrak{B}t + \mathfrak{b} + Aa\cos\zeta}{A+B}$.

Deinde vero cum sit $\frac{ddp}{ddx} = \frac{\sin\zeta}{\cos\zeta}$, ob $ddp = -\frac{Ba\,dd.\sin\zeta}{A+B}$ & $ddx = -\frac{Ba\,dd.\cos\zeta}{A+B}$, erit $\frac{dd.\sin\zeta}{dd.\cos\zeta} = \frac{\sin\zeta}{\cos\zeta}$, seu $\sin\zeta.\,dd.\cos\zeta - \cos\zeta.\,dd\sin\zeta = 0$ cujus integrale est $-\sin\zeta.\,d\cos.\zeta + \cos\zeta.\,d\sin\zeta = \mathfrak{G}dt$ seu $+d\zeta.\sin\zeta^2 + d\zeta.\cos\zeta^2 = +d\zeta = \mathfrak{G}dt$; unde fit $\zeta = \mathfrak{G}t + g$. Quocirca ad quodvis tempus t definietur angulus $ABb = \zeta$, hincque valores litterarum p, x, q & y sicque positio corpusculorum A & B filo inter se connexorum perpetuo assignari poterit. Q. E. J.

Coroll. 1.

8. Quia est $A\,dp + B\,dq = \mathfrak{A}dt$ seu $\frac{A\,dp + B\,dq}{(A+B)\,dt} = \frac{\mathfrak{A}}{A+B}$, centrum commune gravitatis amborum corporum secundum directionem axis Oab uniformiter progreditur;

tur; simili vero modo ob $\frac{A\,dx + B\,dy}{(A+B)\,dt} = \frac{\mathfrak{B}}{A+B}$, quoque ab hoc axe motu uniformi recedet, ideoque conjunctim motu uniformi lineam rectam describet.

Coroll. 2.

9. Quoniam est $\frac{dz}{dt} = \mathfrak{G}$ ac propterea constans, sequitur angulum A B b uniformiter increscere, ideoque filum A B uniformiter in gyrum agi; atque adeo hæc duo corpuscula A & B filo AB connexa perinde movebuntur, ac si virga rigida in ipsorum locum substitueretur.

Coroll. 3.

10. Cum sit $z = \mathfrak{G}t + g$ & $dz = \mathfrak{G}\,dt$, erit $d\,\sin z = \mathfrak{G}\,dt \cos z$, & $dd.\sin z = -\mathfrak{G}^2\,dt^2 \sin z$: unde $ddp = +\frac{Ba\mathfrak{G}^2\,dt^2 \sin z}{A+B}$. Quare cum sit $2A\,ddp = P\,dt^2 \sin z$, fiet $\frac{2ABa\mathfrak{G}^2}{A+B} = P$, quæ est vis qua filum tenditur: ubi notandum est, $\mathfrak{G} = \frac{dz}{dt}$ denotare celeritatem rotatoriam fili AB ad distantiam = 1 relatam. Hinc si f sit altitudo debita celeritati, qua A circa B & vicissim B circa A revolvitur, erit $\mathfrak{G} = \frac{\sqrt{f}}{a}$; fietque tensio fili $P = \frac{2AB}{A+B} \cdot \frac{f}{a}$.

Scho-

11. E
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bis præstabit
corpuscula fil
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hibere voluis
porum exami
mihi proposui
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expedito non
puscula atque

12. S
rint connexa, e
eorum motum d

Sumta
rint corpuscul
ad axem perp
$= y$; $Oc = x$
$= b$, ang. A
$q - p =$
$y - x =$
Ex his motus
posito elemen

Scholion.

11. Etsi solutio hujus problematis facillime ex mechanicæ principiis deduci potuisset, tamen hæc solutio, quamvis sit prolixior, quam natura rei postulat, hunc nobis præstabit usum, ut eadem methodo casus, si tria & plura corpuscula filo fuerint colligata, evolvi queant, quod fieri non posset, si solutionem maxime naturalem & concinnam adhibere voluissemus. Progrediar ergo ad casum trium corporum examinandum, quem Vir Celeb. Daniel Bernoulli mihi proposuit, & in quo fundamentum universæ theoriæ de motu corporum flexibilium est positum. Hoc enim casu expedito non difficile erit, eandem methodum ad plura corpuscula atque ad omnia corpora flexibilia extendere.

Problema. III.

12. Si tria corpuscula A, B, C filo inertiae experte fuerint connexa, eaque super plano horizontali utcunque projiciantur, eorum motum determinare. Fig. 3.

Solutio.

Sumta pro lubitu recta Oa pro axe fixo, pervenerint corpuscula elapso tempore t in situm ABC; ductisque ad axem perpendiculis sit $Oa = p$, $Aa = x$; $Ob = q$; $Bb = y$; $Oc = r$, $Cc = z$; tum vero ponatur $AB = a$, $BC = b$, ang. $ABb = \zeta$; ang. $BCc = \eta$, eritque.

$$q - p = a \sin\zeta; \quad r - q = b \sin\eta$$
$$y - x = a \cos\zeta; \quad z - y = b \cos\eta$$

Ex his motus singulorum corpusculorum ita definientur, ut posito elemento temporis $= dt$, sit

Cele-

Celeritas	in directione Oo	Celeritas	in directione $O\omega$
Corpuſculi A	$=\frac{dp}{dt}$	Corpuſculi A	$=\frac{dx}{dt}$
Corpuſculi B	$=\frac{dq}{dt}$	Corpuſculi B	$=\frac{dy}{dt}$
Corpuſculi C	$=\frac{dr}{dt}$	Corpuſculi C	$=\frac{dz}{dt}$

Denotent jam litteræ A, B, C maſſas corpuſculorum A, B, & C, ſitque P tenſio fili AB, Q tenſio fili BC; unde ſequentes orientur ſingulorum corpuſculorum ſollicitationes.

Sollicitabitur	in directione Oo	in directione $O\omega$
Corpuſculum A	vi $= P \sin\zeta$	vi $= P \cos\zeta$
Corpuſculum B	vi $= -P\sin\zeta + Q\sin\eta$	vi $= -P\cos\zeta + Q\cos\eta$
Corpuſculum C	vi $= -Q\sin\eta$	vi $= -Q\cos\eta$

Ad incrementa velocitatum definienda ponamus corpuſculi A celeritatem in directione Oo debitam eſſe altitudini v, & cum ſollicitetur a vi $= P\sin\zeta$, dum ſpatiolum dp abſolvit, fiet $dv = \frac{P\,dp \sin\zeta}{A}$, uti ex principiis mechanicæ conſtat. Quia autem celeritas eſt $= \frac{dp}{dt}$ faciamus $\frac{dp}{dt} = \sqrt{v}$, erit $v = \frac{dp^2}{dt^2}$, & poſito elemento temporis dt conſtante erit $dv = \frac{2\,dp\,ddp}{dt^2}$, quo valore ſubſtituto habebimus $\frac{2\,dp\,ddp}{dt^2} = \frac{P\,dp \sin\zeta}{A}$ ſeu $2A\,ddp = P\,dt^2 \sin\zeta$.

Hoc

Hoc igitur
definiendo c
2 A ddp :
2 B ddq =
2 C ddr =
Ex his æqua
2 A d
2 A d
quæ bis inte
2 A p·
2 A x·
Cum vero ſit
r = p + a ſi
his valoribus
(A + B·
(A + B·
ex quibus ori
p = 𝔄
q = 𝔄
r = 𝔄

Euleri Op

Hoc igitur modo ſingulorum corpuſculorum accelerationes definiendo obtinebimus ſequentes æquationes.

$$2\,A\,ddp = P\,dt^2 \sin\zeta; \qquad 2\,A\,ddx = P\,dt^2 \cos\zeta$$

$$2\,B\,ddq = -P\,dt^2 \sin\zeta + Q\,dt^2 \sin\eta \qquad 2\,B\,ddy = -P\,dt^2 \cos\zeta + Q\,dt^2 \cos\eta$$

$$2\,C\,ddr = -Q\,dt^2 \sin\eta; \qquad 2\,C\,ddz = -Q\,dt^2 \cos\eta$$

Ex his æquationibus additis naſcuntur duæ ſequentes;

$$2\,A\,ddp + 2B\,ddq + 2\,C\,ddr = 0$$
$$2\,A\,ddx + 2B\,ddy + 2\,C\,ddz = 0$$

quæ bis integratæ dant:

$$2\,A\,p + B\,q + C\,r = \mathfrak{A}t + \mathfrak{a}$$
$$2\,A\,x + B\,y + C\,z = \mathfrak{B}\,t + \mathfrak{b}$$

Cum vero ſit $q = p + a\sin\zeta$; $y = x + a\cos\zeta$; atque porro $r = p + a\sin\zeta + b\sin\eta$ & $z = x + a\cos\zeta + b\cos\eta$, erit his valoribus ſubſtitutis:

$$(A+B+C)\,p + (B+C)\,a\sin\zeta + Cb\sin\eta = \mathfrak{A}t + \mathfrak{a} \quad \&$$
$$(A+B+C)\,x + (B+C)\,a\cos\zeta + Cb\cos\eta = \mathfrak{B}t + \mathfrak{b}$$

ex quibus oriuntur ſequentes determinationes:

$$p = \frac{\mathfrak{A}t + \mathfrak{a} - (B+C)\,a\sin\zeta - Cb\sin\eta}{A+B+C}$$

$$q = \frac{\mathfrak{A}t + \mathfrak{a} + A\,a\sin\zeta - Cb - \sin\eta}{A+B+C}$$

$$r = \frac{\mathfrak{A}t + \mathfrak{a} + A\,a\sin\zeta + (A+B)\,b\sin\eta}{A+B+C}$$

Similique modo reperietur:

$$x = \frac{\mathfrak{B}t + \mathfrak{b} - (B+C)\,a\cos\zeta - Cb\cos\eta}{A+B+C}$$

$$y = \frac{\mathfrak{B}t + \mathfrak{b} + A\,a\cos\zeta - Cb\cos\eta}{A+B+C}$$

$$z = \frac{\mathfrak{B}t + \mathfrak{b} + Aa\cos\zeta + (A+B)\,b\cos\eta}{A+B+C}$$

Inventis litteris $p, q, r,$ & $x, y, z,$ tensiones P & Q duplicimodo exprimentur. Erit enim

$$P\,dt^2 = \frac{2A\,ddp}{\sin\zeta} \quad \& \quad P\,dt^2 = \frac{2A\,ddx}{\cos\zeta}$$

$$Q\,dt^2 = \frac{-2C\,ddr}{\sin\eta} \quad \& \quad Q\,dt^2 = \frac{-2C\,ddz}{\cos\eta}$$

unde fit:

$$ddp\cos\zeta = ddx\sin\zeta \quad \& \quad ddr\cos\eta = ddz\sin\eta$$

substitutis autem valoribus ante inventis erit

$$(B+C)\,a\cos\zeta\,dd.\sin\zeta + Cb\cos\zeta\,dd.\sin\eta = (B+C)\,a\sin\zeta\,dd.\cos\zeta + Cb\sin\zeta\,dd.\cos\eta$$

seu $(B+C)\,a\,(\cos\zeta\,dd.\sin\zeta - \sin\zeta\,dd.\cos\zeta) + Cb\,(\cos\zeta\,dd.\sin\eta - \sin\zeta\,dd.\cos\eta) = 0$

& $Aa\,(\cos\eta\,dd.\sin\zeta - \sin\eta\,dd.\cos\zeta) + (A+B)\,b\,(\cos\eta\,dd.\sin\eta - \sin\eta\,dd.\cos\eta) = 0$

At vero generatim est $\cos m\,dd.\sin n - \sin m\,dd.\cos n = \cos m\,(ddn\cos n - dn^2\sin n) + \sin m\,(ddn\sin n + dn^2\cos n)$, ideoque cum sit $\cos m\cos n + \sin m\sin n = \cos(m-n)$ & $\sin m\cos n - \cos m\sin n = \sin(m-n)$ erit

cos

cos m dd. sin
Cujus redu
(B+C) a d
(A+B) bdd
Integretur
(B+C) ad
(A+B) bdy
Unde partib
(B+C) adζ / Cb
=
seu (B+C) / Cb
Per subtract
rentialibus
(B+C) a dd / C b
(dζ²
Ponatur ζ+
&
abibitque pri

$\operatorname{cof} m\, dd.\operatorname{fin} n - \operatorname{fin} m.\, dd\operatorname{cof} n = ddn\operatorname{cof}(m-n) + dn^2 \operatorname{fin}(m-n)$

Cujus reductionis ratione habita fiet

$$(B+C)\,add\zeta + Cbdd\eta\operatorname{cof}(\zeta-\eta) + Cbd\eta^2\operatorname{fin}(\zeta-\eta) = 0$$

$$(A+B)\,bdd\eta + Aadd\zeta\operatorname{cof}(\eta-\zeta) + Aad\zeta^2\operatorname{fin}(\eta-\zeta) = 0$$

Integretur utraque quoad fieri poteft, erit

$$(B+C)\,ad\zeta + Cbd\eta\operatorname{cof}(\zeta-\eta) + Cb\int d\zeta d\eta\operatorname{fin}(\zeta-\eta) = \text{Conft.}$$

$$(A+B)\,bd\eta + Aad\zeta\operatorname{cof}(\eta-\zeta) + Aa\int d\zeta d\eta\operatorname{fin}(\eta-\zeta) = \text{Conft.}$$

Unde partibus integralibus eliminatis fiet:

$$\frac{(B+C)ad\zeta}{Cb} + d\eta\operatorname{cof}(\zeta-\eta) + \frac{(A+B)bd\eta}{Aa} + d\zeta\operatorname{cof}(\eta-\zeta) = \text{Conft.}$$

feu $\dfrac{(B+C)ad\zeta}{Cb} + \dfrac{(A+B)bd\eta}{Aa} + (d\zeta+d\eta)\operatorname{cof}(\zeta-\eta) = \dfrac{dt}{\sqrt{f}}$

Per fubtractionem vero ex duabus illis æquationibus differentialibus oritur:

$$\frac{(B+C)add\zeta}{Cb} - \frac{(A+B)bdd\eta}{Aa} - (dd\zeta - dd\eta)\operatorname{cof}(\zeta-\eta) + (d\zeta^2 + d\eta^2)\operatorname{fin}(\zeta-\eta) = 0$$

Ponatur $\zeta+\eta = v$ & $\zeta-\eta = u$, ut fit $\zeta = \dfrac{v+u}{2}$ & $\eta = \dfrac{v-u}{2}$

abibitque prior æquatio integrata in hanc:

$$\frac{(B+C)\,a\,dv}{2Cb}+\frac{(B+C)\,a\,du}{2Cb}+\frac{(A+B)\,b\,dv}{2Aa}-\frac{(A+B)\,b\,du}{2Aa}$$

$$+\,dv\,\mathrm{cos}\,u=\frac{dt}{\sqrt{f}}$$

Posterior vero aequatio differentio differentialis in hanc:

$$\frac{(B+C)\,a\,ddv}{2Cb}+\frac{(B+C)\,a\,ddu}{2Cb}-\frac{(A+C)\,b\,ddv}{2Aa}+$$

$$\frac{(A+B)\,b\,ddu}{2Aa}-ddu\,\mathrm{cos}\,u+\tfrac{1}{2}\,(dv^2+du^2)\,\mathrm{sin}\,u=0$$

Ponatur brevitatis gratia:

$$\frac{(B+C)\,a}{C}+\frac{(A+B)\,b}{Aa}=m$$

$$\frac{(B+C)\,a}{Cb}-\frac{(A+B)\,b}{Aa}=n$$

orienturque istae duae aequationes:

$$\tfrac{1}{2}\,m\,dv+\tfrac{1}{2}\,n\,du+dv\,\mathrm{cos}\,u=\frac{dt}{\sqrt{f}}$$

$$\tfrac{1}{2}\,n\,ddv+\tfrac{1}{2}\,m\,ddu-ddu\,\mathrm{cos}\,u+\tfrac{1}{2}(dv^2+du^2)\,\mathrm{sin}\,u=0$$

Ex priori fit $dv=\dfrac{2\,dt:\sqrt{f}-n\,du}{m+2\,\mathrm{cos}\,u}$, qui in altera

$$n\,ddv+m\,ddu-2\,ddu\,\mathrm{cos}\,u+(dv^2+du^2)\,\mathrm{sin}\,u=0$$

substitutus ob $ddv=\dfrac{-n\,ddu}{m+2\,\mathrm{cos}\,u}+\dfrac{4\,dt\,du\,\mathrm{sin}\,u:\sqrt{f}-2n\,du^2\,\mathrm{sin}\,u}{(m+2\,\mathrm{cos}\,u)^2}$

dabit:

— nn

$$\frac{-nnddu}{m+2\cos u} = \frac{4ndtdu \sin u : \sqrt{f} - 2nndu^2 \sin u}{(m+2\cos u)^2} + mddu$$

$$-2ddu\cos u + du^2 \sin u +$$

$$\frac{4dt^2 \sin u : f - 4ndtdu \sin u : \sqrt{f} + nndu^2 \sin u}{(m+2\cos u)^2} = 0$$

seu $(mm-nn)ddu - 4ddu\cos u^2 + \frac{4dt^2 \sin u : f + (mm-nn)du^2 \sin u}{m+2\cos u}$

$$+\frac{4m\,du^2 \sin u \cos u + 4du^2 \sin u \cos u^2}{m+2\cos u} = 0$$

Ponatur $dt = \frac{du}{\sqrt{\omega}}$ erit $ddt = 0 = \frac{ddu}{\sqrt{\omega}} - \frac{du\,d\omega}{2\omega\sqrt{\omega}}$ ideoque $ddu = \frac{du\,d\omega}{2\omega}$, quibus valoribus pro dt & ddu substitutis erit:

$$\frac{(mm-nn)d\omega}{2\omega} - \frac{2d\omega\cos u^2}{\omega} + \frac{4du \sin u : f\omega + (mm-nn)du \sin u}{m+2\cos u}$$

$$+\frac{4m\,du \sin u \cos u + 4du \sin u \cos u^2}{m+2\cos u} = 0 \text{ seu}$$

$$\tfrac{1}{2}(m^2-n^2)\,d\omega - 2d\omega\cos u^2 - \frac{m\omega du \sin u}{m+2\cos u}$$

$$+(m+2\cos u)\omega du \sin u + \frac{4du \sin u}{f(m+2\cos u)} = 0$$

Sit $\cos u = s$ atque ob $du \sin u = -ds$ erit:

$$(m^2-n^2)d\omega - 4ss\,d\omega + \frac{2mm\omega\,ds}{m+2s} - 2(m+2s)\omega\,ds = \frac{g\,ds}{f(m+2s)}$$

seu $d\omega + \frac{2\omega\,ds(mm-(m+2s)^2)}{(m+2s)(mm-nn-4ss)} = \frac{g\,ds}{f(m+2s)(m^2-n^2-4ss)}$

quae aequatio multiplicata per $\frac{mm-nn-4ss}{m+2s}$ fit integrabilis, eritque integrale.

$$\frac{(mm-nn-4ss)\omega}{m+2s} = \int\frac{g\,ds}{f(m+2s)^2} = \frac{4}{g} - \frac{4}{f(m+2s)}$$

hinc erit $\omega = \frac{4f(m+2s)-4g}{fg(mm-nn-4ss)} = \frac{du^2}{dt^2}$ & ob du

$= \frac{-ds}{\sqrt{(1-ss)}}$ fiet $dt = \frac{-ds\sqrt{fg(mm-nn-4ss)}}{2\sqrt{(1-ss)(mf-g+2fs)}}$ seu

$$t = \int\frac{ds\sqrt{fg(mm-nn-4ss)}}{2\sqrt{(mf-g+2fs)(1-ss)}}$$ vel etiam

$$t = \int\frac{du\sqrt{fg(mm-nn-4\cos u^2)}}{2\sqrt{(mf-g+2f\cos u)}}.$$ Hancobrem erit

$$\frac{2\,dt}{\sqrt{f}} = \frac{du\sqrt{g(mm-nn-4\cos u^2)}}{\sqrt{(mf-g+2f\cos u)}}$$ atque adeo

$v = 2\int\frac{dt}{(m+2\cos u)\sqrt{f}} - \int\frac{n\,du}{m+2\cos u}$. Ex unica ergo variabili u, definiuntur t & v, ex hisque porro anguli ζ & η, quibus inventis facile assignantur coordinatae p, q, r & x, y, z

sicque

sicque ad
terminari

13.

directione
ideoque u
rectionem

$= \frac{\mathfrak{B}}{A+}$

tur centru
B, & C uni
priori colli
lictorum.

14.
centrum gr
imprimatur
tis cum id
$\mathfrak{B} = 0.$

15.
efcit & pro

O v π pun

ficque ad datum tempus pofitio corpufculorum A, B, C determinari poterit. Q. E. I.

Coroll. 1.

13. Celeritas centri communis gravitatis fecundum directionem axis $O o$ eft $= \frac{A dp + B dq + C dr}{(A+B+C) dt} = \frac{\mathfrak{A}}{A+B+C}$; ideoque uniformis; fimili modo ejus celeritas fecundum directionem rectæ $O \omega$ normalis ad $O o$ eft $= \frac{A dp + B dy + C dz}{(A+B+C) dt}$

$= \frac{\mathfrak{B}}{A+B+C}$, ideoque pariter uniformis, unde colligitur centrum commune gravitatis trium corpufculorum A, B, & C uniformiter in directum progredi. Quod quidem a priori colligi potuiffet ex natura omnium motuum fibi relictorum.

Coroll. 2.

14. Si igitur fuerit & $\mathfrak{A}$ & $\mathfrak{B} = 0$, tum commune centrum gravitatis quiefcet. Quodfi ergo toti fyftemati imprimatur motus æqualis & contrarius motui centri gravitis tum id revera quiefcet, fietque propterea $\mathfrak{A} = 0$, & $\mathfrak{B} = 0$.

Coroll. 3.

15. Quoniam hoc cafu, quo centrum gravitatis quiefcit & proinde fit $\mathfrak{A} = 0$, $\mathfrak{B} = 0$, ejus diftantia in recta $O o$ a puncto O eft $= \frac{Ap + Bq + Cr}{A+B+C} = \frac{a}{A+B+C}$, &

diftandistantia secundum $O\omega$ a puncto O est $= \frac{Ax+By+Cz}{A+B+C} = \frac{\mathfrak{b}}{A+B+C}$, si fuerit $\mathfrak{a}=0$ & $\mathfrak{b}=0$, tum centrum gravitatis in puncto O quiescet.

Coroll. 4.

16. Cum ergo ob rectam Oo cum puncto O arbitrariam motus semper ad hunc casum reduci possit, ut centrum gravitatis in puncto O quiescat, nullam vim amplitudini solutionis inferendo semper licebit constantes $\mathfrak{A}, \mathfrak{B}$ & $\mathfrak{a}, \mathfrak{b}$ evanescentes assumere: eritque propterea:

$$p = -\frac{(B+C)a\sin\zeta - Cb\sin\eta}{A+B+C}; \quad x = -\frac{(B+C)a\cos\zeta - Cb\cos\eta}{A+B+C}$$

$$q = \frac{Aa\sin\zeta - Cb\sin\eta}{A+B+C}; \quad y = \frac{Aa\cos\zeta - Cb\cos\eta}{A+B+C}$$

$$r = \frac{Aa\sin\zeta + (A+B)b\sin\eta}{A+B+C}; \quad z = \frac{Aa\cos\zeta + (A+B)b\cos\eta}{A+B+C}$$

Coroll. 5.

17. Hinc porro celeritates corporum secundum utramque directionem Oo & $O\omega$ definiri poterunt:

Secun-

Secundum

18. C

momentaneis

vivarum fit

erit ejus diffe

Cum enim

Euleri Opus.

Secundum directionem Oo	Secundum directionem Ow
$\frac{dp}{dt} = \frac{-(B+C)ad\zeta\cos\zeta - Cbd\eta\cos\eta}{(A+B+C)dt}$	$\frac{dx}{dt} = \frac{(B+C)ad\zeta\sin\zeta + Cbd\eta\sin\eta}{(A+B+C)dt}$
$\frac{dq}{dt} = \frac{Aad\zeta\cos\zeta - Cbd\eta\cos\eta}{(A+B+C)dt}$	$\frac{dy}{dt} = \frac{-Aad\zeta\sin\zeta + Cbd\eta\sin\eta}{(A+B+C)dt}$
$\frac{dr}{dt} = \frac{Aad\zeta\cos\zeta + (A+B)bd\eta\cos\eta}{(A+B+C)dt}$	$\frac{dz}{dt} = \frac{-Aad\zeta\sin\zeta - (A+B)bd\eta\sin\eta}{(A+B+C)dt}$

Coroll. 6.

18. Conservatio virium vivarum ex sollicitationibus momentaneis facile colligitur. Cum enim summa virium vivarum sit $= \frac{Adp^2 + Adx^2 + Bdq^2 + Bdy^2 + Cdr^2 + Cdz^2}{dt^2}$ erit ejus differentiale $\frac{2Adpddp}{dt^2} + \frac{2Adxddx}{dt^2} + \frac{2Bdqddq}{dt^2}$

$$+ \frac{2Bdyddy}{dt^2} + \frac{2Cdrddr}{dt^2} + \frac{2Cdzddz}{dt^2}$$

$$= \left\{ \begin{array}{l} P(dp\sin\zeta + dx\cos\zeta - dq\sin\zeta - dy\cos\zeta) \\ Q(dq\sin\eta + dy\cos\eta - dr\sin\eta - dz\cos\eta) \end{array} \right. = 0.$$

Cum enim sit $q - p = a\sin\zeta$, erit $dq - dp = ad.\sin\zeta$

 et

et $(dp - dq)\sin\zeta = -\frac{1}{2}\, ad.\sin\zeta^2$. Similimodo erit $(dx - dy)\cos\zeta = -\frac{1}{2}\, ad.\cos\zeta^2 = +\frac{1}{2}\, ad.\sin\zeta^2$ ob $\cos\zeta^2 = 1 - \sin\zeta^2$: ideoque coëfficiens ipsius P est $= 0$, pariterque ipsius Q. Unde differentiale virium vivarum est $= 0$, ideoque summa virium vivarum constans.

Coroll. 7.

19. Ex iisdem æquationibus solutiones momentaneas exprimentibus concluditur fore:

$$\frac{2Axddp + 2Byddq + 2Czddr}{dt^2} = P(x-y)\sin\zeta + Q(y-z)\sin\eta =$$

$-Pa\sin\zeta\cos\zeta - Qb\sin\eta\cos\eta$, similique modo:

$$\frac{2Apddx + 2Bqddy + 2Crddz}{dt^2} = P(p-q)\cos\zeta + Q(q-r)\cos\eta =$$

$-Pa\sin\zeta\cos\zeta - Qb\sin\eta\cos\eta$, ex quibus sequitur fore:

$$A(xddp - pddx) + B(yddp - qddy) + C(zddr - rddz) = 0$$

cujus integrale est:

$$A(xdp - pdx) + B(ydq - qdy) + C(zdr - rdz) = 0.$$

Coroll. 8.

Fig. 4. 20. Reliquæ constantes, quæ integratione in solutionem introducuntur, ex statu corpusculorum initiali determinari debent. Si igitur assumamus centrum gravitatis in O perpetuo quiescere, atque corpuscula initio omnia in recta Oo sita fuisse, anguli ζ & η ita definiri debent, ut posito $t = 0$ fiant recti. Tum igitur celeritates corporum secundum axem Oo evanescent, at vero secundum $O\omega$ erunt, ut sequitur

dx

$$\frac{dx}{dt} = \frac{(B+C)ad\zeta + Cbd\eta}{(A+B+C)dt};\quad \frac{dy}{dt} = \frac{-Aad\zeta + Cbd\eta}{(A+B+C)dt};$$

atque $\frac{dz}{dt} = \frac{-Aad\zeta - (A+B)bd\eta}{(A+B+C)dt}$.

Scholion.

21. Solutio ergo hujus problematis ab integratione formulæ $t = \int \frac{du\sqrt{fg(mm - nn - 4\,\mathrm{cof}\,u^2)}}{2\sqrt{(mf - g + 2f\,\mathrm{cof}\,u)}}$, & propterea per quadraturam curvæ cujuspiam construi potest, ita ut ad quemvis valorem ipsius t valor anguli u assignetur: quo autem invento nova opus est quadratura ad angulum v determinandum; quamobrem solutio practica hujus problematis maxime est operosa. Dantur tamen nonnulli casus, quibus solutio multo fit simplicior ac tractabilior, quos hic seorsim evolvamus.

Casus. I.

22. Quoniam invenimus inter t & u hanc æquationem

$$2\,dt\sqrt{(mf - g + 2f\,\mathrm{cof}\,u)} = du\sqrt{fg(m^2 - n^2 - 4\,\mathrm{cof}\,u^2)}$$

manifestum est huic æquationi satisfieri, si fuerit:

$mf - g + 2f\,\mathrm{cof}\,u = 0$ seu $\mathrm{cof}\,u = \frac{g - mf}{2f}$; fiet enim u constans, & $du = 0$, unde utrumque membrum evanescit: Sit igitur $u = 2\alpha$, ut sit $\mathrm{cof}\,2\alpha = \frac{g - mf}{2f}$, eritque $m + 2\,\mathrm{cof}\,u = \frac{g}{f}$; ideoque $v = 2\int \frac{dt\sqrt{f}}{g}$, ac propterea $v =$