

On amicable numbers*

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At this time when analysis is uncovering the approach to many profound areas in mathematics, problems about the nature and properties of numbers seem to have been almost totally ignored by most geometers. Indeed, the investigation of numbers is considered by many to add nothing to analysis. But to be sure, the study of numbers in many cases requires much more insight than the subtlest of geometric questions; for this same reason, arithmetic questions seem undeservedly to have been neglected. Indeed, the highest talents, from whom it must be considered that analysis has received the greatest contributions, have judged the properties of numbers to be not unworthy of investing the greatest effort and zeal to unfold them. I understand that even Descartes, with his great and wide knowledge, and who was overtaken by the contemplation of mathematics, was however not equal to overcoming the problem of amicable numbers. After him, much work was accomplished by van Schooten, with extensive study. Two numbers are called amicable numbers if the aliquot divisors of the one summed together produce the other; of this type are the numbers 220 and 284. Of the first, 220, its aliquot parts, that is its proper divisors, are $1+2+4+5+10+11+20+22+44+55+110$, whose sum produces 284: and in this way the aliquot parts of the number 284,

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$1+2+4+71+142$, in turn produce 220. There is no doubt that aside from these two numbers many others, and even infinitely many, could be given which have this same property; however, neither Descartes or after him van Schooten were able to find more than three of these pairs of numbers, and they were not equal to this study although they are seen to have been very devoted to tackling this. There is a method which can be used to generate both numbers in a pair of amicable numbers, so that without much work amicable numbers are able to be found. For this, the numbers are formed by the formulas 2^nxy and 2^nz , where x , y and z denote prime numbers; it is also required for them to be set such that $z = xy + x + y$ is prime, and also such that $2^n(x + y + 2) = xy + x + y + 1$. Therefore for each of the successive different values that the exponents n take, prime numbers x and y are searched for such that $xy + x + y$ makes a prime number, and then the formulas 2^nxy and 2^nz produce amicable numbers. It is easily seen that as the exponents proceed to larger n , soon the numbers $xy + x + y$ will have reached such a size that it will be impossible to discern whether or not they are both prime, with the table of prime numbers not having been extended beyond 100000.

Clearly we should not lightly put aside the question of whether all the amicable numbers can be assumed to be included in these formulas. I have carefully assessed this, and not calling on any tricks, but only using the nature of division, I have obtained many other pairs of amicable numbers, of which I relate 30 notable pairs here; so that their origin and nature would be clearly seen, I express them through their factors. Thus, these amicable numbers are:

I.	$2^2 \cdot 5 \cdot 11$	&	$2^2 \cdot 71$
II.	$2^4 \cdot 23 \cdot 47$	&	$2^4 \cdot 1151$
III.	$2^7 \cdot 191 \cdot 383$	&	$2^7 \cdot 73727$
IV.	$2^2 \cdot 23 \cdot 5 \cdot 137$	&	$2^2 \cdot 23 \cdot 827$
V.	$3^2 \cdot 5 \cdot 13 \cdot 11 \cdot 19$	&	$3^2 \cdot 5 \cdot 13 \cdot 239$
VI.	$3^2 \cdot 7 \cdot 13 \cdot 5 \cdot 17$	&	$3^2 \cdot 7 \cdot 13 \cdot 107$
VII.	$3^2 \cdot 7^2 \cdot 13 \cdot 5 \cdot 41$	&	$3^2 \cdot 7^2 \cdot 13 \cdot 251$
VIII.	$2^2 \cdot 5 \cdot 131$	&	$2^2 \cdot 17 \cdot 43$
IX.	$2^2 \cdot 5 \cdot 251$	&	$2^2 \cdot 13 \cdot 107$
X.	$2^3 \cdot 17 \cdot 79$	&	$2^3 \cdot 23 \cdot 59$
XI.	$2^4 \cdot 23 \cdot 1367$	&	$2^4 \cdot 53 \cdot 607$
XII.	$2^4 \cdot 17 \cdot 10303$	&	$2^4 \cdot 167 \cdot 1103$
XIII. ^a	$2^4 \cdot 19 \cdot 8563$	&	$2^4 \cdot 83 \cdot 2039$
XIV.	$2^4 \cdot 17 \cdot 5119$	&	$2^4 \cdot 239 \cdot 383$
XV.	$2^5 \cdot 59 \cdot 1103$	&	$2^5 \cdot 79 \cdot 827$
XVI.	$2^5 \cdot 37 \cdot 12671$	&	$2^5 \cdot 227 \cdot 2111$
XVII.	$2^5 \cdot 53 \cdot 10559$	&	$2^5 \cdot 79 \cdot 7127$
XVIII.	$2^6 \cdot 79 \cdot 11087$	&	$2^6 \cdot 383 \cdot 2309$
XIX.	$2^2 \cdot 11 \cdot 17 \cdot 263$	&	$2^2 \cdot 11 \cdot 43 \cdot 107$
XX.	$3^3 \cdot 5 \cdot 7 \cdot 71$	&	$3^3 \cdot 5 \cdot 17 \cdot 31$
XXI.	$3^2 \cdot 5 \cdot 13 \cdot 29 \cdot 79$	&	$3^2 \cdot 5 \cdot 13 \cdot 11 \cdot 199$
XXII.	$3^2 \cdot 5 \cdot 13 \cdot 19 \cdot 47$	&	$3^2 \cdot 5 \cdot 13 \cdot 29 \cdot 31$
XXIII.	$3^2 \cdot 5 \cdot 13 \cdot 19 \cdot 37 \cdot 1583$	&	$3^2 \cdot 5 \cdot 13 \cdot 19 \cdot 227 \cdot 263$
XXIV.	$3^3 \cdot 5 \cdot 31 \cdot 89$	&	$3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 29$
XXV.	$2 \cdot 5 \cdot 7 \cdot 60659$	&	$2 \cdot 5 \cdot 23 \cdot 29 \cdot 673$
XXVI.	$2^3 \cdot 31 \cdot 11807$	&	$2^3 \cdot 11 \cdot 163 \cdot 191$
XXVII.	$3^2 \cdot 7 \cdot 13 \cdot 23 \cdot 79 \cdot 1103$	&	$3^2 \cdot 7 \cdot 13 \cdot 23 \cdot 11 \cdot 19 \cdot 367$
XXVIII.	$2^3 \cdot 47 \cdot 2609$	&	$2^3 \cdot 11 \cdot 59 \cdot 173$
XXIX.	$3^3 \cdot 5 \cdot 23 \cdot 79 \cdot 1103$	&	$3^3 \cdot 5 \cdot 23 \cdot 11 \cdot 19 \cdot 367$
XXX.	$3^2 \cdot 5^2 \cdot 11 \cdot 59 \cdot 179$	&	$3^2 \cdot 5^2 \cdot 17 \cdot 19 \cdot 359$

^aTranslator: This is not an amicable pair. Ferdinand Rudio, the editor of this paper for the *Opera Omnia*, notes that this was observed by K. Hunrath in 1909/10.