

# PHYSICAL INVESTIGATIONS ON THE NATURE OF THE SMALLEST PARTS OF MATTER

LEONHARD EULER

1

Variety is a property that is so universal and so essential to all bodies that we would be unable to find two that resemble each other perfectly. In bodies that are of considerable size, the diversity is so manifest that no one can doubt it; in small bodies, those that escape our view, the microscope allows us to discover so many different kinds that several philosophers have come to establish diversity and non-resemblance as a general law of nature. Indeed this diversity is not only found in the form and the arrangement of parts, but, in addition, the less essential qualities differ so considerably that we would be unable to find two bodies that possess the same quality in the same degree. Thus we observe an almost infinite diversity in colors; and we have reason to believe that there are no two colored bodies on earth that have the exact same hue. It would perhaps also be the same for hardness, softness, elasticity, and all of the other qualities that differ almost infinitely in the bodies on which we are able to experiment. It seems that we cannot even exclude size, for although this is not something that we can change in most bodies to our liking, it is difficult, for example, to render the two arms of a balance equally long or equally heavy. Even when they matched in appearance, we are however obliged to believe that there are still some differences between them that are so small that they escape our senses. But negligible difference notwithstanding, nothing prevents these things from varying infinitely.

2

Although this diversity is found as much in the smallest particles of bodies as in the bodies themselves, there is no doubt that the larger the body, the more susceptible it is to variety. For in addition to the diversity that is found in the smallest particles, their arrangement can change infinitely so that even if the smallest particles resemble each other, their various combinations could produce bodies that are very different. Therefore the diversity of bodies results from two sources. One is the diversity of the particles themselves; the other is the variety that is found in their arrangement, and both are capable of producing an infinite number of variations. Even if the smallest particles resemble one another, the diversity of their arrangement alone can furnish an infinite number of completely different bodies. And if there were only one way to arrange and combine the smallest particles, we would have to attribute the diversity of bodies to their intrinsic difference. But as the diversity to which bodies are susceptible increases in proportion to the size

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Translated from the French by Michael P. Saclolo, Department of Mathematics, and Peter Wake, Department of Philosophy, St. Edward's University, Austin, Texas, July 20, 2011.

of the bodies, we have reason to doubt whether the smallest, ultimate molecules<sup>1</sup> of matter were susceptible to some diversity in their state. For since they themselves are not composed of any smaller parts, one of these causes of diversity no longer applies. It is therefore quite an important question, as much in physics as in metaphysics, to know if the smallest particles of matter resemble each other or not. The philosophers are firmly divided on this subject. Some maintain that all these ultimate particles differ so much from one another that no two are perfectly alike. Others to the contrary want them to resemble each other perfectly.

## 3

It would be rash to make a decision on this question; experiments refuse to provide any help in this regard, and reason alone is not sufficient to enlighten us at this point. I shall therefore limit my investigation to only examining the ratio between extension and inertia in the smallest particles of matter. Although it is not possible to push experiments up to that level, I have however noted that common experience aided by incontestable principles of reason can lead us to a sound conclusion. This will then ensure that we discover several other properties of matter about which we have been only too unsure. Newton has shown almost geometrically that the weight of the body is proportional to its inertia. And as the idea of inertia does not differ from that of mass, or the amount of matter of which a body is made, it follows that the weight of each body is proportional to the amount of matter that it contains. Thus if we consider two balls of the same size, one of gold and the other of silver, since the former is heavier than the latter, we must surely conclude that there is more matter contained in the gold ball than in the silver one. As weight is proportional to inertia, it will therefore be an accurate measure of the quantity of matter that makes up a body. This discovery is based partly upon experiment and partly on reasoning. The former has shown us that all bodies fall at the same speed in space devoid of air. Reasoning, in turn, shows us that in order to impart the same motion to different bodies, it is absolutely necessary that the forces be proportional to inertia, that is to say, the quantity of matter in the bodies. In this case, force is gravity, that which gives bodies weight; consequently this gravity will be proportional to the quantity of matter.

## 4

Specific gravity is the ratio between a body's weight and its extension: the greater the ratio the greater the specific gravity [and vice versa]. If we conceive of two bodies of equal extension, for instance a cubic foot each, the ratio between their specific gravities will like be that of their weights; the specific gravity of all bodies is determined in this way. This notion is known throughout the world, and when we say that one material is undoubtedly heavier than another, it must have something to do with the specific gravity. That is to say, if one takes two equal volumes of these two materials, the weight of one will surpass the weight of the other. Thus, everyone understands that when we say that gold is heavier than silver, we are comparing two equal volumes, because no one would admit that an ingot of silver weighing one hundred pounds would be heavier than an ingot of gold weighing fifty. In this sense we say that mercury is heavier than water, that water

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<sup>1</sup>Translators' note: Euler uses the expression "dernières molécules" to convey the idea of the most basic parts of matter. We have chosen to use the English word "ultimate."

is heavier than distilled wine<sup>2</sup>, and in these terms, anyone can see above all that we are supposing two equal volumes. Yet we find such a great variety among bodies in relation to specific gravity that we would have trouble finding two substances that are of equal weight. And even when we are unable to notice a difference between them, it is apparently only for lack of instruments subtle enough to reveal the difference. That is why we have reason to believe that even though two ingots of gold of the same volume seem to us to be of equal weight, their weights are nevertheless not precisely the same, and that if we had finer instruments, we would not fail to observe some difference between them. For although gold is nothing if not a consistent or homogeneous material, one cannot assume that all of its parts have the same specific gravity. This same reasoning can be applied to other metals as well and to all materials that seem homogeneous to us.

## 5

Since we have sufficient grounds to say that all bodies differ in their specific gravity, we shall ask whether the same diversity is not also found among the smallest molecules that make up these bodies. To look at this question as a whole, it seems at first that we should affirm it. We could say that since all bodies differ among themselves according to their specific gravity, this diversity is only due to the parts of which they are made, and consequently these smallest particles themselves must vary infinitely with regards to specific gravity. We could even push the principle of non-resemblance or that of sufficient reason so far that we come to maintain that diversity with regards to specific gravity is found as much in the smallest particles as in large ones. But the plausibility of all these reasons notwithstanding, we would be thoroughly mistaken if we wanted to add faith as well. For I will show that all of the smallest particles which constitute the bodies that surround us are equally heavy, or have the same specific gravity, and I will do so with such clarity that no one will be able to doubt it. One might perhaps initially see this proposition as a great paradox. The metaphysicians who extend universal inequality even to the elements that make up matter will be quite surprised that the identity of specific weight can generally be found, not in the elements, but in all of the smallest particles of matter itself, even those that are still quite far from these elements, as I shall soon show. I hope to prove my claims with enough rigor to avoid having to respond to any objections. And I shall be content to have discovered such a beautiful property of matter, one that can lead to still further discoveries, if we are not hurried in our reasoning.

## 6

Before proving this paradox, I must explain more clearly what I mean by the smallest particles of matter, and this explanation shall lead us to the proof itself. Each body has a certain amount of matter that is specific to it and which constitutes the being of the body, so to speak. These are the parts that move conjointly with the body, and their heaviness produces the weight of the body itself. One must therefore distinguish this matter specific to the body from that which penetrates freely through the pores and fills conjointly with those parts which belong to the space that is occupied by the body. As such everyone can distinguish the parts of air which are found in the pores of a sponge from the parts of the sponge itself.

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<sup>2</sup>Translators' note: ethanol

And as there is no doubt that the world is filled with a fluid matter that is elastic and very subtle, what we call the ether, the pores of the bodies must be penetrated by it. As a result this subtle matter occupies a substantial part of the space that the body seems to fill. Even though this matter is confined by the same boundaries that define the body, we must nevertheless not presume that it belongs to the body itself. That is why I shall call it foreign, to distinguish it from the material belonging to the actual body, as well as the other fluid materials, visible or not, which can be found in the cavity of the pores.

## 7

That the gravity of all the bodies that are scattered about the earth has a physical cause, that there is a mechanical force that pushes them down: this is what I absolve myself from proving here, although the true cause is not known to us in detail. But in general it is certain that there is matter that is extremely subtle, which through its motion is endowed with a force capable of pushing down bodies and producing all phenomena of gravity. It is of little importance to me whether this matter is the ether or not. Its effects clearly show us that it is extremely subtle, since no experiment is capable of allowing us to sense it, or alter its effects. Therefore all bodies, as long as they have weight, will be penetrated by this subtle matter and consequently it will freely traverse their pores. But as pores do not cover bodies in their entirety, and as bodies enclose matter that is specific to them, it is clear that there must be portions of a body devoid of pores and through which this subtle matter which produces gravity cannot pass. And I am speaking of these smallest particles here. I am not saying that these particles do not have pores at all; perhaps they still have some pores, but these are so small that this subtle matter cannot pass through them. These particles are therefore of finite size and are consequently composed of even smaller parts. Thus they are different from those that are included under the category of elements. The term molecule is the most appropriate to designate them. Thus each body is composed of a certain number of molecules which constitute its proper material and they are arranged to form the pores through which the subtle matter that constitutes the cause of a body's heaviness can freely pass.

## 8

The weight of a body is nothing but the sum of all the forces that push down on its molecules. And through what we have proven above concerning its heaviness, it is also clear that the forces which push the molecules must be proportional to the inertia of these molecules or the amount of matter that they contain. Yet in whatever manner we imagine the cause of gravity, as it is the effect of the pressure of a fluid, the force with which each molecule is pushed will always be proportional to the extension or volume of this molecule. For it is a general rule of hydrostatics that fluids act according to volumes: a body submerged in water is constantly pushed by a force equal to that of an equal volume of water, but in an opposite direction. It follows that a body submerged in water loses part of its weight and that this loss is still proportional to the volume. This same law applies to all fluids which act through pressure, whatever variety they may be. And yet everyone who has taken on the task of explaining the cause of heaviness, however different their hypotheses may be, agrees that the ultimate molecules of matter which support the force of

gravity are pushed by forces proportional to their volumes. So two molecules of equal volume will also be equally heavy, and if these molecules are unequal with regard to volume, their weights will differ in the same proportion.

## 9

Having therefore shown above that the weight of molecules is proportional to inertia or the amount of matter that each molecule contains, it follows that the quantity of matter in these molecules is constantly proportional to their volumes in such a way that if the molecules were equal in extension, they will also be composed of an equal amount of matter, and be equal in their weight as well. We call density the ratio between the quantity of matter and its extension, and we say that two bodies are equally dense so long as they contain equal portions of matter while having equal volumes, or what amounts to the same thing, when their weights are in proportion to their volumes. Next, a body is said to be denser when it contains more matter or is heavier but has the same volume [as another]. Therefore all molecules that make up a body assume weight with respect to their volume, as well as with respect to their mass: they will all have the same specific gravity and will all be equally dense. Yet as heaviness, to the extent that we can observe it, is nothing but a property of the bodies that are found on the surface of earth, and given that the weight of the very same body will change if it were to be transported to another place, it is clear that the weight does not mark a fixed property of the body. That is why when I say that the molecules or the smallest particles of bodies assume weight with respect to their volumes, it must be understood that it is on the condition that they are roughly at the same distance from the center of the earth. But when I say that the mass (or the amount of matter) of molecules is proportional to their volumes, it is a general proposition which is no longer attached to a particular situation, since a change of location would not change anything, neither the amount of matter nor the extension of the molecules. We have therefore sufficiently proven the truth that all molecules of a body are equally dense, and that the inequality of density that we observe in all large bodies can be reduced entirely to the molecules that compose all bodies in general.

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Therefore whatever difference there may be between the density and the specific gravity in the bodies that surround us, it is still certain that all the molecules that compose them have the same specific gravity, or to speak more precisely, the same density. Consequently although gold is the heaviest and densest substance that we know, we can nevertheless be assured that its molecules are neither heavier nor denser than those that make up water or air or bodies that are even lighter than these. Thus if gold does not contain a greater number of molecules under the same volume, there must be another reason for why gold is heavier than other bodies. Yet as gold still has a great number of pores, if it were possible to compress it to the point where all of its molecules touched one another and there were no more space between them, it is evident that the gold would become even denser and heavier than it actually is. Yet in this state its specific gravity would be precisely the same as that of the molecules, and it follows from this that the specific gravity of molecules must surpass that of gold. Let us suppose that the pores of gold occupy half of the volume. The specific gravity of the molecules that all bodies are made

of would be two times greater than that of gold. But we have reason to believe that the pores of gold occupy a much larger part of the entire volume than half. Therefore the specific gravity or density of molecules would be several times greater than that of gold. As water is about 19 times lighter than gold, if we were to stop at the first supposition, it would only be one 38th of the volume that water occupies and would be filled with molecules. The rest would either be empty or filled with foreign matter that does not constitute a part of water. And in air, which is 800 times lighter than water, only one 30400th of its volume would contain the matter that is specific to air.

## 11

This reasoning, which we have drawn from the nature of heaviness, only shows us that the molecules of the bodies that surround the earth and which assume weight in relation to it are all equally dense. And one could still doubt whether this same property also extends to bodies that are found in the bowels of the earth or the bodies that constitute other celestial bodies. But as we have no reason to doubt that heaviness observes the same law on all planets that it observes on earth, we must conclude that all bodies on every planet are composed of molecules that are the same with respect to density. By applying my reasoning to all those celestial bodies that are subject to forces that observe a general law, we will that not only bodies on earth but also those found on planets and even comets must all be composed of equally dense molecules. Therefore it prevails everywhere, in all the molecules of those bodies that have the same density. It is all the more surprising then that nature everywhere appears to affect an infinite diversity. But perhaps this uniformity is a necessary consequence of the essence of matter, and if we knew it more perfectly, we would not fail to see that this degree of density is as essential to matter as is the fact that for a triangle the sum of all its angles is equal to two right angles.

## 12

But let us examine more closely the state of those smallest particles of bodies that we have called molecules, those particles which support the impressions of subtle matter, and stand as the cause of heaviness. The following question arises when this matter no longer passes through these molecules, or when these molecules no longer have any pores at all, or, when they do have them, they are so small or so narrow that this subtle matter cannot pass through them: are the molecules of bodies completely solid, or do they have pores? Since all molecules are equally dense, if we say that they have pores, we would be obliged to say that there is in each molecule the same proportion between the space that is filled with matter and that which is occupied by the pores. Yet not only do we see no reason for such a general arrangement, but experimentation teaches us that as soon as there are pores in bodies, the relation between the pores and the solid part varies infinitely. One is even less entitled to say that these molecules are composed of an arrangement of several parts, for it would be inconceivable that the molecules be organized in such a way that the relation between the solid part and the porous part always remains the same. Let us maintain the former and say that all molecules are perfectly solid and completely devoid of pores. They will therefore be small solid masses, and since all of their parts have the same density, one can consider them

to be perfectly homogeneous or composed of similar matter. That is, one cannot imagine any other difference among these molecules, including differences in size and form, for neither of these could ever change in their essence. As for the rest there is no doubt that these particles are extremely small, and that their smallness surpasses our imagination. Although, however, they no longer have any pores which mark a composition of parts, it would be a grave mistake to maintain that these particles were absolutely indivisible. For since they still have finite size, divisibility necessarily applies, even though they are not indeed subdivided.

## 13

What I have just said only concerns those bodies that have weight and their parts, and it might not hold for bodies that have no weight, such as the subtle matter itself that causes heaviness. It initially seems highly probable that there must be a great difference between the matter that causes heaviness by its continual pressure and the one that receives the pressure. However subtle this fluid may be, it is nonetheless material, and if it is the essence of matter to have a certain degree of density, it must be said that the particles of this subtle matter would be as dense as the molecules of bodies. If we wanted then to maintain that everything is completely filled by matter and there is no void at all, space as such would be filled with matter that is equally dense throughout, and even denser than gold. This makes it extremely difficult, so as not to say impossible, to explain motion. For even though there is only a small part of bodies that has weight, and which is felt by phenomena, the other part, due to its great density, would be unable to resist motion; yet as soon as we remove the resistance of weighted bodies such as air, we hardly observe any resistance which diminishes the motion of bodies. This consideration compels us therefore to say that either there is a void in the world, and that the greatest part of space does not contain matter, or that the subtle matter, which causes weight, is something completely different from what makes up bodies that have weight.

## 14

By embracing the first sentiment, we shall gain little. For if we say that the particles of this subtle matter are as dense and thick as the molecules of bodies, then in order to obtain a sufficient void to explain motion we shall be forced to separate the particles of subtle matter so far from one another that, we will no longer be able to conceive of how heaviness could be produced by such matter. For it is indisputable that the fluid that causes gravity must be extremely compressed. But how can one attribute such a state of compression with particles that are dissipated and far removed from one another. Therefore only the other sentiment remains, and in light of this we maintain that the matter that constitutes subtle fluid, the cause of weight, is of a completely different nature from that of which all tangible bodies are made. There are therefore two types of matter, one that furnishes the substance of all tangible bodies, and which has particles of the same, very considerable density, a density that surpasses that of gold by several times. The other kind of matter is that which makes up the subtle fluid that causes gravity and that we call the ether. It is probable that this matter has, uniformly, the same degree of density, but that this degree is incomparably smaller than that of the first kind. Not only does the reasoning drawn from the possibility of motion prove the

extreme rarity of this fecund species of matter, but the propagation of light that is caused no doubt by this very same subtle fluid shows us that its density must be several thousand times smaller than that of air, and consequently several million times smaller than the density of molecules of which large bodies are composed.