Sur l'accord des deux dernieres eclipses du soleil et de la lune avec mes tables, pour trouver les vrais momens des pleni-lunes et novi-lunes

Translation of Leonard Euler's E141 paper from the French

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ON THE AGREEMENT
OF THE LAST TWO ECLIPSES OF THE SUN
AND MOON WITH MY TABLES, FOR FINDING THE ACTUAL TIMES OF THE HALF-MOON AND NEW MOON,

BY MR. EULER

I.

My calculations that I presented in a preceding Memoir* for the Solar Eclipse we had witnessed here on July 25, 1748 agree so well with the observations of the beginning and the end of this Eclipse that it would be most difficult to promise anything better. Following my tables, I had established the beginning of this Eclipse at 10^{h}, 17′, 45″ and the end at 1^{h}, 24′, 0″; as it happens, although the beginning of this Eclipse could not be seen, we were nevertheless certain enough of concluding it at 10^{h}, 18′ by the phase that appeared at 10^{h}, 20′, and the end of this Eclipse was observed at 1^{h}, 24′, 30″. We were no longer mistaken in the starting time of the annulus, which I announced; but its duration, which I had placed at 5′, 10″, was observed to be much shorter at 1′, 20″. This would not appear to favor my tables, although other tables that are considered to be better only showed this Eclipse partially, and their beginning and end times amount to an error of several entire minutes. Yet, toward correcting the error in the duration of the annulus, I must note that in my calculations I had assumed the latitude of Berlin was 52°, 36′; now in fact the last observations that Mr. Kies made with the excellent Quadrant that Mr. de Maupertuis gave to the Academy only gave its elevation to be 52°, 31′, 30″, so I had placed Berlin too far North by 4′, 30″. Just glancing at the chart of this Eclipse published at Nurnberg offers the assurance that if Berlin were situated 4½′ more to the north, the duration of the annulus would have considerably lengthened and would have been very close to my calculation.

II. For the Lunar Eclipse that appeared between the 8^{th} and 9^{th} [*Dec.*] † of August, upon reading the last article in the Astronomical Almanac it would be said that the times put forth under the heading of my tables are not very much in agreement with the observations. The beginning was predicted at 11^{h}, 0′, 14″ and the end at 1^{h}, 14′, 4″ but

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* See the Memoirs of 1747 p. 250 & following. [Translator's Note: This refers to E117]

† I am unsure of the meaning of *Dec.* in the original work.

was observed to begin at 11^h, 5′ and end at 1^h, 18′. I admit that this difference would ruin all the good opinions of my tables that might have been inspired by the agreements with the Solar Eclipse, and that this was very surprising to me since I had rectified my tables for a great number of Lunar Eclipses. I therefore believed that before I judged my tables too harshly, it was necessary to rework my calculations to see if there was not some mistake wedged in it from being too hasty. Therefore, the following are the details of all my calculations.

III. I thus begin by finding the mean opposition time that occurred around August 8, 1748, of which the calculation will follow my printed tables from the Latin Almanac for 1749. As it follows:① (see endnotes)

	☽to☉	Mean Long.: ☉	Mean An.: ☉	Mean An.: ☽	Mean Long.: ☊
1741, 1^d, 20^h, 44′, 15″	6^s	9^s, 12°, 1′, 38″	6^s, 3°, 25′, 46″	9^s, 27°, 15′, 46″	3^s, 4°, 9′, 52″
1748, 13, 3, 52, 28	0	0, 12, 17, 11	0, 12, 9, 51	2, 26, 3, 39	4, 16, 3, 2
July 25, 17, 8, 21	0	6, 23, 44, 49	6, 23, 44, 14	6, 0, 43, 3	0, 10, 56, 47
July 40, 17, 45, 4	6	4, 18, 3, 38	1, 9, 19, 51	6, 24, 2, 28	4, 26, 59, 49
Or Aug. 8, 17, 45, 4					10, 7, 10, 3

July 40th coincides with August 9th, but because this year is a leap year, it is necessary to deduct a day since February was lengthened. Therefore, according to the mean movement, the opposition occurred at Paris in August of 1748 at 8^d, 17^h, 45′, 4″ mean time, and the equation of time is negative 5′, 0″ so the time of the mean opposition would be at Paris in August of 1748 at 8^d, 17^h, 40′, 4″; and for Berlin it is necessary to adjust the longitudinal difference, which is 44′, 36″, resulting in the mean opposition occurring at Berlin in

August of 1748 at 8^d, 18^h, 24′, 40″ true time.

IV. Having for these times the mean anomalies of the Sun and Moon, the eccentric anomalies can be determined, with the help of these tables, by applying the coinciding equations, giving:

The Solar eccentric anomaly 1^s, 8°, 43′, 44″
The Lunar eccentric anomaly 6, 25, 23, 27†

From this, the arguments and equations for the tables of equations can be easily formed.②

Table	Argument	Additive Eq.	Subtractive Eq.
" I	6^s, 25°, 23′, 27″	- - - -	3^h, 44′, 18″
II	1, 8, 43, 44	- - - -	2, 35, 59
III	8, 4, 7, 11	0^h, 8′, 15″	
IV	5, 16, 39, 43	0, 2, 31	
V	2, 29, 30, 38	- - - -	0, 1, 1
VI	10, 12, 3, 10	31	
		+0, 11, 17	– 6, 21, 18
			+ 0, 11, 17
		Total Subtractive Equation	6, 10, 1

It is therefore necessary to subtract 6^h, 10′, 1″ from the mean opposition time to get the true opposition time in orbit.

Mean opposition at Berlin in August of 1748 8^d, 18^h, 24′, 40″
– 6, 10, 1

† This number is written in the order 6, 23, 25, 27 in the original work, but should be 6, 25, 23, 27.

Opposition in orbit at Berlin in August of 1748 8, 12, 14, 39 true time.

V. Now, this time I am looking for the true positions of the Sun and of the Moon and their eccentric anomalies using the mean position of the ascending node.[3]

	Mean Long.: ☉	Mean An.: ☉	Mean An.: ☽	Mean Long.: ☊
Mean ☍ Time				
minus	4, 18, 3, 38	1, 9, 19, 51	6, 24, 2, 28	10, 7, 10, 3
6^h, 10′, 1″	– 15, 12	– 15, 12	– 3, 21, 26	+ 49
☍ Time in Orbit	4, 17, 48, 26	1, 9, 4, 39	6, 20, 41, 2	10, 7, 10, 52
Equation:	– 1, 11, 37	– 35, 56	+ 1, 10, 18	
	4, 16, 36, 49	1, 8, 28, 43	6, 21, 51, 20	
	True Long. ☉	Ecc. An. ☉	Ecc. An. ☽	

Therefore, the true longitude of the Sun is 4^s, 16°, 36′, 49″
The longitude of the Moon in its orbit is 10^s, 16°, 36′, 49″
since we know that in this moment the position of the Moon in its orbit differs by 6 signs from that of the Sun.

VI. Now it is just a matter of determining the true position of the ascending node ☊ with the inclination of the lunar orbit to the ecliptic, which can be found by means of the Lunar Tables that I published in the anthology of my works.[4]

Tables for the ☊ & the Inclination	Argument	Mean Long.: ☊	Inclination
		10^s, 7°, 10′, 52″	
I	6, 21, 51, 20	Eq. + 38	
II	1, 8, 28, 43	Eq. + 4, 30	
☉	4, 16, 36, 49	10, 7, 16, 0	
☊	10, 7, 16, 0		
III	6, 9, 20, 49	Eq.+0, 29, 49	5°, 16′, 39″
		10, 7, 45, 49	
IV	6, 0, 0, 0	Eq. 0, 0, 0	Eq. – 42
V	0, 9, 20, 49	Eq. + 2, 14	Eq. + 36
		10, 7, 48, 3	5, 16, 33
		True Long.: ☊	True Inclination

Therefore, the true longitude of ☊ is 10^s, 7°, 48′, 3″
and the inclination of the lunar orbit is 5, 16, 33.

VII. To have all the elements for calculating the Eclipse, it is still necessary to find the apparent diameters, the horizontal parallaxes, and the hourly movements of both the Sun and the Moon, which can be easily found from the tables attached to the Astronomical Almanac for 1749. The following formulas could also be used where v is the eccentric anomaly of the Moon, and u is that of the Sun.

The apparent diameter of the Sun $= 1933'' - 32''.4\cos u$
The horizontal parallax of the Sun $= 12''$
The hourly movement of the Sun $= 147''.87 - 4''.95\cos u$
For the Moon in opposition:
The app. horiz. diam. of the Moon $= 1892'' - 122''\cos v + 4''\cos 2v$
The horiz. parallax of the Moon $= 3430'' - 222''\cos v + 8''\cos 2v$

$$\text{The hourly movement of the Moon} = 2023''.1 - 258''.3\cos v + 11''.7\cos 2v - 1''.8\cos u + 1''.4\cos(v-u)$$

For the conjunctions, only 2″ need to be subtracted for the apparent diameter, and 3″ for the parallax and the hourly movement.

VIII. Using these formulas, since u = 1, 8, 28, 43 and v = 6, 21, 51, 20, we find:

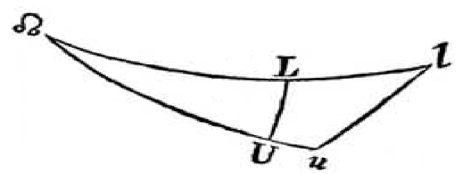
The apparent diameter of the Sun	= 1908″ = 31′, 48″
The horizontal parallax of the Sun	= 12″
The hourly movement of the Sun	= 144″ = 2′, 24″
The app. horiz. diam. of the Moon	= 2008″ = 33′, 28″
The horiz. parallax of the Moon	= 3642″ = 60′, 42″
The hourly movement of the Moon	= 2269″ = 37′, 49″.

The hourly movement of the node is 8″

Furthermore, the sum of the parallaxes is	60′, 54″
which if subtracted by the radius of the Sun	15′, 54″
gives the radius of the umbra to be	45′, 0″.

But Earth's atmosphere augments the umbra by 40″, as can be concluded by observations, which gives the adjusted radius of the shadow to be 45′, 40″.

IX. In the adjacent figure, let ☊Uu be the Ecliptic, ☊Ll the lunar orbit, and ☊ the ascending node. Also, at the moment of opposition in orbit, let U be the center of the shadow and L that of the Moon, and to make the problem more general let ☊U = ☊L = a in which a is the arc found if the longitude of ☊ is subtracted from the position of the Moon; and let ω be the angle for the inclination ☊ of the two orbits. By using some rules of spherical trigonometry we can set up the following relation:



$$\cos UL = \cos\omega \sin^2 a + \cos^2 a = \cos\omega\sin^2 a + 1 - \sin^2 a .$$

Since $1-\cos\omega = 2\sin^2(\tfrac{1}{2}\omega)$, we have

$$2\sin^2 a \sin^2(\tfrac{1}{2}\omega) = 2\sin^2(\tfrac{1}{2}UL)$$

and hence

$$\sin(\tfrac{1}{2}UL) = \sin a \sin(\tfrac{1}{2}\omega) .$$

X. Now we need to find the distance between u and l, the centers of the shadow and the Moon after x hours since the moment of opposition in orbit. To this effect, let m be the hourly movement of the Sun or shadow since the node and n the hourly movement of the Moon. If these hourly movements are adjusted by 8″ as found in the tables given above, we have ☊u = $a + mx$ and ☊l = $a + nx$. Now defining the distance $ul = z$, we have

$$\cos z = \cos\omega \sin(a+mx)\sin(a+nx) + \cos(a+mx)\cos(a+nx)$$

or since

$$\sin b \sin c = \tfrac{1}{2}\cos(b-c) - \tfrac{1}{2}\cos(b+c) ,$$

and

$$\cos b \cos c = \tfrac{1}{2}\cos(b-c) + \tfrac{1}{2}\cos(b+c) ,$$

this equation can be expressed as:

$$\cos z = \tfrac{1}{2}\cos\omega\cos((n-m)x) - \tfrac{1}{2}\cos\omega\cos(2a+(n+m)x)$$
$$+\tfrac{1}{2}\cos((n-m)x) + \tfrac{1}{2}\cos(2a+(n+m)x)$$

or better yet

$$\cos z = \cos^2(\tfrac{1}{2}\omega)\cos((n-m)x) + \sin^2(\tfrac{1}{2}\omega)\cos(2a+(n+m)x)$$

or

$$1-2\sin^2(\tfrac{1}{2}z) = \cos((n-m)x) - \sin^2(\tfrac{1}{2}\omega)\cos((n-m)x) + \sin^2(\tfrac{1}{2}\omega)\cos(2a+(n+m)x).$$

Knowing

$$\cos(2a+(n+m)x) = \cos(2a)\cos((n+m)x) - \sin(2a)\sin((n+m)x)$$

we obtain:

$$^{\dagger}1-2\sin^2(\tfrac{1}{2}z) = \cos((n-m)x) - \sin^2(\tfrac{1}{2}\omega)\cos((n-m)x)$$
$$+\cos(2a)\sin^2(\tfrac{1}{2}\omega)\cos((n+m)x)$$
$$-\sin(2a)\sin^2(\tfrac{1}{2}\omega)\sin((n+m)x)\ .$$

But the angles $(n–m)x$ and $(n+m)x$ are very small so

$$\cos((n-m)x) = 1-\tfrac{1}{2}(n-m)^2x^2,\ \cos((n+m)x) = 1-\tfrac{1}{2}(n+m)^2x^2$$

and

$$\sin((n+m)x) = (n+m)x,$$

which gives

$$4\sin^2(\tfrac{1}{2}z) = (n-m)^2x^2 + 2\sin^2(\tfrac{1}{2}\omega) - \sin^2(\tfrac{1}{2}\omega)(n-m)^2x^2$$
$$-2\cos(2a)\sin^2(\tfrac{1}{2}\omega) + (n+m)^2x^2\cos(2a)\sin^2(\tfrac{1}{2}\omega)$$
$$+4(n+m)x\sin a\cos a\sin^2(\tfrac{1}{2}\omega)$$

or

$$4\sin^2(\tfrac{1}{2}z) = 4\sin^2 a\sin^2(\tfrac{1}{2}\omega) + (n-m)^2x^2\cos^2(\tfrac{1}{2}\omega)$$
$$+(n+m)^2x^2\cos(2a)\sin^2(\tfrac{1}{2}\omega) + 4(n+m)x\sin a\cos a\sin^2(\tfrac{1}{2}\omega)$$

or

$$4\sin^2(\tfrac{1}{2}z) = (2\sin a + (n+m)x\cos a)^2\sin^2(\tfrac{1}{2}\omega) + (n-m)^2x^2\cos^2(\tfrac{1}{2}\omega)$$
$$-(n+m)^2x^2\sin^2 a\sin^2(\tfrac{1}{2}\omega)\,.^{\ddagger}$$

XI. The distance between the centers $ul = z$ will be the smallest if we take the derivative for the value of $4\sin^2(\tfrac{1}{2}z)^{\S} = 0$, which gives

$$(n-m)^2x\cos^2(\tfrac{1}{2}\omega) + (n+m)^2x\cos(2a)\sin^2(\tfrac{1}{2}\omega) + 2(n+m)\sin a\cos a\sin^2(\tfrac{1}{2}\omega) = 0,$$

from where we can derive

$$x = \frac{-(n+m)\sin(2a)\ \sin^2(\tfrac{1}{2}\omega)}{(n-m)^2\ \cos^2(\tfrac{1}{2}\omega) + (n+m)^2\ \cos(2a)\ \sin^2(\tfrac{1}{2}\omega)}$$

and this value when substituted gives:

† The left hand side was mistaken written as $1-\sin^2(\tfrac{1}{2}z)$ in the original document.

‡ This equation in the original document is missing the two at the head of the first term.

§ This term was mistakenly written as $a\sin^2(\tfrac{1}{2}z)$ in the original document.

$$\sin(\tfrac{1}{2}z)=\sin a\sin(\tfrac{1}{2}\omega)\sqrt{\frac{(n-m)^2\cos^2(\tfrac{1}{2}\omega)-(n+m)^2\sin^2 a\,\sin^2(\tfrac{1}{2}\omega)}{(n-m)^2\,\cos^2(\tfrac{1}{2}\omega)+(n+m)^2\,\cos(2a)\,\sin^2(\tfrac{1}{2}\omega)}}\,.$$

Since the terms that are multiplied by $\sin^2(\tfrac{1}{2}\omega)$ are extremely small in comparison to the others, the centers of the shadow and the Moon will approach each other as near as possible x hours after the opposition in orbit when

$$x=\frac{-(n+m)\sin(2a)\,\tan^2(\tfrac{1}{2}\omega)}{(n-m)^2}\left(1-\frac{(n+m)^2\cos(2a)\tan^2(\tfrac{1}{2}\omega)}{(n-m)^2}\right)$$

and the distance $ul=z$ will be:

$$\sin(\tfrac{1}{2}z)=\sin a\sin(\tfrac{1}{2}\omega)\left(1-\frac{(n+m)^2\cos^2 a\tan^2(\tfrac{1}{2}\omega)}{2(n-m)^2}\right).$$

XII. But now let us reverse the case and suppose that the distance $ul=z$ is given, and we must solve for the time at which we would find this distance. Firstly, we find an angle φ given by the relation

$$\cos\varphi=\frac{\sin a\sin(\tfrac{1}{2}\omega)}{\sin(\tfrac{1}{2}z)}$$

and putting back into the equation the value

$$\sin(\tfrac{1}{2}z)=\frac{\sin a\sin(\tfrac{1}{2}\omega)}{\cos\varphi}$$

we will have

$$\begin{aligned}4\sin^2 a\sin^2(\tfrac{1}{2}\omega)\tan^2\varphi&=(n-m)^2x^2\cos^2(\tfrac{1}{2}\omega)\\&\quad+(n+m)^2x^2\cos(2a)\sin^2(\tfrac{1}{2}\omega)+2(n+m)x\sin(2a)\sin^2(\tfrac{1}{2}\omega)\end{aligned}$$

where the last two terms are very small, so we have approximately

$$x=\frac{2\sin a\,\tan(\tfrac{1}{2}\omega)\tan\varphi}{n-m}\,.$$

Therefore, if we suppose that

$$x=\frac{2\sin a\,\tan(\tfrac{1}{2}\omega)\tan\varphi}{n-m}-y\,,$$

we will have:

$$\begin{aligned}0&=-4(n-m)y\sin a\sin(\tfrac{1}{2}\omega)\cos(\tfrac{1}{2}\omega)\tan\varphi\\&\quad+\frac{4(n+m)^2}{(n-m)^2}\sin^2 a\cos(2a)\sin^2(\tfrac{1}{2}\omega)\tan^2(\tfrac{1}{2}\omega)\tan^2\varphi\\&\quad+\frac{4(n+m)}{n-m}\sin a\sin(2a)\sin^2(\tfrac{1}{2}\omega)\tan(\tfrac{1}{2}\omega)\tan\varphi\end{aligned}$$

from where we can derive

$$y=\frac{n+m}{(n-m)^2}\sin(2a)\tan^2(\tfrac{1}{2}\omega)+\frac{(n+m)^2}{(n-m)^3}\sin a\cos(2a)\tan^3(\tfrac{1}{2}\omega)\tan\varphi.$$

From this, we will finally have:

$$x=\frac{\sin a\tan(\tfrac{1}{2}\omega)}{n-m}\left(2\tan\varphi-\frac{2(n+m)}{n-m}\cos a\tan(\tfrac{1}{2}\omega)-\frac{(n+m)^2}{(n-m)^2}\cos(2a)\tan^2(\tfrac{1}{2}\omega)\tan\varphi\right).$$

XIII. This formula by which we have come to express the value of x serves to find the beginning and the end of an Eclipse, or the immersion and emersion if the Eclipse is total. Because the tangent of the angle φ can be taken either positively or negatively, this formula is guaranteed to always give a double value. So to find the moments of the beginning and the end of an Eclipse, we only need to set z equal to the sum of the radii of the shadow and the Moon; and if z is set to be the difference of these radii, the moments of immersion and emersion can be found. Firstly, it can be seen that if either of these cases is possible, it is necessary that

$$\sin(\tfrac{1}{2}z) > \sin a \sin(\tfrac{1}{2}\omega).$$

If $\sin(\tfrac{1}{2}z) < \sin a \sin(\tfrac{1}{2}\omega)$, then this would be a sign that the problem is impossible, because unless the difference were extremely small, it may be destroyed by the small terms neglected in the calculation.

XIV. Let us now apply these formulas to the Lunar Eclipse in question, whose time of opposition in orbit was found in Berlin to be on August 8^{d}, 12^{h}, 14′, 39″ in 1748 true time, which serves as our epoch. The values of the variables needed in the calculation are:

Position of the Moon in its orbit	10^{s}, 16°, 36′, 49″
Position of the ascending node ☊	<u>10, 7, 48, 3</u>
The arc ☊L = ☊U =	0, 8, 48, 46
Therefore we have a =	8, 48, 46
and the inclination or angle of ☊ = ω =	5, 16, 33
and ½ω =	2, 38, 16½
From this we have log sin a	= 9.1852764[5]
log sin(½ω)	= <u>8.6629848</u>
and hence log sin(½UL)	= 7.8482612
Therefore ½UL	= 24′, 14½″
and the distance UL	= 48′, 29″

at the moment of opposition in orbit.

XV. The hourly movement of the Sun = 144″ and the hourly movement of the Moon = 2269″, thus m = 152 and n = 2277, hence $n - m$ = 2125 and $n + m$ = 2429. From this we can find the moment when the centers of the shadow and the Moon are closest by using the formula $x = \dfrac{-(n+m)\sin(2a)\tan^2(\tfrac{1}{2}\omega)}{(n-m)^2}$, having neglected the other term since it is extremely small. The calculation will be

$\log(n+m)$ = 3.3854275		$2a$ = 17, 37, 32[†]
$\log(n-m)$ = <u>3.3273589</u>		$\log\tan(\tfrac{1}{2}\omega)$ = 8.6634454
$\log\dfrac{n+m}{n-m}$ = 0.0580686		
$\log\sin 2a$ = 9.4807506		
$\log\tan^2(\tfrac{1}{2}\omega)$ = <u>7.3268903</u>		

[†] The original was mistakenly written as 17, 36, 32.

$\log(-(n-m)\,x) = 6.8657102$
minus $\underline{4.6855749}$
in seconds: 2.1801353
$\log(n-m) = \underline{3.3273589}$
$\log(-x) = 8.8527764$
$x = -0.07125^{h} = -4.275' = -4', 16''$ [‡] $(x = -0.07131^{h} = -4.279' = -4', 17'')$

Therefore the time of the closest proximity of the centers is found to be for Berlin on August 8^{d}, 12^{h}, 10′, 23″ in 1748 true time. *(August 8^{d}, 12^{h}, 10′, 22″)*

XVI. For the smallest distance between the centers that corresponds to this moment, if it is called z, we already have the value approaching 48′, 29″; but we still need to take away $\frac{(n+m)^2}{(n-m)^2}\sin a\sin(\tfrac{1}{2}\omega)\cos^2 a\tan^2(\tfrac{1}{2}\omega)$, therefore the calculation is:

$\log\left(\frac{n+m}{n-m}\right)^2 = 0.1161372$
$\log\sin a\sin(\tfrac{1}{2}\omega) = 7.8482612$ $\qquad$ $\log\cos a = 9.9948423$
$\log\cos^2 a = 9.9896846$
$\log\tan^2(\tfrac{1}{2}\omega) = \underline{7.3268908}$
5.2809738
minus $\underline{4.6855749}$
0.5953989

which is approximately 4″ when the logarithm is undone.

Thus the smallest distance between the centers is 48′, 25″ which is then subtracted from the sum of the radii 45′, 40″ + 16′, 44″ = 62′, 24″ leaving 13′, 59″ for the totality of the Eclipse, which may be solved by using the rule of three on the following ratio, where 6 is the radius of the Moon:

$$16', 44'' = 1004'' \text{ and } 13', 59'' = 839'' \text{ so } 1004 : 6 = 839 : 5.014$$

The magnitude of this Eclipse was therefore 5.014 digits[†] at 12^{h}, 10′, 23″. *(12^{h}, 10′, 22″)*

XVII. To find when this Eclipse began and ended, we must suppose that z = the sum of the radii, or z = 62′, 24″ and must find an angle φ such that $\cos\varphi = \frac{\sin a\sin(\tfrac{1}{2}\omega)}{\sin(\tfrac{1}{2}z)}$.

Since $\tfrac{1}{2}z$ = 31′, 12″, we have:

$\log\sin a\sin(\tfrac{1}{2}\omega) = 7.8482612$
$\log\sin(\tfrac{1}{2}z) = \underline{7.9578747}$
$\log\cos\varphi = 9.8903865$
Therefore $\varphi = 39°, 1', 8''$
and $\log\tan\varphi = 9.9084037$ $\qquad$ *(9.9086619)*

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‡ The original work had several arithmetical mistakes that carried through the rest of the work from this point. I have kept the original values, but have also included the adjusted values from the *Opera Omnia* in parentheses for mathematical accuracy.

† In astronomy, digits are 1/12 the diameter of the Sun or Moon. This is used to express the magnitude of an Eclipse (for example, an Eclipse of eight digits is one that hides 2/3 the diameter of the disk).

Since the value of x is composed of three members, let us solve each part separately using the following calculation.

$$
\begin{array}{lll}
\log \sin a & = 9.1852764 & \\
\log \tan(\tfrac{1}{2}\omega) & = \underline{8.6634454} & \\
 & \ 7.8487218 & \\
\log(n-m) & = \underline{3.3273589} & \\
 & \ 4.5213628 & \\
\quad\text{minus} & \ \underline{4.6855749} & \\
\log \dfrac{\sin a \tan(\frac{1}{2}\omega)}{n-m} & = 9.8357878 & \\
\log \tan\varphi & = 9.9084037 & (\mathit{9.9086619}) \\
\log 2 & = \underline{0.3010300} & \\
\log \text{PartI} & = 0.0452215 & (\mathit{0.0454799}) \\
\text{PartI} & = \underline{1.1097^{h}\quad} & (\mathit{1.1104^{h}}) \\
\log \dfrac{\sin a \tan(\frac{1}{2}\omega)}{n-m} & = 9.8357878^{\dagger} & \\
\log \dfrac{n+m}{n-m} & = 0.0580686 & \\
\log 2 & = \underline{0.3010300} & \\
 & \ 0.1948864 & \\
\log \cos a & = 9.9948423 & \\
\log \tan(\tfrac{1}{2}\omega) & = \underline{8.6634454} & \\
\log \text{PartII} & = 8.8531741 & \\
\text{PartII} & = \underline{0.0713\quad} & \\
\log \dfrac{\sin a \tan(\frac{1}{2}\omega)}{n-m} & = 9.8357878 & \\
\log \left(\dfrac{n+m}{n-m}\right)^{2} & = 0.1161372 & \\
\log \cos 2a & = 9.9799536 & \\
\log \tan^{2}(\tfrac{1}{2}\omega) & = 7.3268908 & \\
\log \tan\varphi & = \underline{9.9084037} & \\
\log \text{PartIII} & = 7.1651731 & \\
\text{PartIII} & = 0.0015 &
\end{array}
$$

XVIII. Combining these parts according to their signs, and giving $\tan\varphi$ an ambiguous sign, we find the following two values for x.

I. $x = 1.1097 - 0.0713 - 0.0015 = +1.0369$ $\qquad$ ($x = \mathit{1.1104 - 0.0713 - 0.0015 = +1.0376}$)

II. $x = -1.1097 - 0.0713 + 0.0015 = -1.1795$ $\qquad$ ($x = \mathit{-1.1104 - 0.0713 + 0.0015 = -1.1802}$)

Therefore, to get the beginning of the Eclipse, take the time of opposition in orbit and subtract

$$1.1795^{h} = 1^{h}, 10'.770 = 1^{h}, 10', 46'', \quad (\mathit{1.1802^{h} = 1^{h}, 10'.812 = 1^{h}, 10', 49''})$$

and to get the end of the Eclipse, take the same epoch and add

† This value was misprinted as 9.8357474

$1.0369^h = 1^h, 2'.214 = 1^h, 2', 13''$: $\quad$ *($1.0376^h = 1^h, 2'.256 = 1^h, 2', 16''$)*

Time of the opposition, Aug. $8^d, 12^h, 14', 39''$

	– 1, 10, 46	(– *1, 10, 49*)
	+ 1, 2, 13	(+ *1, 2, 16*)
Beginning of the Eclipse	11, 3, 53	(*11, 3, 50*)
End of the Eclipse	13, 16, 52	(*13, 16, 55*)

XIX. Let us gather all we have found, and let us see that according to my tables, the moments of the Eclipse in Berlin on August 8, 1748 true time are:

The beginning	$11^h, 3', 53''$	(*11, 3, 50*)
The largest obscuration	12, 10, 23	(*12, 10, 22*)
The opposition in orbit	12, 14, 39	
The end of the Eclipse	13, 16, 52	(*13, 16, 55*)
The magnitude of the Eclipse	5.014 digits	
and the duration of the Eclipse	$2^h, 12', 49''$	(*2, 13, 5*)

Presently, one can see that this calculation compared to observation is so close that a better conformity can be hardly expected, seeing that we are still not too certain of the umbra's augmentation caused by Earth's atmosphere, and these observations are not sensitive to such precision, which could cause uncertainty of up to a minute, since it is extremely difficult to distinguish the observation of the umbra from that of the penumbra.

Translation Commentary

Notation used:
- ☉ - Sun
- ☽ - Moon
- ☊ - ascending node, the point in the lunar orbit where the Moon crosses from below to above the ecliptic
- ☍ - opposition, ecliptic longitudes are 180 degrees

In the original work, the times are expressed as w^j, x^h, y', z'', which represents *w* days, *x* hours, *y* minutes, *z* seconds, so I changed the notation to w^d, x^h, y', z'', so as to agree with the English. Degrees are represented as $w^s, x°, y', z''$, which represents *w* signs, *x* degrees, *y* arc-minutes, *z* arc-seconds. Signs are used almost exclusively in Astrology. There are 12 signs in the zodiac, each representing 30 degrees of a circle. This helps to clarify the second column in the table at the top of the second page entitled ☽to☉; 6^s represents a syzygy in which the Moon and Sun are on opposite sides (180 degrees) of the ecliptic plane, or a Lunar Eclipse, and 0^s represents a syzygy in which the Moon and Sun are aligned on the same side (0 degrees) of the ecliptic, or a Solar Eclipse.

① The longitudes and anomalies for July 40 / Aug. 8 are found by adding down the columns except for the final column in which the second and third values are added, and then this result is subtracted from the first value to get the final value.

② The arc-seconds in V were originally 33, but should be 38. The signs in VI were originally 0, but should be 10. The additive and subtractive totals listed under their respective columns are found by adding down their respective columns. The total subtraction equation is then the sum of these two values as shown immediately below the table. Also, the arguments are significant in the following manner: I is the Lunar eccentric anomaly, II is the Solar Eccentric Anomaly, III is I + II, IV is I – II, V is 2*I + II, and VI is 2*I – II.

③ The second row in the mean Lunar anomaly is originally marked as operated on by subtraction, but by looking at the values, it is clear to see that this is in fact an additive operation.

④ The arguments are significant in the following manner: I is the true Lunar eccentric anomaly, II is the true Solar eccentric anomaly, the next two are the longitudes of the Sun and ascending node, III is ☉ + ☊, IV is ☍, V is III – IV.

⑤ All of the calculations that use log are using $\log_{10}$. If the resultant values are negative, add 10. When these values are used to derive the values under their respective horizontal bars, then subtract 10. This process is an easy way of calculating the formulas given doing only simple addition and subtraction. With the use of calculators nowadays, this process can be overlooked.

References

[1] L. Euler, *Sur l'accord des deux dernieres eclipses du soleil et de la lune avec mes tables, pour trouver les vrais momens des pleni-lunes et novi-lunes*, Memoires de l'academie des sciences de Berlin **4** (1750) pp. 86-98 (*Opera Omnia*: Series 2, Volume 30, pp. 89 – 100). Available online at the Euler Archive: http://www.math.dartmouth.edu/~euler/

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