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Development of Selected Mathematical Instruments Representing Angular, Logarithmic and Arithmetic Computation

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DEVELOPMENT

OF

SELECTED MATHEMATICAL INSTRUMENTS REPRESENTING ANGULAR, LOGARITHMIC AND ARITHMETIC COMPUTATION

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BY

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CHAPTER I

Angular Computation

Sextant

The Sextant in its earliest known form consisted of divided circles and compasses with simple sights. An early Greek astronomer of the second century after Christ, Claudius Ptolemaeus, or more commonly called Ptolemy, wrote a book entitled Megale Syntaxis tes Astronomias, also known by the Arabic title Almagest. The instrument described in this book was called the Astrolable and was used to measure the angular distance between stars. It was made of two concentric vertical circles, the largest and outer circle was about sixteen inches in diameter with graduated arc; the central ring was movable and carried the two sights.

Later the Cross Staff (Plate I) consisting of simple sighting bars with cross-pieces AB and CD, each graduated to the same scale, was invented to be used in navigation. As one star would be sighted along the bar CD; the other bar AB would be moved perpendicularly along CD until the other star could be seen at A. The length from A to CD was read and the angular distance found on a prepared table.

The most serious defect in the Astrolable and Cross-staff was in taking altitudes. This imperfection was partly corrected by the development of the idea of reflecting the rays of light. Tycho Brahe, a nobleman of Denmark, was one of the first to make any important changes in the Cross Staff by the use of reflection. His main use of this

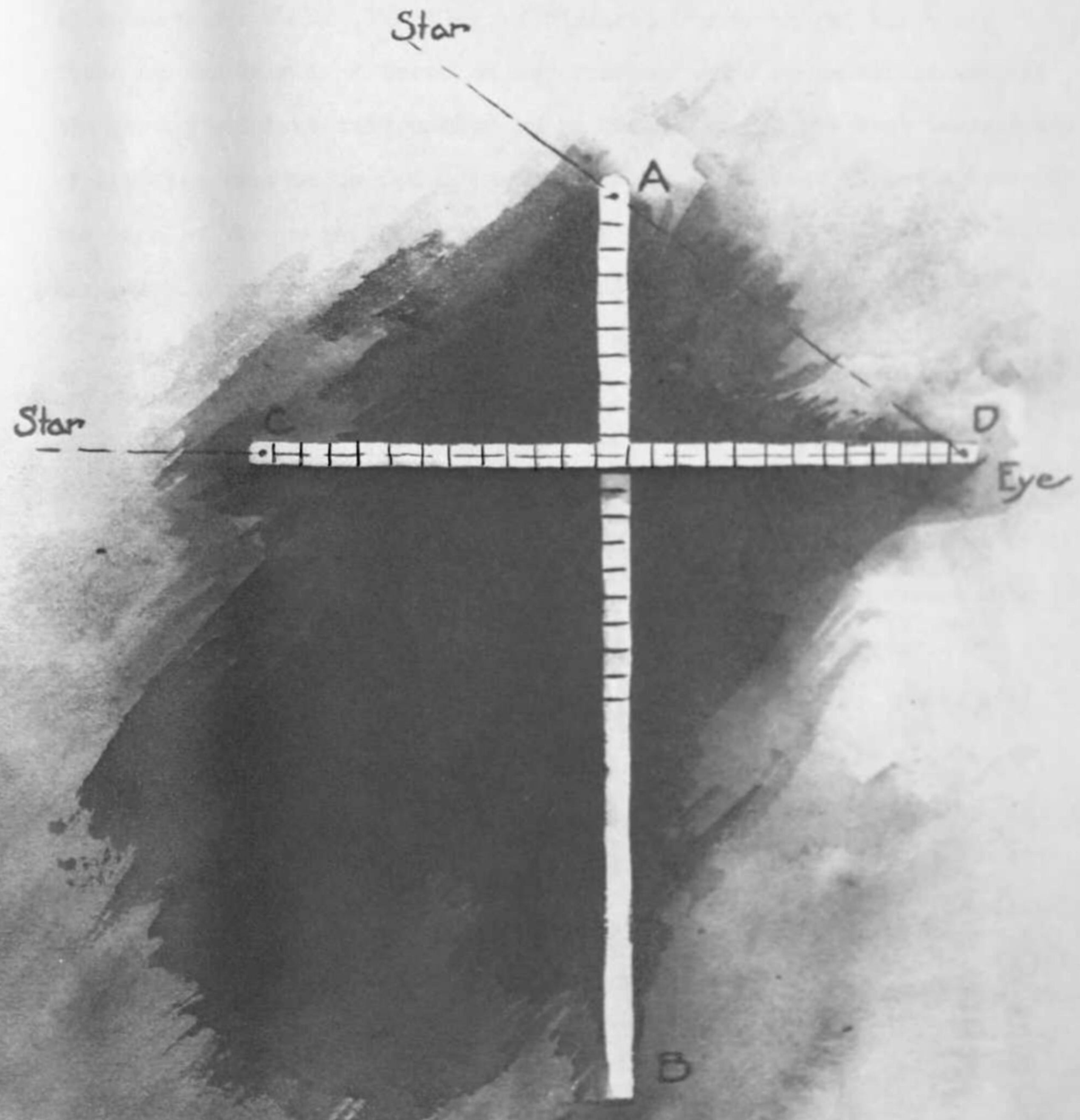


PLATE 1

instrument was in measuring the altitudes of stars. Recognizing the inevitable errors in his instrument, he found the amounts and made allowances for them. The King of Denmark, Frederick II, built for Tycho on the Island of Hveen an observatory where he installed many of the newest and best instruments to be had. Some of the best instruments of all time were to be found in this castle, Uranburg. Tycho developed the idea of the concentric circles by placing them on a staff, strengthening the parts and using a movable radius for the inner circle. (Plate II)

In the year 1608, by chance, a great and very valuable discovery was made by a Dutch spectacle maker, Lippershey. He found that two lenses placed at some distance apart would magnify distant objects as readily as a single lens would that of an object near at hand. He called his instrument a telescope. This instrument revolutionized the study of astronomy and was subsequently improved, making a most important mathematical instrument. The Sextant devised by Hevel,or Helvius, was probably the most practical instrument produced to that time. (Plate III) One star would be sighted thru the stationary sight by use of AC. Another star would then be found by the use of the movable arm AB. The angular distance could then be used in finding the distance of the stars as in Figure 2. Later a telescope was attached at A. In 1609, Galileo Galilei, having learned of this invention, began the building of one for himself which attracted much attention. Later more instrument makers began to use the telescope. However, Hevel's Sextant (Plate III) was probably first to use the telescope.

The next important advance in the development of the telescope

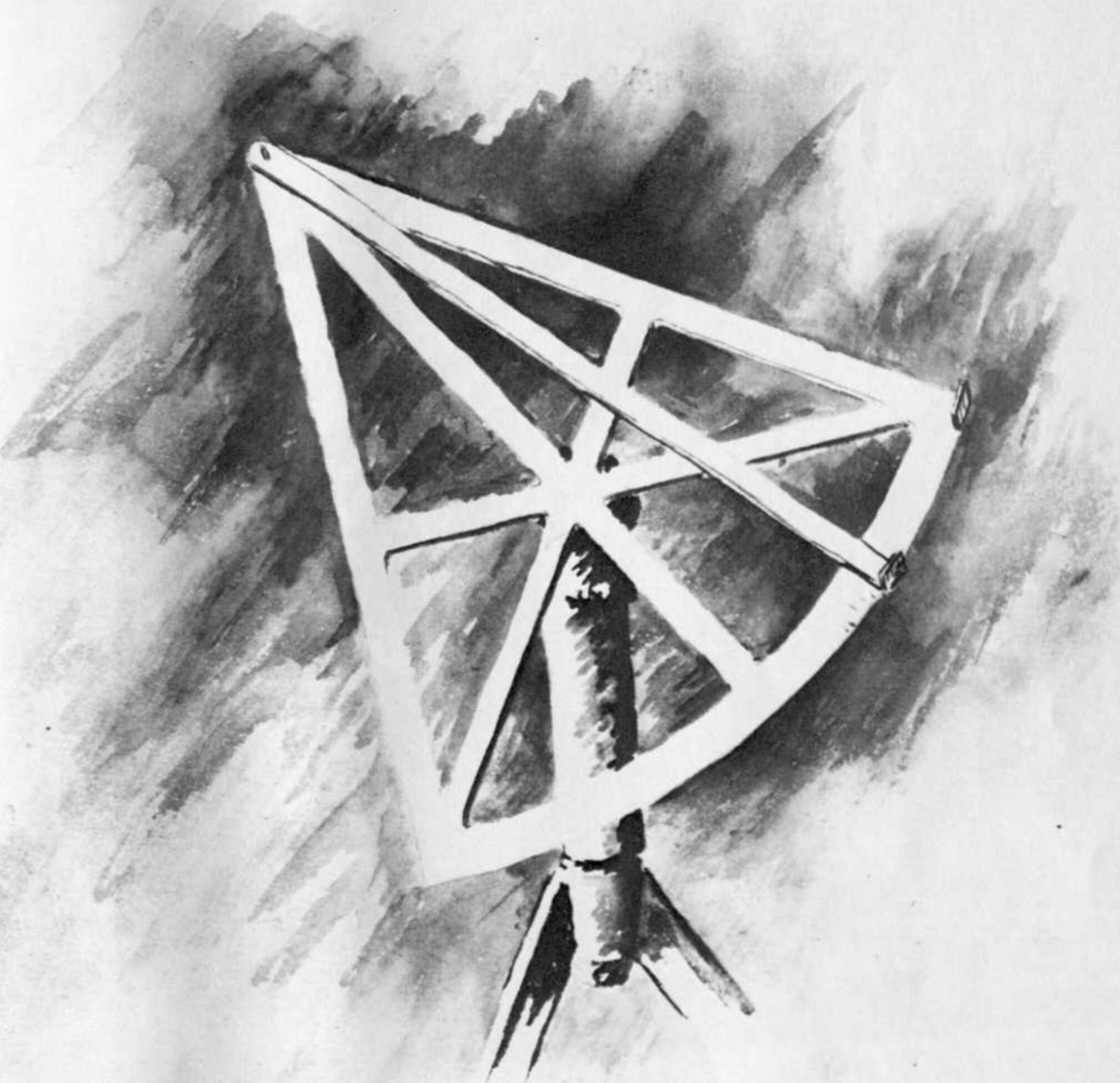


PLATE 2

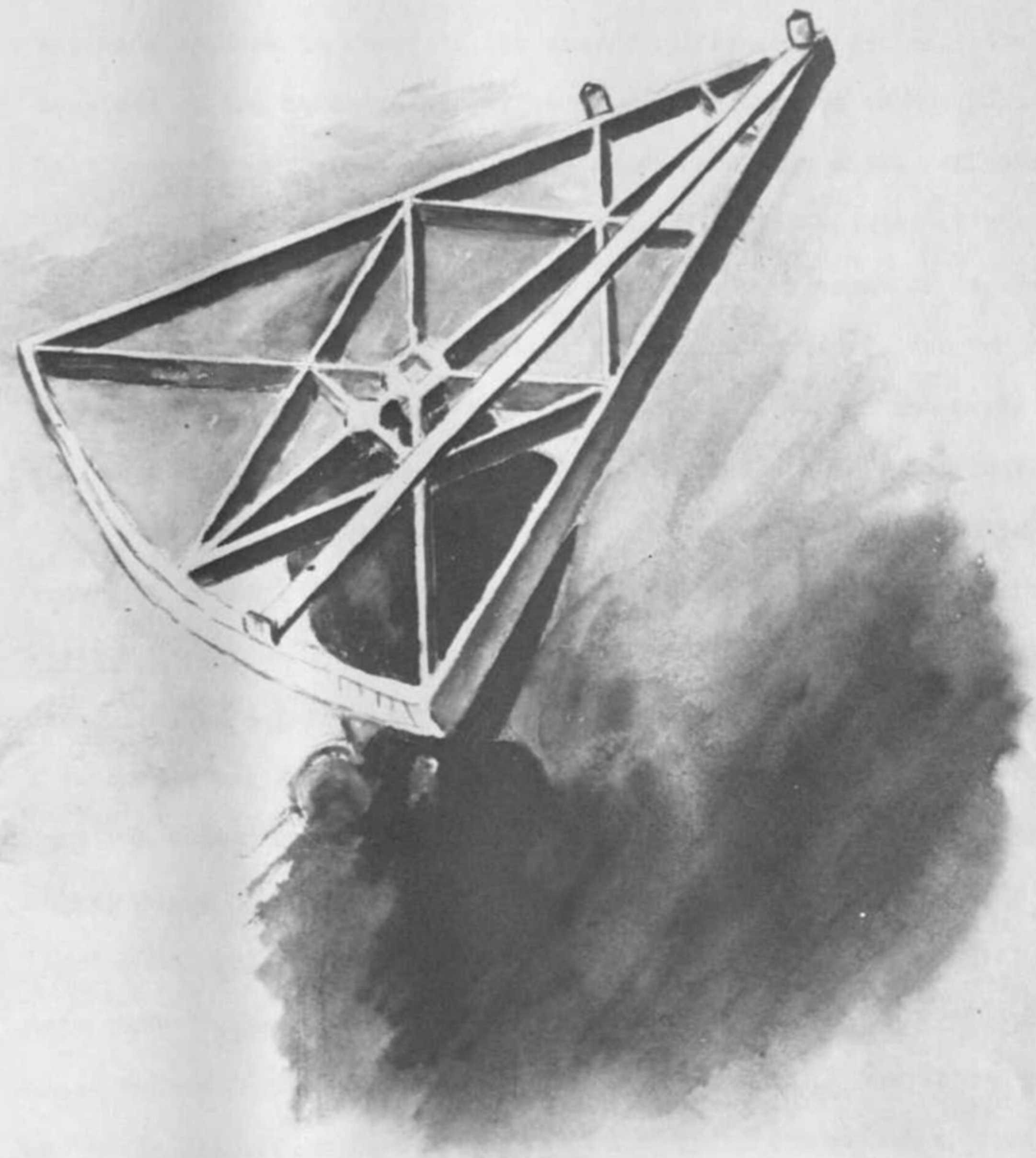


PLATE 3

was made by Hook in 1685, in the use of reflectors. His contribution consists of two definite stages, both fully described in his published Posthumous Works: The first having only one mirror which reflected the light from one object into the telescope which was pointed at the other object. The second used two single reflections by means of two telescopes, the axis of which were the radii of the sextant. The two objects, to be measured could be sighted thru the telescopes and the angle could be found by reading the scale of the angle on the arm of the instrument.

Newton seems to have worked along this line but nothing was known of it until 1742 when his works were printed by Halley in the Philosophical Transactions. The arc of his instrument was equal to one-eighth of a circle and was divided into ninety parts read as degrees. A telescope was fixed on the radius of the sector. The object glass was near the center and had outside of it a mirror inclined at a forty-five degree angle to the axis of the telescope, thus intercepting half the light which would naturally fall on the object glass. One object is seen thru the telescope; a movable radius, having on it a second mirror close to the first, is turned around the center until the second object by double reflection is seen in the telescope along with the first. Halley is better known as "our southern Tycho" because of his valuable observations made with the sextant on the Island of St. Helena. The sextant which he used was five and one half feet in radius.

Before Halley's and Newton's Plans were published, however, the instrument in its present form came into use. John Hadley published May 13, 1731 the description of an octant using double reflection.

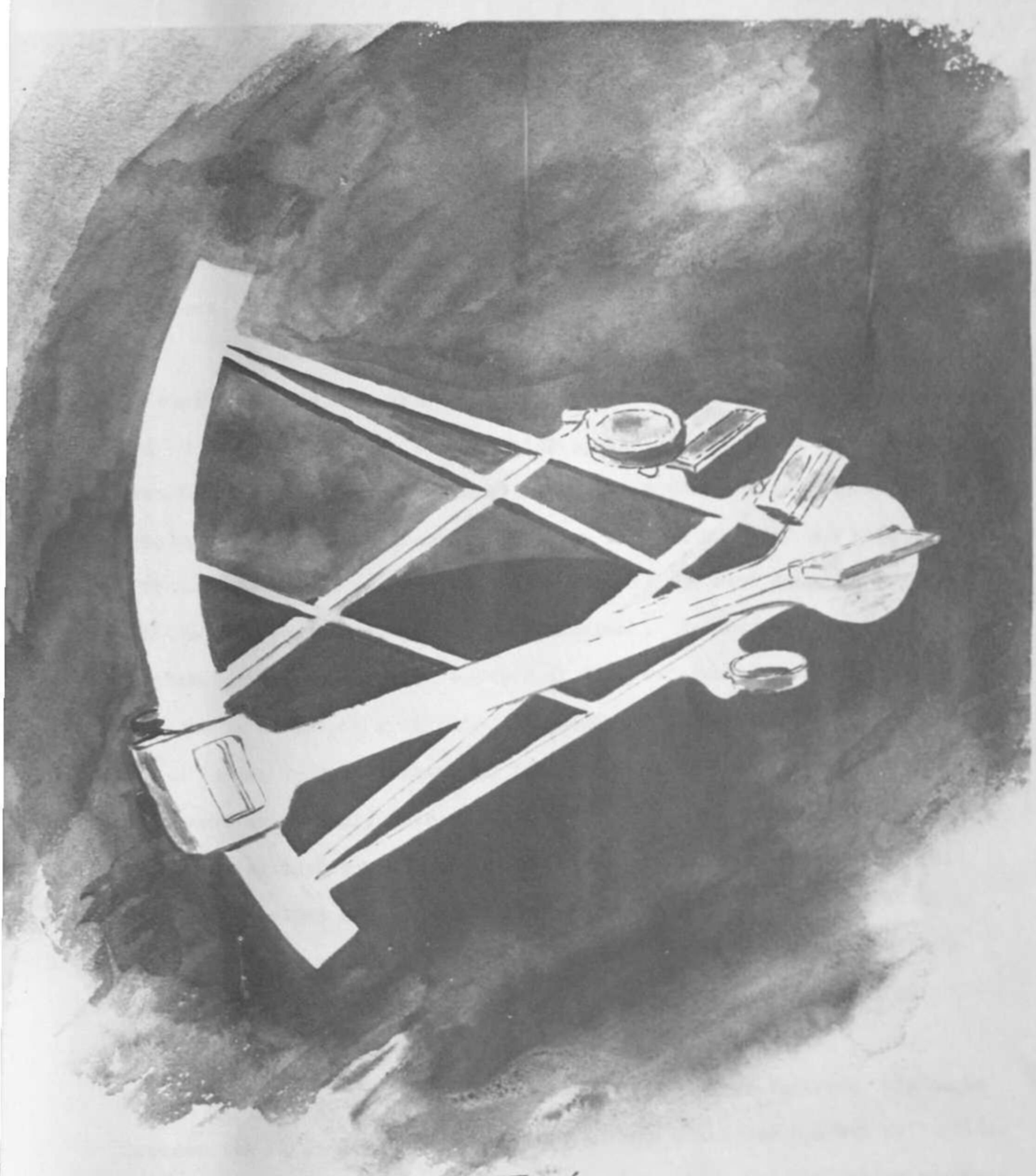
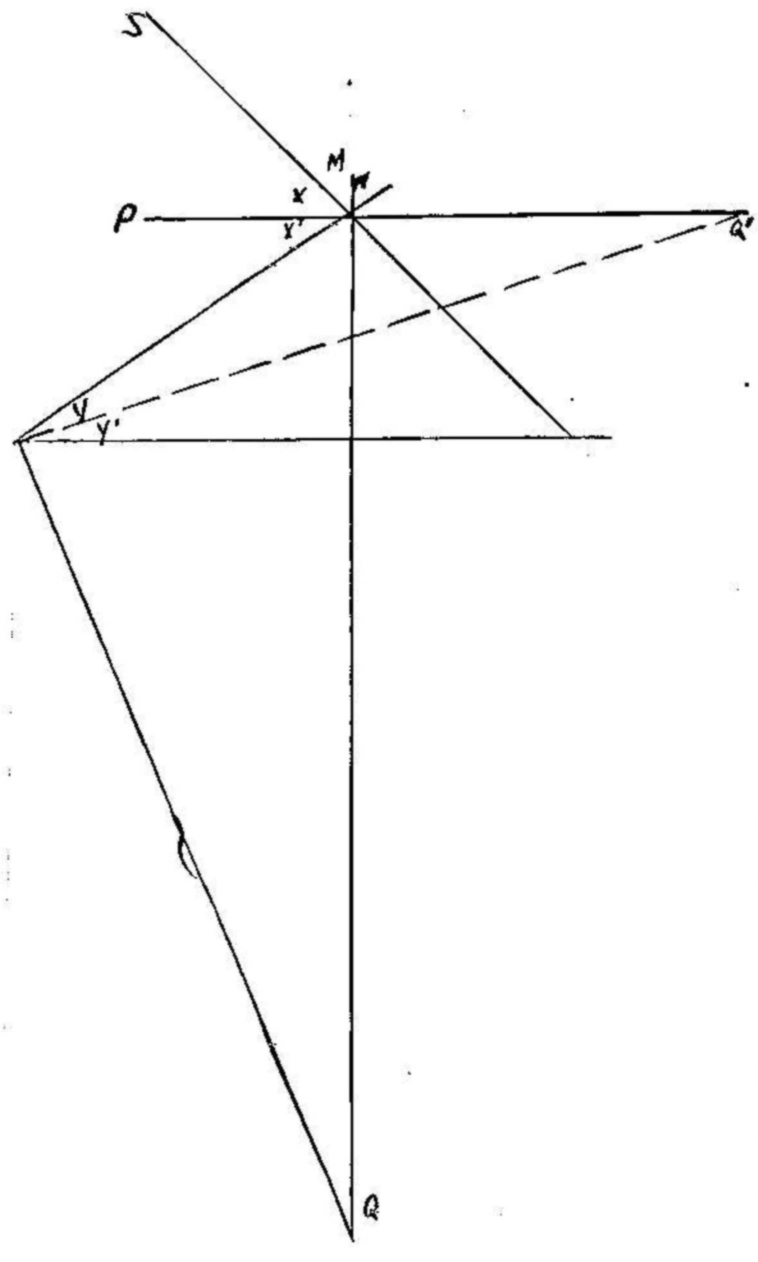


PLATE 4

Two weeks later he exhibited it. It was tried on board the "Chatham", a government yacht, and found to be very satisfactory. (Plate IV) Its range of use was limited to measuring angles up to ninety degrees. The octant was not generally recognized until 1757 when Captain Campbell of the English Navy enlarged the instrument to measure up to 120 degrees, which is its present form.

Thomas Godfrey, of Philadelphia, independent of Hadley and Newton, invented a sextant. In support of a claim of priority presented on January 31, 1734, at a meeting of the Royal Society of London, two affidavits were read attempting to prove that one of Godfrey's quadrants was used about November, 1730, on board a sloop by the name of "Truman" on a trip to Jamaica. Also that it was used again in August of the next year in a trip to New Foundland. Another statement that is probably untrue was that a brother of Godfrey's sold a quadrant at Jamaica to a Captain or Lieutenant Hadley of the British Navy, who brought it to London to his brother, an instrument maker. One reason that this statement is probably untrue is that Hadley was a country gentleman of means and an instrument maker only for pleasure. His brother, George, who always helped him in his laboratory was a barrister and not a member of the Navy. Therefore, I believe that it is more than likely that the sextant of America and England had no connecting link as some of us would like to imagine.

The theorem of the sextant may be stated as follows: The angle between the first and last direction of a ray which has had two reflections in the same plane is equal to twice the angle which the reflecting surfaces make with each other.



By the law of reflection, angle x equals angle x' and angle y equals angle y'.

Angle SMH is an exterior angle of the triangle MHE and is equal to to the sum of the opposite interior angles.

But angle SmH equals 2x and angle MHE equals 2y therefore angle E plus 2y equals 2x or angle E equals 2x minus 2y or 2(x minusy) Angle x equals angle PMH.

Angle y equals angle MHQ' therefore angle E is equal to 2(PMH minus MHQ'). Similarly in triangle HMQ, angle x is exterior angle and equal to the sum of the opposite interior angles.

Angle x equals angle HQ'M plus angle y therefore angle HQ'M equals angle x minus angle y which is also equal to angle PMH minus angle MHQ'. Therefore angle HQ'M equals $\frac{1}{2}$ of angle E which is equal to angle Q. Similarly we could prove that angle Q equals $\frac{1}{2}$

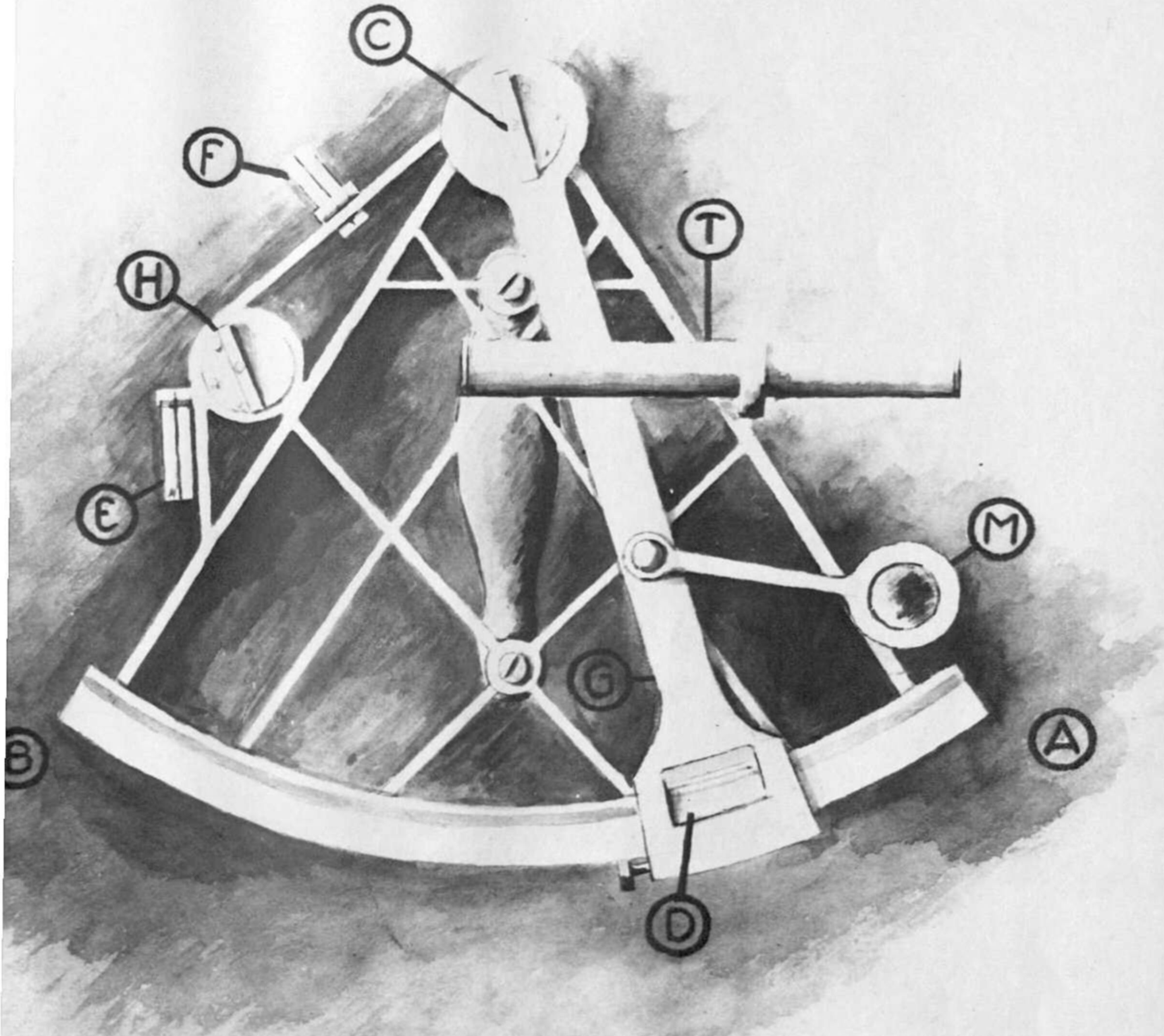


PLATE 5

of angle E, therefore the angle at angle E is equal to two times the angle S at Q and Q' (which was to be proved).

In the description of the instrument which will be given by the use of Plate 5, D is the vernier, AB the scale, the mirror at C is the index glass as it is on the arm of the vernier, the mirror H is on the frame and is called the horizon glass. F and E are sets of colored glasses which may be used when looking at the sun. T is the telescope and M is a small magnifying glass, pivoted at G. The arc AB which is sixty degrees is divided into one hundred twenty equal parts.

The mirror H is only half silvered, the other half being transparent. One star is sighted thru the transparent half of H and the vernier D is moved along AB until the other star is reflected from the mirror C into the mirror at H. As the arc is divided into many parts it is comparatively easy to discover the angle by reading the index at D by means of M.

Owing to the fact that the sextant can be used on a moving object, such as a boat, it is an essential in navigation. The latitude at sea may be determined by this instrument. The observer when using this instrument to obtain the altitude of the sun, sights the horizon thru the transparent part of H, and the vernier D is moved along AB until the sun is reflected from the mirror C into the mirror at H. The sextant is a seafaring instrument, because where it is possible to have a stationary vertex angle as on land, it is not practical. The development of instruments for problems on land has given us the modern transit.

CHAPTER II

Logarithmic Computation

Slide Rule

The Slide Rule, probably the most useful of mathematical instruments, owes a great deal to John Napier, Baron of Merchiston, of Scotland, who invented logarithms in 1614. Logarithms have simplified rapid calculation in mathematics probably more than any other one thing. It was gladly received both on the Continent and in England.

In 1620, we have another invention which lead to the making of the first slide rule. During that year Edmund Gunter, professor of astronomy in Gresham College, London, prepared the logarithmetic "line of numbers" in which the digits from one to ten were placed on a straight line proportional to the logarithms of the numbers. This line, along with lines containing the logarithms of the trigonometric functions, was placed on a ruler called "Gunter's Scale". This scale could only be used by the help of compasses. Products or quotients could then be found by adding or subtracting distances on the scale. The history of the slide rule has been very difficult to trace and many authors have confused the Gunter Scale with the slide rule. Some have thought that Wingate invented the first straight edged slide rule in 1624 because of his booklet published in London in 1628 on the Construction and Use of the Line of Proportion. Cajori, the able historian, secured a reader to examine this work, who found that the "line of proportion" was not a slide rule, but merely a tabular scale in which the numbers were given in spaces on one side of a straight line. On the other side of the line was found the mantissas of the common logarithms of these numbers. This instrument was merely a scale and had no sliding parts.

Wingate was not satisfied with his rule and in 1630 he published his book, Natural and Artificial Arithmetic, giving a description for his slide rule. He placed the logarithmic divisions on two rulers, which would slide one along the other.

In 1632 in an article on the Circles of Proportion and the Horizontal Instrument published in London we find Oughtred working along the same line. De Morgan in the Penny Cyclopaedia and in the English Cyclopaedia on Arts and Sciences tells us of his work. This historian states that Oughtred showed his instrument and notes to a pupil, William Foster, a popular mathematics teacher of London, who translated Oughtred's material and published the above named article on Circles of Proportion and the Horizontal Instrument on the slide rule. In the year 1633 another article simplifying addition was published. DeMorgan quotes from Foster's works the following extract:

> "Being in the time of the long vacation 1630, in the country, at the house of the Reverend, and my most worthy friend and Teacher, Mr. William Oughtred (to whose instruction I owe both my initiation, and whole progresse in these Sciences), I upon occasion of speech told him of a Ruler of Numbers, Sines, and Tangents, which one had bespoken to be made (such as is usually called Mr. Gunter's Ruler) six feet long, to be used with a payre of beame-compasses. He answered that was a poore invention, and the performance very troublesome. But, said he; seeing you are taken with such mechanicall wayes of Instruments, I will show you what devises I have had by mee these many yeares. And first, hee brought to mee two Rulers of that sort, to be used by applying one to the other, without any compasses; and after that hee showed mee those lines cast into a circle or Ring, with another moveable

circle upon it. I, seeing the great expeditenesse of both these wayes, but especially of the latter, wherein it farre excelleth any other Instrument which had bin knowne; told him, I wondered that he could so many yeares conceale such usefull inventions, not only from the world, but from my selfe, to whom other parts and mysteries of Art he had bin so liberall. He answered, the true way to Art is not by Instruments, but by Demonstration; and that it is a preposterous course of vulgar Teachers, to begin with Instruments and not with the Sciences, and so instead of Artists to make their schollers only doers of tricks, and as it were Juglers: to the despite of Art, loose of precious time, and betraying of willing and industrious wits unto ignorance, and idlenesse. That the use of Instruments is indeed excellent, if a man be an Artist; but contemptible, being set and opposed to Art. And lastly, that he meant to commend to me the skill of Instruments, but first he would have me well instructed in the Sciences. He also showed me many notes, and Rules for the use of those circles, and of his Horizontall Instrument (which he had projected about 30 years before) the most part written in Latine, all of which I obtained of him leave to translate into English, and make publique, for the use, and benefit of such as were studious, and lovers of these excellent Sciences".*

In 1638 George Barkham gave St. John's College one of Oughtred's Circles of Proportion made in 1632. This instrument is inscribed with the names of the maker, Elias Allen, and also that of the donor:

"The face of the instrument is engraved with Oughtred's Horizontal Instrument. The back is engraved with eleven Circles as described on the "<u>Circles of Proportion and the Horizontal Instrument</u>". The uses of both Written in Latine by Mr. W. O(ughtred). Translated into English and set forth for the publique benefit by William Foster, London, printed for Elias Allen, maker of these and all other Mathematical Instruments, and are to be sold at his shop over against St. Clement's Church without Temple-barr** in 1632.

"Oughtred's Circles were read by two radical pointers (unfortunately missing in the St. John instrument), attached to the centre of the concentric circles, each charged with a logarithmic scale. These pointers could either

* Cajori - Logarithmic Slide Rule
** The Temple-barr was an arched gateway between Fleet Street and the Strand.

move round to-gether, united by friction, or open and shut by the application of pressure; they were, in fact a pair of compasses, laid flat on the circle, with their pivot at its centre.

"Calling these pointers antecedent and consequent; to multiply A and B, the consequent arm must be brought to point to 1, and the antecedent arm then made to point to A. If the pointers be then moved together until the consequent arm points to B, the antecedent arm will point to the product of A and B.

"Circles:

"First--outside--of Sines--5° 45' to 90° (first 30° divided into 12 parts each which makes 5 minutes) from there to 50° into six parts which makes 10 minutes; to 75° into two parts or 30 minutes after that until 85° --not divided.

"Second--Tangent 5° 45' - 45° (each degree divided into 12 parts or 5 minutes)

"Third--Tangent 45° to 84° 15' (12 parts)

"Fourth--Logarithms--Unequal numbers (unoequall) 2,3,4,5,6,7,8,9,1 (Up to 5 divided into 100 parts)

"Fifth--Logarithms--Aequall numbers. 1,2,3,4,5,6,7, 8,9,1--each divided into 100 parts.

"Sixth and seventh--Logarithms--Divided into degrees--each degree ten parts--each 6 minutes or 10 hundredth parts a piece. Sixth up to 44° 5'. Seventh from 44° 5' to 70°.

"Eighth--Tangent--from 84° - 89° 25'.

"Ninth--Tangent--from 35' to 6° *

During the later part of the Seventeenth Century many men worked on the Slide Rule but only a very few of the changes made were of real merit, Seth Partridge, a surveyor and teacher of mathematics, made the major contribution by the invention of the rule with sliding

* Gunther--Early Science in Oxford

parts. De Morgan says of Partridge's instrument:

> "that the rules were separated and made to keep together in sliding by the hand; perhaps Partridge considered the invention his, in right of one ruler sliding between two others kept together by bits of brass." *

Leadbetter in 1755 published a book in the Royal Gauger in one section of which he discussed the work of Hunt and Everard, both having made a real contribution in the development of the sliding rule. Of W. Hunt's rule he says:

> "Upon Hunt's Sliding Rule there is a line of segments, by which the area of the segment of a circle may be found, as he shows in his Mathematic Companion, pages 168 and 169." **

The complete title of this book gives a good description of the uses for his rule, as follows:

> "A Mathematical Companion, of the Description and Use of a New Sliding-Rule by which many useful and necessary questions in Arithmetick, Interest, Planometry, Astronomy, Fortification, Dialling, etc., may be speedily resolved, without the help of Pen or Compass.------1697." ***

Leadbetter evidently did not know of Wingate and Oughtred when he said that Thomas Everard invented the first slide rule in 1683. In 1696 Everard, along with other scientists, was called before Parliament to make demonstrations of experiments of the cubical contents of a Standard bushel. This shows that he was considered an eminent authority on this science. In Stone's translation from the French of M. Bion's Construction and Principal Uses of Mathematical Instruments is found an excellent description of this rule. Of it he says:

> "This instrument is commonly made of Box, Exactly a foot long, one inch broad, and about six-tenths of an

* - Cajori, Slide Rule--16 ** - Cajori *** - Cajori

inch thick. It consists of Three Parts, viz., A rule, and two small Scales or Sliding-Pieces to Slide in it; one on one Side and the other on the other; so that when both the Sliding-Pieces are drawn out to their full extent the whole will be Three feet long.

"On the first broad Face of this Instrument are four lines of numbers; the first Line of Numbers consists of two Radius's, and is numbered 1,2,3,4,5,6,7, 8,9,1, and then 2,3,4,5, etc., to 10. On this Line are placed four Brass Center Pins, the first in the first Radius, at 2150.42, and the third likewise at the same numbers taken in the second Radius, having M.B. set to them; signifying that the aforesaid Number represents the Cubic Inches in a Malt Bushel: the second and the fourth Center pins are set at the Number 282 on each Radius; they have the Letter A set to them, signifying that the aforesaid Number 282 is the Cubic Inches in an Ale Gallon----the second and third Line of Numbers which are on the Sliding Piece----are exactly the same as the first Line of Numbers: They are both, for Distinction, Called B. The little black Dot, that is hard by the division 7, on the first Radius, having Si set after it is put directly over 1707 which is inscribed in a Circle whose Diameter is Unity, hard by 9, often which is writ Se, is set directly over 886, which is the Side of a square equal to the area of a circle whose Diameter is Unity. The black Dot that is nigh W, is set directly over 231, which is the Number of Cubic Inches in a Wine Gallon. Lastly, the black Dot by C, is set directly over 3.14, which is the Circumference of a Circle, whose Diameter is Unity. The fourth Line on the first Face, is a broken Line of Numbers of two Radius's, numbered 2,10,9,8,7,6,5,4,3,2,1,9,8,7,6,5,4,3, the number 1 is set against M.B. on the first Radius. This Line of Numbers hath Md set to it, signifying Malt Depth." *

Many persons have given the honor of the change in the slide rule to Pearson, who wrote about a Century after Everard's time. Already we have noticed in the study of Everard's slide rule that the logarithmic line had been inverted; the only difference in the two rules being that Pearson inverted the slider while Everard inverted the line.

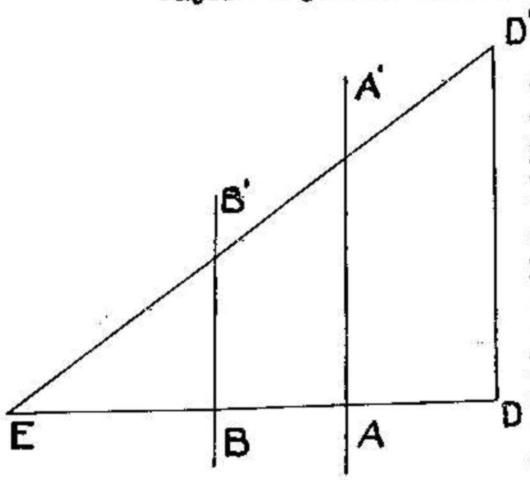
The first person using the slide rule to solve numerical equations successfully was Sir Isaac Newton. The following is a translation

* - Cajori--Log. Slide Rule

of a part of a letter written by Oldenberg to Leibnitz in June, 1675:

> "Mr. Newton, with the help of logarithms graduated upon scales by placing them parallel at equal distances or with the help of concentric circles graduated in the same way, finds the roots of equations. Three rules suffice for cubics, four for biquadradics. In the arrangement of these rules, all the respective coefficients lie in the same straight line. From a point of which line, as far removed from the first rule as the graduated scales are from one another, in turn, a straight line is drawn over them, so as to agree with the conditions conforming with the nature of the equation; in one of these rules is given the pure power of the required root". *

Cajori explains the method used in this manner:



> "In proving a cubic equation x ax bx - c, three rules, A,B,D, logarithmically graduated, are placed parallel and equadistant. Find the numerical value of b on the rule B; the numerical value of a on the Rule A and the 1 find on D. Arrange these three points on a straight line B,D,A. select E in this line so that BE BA. A line ED' passed about E until the numbers B',A', and D' are equal to c. The D' is equal to X' and x can be found. The length of the rule B which extends below B is the log. b. We will assume that BB' log. x and it then follows that the rule belowB' is equal to log. b log. x - log. bx. AA' 2 log x, then the part of AA' below the point A' is log. ax . **

The slide rule was little known on the Continent. A German writer by the name of Biler invented an instrument he called *Instrumentum Mathematicum Universale*. He described it in a publication which he brought out in 1696 under the title: *Descripio Instrumenti Mathematici Universalis, quo mediante omnes proportiones sine circino atque*

* - Cajori, Page 21

** - Cajori, Page 21

calculo methodo facillima inveniuntur. This rule was semicircular in form with sliding concentric semicircles.

Stone in his Dictionary of 1743 describes in detail the use of the early rules. From this article, Cajori gives the following quotation:

> "Take as many Gunter's Lines, (upon narrow rules) all of the same Length, sliding in Dove-tail Cavities, made in a broad oblong Piece of wood, or Metal, as the Equation whose Roots you want the Dimensions of, having a Slider carrying a Thread or Hair backward or forwards at right angles over all these Lines, and let these Gunter's consist of two single ones, and a double, triple, quadruple, etc., one fitted to them; that is, let there be a fixed single one a top, and the first sliding one next that, let be a single one, equal to it, each number from 1 to 10. Let the second sliding one be a double Line of Numbers, numbered 1,2,3,4,5,6,7,8, 9, to 10, in the Middle, and from 1 in the middle to 1,2,3,etc., to 10, at the End. Let the third sliding one be a triple Line of Numbers, numbered 1,2,3,4,5,6, 7,8,9,1, and again 2,3,4, etc., to 10, and again 2,3,4, etc., to 100 at the End. The distance from 1 to 1, 1 to 10, and 10 to 100 being the same, let the fourth sliding one be numbered 1,2,3,4,5,6,7,8,9,1; and again 2,3,4, etc., to 10; and again 2,3,4, etc., to 100; and again 2,3,4, etc., to 1000. The distance from 1 to 1, 1 to 10, 10 to 100, and 100 to 1000, being the same, and so on.
>
> "This being done take the Coefficient prefixed to the single value of the Unknown Quantity upon the fixed single Line of Numbers; the Coefficient of the Square of the Unknown Quantity, upon the double Line of Numbers; the Coefficient of the Biquadrate of the Unknown Quantity, upon the quadruple Line of Numbers, and so on. And the Coefficient of the cube of the unknown Quantity upon the triple line of numbers; the Coefficient of the first or highest term (being always Unity) take upon that Line of Numbers expressed by its Dimension, that is, if a square upon the double Line; a Cube, upon the Triple Line, etc. I say, when this is done, slide all these Lines of Numbers so, that these Coefficients be all in a right line directly over one another, and keeping the Rulers in this Situation, slide the Thread or Hair in such manner, that the Sum of all the Numbers upon the fixed single Line, the double Line, the triple Line, etc.,

which the Thread or Hair cuts, be equal to the Known Term of the Equation, which may be readily enough done with a little practice; and then the number under the Thread upon that Line of Numbers of the same hame with the highest Power of the unknown Quantity of the Equation, will be the pure Power of the unknown Quantity, whose Root may be had by bringing Unity on the Single Sliding-Line directly over Unity upon this Line. After this, if you divide the Equation by this Root, you will have another, one Dimension less;and thus you may proceed to find a Root of this last equation; which done, if it be divided by this last Root, you will get an Equation two Dimensions less, and by Repetition of the Operation you will get a third Root, and so fourth, fifth, etc., if the given Equation has so many, and if any of the intermediate Terms are wanting, the Gunter's express'd by the Dimensions of those Terms, must be omitted." *

Everard's Rule was undoubtedly the most commonly used at that time; since then many persons have made very important modifications. Charles Leadbetter fully describes his alterations in his articles in the Royal Gauger of 1755. Everard's rule had two sliders, to which Leadbetter added a third, narrower than either of the other two sliders, which he placed between the other two rules. This slider contained a line of numbers of double radius.

Mr. J. Vero made other very important changes in the rule as follows: The length of one foot had only one radius of the line of numbers and the two sliders worked together, thus making the dividing power twice as high as in Everard's Rule.

John Robertson, at one time Librarian of the Royal Society of London, improved Gunter's scale for the purpose of navigation. One of the most interesting of his changes was the "runner" or "index". He was probably the first to make any practical use of this runner even

* - Cajori, page 27

though others had some knowledge of it. Many eminent authorities hold that the runner had not been used successfully earlier than the nineteenth century.

George Adams, manufacturer of mechanical instruments for King George III, designed the spiral rule, in 1748, which contained ten spiral windings. This type of rule has been probably one of the most popular up to the present time.

William Nicholson, Editor of Nicholson's Journal, is easily the leading figure in slide rule evolution produced in the eighteenth century. His discoveries were not fully appreciated until nearly a century later when his improvements received a more cordial reception. Of all the different types planned by him, he gave the Spiral Rule the preferred place.

The Slide Rule did not come into its own until about 1881 when it appears to be in general use in mathematical calculation. It was during this year that Edwin Thacher patented his Cylindrical Slide Rule. Since that time interest has steadily increased, particularly in America.

Probably the most popular and satisfactory slide rule in general use today is the one designed by Mannheim, a French Army officer, about 1850, which is the straight slide rule with runner. While the runner had been invented in England about two hundred years earlier, that country was much slower to adopt its use than France, Italy and Germany.

The upper scale on the rule and the upper scale on the slide are identical, each having a double graduation from 1 to 10. The lower scale on the slide and the lower scale on the rule are also marked identically, both having a single graduation from 1 to 10.

The near future is very likely to witness several very important improvements to conform with the criticisms of a number of eminent mathematical scholars to give the Slide Rule a more general use.

CHAPTER III

ARITHMETIC COMPUTATION

The methods and forms of rapid arithmetic calculation have had a very interesting development. There are two general types; mechanical forms, and the development of tables. In connection with the study of Calculation, a knowledge of the origin of figures may be desirable.

Before the method of making marks for records was discovered, the fingers served both for numerals and as a means of expressing them, One was represented by the use of the first finger; two by the use of two fingers; three by the use of three fingers, and so on, up to five. Similarly one could count up to ten by the use of both hands. Thus it was comparatively simple to count to ten but hard to count higher numbers. At first this was overcome by using two men, one to count the units up to ten on his fingers, and the other to count these groups of ten. Later the method became more complex, making it possible to count as high as nine thousand.

Another comparatively early method of computation was the use of stones. This method was very similar to that of finger reckoning. As in counting by the use of one man, the stones up to ten would be placed in one pile to represent the tens place, thus they would get their hundreds and even much higher places. This was probably the earliest method of counting armies, cattle, slaves, etc. The tally stick was also used in very much the same method, or by cutting notches in one stick for the tens, in another for the hundreds, and so on.

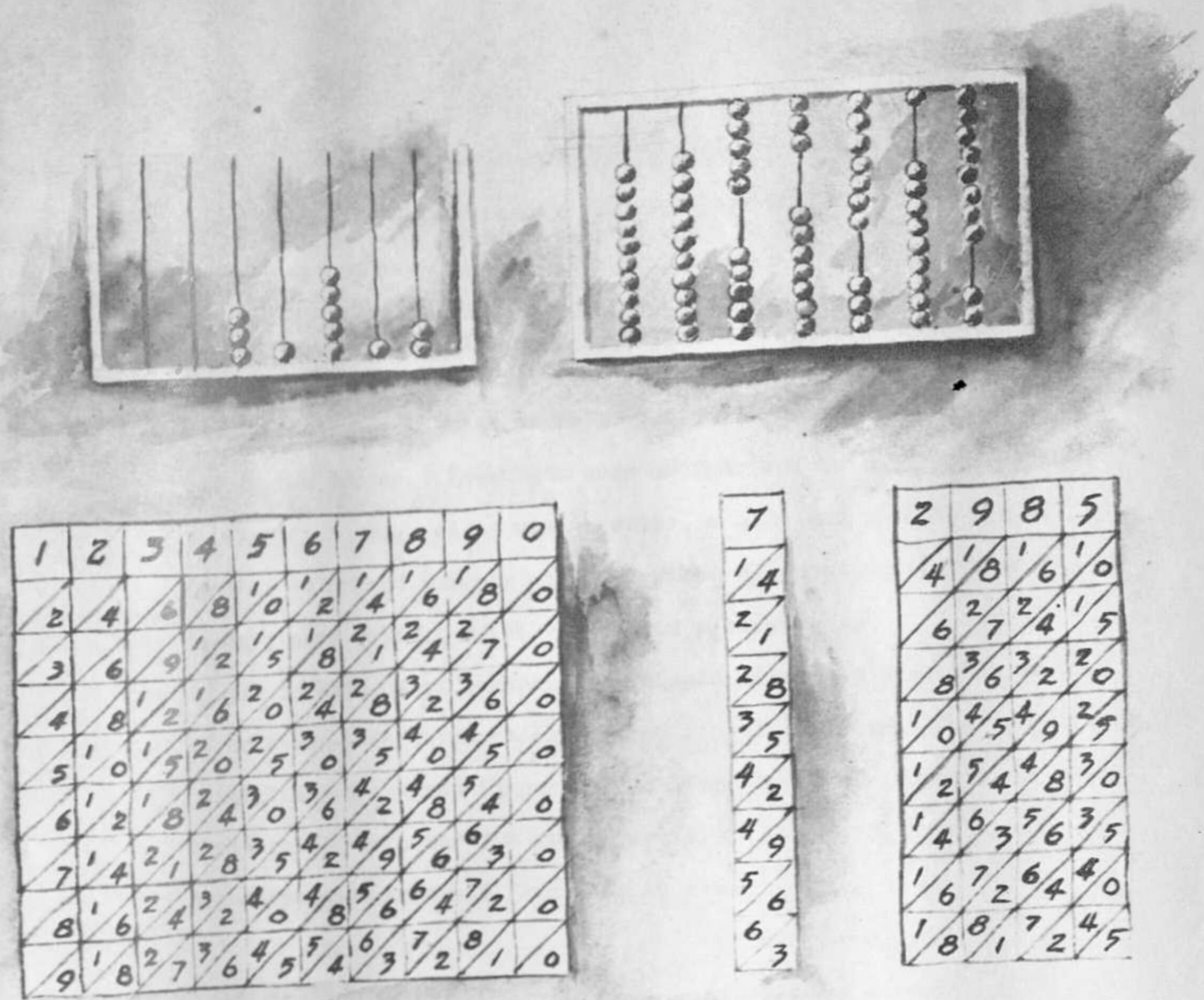
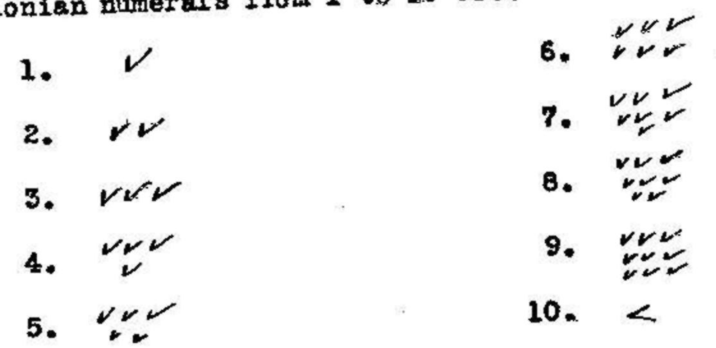


PLATE 6

A method used in the south today, that of the knotted cord, might be mentioned. Probably the first record is where we read of it being used in China in the sixth century B.C. Knots were tied on the strings following the same plan as that of the tally sticks and the stones. It was not only used for recording numbers but also to give the dates. Even as late as 1872 we find it being used in India in taking the census. The knots were on four colors of cord; the black to signify a man; red, a woman; white, a boy; and yellow, a girl. Another very good example of this method of counting is the rosary of to-day; each bead of which represents a prayer.

Methods of recording developed along the same idea as that of counting. One line stood for one finger, two lines for two fingers, three lines for three fingers, four lines for four fingers, and some distinctive character for five fingers.

The Babylonian numerals from 1 to 10 are:

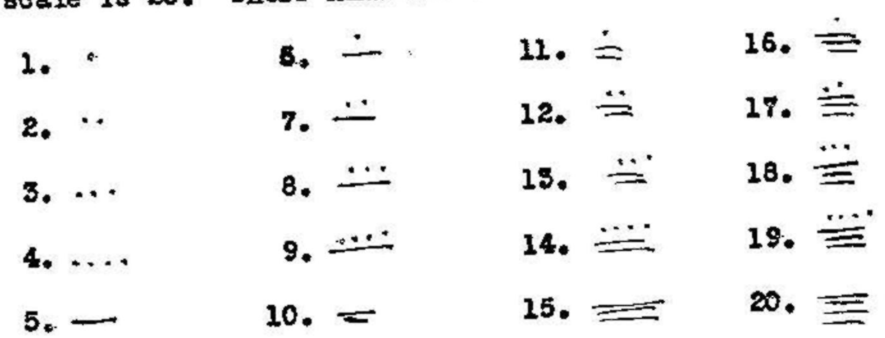


The Chinese numerals from 1 to 10 are:

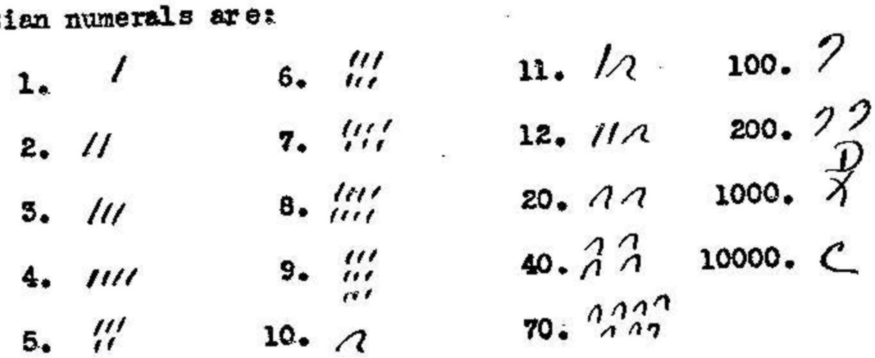
1. 一
2. 二
3. 三
4. 四
5. 五
6. 六
7. 七
8. 八
9. 九
10. 十

The Sankrist of India numerals from 1 to 10 are:

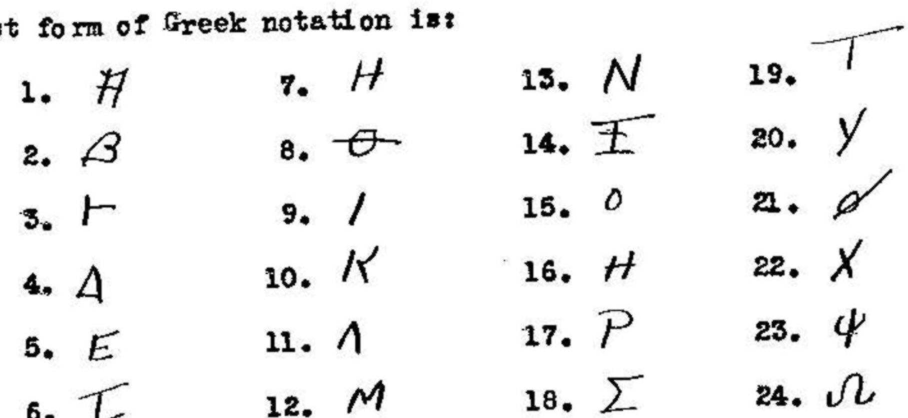
The Maya scale is 20. Their numerals are:



The Egyptian numerals are:



The oldest form of Greek notation is:



The characters for the Roman Numerals are:

1.	I	100.	C
5.	V	500.	D
10.	X	1,000.	M
50.	L		

Of these I, II, III, IIII, are clearly of primitive character. Of the rest X is the earliest and is probably I crossed. C stands for centum and M for mille. L and D are thought to represent the upper halves of the early symbols for C and M.

These systems, as can easily be seen, were unsuited to purposes of calculation, therefore it was necessary to use some form of mechanical computation. The earliest known instrument of calculation is the Abacus. The date of its origin is not known, but it was probably used by the Egyptians as early as 460 BC. The earliest meaning for the term abacus was a table covered with sand. In its simplest form, the abacus is made of a wooden board with a number of grooves cut in it, or a table covered with sand in which grooves are made with the fingers. As many counters or pebbles as there are units are put on the first groove, as many on the second as there are tens, and so on. In order to add, for each object a pebble is placed on the first groove; as soon as there are ten pebbles there, they are taken off and one put on the second groove; as soon as there are ten on the second groove, they are taken off and one put on the third groove; and so on. There were many forms in which it was used, such as the dust abacus and the counter abacus. It was later made with a number of parallel wires or strings, as in the Aztec Quipus or Chinese Swan- Pan, or the counter abacus. Beads disks or counters were strung on the wires, one line standing for unity, the next for tens, and so on.

The Chinese Swan-Pan was probably used as early as 1000 B.C. The instruments varied in width, some having few and others many rods; the more numerous the rods with their accompanying beads, the greater their capacity. The size most generally used had fifteen rods. The highest number that could be run up on such an instrument was 999,999,999,999,999. (Figure I, Plate 6) illustrates one with seven strings.

We find a great deal of improvement made in the Russian Tschotii (Figure II, Plate 6). The wires are set in a rectangular frame and nine or ten beads are permanently threaded on each wire which is somewhat longer than is necessary to hold them. As many beads as there are units, tens, hundred, etc., were pushed to the upper part of the frame and could easily be read. It was possible to add, subtract, multiply and divide by the use of this instrument.

The old methods, as finger reckoning, use of tally sticks, and the abacus, were used until the development of counters. Later these counters bore numbers, and were attached to rods, disks and cylinders, which could be moved to indicate the desired results. An example of this type was a set of rods invented by Napier in 1617. The rods were rectangular slips of bone, wood, metal or cardboard, each of which were divided by cross lines into nine little squares, usually about three inches long and one-third of an inch across. In the top square one of the digits was engraved, and the results of multiplying it by 2,3,4,5,6,7,8, and 9 were respectively entered in the eight lower squares. Figure 3, Plate 6, shows ten such rods side by side. When the number is greater than 9 the number in tens place is placed in the upper part of the square. The seventh slip is shown in Figure 4, Plate 6. For an example, we may multiply 2985 by 317. The rods headed by 2, 9, 8, 5, are taken out and put side by side as shown in Figure 5, Plate 6. First multiply 2985 by 7 as:

```
   2985
      7
   ----
     35
    56
   63
  14
  -----
  20895
```

However, on the seventh line of the Figure 5, Plate 6, we find the two numbers 1653 and 4365. If we add these according to the diagonals, we find:

```
  1653
   4365
  -----
  20895
```

Thus we can multiply by 7, 1, and 3 and arrange them in the following order:

```
     2985
  20895  /7
   2985 /1
  8955 /3
  ------
  956245
```

However, in the year 1614, we note a discovery that changes the entire trend of thought, that of logarithms by John Napier, Baron of Merchiston, Scotland. The theory is as follows:

------------P

T--------------Q---------------S

------------P'

T'--------------Q'---------------S'

TS is the straight line of definite length; T'S' extends to the right indefinitely. The moving points P and P' start from T and T' with equal speed. P' continues at the same speed but P is retarded so that its distance is always proportional to its distance from S. When P is at the position Q the logarithm of QS is represented by the length T'Q' on the other line. There is one very noticable peculiarity in Napier's Logarithms. This logarithm increases as the numbers decrease and any numbers exceeding TS have negative logarithms. The difference between Napier's logarithms and of the logarithms to the base "e" may be expressed by the following formula:

$$\text{Nap. Log. } y' \text{ equals } 10^7 \text{ nat. log. } \frac{10^7}{y}$$

In 1620 Gunther made a "line of numbers" which lead directly to the making and development of the slide rule. (Chapter II)

Many attempts have been made to perform calculations mechanically. Napier's rods succeeded in multiplication. Others have tried to perform the four operations by machinery alone.

In 1642 Blaise Pascal, the French philosopher, produced a model which while far from perfect came very near the desired end. His machine was constructed on the principal of wheels side by side on the same axis, and upon the circumferences of each were marked the first nine numerals. These wheels were so arranged that one turn of the first wheel caused the next wheel to go through a tenth of a revolution, and so on.

In 1820, the French mathematician, M. Thomas de Colmar, invented the first practical machine for rapid and accurate addition, multiplication, subtraction and division. Multiplication was accomplished by repeated addition. The machine was operated by means of a number of pointers, and a crank.

In 1822 Charles Babbage, an Englishman, evidently ignorant of the Frenchman's invention because of his desire to keep the discovery a secret from the rest of the mathematical world, produced a machine about the size of a small square piano, which had sufficient merit to attract the attention of the English Government, which in 1823, began supplying funds for the advancement of this work. For ten years a large amount of money was expended but the instrument was never completed. Parliament finally refused any further grants. This instrument could work logarithms, but could not perform arithmetical operations. From that time on many instruments have been constructed for purely commercial purposes.

It was only in recent years, in fact, only since the adoption of depressable keys, that successful machines for rapid calculation have been devised. There were several different machines utilizing keys previous to the experiment of Dorr E. Felt of Chicago, who, commencing in 1884, developed what is the present day Comptometer.

Previous to Felt's experiment, Frank S. Baldwin, secured a patent on a machine that was a forerunner of a line of succesful modern calculating machines, including those of the Monroe Calculating Machine

Company and with some variations, the Brunsviga and other machines of the Odhner type.

The first known depressable key, crank-operating machine made to add columns of figures and to print numerical items as they were added was patented in the United States in 1885 and in England in 1893, by Henry Pottin, a Frenchman.

In 1888 William Seward Burrough patented a machine somewhat similar to the design of Baldwin, which was designed to show only the final results of the calculation. Later another patent was taken out by him to combine the recording of numerical items and totals in one machine. These early Burroughs Machines were the foundation of the American Arithmometer Company, the name of which was later changed to the Burroughs Adding Machine Company.

The first combined typewriter and adding machine was patented in 1888 by A. C. Ludlum.

The first machine to apply the multiplication table and thus employ multiplication directly, was invented by M. Bollee, a Frenchman, in 1889 and was the forerunner of the present day "Millionaire" invented in 1892 by Otto Steiger of Switzerland.

It is stated that in 1892 there were only five hundred calculating machines in use in the entire world, but from then on the marketing possibilities encouraged development of the many makes of machines on the market today.

Present day adding machines are divided into three general types: listing, or those machines which print a record of items and totals; <u>non-listing</u>, or those machines having no printing mechanism; and <u>computing</u>, or non-listing machines designed for high speed calculation.

Subtraction is accomplished by addition, the same as in multiplication and division. Nearly all makes of adding machines have on their key tops the complementary numbers. These are small digits located beside the large numbers used for addition. Their relation to the large digit is such that a total of the two numbers on each key is nine. For an example, suppose we were to subtract 27 from 83. We would first add 83 into the machine and then press the keys marked in large figures 73 (the complement of 27) thus giving a number of 156. When the co-digit zeros (large figure 9, small figure 0) are set up on the keys left of the other numbers the one is transferred all the way across the key-board, either to the last column or entirely out of the machine. Thus we might have a number such as 100056. When the subtraction operation is repeated the procedure becomes division. Suppose for example that we wished to divide 3 into 15. We would first add 15 into the machine and then press the key marked 7, the complement of 3. After adding 7 five times we get 50 which shows that 3 is one-fifth of 15. Thus:

```
15
 7  co-digit for 3
--
22
 7
--
29
 7
--
36
 7
--
43
 7
--
50
```

Multiplication is repeated addition. For example to multiply 3 times 4 we would add 3 four times.

As these adding and calculating machines are large it is very difficult to move them from place to place. However, the Computer Manufacturing Company of San Francisco is making a small machine which is five inches in diameter. The Ross Rapid Computer, by an arrangement of scales, can multiply and divide but as in the slide rule it does not add. It is operated by the turning of two dials.

Another instrument that would come under the head of mechanical arithmetic computation is that of the cash register invented by James Ritty of Dayton, Ohio, in 1879. His original machine was developed from a revolution counter on a steamship engine. However, the credit for the development of this instrument to its present form is due to John H. Patterson, the founder of the National Cash Register Company. Mr. Patterson was attracted to the value of Ritty's register and in 1882 he purchased two at fifty dollars each for use in the store of a mine of which he was the proprietor. These registers, designed to record sales by punching holes in a roll of paper, did not have any adding mechanism. The paper roll was divided into columns corresponding to the key denomination, and to find the totals of a transaction it was necessary to add the number of holes punched in each column of the paper and multiply it by the denomination of the column.

Mr. Ritty sold his business to the founders of the National Manufacturing Company of Dayton, Ohio. The first adding register, later called the detail adder, was made in 1883. Later when more stock was issued, John Patterson and his brother bought 50 shares. From then on the company made a great deal of progress and was later named the National Cash Register Company. From then until 1924 over two million registers had been sold. The original function of the cash register was to prevent dishonesty among clerks. This is still one of the functions, but the principal use is for keeping business records accurately.

The Detail Adder is a non-printing register having counters connected with each key so that the totals for each key is recorded separately. For example, four depressions of the 5¢ key would be shown on the counter for that key as 20¢, also four depressions of the 10¢ key would be recorded on the counter for that key as 40¢, etc. The Total Adder is the present form of registers. The totals of sales are carried and added. These machines may be equipped with or without resetting devices. The Tied-up Counter is impossible to be cleared for the adding wheels can not be turned back to zero. The totals of one day's sales can only be found by subtracting the totals for the day before from the present total. The only way of resetting this type would necessitate taking the machine apart. The Free Counter type may be reset at any time desired. There are many types of resetting devices but the most common is by the use of a key.

A comparatively new device is that of the Combination Register invented about 1919. It is made of an adding machine, mounted on a cash drawer cabinet, and having a means of automatically opening the cash drawer as each sale is registered. Beside the economy of time by the combination of the adding machine and cash register there is the added advantage of the printed detailed record slip. All instruments are designed to prevent dishonesty by enforcing registration of each transaction. They are equipped with locks to prevent the taking or seeing of totals by an unauthorized person. Another advantage is that the adding machine may be removed from the cash drawer when needed for adding purposes. The machine may be cleared when it is to be used for simple adding machine work.

Another comparatively new development in the field of mechanical arithmetical computation is that of the Bookkeeping Machine. The method used in the machine follows the established rules, for example, an old debit balance, plus a debit or minus a credit, equals a new balance. In the first operation the printing point is at the old balance column. The old balance is set up on the keyboard and when the motor bar is depressed the amount is imprinted and added in the counter. The carriage then moves automatically to the next, or debit column. The amount of debit is then set up on the keyboard, the motor bar pressed, printing the amount in the debit column, and adding it to the old balance on the counter. Then the amount of credit is set up and printed, then the subtracting device causes the amount to be subtracted out of the counter, the remainder being the new balance. This new balance may then be printed in its column which clears the counter. This may be illustrated by the accompanying drawings:

ACCOMPANYING DRAWINGS

FIRST

Old Bal.	Debits	Credits	New Bal.
150			
Printing Point			
Counter			
00,150			

SECOND

Old Bal.	Debits	Credits	New Bal.
150	100		
	Counter		
	00,250		

THIRD

Old Bal.	Debits	Credit	New Bal.
150	100	25	
		Counter	
		00,225	

FOURTH

Old Bal.	Debits	Credit	New Bal.
150	100	25	00,225
			Counter
			00,000

The development of tables for rapid calculation has made equal progress with that of the mechanical instruments. The tables of height, weight and age had a very interesting evolution and have been published after a most careful and scientific investigation by such authorities as Dr. Bird T. Baldwin of the University of Iowa and Dr. Thomas D. Wood of Columbia University, whose studies have extended over a period of years. The tables of height, weight and age produced by these scientists have been revised and simplified by Mrs. Louise McCain Ross, a graduate of the University of California and the University of California Hospital, who, using their findings as a basis, has devised a much more usable set of measurements, and are presented here as an example of pioneering in the field of present day problems. (Plate 7) Her system has been accepted by the San Diego City Schools for use in physical education and health work in all the city schools. The following are a few of the rules prepared by Mrs. Ross for the use of these tables:

1. Weight the child without shoes, sweater or coat.

2. Have him stand before the chart, good posture, no shoes. Obtain height, using the central column. of figures.

3. Find the age--nearest birthday.

4. For Boys--use columns to the left of central column.

5. For Girls--use the columns to the right.

6. Opposite each inch will be found the normal weight for that height and age.

7. Where the weight is not given directly--estimate by striking an averagebetween the two nearest figures.

#	Age	HEIGHT (inches)	AGE	Wt — WEIGHT (lbs) GIRLS
171	19			
170	18			
164	16			
160	15	74		
167	19			
164	18			
160	16			
157	15	73		
163	19			
158	18			
156	17			
153	15	72		
159	19		18	145
154	18		17	144
151	16		15	140
148	14	71	14	138
155	19		18	144
148	17		16	140
144	15		15	138
143	14	70	14	136
152	19		18	142
149	18		17	140
143	16		15	137
137	14	69	14	135
147	19		18	138
141	17		16	136
137	16		14	133
134	14	68	13	131
142	19		18	135
136	17		16	133
130	15		14	130
124	13	67	13	128
139	19		18	130
132	17		16	128
125	15		15	125
119	13	66	13	124
134	19		18	126
127	17		16	123
120	15		14	121
114	12	65	12	118
130	19		18	123
117	16		16	120
113	14		14	117
109	12	64	12	114
127	19		18	120
118	17		16	117
108	14		14	112
105	11	63	12	110
116	18		18	118
104	15		15	113
102	13		13	106
100	11	62	11	104
106	17		18	116
100	15		16	112
97	13		14	105
95	11	61	11	99
96	16		18	111
94	14		16	108
92	13		12	95
91	10	60	10	91
90	16		17	104
90	14		15	100
89	12		13	92
87	10	59	10	87
87	15		16	101
85	13		14	93

PLATE 7

92	15		12	[illegible]
91	10	60	10	91
90	16		17	104
90	14		15	100
89	12		13	92
87	10	59	10	87
87	15		16	101
85	13		14	93
84	11		12	86
83	9	58	10	84
83	15		15	92
82	13		13	84
81	11		11	82
79	9	57	9	80
80	15		14	83
78	13		12	79
77	10		10	78
75	8	56	9	76
74	14		14	78
74	12		12	75
73	10		10	74
72	8	55	8	72
72	14		15	73
71	12		12	71
70	10		10	70
70	8	54	8	69
68	13		13	71
67	11		11	68
67	9		9	67
66	7	53	7	66
64	13		12	67
64	11		10	64
64	9		8	64
63	7	52	7	63
	12		12	65
61	10		11	63
	8		9	61
	7	51	7	59
58	12		12	62
58	10		10	59
58	8		8	57
57	6	50	6	56
	11		11	56
55	9		9	55
	7		7	54
	6	49	6	54
53	10		11	53
53	8		9	52
53	7		7	52

	12		12	65
61	10		11	63
	8		9	61
	7	51	7	59
58	12		12	62
58	10		10	59
58	8		8	57
57	6	50	6	56
	11		11	56
55	9		9	55
	7		7	54
	6	49	6	54
53	10		11	53
53	8		9	52
53	7		7	52
52	6	48	6	52
50	10		10	50
50	8		8	50
50	6		6	50
49	5	47	5	49
48	9		9	48
48	8		7	47
48	6		6	47
47	5	46	5	47
	9		9	
46	7		7	45
	6		6	
	5	45	5	
	8		8	
44	7		7	42
	6		6	
	5	44	5	
	8		8	
41	7		7	41
	6		6	
	5	43	5	
	8		7	
39	7			39
	6		6	
	5	42	5	
	7		7	
38				37
	6		6	
	5	41	5	
	6		7	
36				36
	5		6	
		40	5	
	6		6	
35				34
	5		5	
		39		
	6		6	
34				33
	5		5	
		38		

8. Record all measurements at once--giving the child necessary instructions at the time of weighing or holding over such children as need it, for special instructions.

Mrs. Ross has also made another interesting contribution in the matter of age, weight and height in devising a table (Plate 8) for the finding of the percentage of gain or loss in weight. For example, suppose a child with the normal weight of 105 and the actual weight of 115 has been measured. In order to find the percentage of gain, first turn the pointer to 105 on the outer circle and since 10 is the difference between the actual and normal weight, find 10 on the pointer and then read the percentage on the inner circle opposite 10. Percentages are figured on the normal weight rather than on the actual weight.

Tables for use in mathematics have also been devised. Probably one of the most outstanding of which is Dr. A. L. Crelle's *Calculating Table* which consists of the products of every two numbers from 1 to 1,000 and their application to the multiplication and division of all numbers over 1,000.

The *Engineers Manual*, by Ralph G. Hudson and Joseph Lipka, giving the principal formulas and tables of Mechanics, Mathematics, Heat, Electricity and Hydraulics may be cited as a good example of some of the many collections of tables in use today. The purely mathematical section of this book has been published separately in a much smaller and more usable edition under the name of the *Mathematical Manual*. This manual contains the following: the common logarithms; numbers from 1 to 1,000 with their squares, cubes, square roots; recipricals, circumferences, and circular areas; degrees to radians; natural sines, cosines,

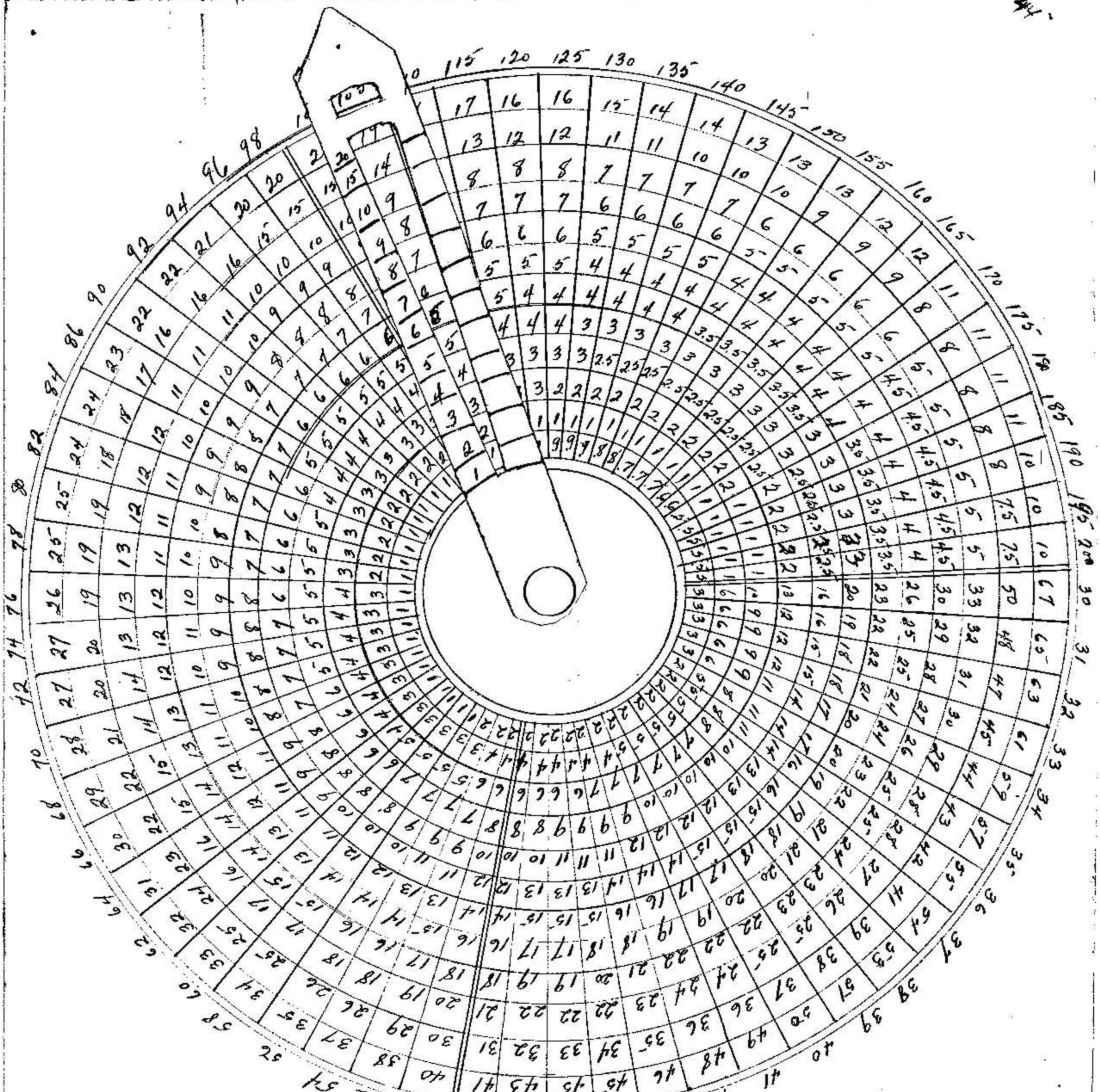


PLATE 8

and tangents; common logarithms of sines, cosines and tangents; hyperbolic sines, cosines, and tangents; values of the functions c^x and c^{-x}; decimal equivalents of fractions; and the length of arc, length of chord, height of sgment and area of segment subtending an angle (ϕ) in a circle of Radius (R).

These types of tables have contributed largely to the simplification of mathematical calculations. And while their development is not as apparent as that of the mechanical devices yet they represent undoubtedly some of the greatest of the contribuitons to the science of mathematics.

CONCLUSION

Mathematics, the oldest science, is the mother of all sciences. All scientific research and development, either directly or indirectly, owes its standing to the products of mathematical accuracy. Each person having any part in the development of a mathematical instrument has been working first of all for accuracy and simplicity in the result and in the instrument itself. It was not always possible to produce an instrument that would work with sufficient exactness, because many times atmospheric and other conditions have made it impossible for even the most accurate calculator to get a true register. As in the case of Tycho Brahe, the scientist must find the percentages of error and make allowances for them, which generally entails a great amount of mathematical calculations. After the instruments were perfected to a good state of precision, the desire for the simplification of the mathematical processes have received greater attention. This, along with the desire for time saving appliances, seem to have an important place in the making of up-to-date instruments. Undoubtedly, the science of mathematics is still in its infancy and is destined to continue to be one of the major factors of all progress.

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