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Factors underlying high school mathematics teachers' perceptions of challenging math tasks

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FACTORS UNDERLYING HIGH SCHOOL MATHEMATICS TEACHERS' PERCEPTIONS OF CHALLENGING MATH TASKS

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DEDICATION

For Hallie and Emoree. Thank you taking this dissertation journey with me. May you each find joy in your own academic endeavors and may you always know that I love you so.

For my mom and dad. Without your support and love, this journey would not have been possible. I love you both.

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Factors Underlying High School Mathematics Teachers' Perceptions of Challenging Math Tasks

Abstract

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In this confirmatory factor analysis, factors previously identified to explain the variability in Middle School Mathematics Teachers' perception of the Common Core State Standards of Mathematics were considered as factors hypothesized to effect high school math teachers' perceptions of challenging math tasks (CMTs). The factor of student characterization (i.e., disposition, academic preparation, and student behavior) was additionally considered as a factor hypothesized to explain teachers' perceptions of CMTs, as well as site-based variables (i.e., curriculum, assessment and evaluation, professional development, and collaboration). In addition, teachers' understanding of the importance of the mathematical practice standards and teacher familiarity with enacting CMTs were factors considered in the model. The original septenary factor structure was modified and good model fit was achieved. In addition to the confirmatory factor analysis model which provides a structure for considering teachers perceptions of CMTs, descriptive statistics are presented from the survey developed that captured teachers' perceptions of CMTs relative to their sites.

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Chapter 1: Introduction

"The national average wage for all Science, Technology, Engineering, and Math (STEM) occupations [in 2015] was $87,570, nearly double the national average wage for non-STEM occupations ($45,700)" (Fayer, Lacey, & Watson, 2017, p. 6). Additionally, from 2009 to 2015, STEM occupational growth (10.5%) outpaced non-STEM occupational growth (5.2%) (Fayer et al., 2017, p. 7). Rigorous high school mathematical preparation is viewed as a "gate-keeper" to the more lucrative and plentiful STEM occupational opportunities available through successful enrollment and completion of postsecondary STEM degree programs (Iskander, Kapila, & Kriftcher, 2007). In order to better prepare high school students for college STEM programming and eventual STEM careers, improvement in high school students' mathematical learning outcomes has been targeted in part through an evolving standards movement. In one national survey, 71% of 12,000 mathematics teachers (grades 1-12) indicated that the Common Core State Standards of Mathematics (CCSSM) were "extremely important" because the CCSSM "reflect the knowledge and skills students will need for success in college and careers" (Cogan, Schmidt, & Houang, 2013, p. 5). However, standards have not significantly improved learning outcomes in the K-12 math pipeline (Asempapa, 2017). Reform in mathematics education requires reform at the transactional level between student and teacher (Boaler, 1998), in addition to clear standards to inform instruction. The National Council of Teachers of Mathematics (NCTM) publication (2014), *Principles to Practice*, outlined eight mathematics teaching practices intended to "provide a framework for strengthening the teaching and learning of mathematics" (p. 9) at the transactional level.

This study aimed to examine factors underlying teachers' perceptions of one of the eight teaching practices espoused by the NCTM in *Principles to Practice* (2014): implementing

Challenging Math Tasks (CMTs). Implementing math tasks that promote reasoning and problem solving "form the basis for students' learning" (Doyle, 1988, p. 9). Mathematical tasks are the building blocks used to construct students' mathematical knowledge, so teachers' perceptions of these building blocks effect instruction at the transactional level.

Background

The mathematical standards movement in the United States (U.S.) was launched with the publication of the *Curriculum and Evaluation Standards for School Mathematics* by the NCTM in 1989 (Schoenfeld, 2015). The standards movement became even more real for educators, however, with the No Child Left Behind Act of 2001 (2002) which required challenging state standards in mathematics accompanied by systematic student assessment. By 2010, state standards were replaced by more rigorous standards, the CCSSM. Today, "forty-two states, the District of Columbia, four territories, and the Department of Defense Education Activity (DoDEA) have adopted the Common Core State Standards" (Achieve, 2013, p. 1). We now have nearly nationally adopted mathematical standards.

Standards alone do not seem to be remedy enough to produce mathematically capable high school graduates. *The Nation's Report Card* (2015) published by the National Center for Education Statistics (NCES) indicated that "twenty-five percent of twelfth-grade students performed at or above the Proficient level in 2015" (para. 3) in mathematics. In other words, only a quarter of graduating seniors are mathematically prepared to meet the standards set forth by the Department of Education on the National Assessment of Educational Progress.

The mathematical reform movement charges classroom teachers with implementing standards based mathematical instruction differently, touting the implementation of less procedural instructional methodologies and more opportunities for students to experience "doing mathematics" (Spillane & Zeuli, 1999, p. 5). Spillane and Zeuli (1999) indicated that "core dimensions of instruction will have to change substantially, especially the mathematical tasks (i.e., the problems, questions, and exercises students work on) and classroom norms (i.e., the ways teachers and students interact with each other about mathematics)" (p. 4). In the next section, definitions and some acronyms specific to this study are provided.

Definitions

The acronym that will be used for challenging tasks or challenging mathematics tasks is CMTs. "Challenging tasks are complex and absorbing mathematical problems with multiple solution pathways, whereby the whole class works on the same problem" (Russo & Hopkins, 2017, p. 31). A sample CMT can be found in Appendiz B. In the educational landscape today, regardless of whether or not teachers are using CMTs, most high school mathematics teachers are using the CCSSM to guide instruction.

> At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, modeling and probability and statistics), showing the body of knowledge students should learn in each category to be college and career ready, and to be prepared to study more advanced mathematics. (Common Core State Standards Initiative [CCSSI], 2012, p. 2)

The CCSSM help teachers to frame content that is appropriate for each high school course (e.g., Algebra 1, Geometry, or Precalculus). Within the CCSSM, a list of eight mathematical habits of mind that teachers should help to instill in students is provided. These eight mathematics practice standards (MPS) "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (CCSSI, 2012, para. 1) and are displayed in Table 1 along with a summary of the key terms, definitions, and acronyms used in this study.

Table 1

Table of Key Terms, Acronyms, and Definitions

Term	Acronym	Definition
Confirmatory Factor Analysis	CFA	"Uses previous research and relevant theory to decide in advance what the factors or constructs are that underlie the measures. The fit statistics then provide feedback concerning the adequacy of the model explaining the data" (Kieth, 2015, p. 333).
Challenging Mathematical Tasks	CMTs	"Challenging tasks are complex and absorbing mathematical problems with multiple solution pathways, whereby the whole class works on the same problem" (Russo & Hopkins, 2017, p. 31).
Common Core State Standards of Mathematics	CCSSM	A standards document that outlines what students should learn in each high school course, in order to be college and career ready.
Mathematical Practice Standards/ Standards for Mathematical Practice	MPS/SMP	Eight habits of mind educators of mathematics are encouraged to develop in their math students which include the following: 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Mathematics Task Framework	MTF	A four-phase model of the transmogrification of mathematical tasks from selection or creation, to setup, implementation, and lastly, student learning.
Science, Technology, Engineering, and Math	STEM	Acronym standing for Science, Technology, Engineering, and Math
The Organisation for Economic Cooperation and Development	OECD	Multi-governmental agency that works for economic collaboration.

Description of the Research Problem

Despite the adoption of new mathematical standards at a nearly national level (Marrongelle, Sztajn, & Smith, 2013) and specific rhetoric on how to best implement the more rigorous Common Core Mathematical Standards through the effective implementation of CMTs (NCTM, 2014), some mathematics teachers remain reluctant to implement CMTs (Russo & Hopkins, 2017). Difficulties associated with the acquisition (Monarrez & Tchoshanov, 2017), setup, and implementation of CMTs is relatively well documented in the literature (Stein, Grover, & Henningsen, 1996); however, secondary teachers' overall perceptions of CMTs under the CCSSM, nor the factors mitigating teachers' perceptions of them, are not well documented (Candela, 2015).

The educational reform movement's evolving agenda to move teaching beyond a focused presentation of mathematical content can be seen with the 1989 and 2000 releases of the NCTM's *Curriculum and Evaluation Standards for School Mathematics* which aimed to position educators as brokers of the mathematical habits of mind (Schoenfeld, 2015). Likewise, the CCSSM detailed similar habits of mind beneficial for doing mathematics, calling these habits collectively, mathematical practices. CMTs provide a rich opportunity for students to engage in practicing mathematical habits of mind, yet the natural propensity of educators to minimize the challenge for students as they engage in solving CMTs is well documented and effectively minimizes students' opportunities to hone their problem-solving skills and mathematical habits of mind. Teachers' propensity to minimize challenge by "spoon feeding" allows classrooms to efficiently and effortlessly move through content. "When novel work [where students must determine the strategy] is being done, activity flow is slow and bumpy" (Doyle, 1988, p. 174). Effectively facilitating student learning through the use of CMTs requires a special tacit knowledge for teaching, known as mathematical-task knowledge (Chapman, 2013), and perhaps an acute awareness of the opportunity CMTs provide for intentionally developing mathematical habits of mind.

Effectively facilitated student learning of mathematics using CMTs positively impacts student learning outcomes in mathematics (Stein & Lane, 1996). Additionally, mathematical task professional development (PD) has been constructive in improving teachers' effective use of CMTs (Boston & Smith, 2009). Mathematical task PD is an underlying factor in middle school teachers' perceptions of the CCSSM. Research has not established general mathematical PD to be an underlying factor in teachers' perception of CMTs. Implementing more specialized math PD compared to more general math PD may be critical to the mathematical reform movement, as "teachers affect tasks, and thus students' learning" (Doyle, 1988, p. 169).

It would seem that teachers' perceptions of CMTs might serve as a barometer to measure the degree of effective instruction students experience daily. Assessing the effective regular use of CMTs as deployed in classrooms is not a simple undertaking (Doyle, 1988); whereas, capturing teachers' perception of CMTs is a less strenuous exercise. There are many factors that mitigate teachers' instructional realities (Doyle, 1988). Knowing the factors *a priori* that mitigate teachers' perceptions of CMTs under the CCSSM informs targeted reform. The factors underlying secondary mathematics teachers' current perceptions of using CMTs under the CCSSM has not yet been investigated extensively. Middle school teachers' perceptions of the Common Core Mathematics Standards have been studied (Davis, Drake, Choppin, & Roth McDuffie, 2014), but Davis et al. indicated that more research is needed. Additionally, Davis et al. (2014) asserted that "future surveys addressing mathematics teachers' perceptions about the CCSSM and the instructional environment should create additional survey items to further understand these factors" (p. 23). This research aimed to highlight factors that mitigate teachers' perceptions of CMTs at the site level.

What is certain is that with nearly national standards accompanied by proper instructional tacit knowledge (i.e., properly facilitated mathematical tasks) there is the potential for an unprecedented opportunity to improve learning outcomes for students of mathematics across the nation potentially leading to a stronger American work force (Cogan et al., 2013). Toward this end, teachers' perceptions of CMTs are important given the premise that students' math experiences, or lack thereof, are a function of teachers' perceptions of CMTs.

As teachers are fast approaching the first decade of instruction under the CCSSM, teachers' instructional environments have likely begun to stabilize with respect to their understanding and implementation of the CCSSM. Teachers' perceptions are of critical importance as they give us insights into the current state of mathematics instruction and the instructional environment, which are inextricably tied. In this study's review of related research, no survey research found specifically sought to investigate the factors underlying secondary teachers' perceptions of implementing CMTS in the Common Core Era given teachers' instructional environment at the high school level. Capturing teacher perception now, before the onslaught of a new set of standards, provides insight into the evolution of teachers' perceptions of CMTs relative to standards.

New standards seem inevitable; the current President of the United States stands in opposition to the CCSSM (Saul, 2016). Yet, "both those who support and oppose the Common Core generally agree with the main objective: prepare students to compete in the ever-changing job market and the global economy" (Burks et al., 2015, p. 255).

Conceptual Framework

Shulman (1987) emphasized reflection as a component foundational to educational reform. Self-reflection becomes especially important given that "richly developed portrayals of expertise in teaching are rare" (Shulman, 1987, p. 1). Therefore, to help facilitate educators' self-reflection on the best practice of using CMTs, a conceptual framework was selected that has been reportedly effective in assisting teachers with reflecting on their practice of using CMTs: the mathematics task framework (MTF). Researchers Smith and Stein (2011) indicated that "in our five years of experience with middle school [mathematics] teachers in the QUASAR Project…we have seen how focusing on mathematical tasks and their phases of classroom use can assist teachers in the reflection process" (p. 9). Smith and Stein (2011) are credited with the creation of the MTF which provides a four-phase glimpse of the transmogrification of mathematical tasks from selection or creation, to setup, implementation, and lastly, student learning.

The first phase, tasks as they appear in curricular/instructional materials, has long been considered a starting point. Shulman (1987) remarked that "most teaching is initiated by some form of 'text'" (p. 14). Teachers then must plan and determine how a task will be setup, the second phase of the MTF. The third phase, tasks as implemented by students, is regarded as particularly influential on student learning outcomes (Smith & Stein, 2011). The last phase in the MTF is student learning. Teachers' mathematical-task knowledge for teaching (Chapman, 2013) is utilized at each phase of the MTF. The MTF and the associated pedagogical content knowledge required at each phase is discussed in more detail in Chapter 2. The MTF was the lens through which the results of this quantitative investigation were interpreted.

Purpose of the Study

This study adds to the literature by examining whether a hypothesized set of constructs serve as factors that underlie high school mathematics teachers' perceptions of challenging mathematical tasks. Specifically, seven factors were hypothesized to explain teachers' perceptions of CMTs.

Research Question and Hypotheses

The two primary research questions of this study were:

1. Is a proposed septenary factor structure a good fit for understanding the variability in teachers' perceptions of CMTs?
2. Do teachers perceive the CMTs available to them to be well-aligned to the common core state standards?

Seven hypotheses follow that are related to the seven factors that were under investigation:

H_1. Perceived access to CMTs is a factor that explains the variability in teachers' perceptions of CMTs.

H_2. Teachers' understanding of the importance of CMTs as a means to practice the MPS is a factor that explains the variability in teachers' perceptions of CMTs.

H_3. Site-based assessment and teacher evaluation is a factor that explains the variability in teachers' perceptions of CMTs.

H_4. Site-based CMT PD is a factor that explains the variability in teachers' perceptions of CMTs.

H_5. Teachers' familiarity and level of self-reported preparedness to implement CMTs is a factor that explains the variability in teachers' perceptions of CMTs.

H_6. The amount of site-based collaboration around CMTs is a factor that explains the variability in teachers' perceptions of CMTs.

H_7. Teachers' characterization of students is a factor that explains the variability in teachers' perceptions of CMTs.

It was the researcher's assumption that the MTF was an appropriate conceptual framework for analyzing the survey data collected in pursuit of answering the research question. Confirmatory factor analysis (CFA) was performed in an effort to develop a parsimonious list of factors that might help to explain teachers' perceptions of challenging math tasks that relate back to the MTF.

Description and Delimitations of the Study

Survey data were collected that asked secondary mathematics teachers to report their perceptions of their instructional environments and CMTs using a Likert-type scale. The survey included questions that explored secondary mathematics teachers' perceptions of (a) curricular offerings (from primary and supplementary resources) of mathematical tasks, (b) site-based efforts to setup and collaborate around the CMTs and the CCSSM, and (c) site-based efforts to help teachers effectively implement CMTs and the CCSSM. Each survey question represented a variable that was hypothesized to reflect the underlying structure of teachers' perceptions of CMTs. Factors previously identified to explain middle school mathematics teachers' perceptions of the Common Core mathematics standards were taken into consideration when constructing the survey items in this study (Davis et al., 2014).

Outside organizations who regularly interact with secondary mathematics teacher populations were contacted as potential participant pools. Some of these included San Joaquin County Teachers' College; county offices of education (e.g., San Joaquin County Office of

Education); The Silicon Valley Math Initiative; local unified school districts; College Preparatory Mathematics, LLC; and the Monterey Bay Area Math Project (MBAMP). Based on the participant survey return rate, the number of survey items that may be included in the analysis was determined. The number of participants should be "10 to 20 participants for each independent variable" or more (Keith, 2015, p. 203), so ideally, this study would include 70-140 participants. A study sample of 102 participants was achieved.

Descriptive statistical summaries of the collected ordinal data were created. The survey data were additionally analyzed using exploratory factor analysis, an inferential statistics technique used to determine the underlying constructs that might help to explain teachers' perceptions of CMTs under the CCSSM. Using factor analysis, researchers pin-point independent variables that correlate collectively. These highly correlated variables or "super variables" (Tabachnick & Fidell, 2013, p. 3) are the underlying factors that serve as independent variables that can be used to explain a concept under study (Ho, 2014; Tabachnick & Fidell, 2013), which in this case was secondary mathematics teachers' perceptions of CMTs. Teachers' perception of CMTs served as the dependent variable. In identifying the underlying factors, the statistical significance of the individual factors guided the analysis.

Significance of the Study

The results of this study provide school district personnel and PD practitioners additional insight into teachers' perceptions surrounding the implementation of CMTs under the CCSSM, aiding effective decision making, with the goal of improving student learning outcomes in mathematics. Given that the CCSSM are different from previous standards in that they are more rigorous and include mathematical process standards in addition to content standards, research has indicated that teachers would need "to make significant changes in their instructional practices" (Burks et al., 2015, p. 255). Among the changes advocated by the NCTM (2014) is the use of CMTs. Measuring teachers' perceptions of CMTs and the factors underlying teachers' perceptions given their instructional environments under the current CCSSM, allows reformers, which includes teachers themselves, to examine the degree to which the use of CMTs may have infiltrated classrooms. To hone students' mathematical proficiency requires the inculcation of CMTs "to ensure that students have the opportunity to engage in high-level thinking, teachers must regularly select and implement tasks that promote reasoning and problem solving" (NCTM, 2014, p. 17). Teachers' positive perceptions of a concept, such as the use of CMTs, is more likely to indicate or lead to its enactment. Spillane and Zeuli (1999) indicated that identifying "patterns of instructional practice in the wake of reform is important if we are to better understand the relations between reform and teaching" (p. 1) and identified "academic tasks" (p. 2) as a dimension of instruction that is especially reform resistant.

Chapter Summary

Global competitiveness requires a thriving STEM workforce. High school mathematical preparedness, such as performing well in the more rigorous high school mathematics courses (e.g., precalculus and trigonometry) predict student success in college-level STEM courses (e.g., calculus) which ultimately effects a student's chance of completing a STEM degree (Iskander et al., 2010; Redmond-Sanogo, Angle, & Davis, 2016). Performing well in the more rigorous high school mathematics courses is facilitated by rigorous content standards and quality instruction in a students' K-12 mathematics program, which ideally includes exposure to CMTs. Teachers' perceptions of CMTs effect whether or not students are exposed to them, which provide students with richer learning environments.

Chapter 2 discusses content standards relative to the disciplines addressed in STEM education, while positioning the disciple of mathematics within the broader conversation around STEM education and the STEM workforce. Specific characterizations of the CCSSM (i.e., rigorous and excessive) are discussed as captured from existing research. Evidence from the research suggests that the CCSSM are more rigorous than prior standards, but not necessarily more coherent. Previous research suggests that teachers perceive the availability of CMTS aligned to the CCSSM to be an issue at some high school sites. References to qualitative research that depicts a need for Common Core aligned mathematical tasks at the high school level are highlighted. A formal definition of CMTs is provided. In addition, a hierarchical structure for the classification of math tasks is offered. The MTF, the conceptual framework selected for this study, is reintroduced with a detailed description of the pedagogical content knowledge required at each phase of the MTF. Emphasis on the "teacher-talk" that emerges, as found in existing literature, to describe the pedagogical content knowledge ideal at each phase of the MTF is summarized. Finally, teacher PD is discussed generally, followed by a specific discussion of math task PD.

Chapter 3 provides a detailed explanation of the quantitative methodology employed in this study. First, the requirements, assumptions, and basic procedures of CFA are presented. The steps include selection of variables and the "computation of the correlation matrix for all variables, extraction of initial factors, and rotation of the extracted factors to a terminal solution" (Ho, 2014, p. 240). For each step, the Statistical Package for Social Science (SPSS) selections and outputs are discussed. CFA involves interpretation and this is also discussed in Chapter 3.

In Chapter 4, student demographic data and some analysis of how this study's sample poplution of teachers compares to the National Secondary Math Teacher Population as reported by the NCES. In addition, student demographics as reported by teacher participants are presented. Survey response data is reported in eight sections, one section for each factor considered in the model, with general comments regarding the survey data. Last, the statical reportings from the original proposed model to the final design are presented. Summary fit statistics and a discussion of the model's convergent and divergent validity is provided.

Chapter 5 contains a general discussion of the study's finding and conclusions, with implications for practice and further research.

Chapter 2: Literature Review

For some time, the concern regarding the supply of American workers in Science, Technology, Engineering, and Math (STEM) fields has been publicized. As the U.S. increasingly becomes a technology-based society, the need for STEM workers is anticipated to become increasingly important to national prosperity. Yi and Larson (2015) reported, "as our society relies further on technology for economic development and prosperity, the vitality of the STEM workforce will continue to be a cause for concern" (p. 11). To ensure an adequate workforce, research has focused on the STEM pipeline from high school through post-secondary education.

Secondary math course taking and math ability, as well as students' confidence in their mathematical abilities, contribute to the number of students enrolled in post-secondary STEM programs (Chen & Weko, 2009; Moakler & Kim, 2014). Research indicates that a strong mathematical foundation contributes to students' success in STEM majors. Thus, secondary mathematics teachers are encouraged to implement Challenging Math Tasks (CMTs) while maintaining the productive struggle experienced by students. The National Council of the Teachers of Mathemaitcs (NCTM) (2014) holds both the implementation of rich math tasks and supporting students in productive struggle among the eight principles for effective teaching and learning.

Implementation of CMTs where rigor is maintained has been shown to improve student performance on mathematics assessments (Stein & Lane, 1996). Student mathematical learning outcomes have also been shown to be a function of the rigor of the mathematical text selected (National Center for Education Statistics [NCES], 2013). Yet, some curricula do not include rich math tasks aligned to the nearly nationally adopted Common Core mathematics standards and some teachers do not utilize CMTs as a component of instruction.

Educators have communicated that the Common Core State Standards of Mathematics (CCSSM) are more rigorous than previous math standards and that the new standards are still “a mile wide, and an inch deep” (Cogan et al., 2013, p. 3). While teachers’ perceptions of the CCSSM have been investigated, teachers’ perceptions of mathematical tasks under the CCSSM has been only minimally studied and primarily through Professional Development (PD) addressing the implementation of CMT.

This literature review examines the CCSSM, mathematical-task knowledge (Chapman, 2013), a form of pedagogical content knowledge (Shulman, 1987), using the mathematical task framework (Smith & Stein, 2011). PD for teachers of mathematics and what is currently known about teachers’ perceptions of CMTs are explored. Using quantitative factor analysis, Davis et al. (2014) found PD to be a factor contributing to the variability in mathematics teachers’ perceptions of the Common Core mathematics standards at the middle school level. They indicated that other populations, such as elementary and high school teachers, should be studied. Would PD be a statistically significant underlying factor in teachers’ perceptions of math performance tasks at the secondary level of instruction? Davis et al. (2014) did not include any survey items addressing middle school math teachers’ perceptions of math performance tasks under the CCSSM.

This study explored high school mathematics teachers’ perceptions of CMTs and the instructional environment using the math tasks framework. By highlighting both the mathematical-task knowledge necessary to implement math tasks and teachers’ perceptions of implementing CMTs, this study contributes to the conversation on how to support math teachers’ integration of CMTS as a regular instructional practice in the delivery of the Common Core mathematics standards.

The Importance of STEM

We are entering into a new industrial revolution, called Industry 4.0 or the Fourth Industrial Revolution, with implications for the skills required of workers. This new revolution notably demands STEM skills. Many jobs that have previously been available to lower-skilled workers will be outsourced to other countries or to machines, but with a new opportunity for skilled STEM workers. “A single factory may need fewer people to run it; however, as with past industrial revolutions, the increases in productivity should create new markets, new businesses, and new factories that increase demand for skilled labor” (Baldassari & Roux, 2017, p. 21).

Increased access to new opportunities and a higher level of financial compensation are benefits those with STEM “know-how” enjoy, as evidenced by the U.S. Department of Commerce’s, *STEM Jobs: 2017 Update* (Noonan, 2017). This update highlighted the higher wages enjoyed by STEM employees regardless of level of STEM educational attainment; “STEM workers command higher wages, earning 29 percent more than their non-STEM counterparts in 2015” (Noonan, 2017, para. 4). In addition to higher wages obtained for those in STEM occupations, those employed in STEM occupations experienced lower overall employment rates from 1994 to 2015. Job growth in STEM fields has outpaced growth in non-STEM fields and is expected to continue to do so. In the *STEM Jobs: 2017 Update*, growth in STEM employment is projected to grow 8.95% from 2014 to 2024, and 6.4% for non-STEM employment during this same period (Noonan, 2017). STEM know-how benefits individuals in terms of financial security. Collectively, a higher level of STEM know-how translates favorably into national financial security, “that is central to our economic vitality” (Noonan, 2017, p. 12). The Organisation for Economic Cooperation and Development (OECD) (2012) indicated that:

> using economic modelling to relate cognitive skills – as measured by PISA and other international instruments – to economic growth shows (with some caveats) that even small improvements in the skills of a nation’s labour force can have large impacts on that country’s future well-being. (p. 38)

The Importance of STEM Education

To push the American employment workforce to a ready stance of contributing to STEM fields, educating students toward the goal of being STEM-ready naturally emerges. “In turn, strengthening the STEM workforce requires investments in STEM education based on the best empirical evidence” (Cannady, Greenwald, & Harris, 2014, p. 444). The promotion of STEM in K-12 education was advocated by President Obama. In 2009, President Obama “launched the Educate to Innovate initiative to move American students from the middle to the top of the pack in science and math achievement” (The White House, 2013, para. 2). Presidential support of K-12 STEM education continues today under the Trump administration; however, with an increased focus on computer science (The White House, 2017).

STEM aims to unify the distinct curricular areas of science, technology, engineering and mathematics into a more cohesive whole. While the STEM acronym is relatively new, the ideal of cross-curricular instruction has been lauded as advantageous for a long time. “It has long been argued that connecting mathematics and science can benefit learning” (Baxter, Ruzicka, Beghetto, & Livelybrooks, 2014, p. 109).

STEM and Math Instruction

Secondary students’ mathematical proficiency and course taking has been considered a barometer of STEM preparedness (Cannady et al., 2014). To enter the post-secondary STEM career pipeline, students must enroll in post-secondary STEM programs. Factors contributing to the decision to pursue a STEM major are multifaceted. Moakler and Kim (2014) found that academic confidence level was a predictor of STEM career choice; however, they reported that mathematics confidence level was even a better predictor of whether or not incoming college freshman would enroll in STEM undergraduate programs. Sadler, Sonnert, Hazari, and Tai (2014) found that high school participation in calculus, but not necessarily Advanced Placement Calculus, positively predicted an interest in a STEM field of study at the college level.

While interest in pursuing a STEM career is tied to mathematics confidence and high school course taking, a students' ability to successfully move through the mathematics pipeline has also been found to be related to a students' mathematical preparedness. Students must successfully complete college STEM programming in order to gain entrance into a lucrative and rewarding STEM field. Hinojosa, Rapaport, Jaciw, LiCalsi, and Zacamy (2016) in their review of research exploring the foundations of the future STEM workforce, found seven studies that explored whether ACT and SAT math scores helped to ensure successful completion of STEM degrees. "All [seven] found that students with higher SAT or ACT math scores were more likely to achieve postsecondary STEM success" (Hinojosa et al., 2016, p. 10).

Evidence of mathematical preparedness as a factor which contributes to students' success in postsecondary STEM programs is well documented. Yet, "it is clear that resolution on how STEM education fits with our goals for mathematics education still lacks clarity in the minds of many" (Larson, 2017, para. 4). Policy makers, education leaders and industry leaders clamor to integrate STEM into high school mathematics programming while educators complain that there is insufficient instructional time to cover course content outlined in the Common Core standards or modified versions of those standards (Bowman, 2015; Dutcher, 2017). As Larson (2017), the president of NCTM, remarked, "the possibility that we might neglect the full development of students' mathematical understanding in order to integrate STEM 'activities' into an already over packed curriculum is real" (para. 16). He argued that a mathematics education that incorporates the mathematical habits of mind is STEM education, cautioning educators to not sacrifice mathematical competencies outlined in the mathematical content standards to incorporate "STEM-activities" to "STEM-up your classrooms" (Larson, 2017, para. 20).

Content Standards

Content standards and assessments do not currently lend themselves to encouraging educators to implement or craft cross-curricular STEM lessons. Engineering by definition is cross-curricular, yet a curricular structure seems to be less available to educators, as evidenced by a lack of standardization of engineering at the national level. As Reeve (2015) indicated, "all the STEM areas except engineering have national content standards that are used to identify what is important to teach" (p. 12). The other STEM disciplines' standards that are not naturally cross-curricular minimally emphasize a cross-curricular education. Focused content standards serve multiple purposes in preparing students to be college and career ready.

The OECD (2012) outlined some of the helpful ways content standards proved beneficial in OCED member countries. Content standards:

> establish rigorous, focused and coherent content at all grade levels; reduce overlap in curricula across grades; reduce variation in implemented curricula across classrooms; facilitate co-ordination of various policy drivers, ranging from curricula to teacher training; and reduce inequity in curricula across socio-economic groups. (Organisation for Economic Cooperation and Development [OECD], 2012, p. 48)

Educators at large seem to agree that standards alone will not suffice to improve student learning outcomes; the "CCSS needs to be viewed as one part of the U.S. reform effort" (Frye, 2015, p. 535). The power of multiplicity is evidenced by the fact that when content standards are coupled with testing accountability measures, students' performance outcomes have been shown to be higher than in educational systems that do not accompany standards with accountability assessments (OECD, 2012).

The ideal of rigorous national content standards has enjoyed presidential support across party lines for some time (Frye, 2015). The Common Core State Standards (CCSS) has met opposition, perhaps most notably from President Trump. While campaigning, President Trump indicated that “I believe Common Core is a very bad thing” (Saul, 2016, para. 15). During a victory speech he remarked that “we are getting rid of Common Core” (Huseman & Moser, 2016, para. 2). President Trump’s fervor to eliminate the CCSS seemed to notably diminish after the election. Reporters Freedberg and Harrington (2017) noted that “since repeatedly calling for the Common Core to be abolished during the presidential campaign, Trump has not mentioned it since becoming president” (para. 13). However, Trump did later reaffirm a distain for the CCSSI; “Common Core to me, we have to end it” (Berry, 2017, para. 2). However, direct involvement in states’ individual educational policies (i.e., adoption of content standards) is not a power bestowed upon the White House, based on the 10th Amendment. The 10th Amendment states that “the powers not delegated to the United States by the Constitution, nor prohibited by it to the states, are reserved to the states respectively, or to the people” (U.S. Const. amend. X); the federal government does not have jurisdiction over state education. Yet, the educational lines of jurisdiction may have become less crisp over time, due to the power of influence (Frye, 2015). The effect of the Trump administration’s influence on educational reform measures, such as a nationalization of content standards, is yet to be seen.

Common Core Mathematics Standards

The national mathematics standards, the CCSSM, were drafted in 2009 and are now adopted by a majority of states in the U.S. (CCSSI, 2010). The CCSSM have been documented as “disappointing in that they fell short of any real encouragement to integrate subject matter within the content standards” (Capraro & Nite, 2014, p. 6). Standards are not targeting curriculum integration (Asghar, Ellington, Rice, Johnson, & Prime, 2012; Capraro & Nite, 2014). Consequently, some researchers blame standards and assessments for the lack of motivation among educators to implement cross curricular lessons (Asghar et al., 2012).

The CCSSM documentation indicated that “the standards encourage students to solve real-world problems” (CCSSI, 2010, para. 4). Real-world problems are likely STEM-type problems. In addition to emphasizing problem solving, the CCSSM emphasize mathematical habits of mind, “with longstanding importance in mathematics education” (CCSSI, 2018, para. 1). Eight mathematical habits of mind are highlighted in the CCSSM and are referred to as the standards of mathematical practice (SMP). Alongside these SMP, the Common Core State Standards outline specific content standards at the high school level by domain. The content standards for each high school course are divided into five domains: algebra, geometry, functions, probability and statistics, and number and quantity, with a sixth domain, modeling, which is taught in each of the five subsequent domains. Davis et al. (2014) indicated that “more needs to be known about how teachers and districts perceive the CCSSM” (p. 14).

Mathematical tasks provide students an opportunity to problem solve in context while engaging in the SMP. The Common Core mathematics standards make mention of mathematical tasks just once, in the introduction to the standards. The standards make specific reference to mathematics tasks indicating their usefulness in assessment: “mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness” (CCSSI, 2018, p. 4).

In the Common Core mathematics standards, the use of rich math tasks is not lauded as an instructional tool by which teachers could engage students in cross-curricular experiences with an emphasis on doing the math. Rather, the standards seem to imply that the power of tasks lies in their usefulness as tools to assess students understanding and procedural skills. Rich mathematical tasks could be conceived to be cross-curricular in context or simply mathematics situated within a scenario.

Excess. Prior to the nationalization of standards that took place with the adoption of the Common Core mathematics standards, educators complained that state standards were "a mile wide and an inch deep," so to speak (Flick & Kuchey, 2010; Porter, McMaken, Hwang, & Yang, 2011). Some educators were hopeful that the instructional expectations would be more focused at each grade level and allow for more in-depth exploration of concepts under the Common Core mathematics standards. Porter et al. (2011) using a "nationally recognized content analysis procedure" found that the new Common Core mathematics standards were "somewhat more focused" (p. 114).

Educators still complain of excess (Bowman, 2015), likely indicating that the slimming of the standards has been insufficient. However, Bowman, a high school mathematics educator, advocated that educators "assign a few thoughtful problems that incorporate more than one concept" (2015, p. 95) as a panacea for dealing with the "excessive amount of concepts" (2015, p. 95). McDuffie et al. (2017) captured the teacher sentiment of excess when he recorded one middle school mathematics teacher's words repeated below:

> The content is shifting. When we were originally told about the Common Core, we were told that we were going to have much fewer topics to cover, and we were going to be able to go a lot deeper. So far, we have been rushing and trying to get through content quickly so that we can make sure that we make it to what we need to cover before the state exam. (McDuffie et al., 2017, p. 162)

Math tasks have been described as a tool which brings focus to learning for students and teachers alike (Davis et al., 2014; Foster & Noyce, 2004) and it seems that teachers' careful selection and use of mathematics tasks may be helpful in addressing the CCSSM.

Rigorous. The Center on Education Policy (Rentner, 2013) indicated that the CCSSM are thought to be more rigorous than prior state standards by many analysts. Choppin, Davis, Drake and Roth McDuffie (2013) found that 85.5% of middle school mathematics teachers viewed the CCSSM as more rigorous than their previous state standards, and 87.5% viewed the CCSSM practice standards as more rigorous than their previous state standards. Davis et al. (2014) found that teachers' perceived rigor of the CCSSM was a factor that helped to explain the variability in teachers' perceptions of the CCSSM. They indicated that other teacher populations' perceptions of the CCSSM should be studied to determine the extent to which they might be different. There seems to be a lack of research of secondary mathematics teachers' perceptions of the CCSSM. There does seem, however, to be consensus that a learning curve exists for secondary mathematics educators, especially those who have taught more procedurally in the past. The CCSSM ask teachers to implement the kind of teaching that will allow for both procedural fluency and conceptual understanding (Schoenfeld, 2015). "Part of the challenge is dealing productively with student approaches—both correct and incorrect—as students grapple with complex tasks" (Schoenfeld, 2015, p. 189).

Common Core Aligned Curriculum

Teacher leaders and secondary mathematics educators have expressed concern with the mismatch between the primary adopted curricula and the CCSSM and have communicated that mathematics tasks served as "a means for augmenting those materials" (Johnson, Severance, Penuel, & Leary, 2016, p. 171). Slavit and Nelson (2010) reported on one secondary teacher professional learning community (PLC) of Integrated Math 2 teachers who felt that their primary curriculum “lacked a great deal of rich tasks” (p. 205). To provide students with problem solving opportunities this PLC specifically sought out math tasks that met the following characteristics (Slavit & Nelson, 2010):

> (a) “open-ended” with multiple possible strategies
> (b) allow students to “muddle” or “struggle”
> (c) elicit conversation and thinking that are “appropriate for group work,” and
> (d) have “important and relevant” content. (p. 205)

This teacher group demonstrated a well-developed understanding of mathematical-task knowledge in their selection of the criteria indicated above. While the teachers’ requirements were brought forth before the implementation of the CCSSM, the cry for selecting “better problems” during implementation of the Common Core was evident.

Bowman (2015) indicated that teachers should give students “better problems to do during the day that require thinking, reasoning, and persistence” (p. 95). Giving students such problems inevitably requires a selection process that may be from sources outside an educator’s primary curriculum. To what extent secondary teachers feel their primary curricula provide appropriately rigorous math tasks aligned to the standards is not clear. Over half of the 366 middle school mathematics teachers surveyed in one study reported “using digital and/or electronic supplemental resources daily or weekly for practice or remediation, while 39.3% reported using electronic resources for supplemental inquiry-based activities” (McDuffie et al., 2017, p. 153). This might suggest that supplementation at the high school level is also occurring.

Bowman (2015) indicated that “as educators, we are continually reminded that the textbook is a resource and not the curriculum” (p. 91). Selecting and/or developing tasks is part of a teacher’s mathematical-task knowledge (Chapman, 2013). Evidence suggests that secondary mathematics teachers do not find all the high-level tasks in their primary curricula but must develop skill in seeking out or developing high-level, standards-aligned tasks (Chapman, 2013). "They [teachers] pointed out that there was no one resource available that provides all of the ELAs [Exploratory Learning Activities] applicable to the curriculum, justifying the need for them to create or find them" (Chapman, 2013, p. 3). Sullivan and Mornane (2014) indicated that "it seems that teachers appreciated the suggestions of challenging tasks that aligned with the content of their teaching" (p. 211). Selecting or designing challenging tasks seems to be necessary to meet the content and rigor requirements of the CCSSM and may be an obstacle for educators as their primary curricula require task supplementation.

Challenging Math Tasks

"Challenging tasks are complex and absorbing mathematical problems with multiple solution pathways, whereby the whole class works on the same problem" (Russo & Hopkins, 2017, p. 31). Sullivan and Mornane (2014) shared a similar description of challenging tasks from the perspective of what teachers must know to teach such tasks, indicating that teachers should be able to "recognize that a challenging task can be solved in different ways, and that students' emerging intuitive approaches are as much part of learning mathematics as algorithmic routines" (p. 195). Textbook questions that follow an example or description of a particular mathematics concept might not be considered a CMT as a solution path is already identifiable. CMTs require students to select a solution pathway. Boaler (1998) has shown that students who are routinely shown textbook problems may struggle when tasks do not provide a cue. CMTs, therefore, are open-ended problems where students must decide what method or methods to employ to solve the problem. Learning to select a strategy is part of the learning goals of CMTs.

Smith and Stein (1998) developed a hierarchical structure for classifying mathematics tasks based on cognitive demand. They defined four levels of cognitive demand, with two lower level classifications, memorization and procedures without connections, and two higher level classifications, procedures with connections and doing mathematics. CMTs fall into the higher-level classifications, procedures with connections and doing mathematics, requiring a higher level of cognitive demand.

Timeline

Through the writings of the NCTM, the evolution of the specificity of the role of math tasks can be seen. Schoenfeld (2015) noted how the standards movement "can be dated back to the National Council of Teachers of Mathematics' production of the *Curriculum and Evaluation Standards for School Mathematics* in 1989" (p. 183). The NCTM's (1989) description of instructional practices considered as imperative to a positive learning environment never explicitly mentioned math tasks; however, there are implicit references to the use of math tasks in the classroom. For instance, the NCTM advocated for a "problem-solving approach to instruction" (NCTM, 1989, p. 20). The NCTM's position on math tasks became more formalized in their 2014 publication, *Principles to Actions*. However, it is important to note that the term *task* was reserved for assessment in the NCTM (1989) publication. The broadening of the use of the word task, from being used for assessment only (both formative and summative) to being used both in instructional and assessment descriptions, may mark instructional shifts taking place on a larger scale. Research on best practices in implementing mathematical tasks has spanned over three decades (Johnson et al., 2016; NCTM, 1989), and an evolution of attention to math tasks over that period is seen in the emergence of the use of the term as well as the NCTM's declaration of mathematics tasks as one of eight mathematics teaching practices (NCTM, 2014).

The NCTM, in *Principles to Actions* (2014), indicated that standards alone will not lead us to "realize the goal of high levels of mathematical understanding by all students" (p. vii). Realizing the goal of high mathematical understanding by all students would necessitate teachers to incorporate a myriad of practices to improve learning outcomes. Implementing math tasks that promote reasoning and problem solving is necessary to move in the direction of creating a populous that has a high level of mathematical understanding. Successful enactment of mathematics tasks, however, requires mathematical-task knowledge.

Mathematical-Task Knowledge

Collectively, math content knowledge married to pedagogical knowledge has been referred to as pedagogical content knowledge (PCK) (Shulman, 1987). Thus, mathematical-task knowledge is a type of PCK. Chapman (2013) coined the phrase "mathematical-task knowledge" (p. 1) but did not directly indicate that it was a sub-class or type of PCK.

Math task framework. The MTF (Smith & Stein, 2011) serves as a tool to invite teachers into reflection around their mathematical task experiences as they debrief with other teachers implementing mathematics tasks. Smith and Stein's (2011) work spanned over a five-year period as they conducted research as part of a "national reform project" (p. 9), called QUASAR. During this time, the utilization of the MTF led Smith and Stein (2011) to the conclusion that the MTF "can give teachers insight into the evolution of their lessons" (p. 11).

One of the objectives of this study was to measure teachers' perceptions of math tasks under the CCSSM. Another objective was to explain variability in those perceptions to determine where high school mathematics teachers might be in the evolution of mathematics reform in terms of mathematics tasks. The MTF served as the conceptual framework for this survey investigation. Figure 1 is a reproduction of the MTF.

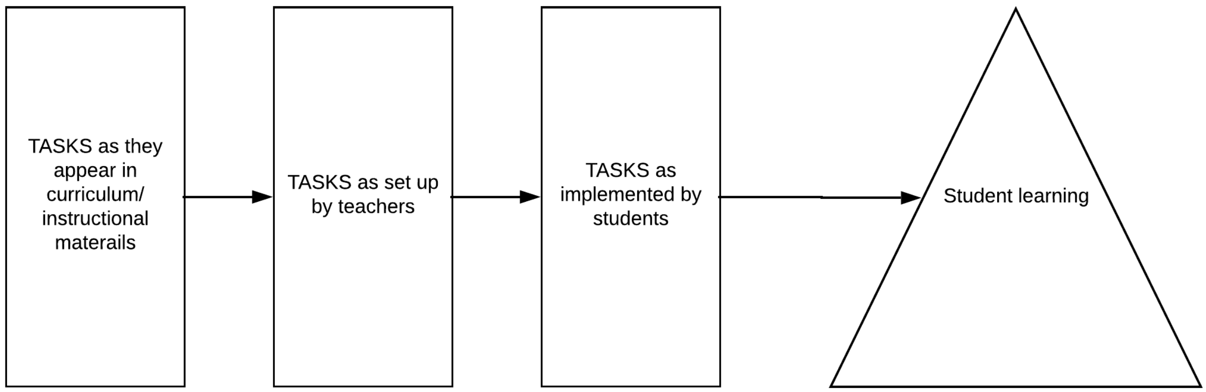

Figure 1. Mathematical tasks framework (Smith & Stein, 2011).

The MTF consists of four distinct parts: tasks as they appear in curricular/instructional materials, tasks as setup by teachers, tasks as implemented by students, and student learning. The first three blocks of the MTF pertain to tasks and each of these task blocks has a list of teacher knowledge designed to ensure the successful enactment of math performance tasks as an instructional practice so that the last phase in the MTF, student learning, is optimized. Teachers have control over the first three blocks of the MTF based on their mathematical-task knowledge, which can be conceptualized further as existing in two parts as indicated by Chapman (2013). Chapman (2013) did not make reference to the MTF but did indicate two areas of teacher knowledge around math tasks: teacher knowledge required in "selecting and developing tasks" (p. 1) and the ability to "optimize the learning potential of such tasks" (p. 1). These two areas of teacher knowledge split the MTF into two halves as illustrated in Figure 2. Teachers begin their work utilizing their knowledge in selecting and developing tasks as they intend them to be setup, encompassing the first two stages in the MTF. This is followed, ideally, by using optimized learning potential task knowledge to help students be successful as active learners, as students implement the tasks. An example of optimized learning potential task knowledge is teachers' knowledge of classroom management as applied to orchestrating mathematical tasks.

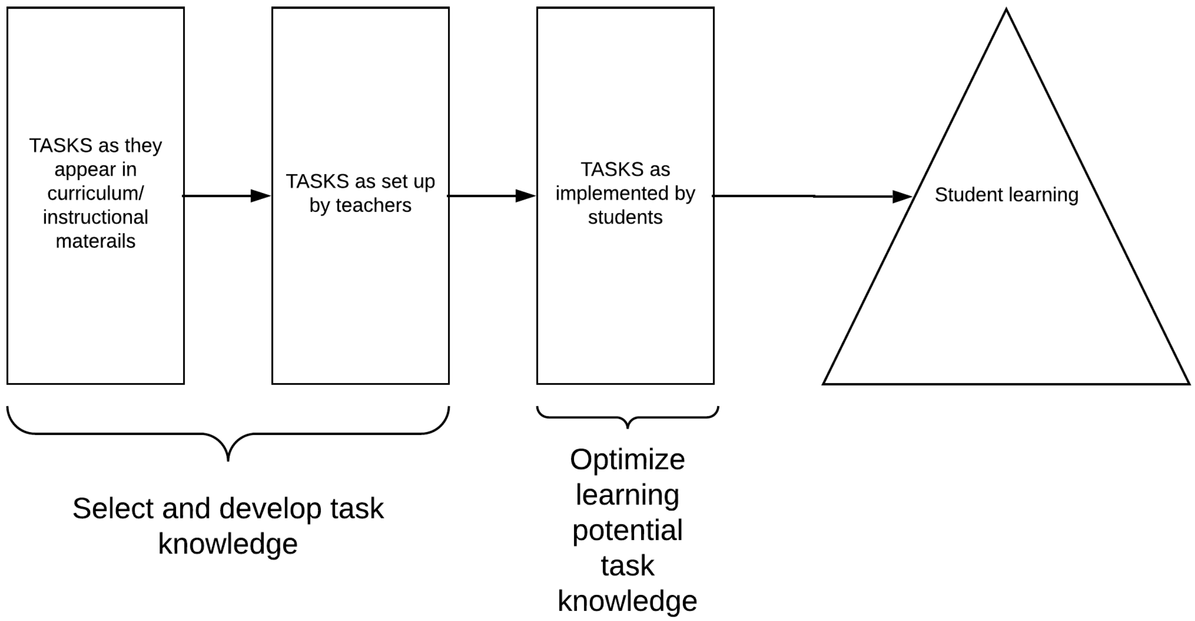

Figure 2. Select, develop, and optimize learning potential task knowledge. Adapted from Smith and Stein (2011).

Select/design and develop or setup task knowledge. Teachers have agreed that augmenting their primary curricula with cognitively CMTs is a way to enact some of the CCSSM reforms measures (Johnson et al., 2016). Teachers require *select and develop task knowledge* in order to first make a selection and/or design a task and then conceptualize a myriad of other considerations to more fully develop the task using *develop task knowledge*. Teachers with a strong select and develop task knowledge will be more likely to enact CCSSM aligned tasks. First there is the selection (or design) process. Second, teachers must develop the tasks for delivery after identifying the task(s) to be utilized. Third teachers bring tasks to life utilizing optimize learning potential task knowledge. Throughout this process teachers benefit from a strong mathematics content knowledge that allows them to grasp multiple solution pathways, aspects of conceptual development, and procedural understandings that might facilitate solutions.

A Need for Tasks That Align to the CCSSM

Johnson et al. (2016) worked with a group of 12 district Algebra 1 teachers from nine different schools that came together to move from *what is* to what *ought to be*. Initially, the district Algebra 1 teachers had a curriculum that did not align to the CCSSM and lacked "sufficient high-quality tasks" (Johnson et al., 2016, p. 176), with a preferred future state of "a revised Algebra 1 curriculum" with "high-quality mathematical tasks aligned to the CCSSM" (Johnson et al., 2016, p. 176). There is evidence that suggests that teachers across the nation, such as those described above, are in need of challenging math tasks that align with the CCSSM (Dossey, McCrone, & Halvorsen, 2016). However, none of the reviewed research quantified high school teachers' perceptions of the degree to which teachers felt their primary curricula offered a satisfactory selection of CCSSM-aligned tasks that offered students an opportunity to engage in higher-level thinking. A middle school survey conducted by McDuffie et al. (2017) indicated that 39.3% of middle school mathematics teachers were using supplementary inquiry-based activities at least weekly.

Selecting tasks is no small task. Selection of CCSSM aligned challenging tasks is no small task. Slavit and Nelson (2010) described a group of high school teachers' work in "negotiating the structure of mathematical tasks" as an "extensive effort" (p. 202). Research undertaken before the enactment of the CCSSM focused primarily on the importance of selecting tasks of a higher cognitive demand (Boston & Smith, 2009; Smith & Stein, 1998). Under the new CCSSM it seems reasonable to believe that teachers are now looking to optimize student learning by paying attention to the standards being addressed in mathematics tasks as well as the level of cognitive demand (Kanold & Larson, 2015, p. 25), which may confound a search for an appropriate task. It seems logical to believe that teachers should first seek out tasks aligned to the standards and then conduct an analysis of the cognitive demand, as suggested by Kanold and Larson (2015), to ensure an adequate proportion of mathematical tasks at both the lower and higher levels of cognition. One recommendation is to maintain a ratio of 3:1; that is to say that there would be three lower level tasks for each higher-level task "for the balance of the unit" (Kanold & Larson, 2015, p. 25).

Teachers sometimes lack knowledge of the specific CCSSM content designated to the specific courses they teach. For instance, students enrolled in an Algebra 1 course, which is a common freshman course of study, typically study quadratic equations, among other mathematics topics. Yet, teachers teaching Integrated Math 1, also a typical freshman course, would not be teaching quadratic equations. The CCSSM designates what content is to be covered in each course. Teachers must take the time to be conversant in the content standards so that appropriate content and tasks aligned to the appropriate standards are implemented. Teacher familiarity with the CCSSM content standards is a component of teachers' select math tasks knowledge.

Teachers knowledge of students' mathematical abilities (Rhoads & Weber, 2005; Smith & Stein, 2011) and language development (Johnson et al., 2016) has also been espoused as a consideration in selection of tasks, as well as an awareness of students' interests (Johnson et al., 2016; Rhoads & Weber, 2005). However, use of cognitive demand as a primary consideration in task selection has a stronger foundation in research compared to some of the other considerations above as contributing to a teacher's select math task knowledge (Johnson et al., 2016).

Task as Setup by Teachers

Once a task is selected or designed, a teacher then moves into applying develop/setup task knowledge, which corresponds to the second block in the MTF, tasks as setup by teachers. In the description of instructional theory that follows, many of the theoretical considerations that could be generalized to any math task is discussed. The lack of formalized theory associated with math task setup knowledge results in teachers' or researchers' descriptive language being adopted. Asikainen, Pehkonen, and Hirvonen (2013) found that expert Finish mathematics teachers who served as mentors "conceptualized teacher knowledge in mathematics mostly in their own terms" (p. 88). Teachers refer to modifications made to tasks or other instructional supports put in place to support student success with mathematical tasks as scaffolding, but as they put their pedagogical knowledge to work, they often resort to the use of *their own terms*.

Prescribed or un-prescribed. A mathematics task can be broken down for student comprehension either verbally or by actually segmenting a task into subparts in print. Teachers must apply setup task knowledge to decide the degree of scaffolding that will be provided in terms of how a task might be segmented into subparts. Sullivan and Mornane (2014) described the subparts that a task might be dissected into as "small, safe, carefully sequenced, and well supported micro steps" (p. 193). However, research suggests that when teachers do not segment a task, students may have an opportunity to learn to plan a course of attack, instead of being given one (Sullivan & Mornane, 2014). The term "prescribed or un-prescribed" was coined by Integrated 2 teachers who collaborated around the development and use of rich math tasks (Slavit & Nelson, 2010, p. 208). Slavit and Nelson (2010) described the meaning of prescribed or un-prescribed as used by teachers to signify "the degree to which a task should be 'broken down' into specific steps or presented as a 'large problem'" (p. 208).

Johnson et al. (2016) indicated that the group of Algebra 1 teachers they collaborated with “were particularly concerned with using certain cognitively demanding task ‘as is’ with students whom they judged to be of lower ability” (p. 179). It could be that this group of teachers might be considering segmenting tasks. Many task accommodations are conceivable, some of which are discussed here. Yet, it does seem that teachers see leaving a task *as is* as an option, which may be afforded to certain student groups and not to others.

Connected or disconnected/Teach-first or task-first. Another aspect of develop/setup math task knowledge exists in determining the positioning of a task within the curriculum line-up. One study by Russo and Hopkins (2017) framed the question of positioning of mathematics tasks as either *teach-first* or *task-first*. The teach-first approach, where content knowledge and/or conceptual modeling are presented before assigning a task was seen by teachers as generally more efficient, given time constraints. Teachers additionally viewed the teach-first approach as allowing “less confident and lower performing students to be successful” (Russo & Hopkins, 2017, p. 39). The task-first approach was “viewed as engaging and empowering for students, providing an opportunity to build student persistence whilst fostering student mathematical creativity” (Russo & Hopkins, 2017, p. 43).

While Russo and Hopkins’ investigation of teach-first or task-first was conducted at the elementary level, their work points to the conversation that exists around the timing of a CMT in the instructional line-up. High school teachers have been recorded as discussing the timing of tasks. When a group of Algebra 1 high school teachers discussed cognitive demand, they indicated that “their ratings would depend on where in the curriculum they might use the task” (Johnson et al., 2016 p. 179). In one account of practice, “the degree to which a task should mesh with the current instructional topic or be used as a review or foreshadowing of different content" was termed "connected" or "disconnected" by Integrated 2 teachers who collaborated around the development and use of rich math tasks, as reported by Slavit and Nelson (2010, p. 208). The degree to which a task introduces content or is used in a formative or summative manner is an instructional consideration. Variation of the degree in which a task is connected or disconnected to course content may allow for different potential benefits to be realized.

Language support. Form and function, which might be thought of as an English Language Arts construct, has made its way into the scaffolding decisions made by mathematics educators, but is less well documented in the research, as noted by Johnson et al. (2016). In documenting the tensions that arose as teachers attempted "to revise rubrics that assessed the language demands of mathematics tasks" (p. 178), Johnson et al. (2016) found the goal of assessing the language demands of math tasks to be "one of the most prominent project tensions" (p. 178). Researchers unfamiliar with the constructs of form and function, initially proposed "demand and access," but teachers, with experience with English Language Arts preferred the more familiar term, form and function (Johnson et al., 2016, p. 178). Again, we see the negotiation of vocabulary to describe instructional decision making, as teachers aim to develop scaffolding support in the form of rubrics for use with CMTs. Form is the smaller details of writing, such as grammatical rules, sentence structure, and specific vocabulary; whereas, function describes the purpose of the writing (i.e., to persuade, to sequence events, to explain, or to summarize). Teachers' concern with supporting English Language Learners as they tackle CMTs is warranted so that students can fully engage in sense making. Moschkovich (2013) emphasized that teachers of English language learners should "use high-cognitive-demand math tasks and maintain the rigor of mathematical tasks throughout lessons and units" (p. 49), also noting that teachers must "address much more than just vocabulary" (p. 50). When teachers are attending to vocabulary, they are attending to form, which is only one ingredient in supporting students in terms of comprehension of mathematical tasks.

Below is a picture captured during a break-out session, "Team Time," at the NCTM Innov8 conference in Las Vegas in 2017. This picture was taken with permission from an anonymous teacher group as they were asked to identify a major area of focus for teaching and learning that they hoped to address in part through participation in the NCTM Innov8 Conference. Teachers of mathematics are attending to form when they intentionally target vocabulary development, as demonstrated in Figure 3.

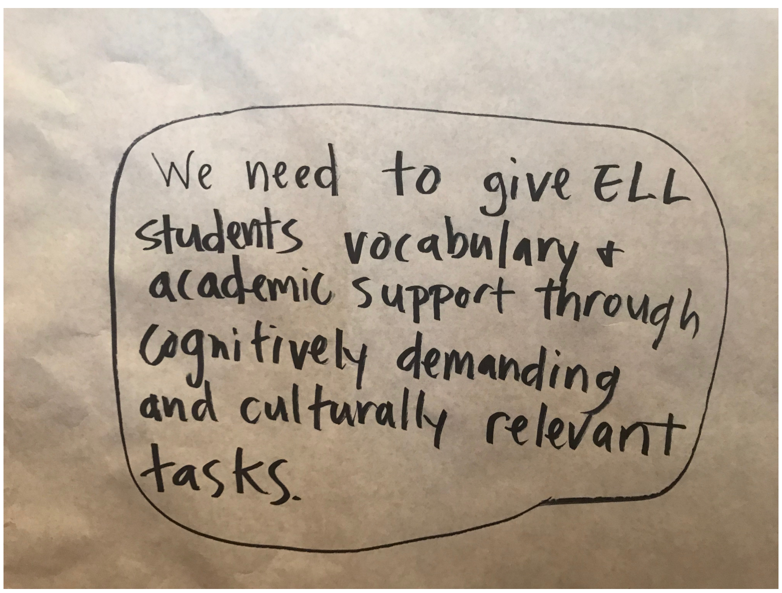

Figure 3. 2017 NCTM Innov8 conference artifact.

Assisting English language learners as they work to comprehend mathematical tasks may require additional scaffolding to make the mathematics more accessible. Develop math task knowledge, therefore, is comprised in part of ones' knowledge of being intentional in attending to reading comprehension and strategies that support reading comprehension.

Optimize learning potential task knowledge. The same math task can be implemented very differently depending on the math task knowledge of the teacher and the dispositions, mindsets, and mathematical abilities of students. As Johnson et al. (2016) noted, “particularly relevant are teachers’ instructional realities that can make task implementation difficult” (p. 182). Gaining optimize the learning potential math task knowledge has been documented and popularized through the case study description of Ron Castleman (an alias) by Smith and Stein (2011). Ron’s self-acknowledged departure from maintaining the cognitive demand of a task followed by his successful implementation of the same type of math task was made possible through the acquisition of new pedagogical knowledge. Smith and Stein (2011) documented how Ron diminished the conceptual learning that was intended when he failed to let students grapple with the mathematics. Students prompted him for answers and students had been reciting the all too common question, “How do you do this?” Ron caved-in, providing students a best first step in a solution pathway, thus degrading the challenge of the task so significantly that the actual diagram associated with solving the task became non-consequential. Giving away a first critical step, or over-scaffolding with a step-by-step set of directions, were found to be among the missteps of teachers who aimed to implement CMTs, but failed to do so. Ron, after gaining additional understanding of his own practice and actions, gained optimize math task knowledge (Smith & Stein, 2011).

Boston and Smith (2009) showed that through a PD intervention, teachers could acquire optimize math task knowledge. Ten secondary mathematics teachers served as a control group in their experimental design. The NCTM (2014) charged educators with learning to “support productive struggle in learning mathematics” (p. 8) as one of eight effective teaching strategies it espoused.

Not over-scaffolding instruction during implementation has been found to be a critical first step in ensuring that students have access to CMTs. Being afforded the opportunity to grapple with mathematics opens the door to new learning opportunities, yet other considerations have been noted as well. For examples, knowing how to enact strong classroom management during the implementation phase of a mathematical task has been considered as part of optimize math task knowledge (Smith & Stein, 2011).

Teacher Professional Development

Researchers have posited that developing teachers professionally will improve the quality of mathematics and science instruction. In order to improve instruction, the PD of teachers has been lauded. Consequently, "nearly every teacher participates in some form of continuing education every year" (Hill, 2007, p. 124). Along with the universal participation of teachers in PD, the scale of professional development has been altered. Moving to national standards for mathematics and science has driven the enactment of PD from a local control context, to looking at PD as having a larger scale context (Marrongelle et al., 2013). Improving instruction in math and science has specifically been targeted. Researchers see teacher PD centered around the CCSSM as critical to their enactment (Marrongelle et al., 2013), going as far as to say that without it, "more challenges and disappointment than actual changes" (Marrongelle et al., 2013, p. 202) would be evident.

Quality professional development. Teacher PD facilitates quality instruction. Evidence that longer PD is a better professional development model is well documented (Hill & Ball, 2004). For instance, in one year-long, elementary teacher PD opportunity, teachers' mode of delivery of content to students, as well as teachers' math content knowledge and teachers' beliefs towards mathematics as a content area, was effectively changed (Polly, Neale, & Pugalee, 2014).

Changes in teachers' perspectives about teaching and learning mathematics is something that researchers find occurs with more extensive PD that spans multiple years (Polly et al., 2014). Scher and O'Reilly (2009) found that "math-focused interventions that take place over multiple years have a pronounced effect on student achievement than interventions occurring over only 1 academic school year" (p. 235). However, the results for science were less pronounced. Relatedly, research by Chval, Abell, Pareja, Musikul, and Ritzka (2008) found that mathematics and science teachers prefer PD that relates directly to their course and grade of instruction.

Types of professional development. Numerous forms of teacher PD exist that can lead to instructional shifts or changes of perceptions by teachers. Teachers may even develop independently of PD (Chapman & Heater, 2010). The process towards change is not documented as a straight-forward process. Chapman and Heater (2010) wrote that "since it is the teacher who eventually determines the level of change he or she desires, planned interventions can be problematic if they do not match the level in which the teacher is interested" (p. 457). Yet, eliciting teachers' PD needs directly from teachers has been reported to be of little consequence. Beswick (2014) argued that "the kinds of things that teachers expressed" (when surveyed about their professional learning goals) "could readily have been predicted from the initial conversations with informed people and the research literature" (p. 102). On the other hand, Bostic and Matney (2013) indicated that elementary and middle school teachers' self-reported "needs for CCSSM-focused PD aligned with their students' prior high-stakes test performance" (p. 17), giving validity to teachers' instructional and PD perceptions. It could be that teachers are more in touch with their PD needs at a specific, standards/curriculum-oriented level than at the more general, over-arching, pedagogical level. Bostic and Matney's research did not address secondary mathematic teachers' perceptions of their content PD needs relative to the CCSSM.

Partnerships with university faculty and others. Suppling middle school and high school teachers with access to STEM knowledge and or tools has been accomplished through some collaborative projects between university faculty and middle school and post-secondary teachers (Beaudoin, Johnston, Jones, & Waggett, 2013; Iskander, Kapila, & Kriftcher, 2010; Kazempur & Amirshokoohi, 2014), with favorable results as reported by the teacher-participants. However, one collaborative teacher inquiry around the development and use of rich mathematical tasks documented that insider-status may be of advantage when facilitating site-based professional learning communities' (PLC) inquiries into practice. In this case study, a regional mathematics specialist assigned to facilitate commented: "I think that I, or we, still have lots to learn about facilitating PLCs, especially if you aren't a part of the school district community" (Slavit & Nelson, 2010, p. 205).

Professional development using inquiry-based instruction. Researchers examining STEM instruction or delivering PD have advocated for an inquiry-based instructional model (Kazempour & Amirshokoohi, 2014; Reeve, 2015). One common form of inquiry-based instruction is project-based learning (PBL).

> Project-based learning (PBL) is a model for classroom activity that shifts away from the classroom practices of short, isolated, teacher-centered lessons and instead emphasizes learning activities that are long-term, interdisciplinary, student-centered, and integrated with real world issues and practices. (Holbrook as cited in Capraro & Jones, 2013, p. 51).

A common implementation of STEM PD found in the case studies examined incorporated the use of PBL as an instructional strategy through which STEM lessons might be deployed to students (Asghar et al., 2012; Han, Yalvac, Capraro, & Capraro, 2015). While PBL falls under the umbrella of inquiry-based instruction, other entry points used in PD have included the scientific method (Kazempour & Amirshokoohi, 2014). The incorporation of math performance tasks has been a more recent approach to teaching problem solving and critical thinking.

Professional development as a change agent. PD has been shown to change teachers' beliefs and perceptions, instructional practices, and their content knowledge. A quantitative study established a link between middle school teacher's perceptions of the Common Core math standards and the level of PD teachers were exposed to. In the quantitative study by Davis et al. (2014), exploratory factor analysis was used to discover that the "degree to which professional development is supported" (p. 19) and "extent of district professional development support" (p. 20) are factors that account for the variability in middle school math teacher's survey responses regarding their perceptions about the Common Core mathematics standards and the instructional environment. Davis et al. (2014) suggested that further research be conducted to see if high school teachers' data would be "similar to or different from middle school mathematics teachers" (p. 23). Davis et al.'s research contributes to the field in that it supports the position that at the middle school level PD invokes change. While one might expect similar outcomes for middle school and high school teachers participating in PD, research outcomes after PD indicate that outcomes may not be similar in the way they effect teachers' beliefs and perceptions, pedagogical approaches, and content knowledge.

The outcomes for students may also vary from the middle school to high school level. However, few research studies have focused on student outcomes as a consequence of PD. "Little work has empirically examined whether, or to what extent, professional development activities influence student learning" (Foster, Toma, & Troske, 2013, p. 256). Research funded through the National Science Foundation targeting mathematics and science PD for teachers used student test scores to measure the benefit of PD in these subject areas (Foster et al., 2013).

Student outcomes on mathematics and science assessments suggest that the intervention had "positive and significant effects for math achievement in the elementary and middle schools but not at the high school level" (Foster et al., 2013, p. 264).

Math teachers and STEM professional development. Many have advocated for the curricular implementation of an integrated STEM education; however, researchers have not provided extensive information on how to assist teachers who might deploy STEM. "The press to integrate mathematics and science comes from researchers, business leaders, and educators, yet research that examines ways to support teachers in relating these disciplines is scant" (Baxter et al., 2014, p. 102). A specific lack of written work that addresses STEM PD for teachers has been documented by Asghar et al. (2012).

The goal of Beaudoin et al.'s (2013) research was to provide a template for university faculty members who may wish to provide support to secondary teachers in implementing STEM lessons. Mathematics and science teacher participants attended a local university led STEM PD training for teachers. Eight PD elements were compared using a Likert-type scale survey. Beaudoin et al. (2013) found that teachers valued time to work together to develop lessons above all else. Other researchers have also documented collative time as highly valued by teachers during PD (Kazempour & Amirshokoohi, 2014). Mathematics teachers were particularly "interested in demonstrations of lessons that integrated other content areas" (Beaudoin et al., 2013, p. 355). Research has shown teachers to be particularly interested in model lessons that are interdisciplinary (Asghar et al., 2012; Beaudoin et al., 2013) and inquiry-based (Kazempour & Amirshokoohi, 2014).

Targeted secondary STEM PD for teachers has proved challenging compared to elementary or middle school enactments of STEM PD. Even with targeted STEM PD, teachers find implementing STEM PBL difficult at the secondary level. In one case study, veteran teachers in science and mathematics experienced 10 sessions of PD annually, for a period of three years focused on the pedagogy of STEM PBL. Teachers' implementation of STEM PBL led researchers Sunyoung, Yalvac, Capraro, and Capraro (2015) to remark that "teacher-driven PDs should be designed to decrease the gap between knowing and doing" (p. 73). The researchers also indicated that without unannounced observations it was not possible to assess whether STEM PBL PD altered the teachers' daily instructional practices.

Teacher resistance to implementing STEM PBL has been reported by research that examined the implementation of STEM PBL at the secondary level (Asghar et al., 2012; Sunyoung et al., 2015). Baxter et al. (2014) examined the effects of STEM PD on elementary teachers' instructional practices. Their PD targeted "connecting these (science and math) disciplines" (Baxter et al., 2014, p. 102). The largest change reported by teachers was "how often students shared their strategies during mathematics instruction" (Baxter et al., 2014, p. 109). Mathematics instruction benefited from the science and math interdisciplinary PD. Implementing math performance tasks requires less training than implementing STEM cross-curricular PBL-type lessons, but still allows for problem solving experiences that may incorporate a cross-curricular context.

Common Core mathematics teacher professional development. We have seen the advent of a new list of ingredients that have emerged on the scene as important components to CCSSM teacher PD. Marrongelle et al. (2003) summarized some of these emerging components. Their summary included attention to discourse, student thinking, formative assessment, and CMTs. The NCTM lists in *Principles to Actions* (2014) eight topics as central to effective teaching and learning, including: establish mathematics goals to focus learning, implement tasks that promote reasoning and problem solving, use and connect mathematical representations, facilitate meaningful mathematical discourse, pose purposeful questions, build procedural fluency from conceptual understanding, support productive struggle in learning mathematics, and elicit and use evidence of student thinking. Of these topics, implementing tasks that promote reasoning and problem solving seem closely aligned with the goal of preparing students to solve cross-curricular, contextualized problems, or STEM problems. Tasks could take on a context outside of mathematics, but do not necessarily need a context at all to allow students to engage in problem solving. "Not all tasks that promote reasoning and problem solving have to be set in a context" (NCTM, 2014, p. 20). This is relevant as creating critical thinkers who solve real world problems will be important as the occupational landscape changes (Bowman, 2015; Noonan, 2017).

PD has been shown to move teachers away from siloed, standards-based instruction. In one PD research project, "a shift, from a simple focus on coverage of isolated standards and topics through disconnected lessons, to creating thematic and contextualized units that would encompass a more meaningful and interconnected web of concepts" was reported by teacher participants (Kazempur & Amirshokoohi, 2013, p. 302). Research examining PD has been criticized for not focusing on changes in teacher practice, and making recommendations that "are reasonable, but are supported by little more than anecdotal evidence" (Scher & O'Reilly, 2009, p. 216), rather than supported by student learning outcomes. While the implementation of math PD for teachers has been advocated, research suggests that more targeted, non-cross curricular, teacher PD may translate more favorably into higher student assessment scores than teacher PD that has a dual focus of math and science. Scher and O'Reilly (2009) concluded that "interventions that focus on either math only or science only tend to have larger impacts on student achievement than those that focus on both math and science" (p. 235). Yet, "the press to integrate mathematics and science comes from researchers, business leaders, and educators" (Baxter et al., 2014, p. 102).

Math task teacher professional development. PD implementations that focus on mathematical tasks are not uncommon (Boston, 2013; Foster & Noyce, 2004). Mirza and Hussain (2014) engaged in an action research project investigating the use or rich math tasks which ultimately served as teacher PD as they investigated "the impact on learning and motivation by using rich tasks in the presence of cooperative learning" (p. 36) and found that rich tasks provided students with the following:

a. Motivation and enthusiasm.
b. Different levels of challenge despite the learner's level.
c. Extension opportunities to those who needed and demanded them.
d. Opportunities for collaboration and discussion in the form of group work.
e. Encouragement to learners to develop confidence and independence and to become critical thinkers. (p. 36)

Mirza and Hussain (2014) documented Mirza's own teacher perceptions of the benefits of using CMTs. Self-realization through action research, in this instance, served as a form of teacher PD. Others have discussed the PD that emerges from teachers doing mathematics tasks, discussing mathematics tasks, and reviewing student responses (Foster & Noyce, 2004). Foster and Noyce (2004) posited that teacher collaboration that is focused on CMTs leads to improvement in instruction, as a result of increases in math content knowledge. "Uncovering these gaps in teachers' content knowledge is central to improving instruction" (Foster & Noyce, 20014, p. 373). Boston (2013) showed that math PD specifically implemented to encourage the use of CMTs and also aimed at supporting teachers in the maintenance of a high-level of rigor during the implementation phase, resulted in just that. “Teachers in the project significantly increased the use of cognitively challenging instructional tasks in their classrooms and their ability to implement these tasks in ways that maintained students’ opportunities for thinking and reasoning” (Boston, 2013, p. 8). Researchers have commented on math tasks ability to focus an individual or group on a mathematical concept (Bowman, 2015; Foster & Noyce, 2004). “The use of instructional tasks has been a focus of teacher professional development” (Davis et al., 2014, p. 173). As such, PD opportunities that are centered on mathematics tasks continue to be a focus of teacher PD that moves educators’ instructional practices forward.

Teachers’ Perceptions

Teachers’ perceptions are often studied through qualitative research. For instance, Bruce-Davis et al. (2014), examined STEM teachers’ perceptions of curricular and instructional strategies and practices using a qualitative lens. Their research, from within STEM high schools, indicated that teachers participated in professional learning communities, but they did not link teachers’ perceptions directly to PD. They did indicate, however, that “quantitative studies, such as survey research of curricular and instructional practices at STEM high schools, will also yield useful exploratory information” (Bruce-Davis et al., 2014, p. 297).

All math teachers are ideally STEM teachers. High school math teachers’ perceptions of CMTs could be studied further through quantitative analysis. The effects of teacher PD as well as other characteristics of the instructional environment on teachers’ perceptions of all phases of implementing mathematical tasks is of specific interest in this work. Does teacher PD and other characteristics, such as the degree to which digital/electronic resources are used or read, the perceived rigor of the CCSSM, or the frequency of teacher collaboration account for the variability in teachers' perceptions of enacting challenging performance tasks under the CCSSM?

Using a quantitative lens (factor analysis), Davis et al. (2014) found PD to be a factor contributing to the variability in mathematics' teachers' perceptions of the Common Core mathematics standards at the middle school level. They indicated that other populations, such as elementary and high school teachers, should be studied. Would PD be a statistically significant underlying factor which explains the variability in teachers' perceptions of CMTs at the secondary level of instruction? Davis et al. (2014) did not include survey items addressing teachers' perceptions of math performance tasks as a component of Common Core mathematics standards based instruction. Variability in teachers' perceptions of math performance tasks as an integral approach to teaching the CCSSM may be related to the instructional environment.

This study sought to quantitatively explore the role of teacher PD and other characteristics of the instructional environment on teachers' perception of CMTs. Exploring teachers' perceptions is common. Calling for research around "how teachers respond to the CCSSM and the professional development related to the CCSSM" is not a new proposal (Marrongelle et al., 2013, p. 208). Hill and Ball (2004) indicated that "typically, professional development studies" ask "teachers to report on the extent of perceived learning or change in practice as a result of the professional development encounter" (p. 347). Teachers' perceptions of the specific CCSSM standards that require additional instructional focus have been favorably correlated to their students' assessment data (Bostic & Matney, 2013). This provides credence to some PD practitioners' exercise of assessing teachers' perceptions of PD needs prior to designing PD sessions.

Are instructional environment characteristics factors that are found to affect teachers' perceptions of the CCSSM also characteristics that explain the variability of teachers' perceptions of CMTs? For instance, does readily available digital access to math performance tasks in teachers' instructional environments play a role in their perceptions? Davis et al. (2014) did not find primary textbooks to play a role in explaining variability in middle school teachers' perceptions of the CCSSM. Rather, the "extent to which digital/electronic curriculum resources are used" (Davis et al., 2014, p. 20) emerged as a factor that explained variability in teachers' perceptions of the CCSSM. The lists below show the factors identified by Davis et al. (2014) as factors in teachers' perceptions of the CCSSM as collected from two different surveys. From Survey 1 the following five factors were identified:

Factor 1. Extent to which digital/electronic curriculum resources are used.

Factor 2. Degree to which PD is supported.

Factor 3. Level of required CCSSM mathematical processes and perceived rigor.

Factor 4. Extent to which digital/electronic curriculum resources are read.

Factor 5. Frequency of teachers' collaborative instructional planning.

From Survey 2 the following five factors were identified:

Factor 1. Extent to which digital/electronic curriculum resources are used.

Factor 2. Extent to which CCSSM requires mathematical processes.

Factor 3. Extent to which state assessment and teacher evaluation influence classroom practices.

Factor 4. Extent of district PD support and CCSSM familiarity.

Factor 5. Extent of teacher familiarity with and preparation for the CCSSM.

Factor 6. Level of perceived rigor within the CCSSM.

Factor 7. Extent of alignment among state SBAC assessments, CCSSM, and classroom assessments.

Survey research investigating secondary mathematics teachers' perceptions of mathematical tasks under the CCSSM was not located in my review of the literature. Literature related to CMTs and implementation of the Common Core mathematics standards was reviewed with an additional focus on the MTF and mathematics PD.

This study employed survey items similar to those above in studying teachers' perceptions of CMTS under the CCSSM. Educators tend to teach to the test (Guerrero, 2014; Schoenfeld, 2015), so variability in teachers' perceptions about the use of CMTS may be explained in part by their perceptions of assessments.

Chapter Summary

Researchers have suggested that the MTF is a tool that has successfully contributed to teachers' acquisition of math task knowledge (Boston & Smith, 2009), a form of pedagogical content knowledge, as the MTF gives teachers a tool by which to reflect on practice. Boston and Smith (2009) specifically used the MTF and were able to garner measurable increases in the number of CMTs selected with measurable improvements also in teachers' ability to implement them.

This study aimed to specifically look at environmental variables that may be factors in teachers' perceptions of using CMTs, taking the three stages of the MTF into consideration, and acknowledging that at each phase (select, setup, and optimize), specific math task knowledge is required. As such, environmental variables that might be related to one or more of the stages of the MTF were considered. More needs to be known about how secondary teachers perceive the use of CMTs in order to support their implementation through each stage of the MTF, alongside standards that teachers seem to perceive as more rigorous than prior standards, yet are still very broad. Research has shown that the use of challenging math tasks has resulted in higher student gains on achievement tests and more favorable student dispositions. Understanding teachers' perceptions will allow for a more well-informed collective effort to encourage the effective use of CMTs. Armed with appropriate math task knowledge and a better understanding of teachers' perceptions, educators will be better positioned to help move students through the STEM pipeline.

Chapter 3: Methodology

This study aimed to examine the degree of fit of a proposed model with a septenary factor structure hypothesized to model secondary mathematics teachers' perceptions of Challenging Math Tasks (CMTs). The first section of this chapter introduces the methodology that was employed in this study, followed by a description of the target population to whom the web survey was distributed. In the instrumentation section, the indicator variables for each construct are provided with explanation. The data analysis section of the paper coves data screening, general setup of the structural equation model in AMOS, and a discussion of model fit. After the data analysis section, convergent and divergent validity is discussed before addressing the limitations of the study.

Research Questions and Hypotheses

The two primary research questions of this study were:

1. Is a proposed septenary factor structure a good fit for understanding the variability in teachers' perceptions of CMTs?
2. Do teachers perceive the CMTs available to them to be well-aligned to the common core state standards?

Seven hypotheses follow that are related to the seven factors that were under investigation:

H_1. Perceived access to CMTs is a factor that explains the variability in teachers' perceptions of CMTs.

H_2. Teachers' understanding of the importance of CMTs as a means to practice the MPS is a factor that explains the variability in teachers' perceptions of CMTs.

H_3. Site-based assessment and teacher evaluation is a factor that explains the variability in teachers' perceptions of CMTs.

H_4. Site-based CMT Professional Development (PD) is a factor that explains the variability in teachers' perceptions of CMTs.

H_5. Teachers' familiarity and level of self-reported preparedness to implement CMTs is a factor that explains the variability in teachers' perceptions of CMTs.

H_6. The amount of site-based collaboration around CMTs is a factor that explains the variability in teachers' perceptions of CMTs.

H_7. Teachers' characterization of students is a factor that explains the variability in teachers' perceptions of CMTs.

Methodology

This study examined a hypothesized model underlying a single latent dependent variable: high school mathematics teachers' perception of CMTs under the Common Core State Standards of Mathematics (CCSSM). It was assumed that there is a causal relationship between high school mathematics teachers' perception of CMTs under the CCSSM and their instructional environment, as well as teachers' pedagogical content knowledge. Teachers' perceptions of CMTs were examined using a non-experimental survey design. It would not be possible to control the multiple and multi-faceted independent variables hypothesized to effect teachers' perceptions of CMTs, so a non-experimental approach was necessary.

Constructs found to be factors in middle school mathematics educators' perceptions of the CCSSM by Davis et al. (2014) using Exploratory Factor Analysis (EFA) were hypothesized as constructs of interest to be included in the survey instrument designed by the researcher. These included curricula, MPS, state assessments and teacher evaluation, PD, and familiarity and preparation. Additionally, student characterization was added as a potential construct. Student characterization was not found to be an underlying factor in teachers' perception of the CCSSM in the survey research undertaken by Davis et al. (2014); however a review of the research emphasized that "particularly relevant are teachers' instructional realities" (Johnson et al., 2016, p. 182), of which student behavior/disposition and academic preparedness are considered.

Quantitative Confirmatory Factor Analysis (CFA) was employed to determine the appropriateness of the selected factor structure. Confirmation or rejection of the validity of the underlying constructs delineated in the designed instrument was possible using CFA. Additionally, less statistically sophisticated analysis of the survey data included simple descriptive statistics including means, standard deviations, and frequency distributions.

Description of participants. The target population under study was high school mathematics teachers, who presumably were at least 18 years of age, given the time required to ascertain appropriate credentialing (i.e., a 4-year college degree). Teachers who teach intervention or core classes in mathematics at the high school level were targeted. This study also targeted teachers with sufficient credentials to teach high school mathematics but the population under study was not constrained to mathematics teachers with a single subject teaching credential in mathematics. For example, in California, there are multiple credential options that might allow a teacher to be placed in a California mathematics classroom (Clark, 2009). The consent form used to communicate to teacher participants the voluntary nature of the survey and some basic information about the study can be found in Appendix A.

The sample size required for CFA is related to the number of constructs (factors) and variables in the design. Myers, Ahn, and Jin (2011) summarized rules of thumb for sample size in CFA indicating $N \geq 200$, $N/p \geq 10$, and $N/q \geq 5$, where N is the sample size, p is the number of variables (i.e., survey items) in the model, and q represents the number of proposed factors in the hypothesized model. With seven factors and three questions per factor (21 survey questions), a sample size of $N \geq 210$ is required by the second rule of thumb. Hair, Anderson, Tatham, and Black (1998) indicated a minimum ratio of at least five participants per survey question, noting that "the absolute minimum sample size must be at least greater than the number of covariances or correlations in the input data matrix" (p. 604). Assuming all the variables are retained, a minimum of 105 participants would be necessary using this minimum ratio. The rule of thumb is "10 to 20 participants for each independent variable" (Keith, 2015, p. 203). This higher limit would require 420 participants. Given the more exploratory nature of this study, the goal was to pursue a sample size of at least 10 participants per survey question or a minimum of 210 participants. A sample size of 102 was achieved.

Data collection. Outside organizations that support teachers of mathematics were contacted and asked to invite their secondary mathematics teachers to participate in the online CMT survey. The survey was created with SurveyMonkey and distributed via email. The intent was to request an opportunity to share this research with secondary teachers and invite survey participation directly when possible to encourage participation. Most of the organizations used to solicit participants were those with which the researcher had already developed a relationship as a member, employee, or provider of mathematics PD. However, an invitation to participate was also emailed directly to secondary mathematics teachers. Organizations with access to secondary teacher populations that were contacted for participation can be found in the list below.

- Monterey Bay Area Math Project
- San Joaquin County Office of Education

- San Joaquin County Teachers' College
- Silicon Valley Math Initiative
- College Preparatory Mathematics
- Asilomar Mathematics Conference 2018
- Khan Academy
- Pajaro Valley Unified School District
- Stockton Unified School District

The survey required teachers to make a selection for each question to ensure a complete data set. In addition to questions that pertained to the constructs being explored, general demographic data were collected including sex, age, race, number of years teaching mathematics, highest degree, state of employment, best estimate for the proportion of students receiving free and reduced lunch at your school, best estimate for the percent of students classified as English language learners across all classes taught).

Instrumentation. The online survey consisted of seven constructs with three variables per construct, for a total of 21 questions. Below is a list of the proposed constructs with each constructs' associated questions (see Figure 4). Respondents were asked to answer each question indicating whether they strongly disagree, disagree, agree, or strongly agree. A search for a valid instrument that had been shown to be reliable and valid in measuring teacher perception of CMTs was undertaken by searching the Internet and University of the Pacific library resources. One applicable instrument, the Mathematics Teaching Practice Inventory (MTPI) (Huinker & Hedges, 2015), was located which contained a construct that was found to measure teachers' perception of their implementation of CMTs, which is effectively a measure of a teachers' familiarity and preparation from an experiential perspective. As such, the original intent of the familiarity and preparation section of the survey was modified to take on a more realistic depiction of familiarity and preparation through the lens of first-hand experience. Only a selection of the indicators from the MTPI task construct were retained (see Figure 4).

Construct 1: Curriculum

- My district adopted primary curriculum and supplemental resources include CMTs aligned to the CCSSM that afford students an opportunity to engage in problem solving.
- My district provides a selection of CMTs that can be solved in multiple ways.
- My district introduces educators to CMTs, which may come from online resources, that allow students to use multiple representations (i.e. tables, graphs, and equations) when solving.

Construct 2: Mathematical Practices

- CMTs give students an opportunity to make sense of problems and persevere in solving them.
- CMTs are an important tool which allow students an opportunity to engage in the MPS.
- Procedural exercises do not allow students sufficient practice with the MPS but CMTs do.

Construct 3: Assessment and Teacher Evaluation

- State SBAC assessments contain CMTs which encourages me to use CMTs.
- Teacher evaluation encourages me to use CMTs.
- District level testing encourages me to use CMTs.

Construct 4: Professional Development

- District PD has addressed the importance of maintaining the cognitive demand as students work CMTs.
- My district has provided PD opportunities where I am able to work CMTs alongside my colleagues.
- District PD has addressed implementing (facilitating) CMTs with students (e.g., supporting productive struggle, classroom management, collaborative groupings, academic discourse).

Construct 5: Familiarity and Preparation (Implementing CMTs)

- I encourage my students to use varied approaches and strategies to solve math problems.
- I purposefully select math tasks that build on and extend student learning from our previous work.
- I give my students math tasks on a regular basis that require a high level of cognitive demand.

Construct 6: Collaboration

- When planning for instruction, I regularly discuss CMTs with colleagues.
- I regularly discuss the scaffolding of CMTs with colleagues.
- My district prioritizes collaboration time to score students' work on CMTs.

Construct 7: Student Characterization

- Student behavior/disposition effects the regularity with which you use CMTs.
- Students' prior academic preparation effects the usefulness of CMTs.
- Students' disposition towards engaging in productive struggle effects the regularity with which you use CMTs.

Figure 4. Instrumentation constructs.

Data Analysis

Default model analysis. The collected data were imported into SPSS 21.0 and were then screened. Data screening consisted of a visual inspection of the data to identify data-entry patterns that are highly improbable, such as responses with “Agree” for all survey items. After importing the data to SPSS, the data could be accessed from within AMOS. The SPSS variable data were linked to the hypothesized structural equation model which was constructed graphically within AMOS. The measurement theory proposed to explain teachers’ perceptions of CMTs were represented in AMOS using a path diagram (Hair, Black, & Babin, 2010). Rectangles representing the survey indicator variables were linked to the latent constructs hypothesized: curriculum, MP, assessment and evaluation, PD, familiarity and preparation, collaboration, and student characterization, as appropriate. An error term was associated with each indicator variable. The measurement theory was represented such that construct indicators were associated with the appropriate latent variables and construct correlations could be indicated, but at the expense of the validity of the default model (Hair et al., 2010). An arrow from each latent variable to the constructs’ indicator variables were drawn, indicating that the latent variable was a function of the indicator variables. In this same respect, the latent variable, teachers’ perceptions of challenging math tasks, is hypothesized to be a function of the seven hypothesized constructions.

After linking the variable data to the AMOS structural equation model, one can proceed with CFA, which requires obtaining the desired statistical outputs and interpreting those outputs for the default model’s statistical outputs. The standardized factor loadings should be statistically significant. Standardized factor loadings allow loadings to be comparable between factors. When creating a path model within AMOS, one of the factor loadings was set to 1 (Hair et al., 1998), so loadings would be comparable.

The default model analysis assumed correlation between factors, as well as within-factor correlation (correlation between the variables). Another assumption was multivariate normality, if when not adhered to can lead to a failure of the chi-squared goodness of fit test (Ho, 2014), which is discussed in the next section.

Model fit.

The χ^2 goodness of fit test. The χ^2 goodness of fit test was used to help determine whether the researcher should reject the null hypothesis that the underlying data fits the proposed model. "Chi-square (χ^2) is the most commonly reported measure of fit" (Keith, 2015, p. 294). In AMOS, χ^2 was reported. AMOS also reported the degrees of freedom that are necessary to determine whether to accept or reject the null hypothesis. The degrees of freedom can be computed by first determining the number of moments in the correlation matrix. The number of moments is related to the number of indicator variables (survey questions). The number of moments is computed using Equation 1 below, where p is the number of moments and N is the number of variables.

$$p = \tfrac{1}{2}(N + 1)(N) \qquad (1)$$

The number of parameters in the model (regression weights plus error terms), q, should be less than the moments computed. The degrees of freedom (df) are computed by subtracting the number of parameters from the number of moments. The degree of freedom is a measure of the parsimony of the model and so it is preferred that the df be large (Keith, 2015). Equation 2 for computing the degrees of freedom.

$$\mathrm{df} = p - q \qquad (2)$$

For the default model, the number of moments, p, is ½(22)(21) = 231. Subtracting the number of parameters (regression weights plus error terms) from the moments, gives the degree of freedom, 231-50, 208, in this instance. In AMOS, the degrees of freedom are reported as DF, λ^2 is reported as CMIN, and NPAR represents the number of parameters, q, being estimated (Keith, 2015). A comparison of CMIN to the critical λ^2 value allows the rejection of the null hypothesis in favor of the alternative hypothesis, or the acceptance of the null hypothesis and the rejection of the alternative hypothesis. The CMIN (from AMOS) should be less than the critical λ^2 value, with probability and degrees of freedom considered, if the null hypothesis is to be accepted.

RMSEA. In addition to the chi-square goodness of fit test, the reported root mean-square error of approximation (RMSEA) indicates the quality of fit additionally, or badness of fit (Ho, 2014), with zero representing best fit. RMSEA should be no greater than .1 (Ho, 2014), with a good approximate fit being indicated with a RMSEA value below .05 (Keith, 2015). Table 2 below summarizes fit statistics used to explore model fit.

Table 2

Model Fit Values

Test	Critical Value	Reference
Chi-square goodness of fit (CMIN in AMOS)	$\chi^2(N, df)$	(Ho, 2014)
CMIN/df	Should be between 1 and 5.	
RMSEA Root Mean Square of Approximation	< .05 (good) .05 to .08 (acceptable fit) .08 to 0.10 (mediocre fit) > 0.10 (poor fit)	(Keith, 2015) (Ho, 2014)
SRMR Standardized Root Mean Square Residual	>.06	(Keith, 2015)
AIC Akaike Information Criterion	Smaller is better if comparing competing model.	(Keith, 2015)
Incremental Fit Indices		
TLI Tucker-Lewis Index	> .9	(Ho, 2014)
CFI Comparative Fit Index	> .9	(Ho, 2014)

As model modifications were made, a table of model statistics including $\Delta\lambda^2$ and Δdf were generated to characterize each models' fit. In addition, path diagrams were provided for each model along with descriptions of modifications made to the default model. The final step of the analysis was to decide whether it would be appropriate to accept or reject the null hypothesis. It may be possible to accept the null hypothesis with modifications to the default model.

Validity

Modification indices and default model modification. If the null hypothesis for the default model is rejected, the modification indices may suggest an alternative model with a lower chi-square goodness of fit value with higher loadings that are more appropriate by allowing error terms to correlate. However, freeing paths should be avoided in most CFAs (Hair et al., 2010). Evidence of error covariance (between error correlations or within-error correlation), as well as inter-construct correlation, pose threats to construct validity. Two different types of threats to validity, convergent validity and divergent validity, are explored more specifically.

Convergent validity. Construct items should be shown to “share a higher degree of variance in common” (Hair et al., 2010, p. 669). Construct validity measures help determine the extent to which an instrument’s construct items successfully measure a theoretical construct. Four values: Cronbach’s alpha, factor loadings, average variance extracted (AVE), and construct reliability (CR), are commonly used to test convergent validity (Hair et al., 2010). Rules of thumb for establishing construct validity using these four measures and the source of the rules of thumb to be utilized are listed in Table 3.

Table 3

Construct Reliability Values

Test	Critical Value(s)	Reference
Cronbach’s Alpha (taken from SPSS) Note: Also called coefficient alpha.	>.7 (Good) > .6 and < .7 (Acceptable)	(Hair et al., 2010)
AVE (Calculated in Excel) Average Variance Extracted	> .5	(Hair et al., 2010)
CR (Calculated in Excel) Construct Reliability	> .7 > .6 and < .7 (Acceptable)	(Hair et al., 2010)

The average variance extracted (AVE) (see Equation 3) and the construct reliability (CR) (see Equation 4) will both be calculated for each construct in Excel using the formulas below, the standardized factor loadings, L_i, and the number of items per construct, where the error variance is calculated from $e_i = 1 - L_i^2$.

$$AVE = \frac{\sum_{i=1}^{n} {L_i}^2}{n} \tag{3}$$

$$CR = \frac{(\sum_{i=1}^{n} L_i)^2}{(\sum_{i=1}^{n} L_i)^2 + \sum_{i=1}^{n} e_i} \tag{4}$$

The factor loadings, L_i, called standard regression weights in AMOS, were extracted.

Divergent validity. The test for divergent validity requires that the AVE estimates determined for each construct during the test for convergent validity are greater than the square of the standardized interconstruct correlations, which are provided by AMOS in text output (Hair et al., 2010). Divergent validity ensures that correlation within a construct is stronger than across constructs. What is described is, in essence, the Fornell-Lacker criterion.

Limitations

The original eight factors considered as potential constructs were taken from a study of teachers' perceptions of the CCSSM that used EFA, not of teachers' perceptions of CMTs under the CCSSM. Therefore, the hypothesized constructs composition in this study was originally theorized using inductive logic. Inductive logic makes broad generalizations from specific observations. Given that factors had already been identified that affect teachers' perceptions of the Common Core mathematics standards at the site level, these same factors could affect teachers' perceptions of CMTs under the CCSSM at the site level.

After a review of the literature on CMTs, deductive logic was employed. Two factors from the Davis et al. (2014) study were eliminated. One of the original factors from Davis et.al. (2014), level of perceived rigor within the CCSSM, was not used as a construct since by definition CMTs are rigorous. In addition, the EFA factor: alignment among state SBAC assessments, CCSSM, and classroom assessments, was not considered relative to CMTs as there was evidence to suggest that there was a lack of CMTs aligned to the CCSSM. Instead, the first construct was modified to accommodate this factor. The first construct relates to the accessibility of CMTs aligned to the CCSSM relative to site-based curriculum. A construct was added, teacher's characterization of students, as the literature seemed to indicate that student characterization may play a role in teachers' perceptions of CMTs. The conceptualization of constructs should be reviewed by other experts and focus groups so that the indicators might be refined. So, while expert review and focus group review has yet occurred, attending to the review of constructs and indicators is still attainable.

Another limitation of this study is the brevity of the survey. A short survey was elected as it would require less time for respondents to complete. This brevity is a limitation of the study design as short surveys are normally less reliable (Morgado, Meireles, Veves, Amaral, & Ferreira, 2017). Finally, in terms of scale, the proposed items do not attend to a balance between items that access both positive and negative beliefs. The researcher's intent was to make the survey as easy to respond to as possible; however, with expert input, the decision to create a balance between positively-worded and negatively-worded indicators may be pursued, which could make the instrument more reliable in theory.

Chapter Summary

In the words of Doyle (1988), "teachers affect tasks, and thus students' learning" (p. 169). It was posited that high school teachers with a positive perspective of CMTs are more likely to allow all students the opportunity to engage in practicing authentic problem solving via CMTs, affording students an opportunity to hone their mathematical habits of mind. In other words, teachers who have a positive perspective of CMTs seem more likely to afford students an opportunity to learn from rich tasks. This work hypothesized a causal relationship between teachers' perceptions of CMTs and their educational learning communities' site-based characteristics (i.e., curriculum, assessment and evaluation, PD, and collaboration), teachers' own CMT PCK (i.e., knowledge of the interplay between CMTs and MPS, familiarity with CMTs, and preparation towards using CMTs), as well as teachers' characterization of students.

Schoenfeld (2015) optimistically discussed how new CCSSM state test consortiums' (Smarter Balanced Assessment Consortium and The Partnership for Assessment of Readiness for College and Careers) math tasks might be different with the implementation of the Common Core mathematics standards, incorporating assessment of the mathematical practices alongside

CMTs. He indicated that as a result of WYTIWYG (what you test is what you get), teachers' instructional practices might shift. Will the variability in teachers' perceptions of CMTs be tied to testing? This research explored student assessment and evaluation as a construct under investigation, as well as teacher evaluation. Additionally, the phasing in of the CCSSM before curriculum resources (i.e., CMTs) were made available (Dossey et al., 2016) may have translated into a less favorable teacher perception of CMTs. Sites variability in the availability of CMTs aligned to the CCSSM may affect teachers' perception of CMTs and was also a construct under investigation. Teachers' perceptions of CMTs may also be affected by their overall level of familiarity and preparedness to use CMTs and the PD and collaboration taking place at their site. Realistically, these last three items would seem to cross correlate. Teachers' characterization of students was also considered.

Structural equation modeling (SEM), more specifically CFA, was utilized to investigate the legitimacy of the seven underlying factors hypothesized to explain teachers' perceptions of CMTs. CFA is a type of SEM that "is applied to test the extent to which a researcher's *a priori*, theoretical pattern of factor loadings on pre-prescribed constructs represent the actual data" (Hair et al., 2010, p. 671). This work aimed to test whether measurement data fit the model identified *a priori* to explain variability in teachers' perception of CMTs.

Chapter 4: Data Analysis

Background Descriptive Statistics

The first section of the online survey collected participant consent and the second section provided background information about the study that included key terms, acronyms, definitions, and a brief narrative. The third section of the survey instrument contained seven items designed to capture the demographics of the sample of high school math teachers who participated. All subjects participated voluntarily and Table 4 shows a summary of their demographics.

Table 4

Participant Demographics

Demographic Categories	n	Percentage
Gender		
Male	46	46.46%
Female	53	53.54%
Age		
21 to 29 years	18	18.00%
30 to 39 years	21	21.00%
40 to 49 years	23	23.00%
50 to 59 years	22	22.00%
60 years or older	16	16.00%
Ethnicity		
White or Caucasian	74	74.75%
Black or African American	2	2.02%
Hispanic or Latino	10	10.10%
Asian or Asian American	9	9.09%
American Indian or Alaska Native	0	0%
Native Hawaiian or other Pacific Islander	2	2.02%
Another race	2	2.02%
High School Mathematics Teacher		
Yes	95	94.06%
No	6	5.94%
Years Teaching		
Less than 1 year	3	2.97%
At least 1 year but less than 3 years	11	10.89%
At least 3 years but less than 5 years	11	10.89%
At least 5 years but less than 10 years	12	11.88%
10 or more years	64	63.37%
Highest Degree		
Bachelor's	34	33.66%
Master's	62	61.39%
Doctoral	5	4.95%
Current State in Which Participant Teaches		
Arizona	2	1.98%
California	76	75.25%
Colorado	2	1.98%
Illinois	2	1.98%
Indiana	1	.99%
Kentucky	1	.99%
Massachusetts	2	1.98%
Minnesota	1	.99%
Missouri	1	.99%
New Jersey	3	2.97%
New York	4	3.96%
North Carolina	1	.99%
Oregon	1	.99%
Pennsylvania	1	.99%
South Carolina	1	.99%
Washington	2	1.98%

Note. N = 102.

The majority of this study's participants were White or Caucasian (75%) and female (53%), which parallels national statistics, with a reported 81.5% of secondary mathematics teachers being White and 52.6% of secondary mathematics teachers being female (NCES, 2016). This study's Asian or Asian American sample (9.09%) was greater than the national average of 4.1%, while the Black or African American teachers were underrepresented in this study at 2.02%. Nationally, the percentage of Black or African American mathematics teachers at the high school level is 6.4%, as reported by the NCES in 2016. Hispanic or Latino mathematics teachers were also overrepresented in this study at 10.10% compared to the national average of 6.1% (NCES, 2016). However, it is pertinent to note that the four major ethnic groups who teach secondary mathematics as presented by the NCES were also the four major groups represented in this study.

In terms of the age of the teachers in this study, this study seemed to attract a larger percentage of older teachers compared to the age distribution reported by the NCES (2016). Sixteen percent of the teachers who participated in this study where 60 years or older compared to the national average of 8.3% (NCES, 2016). Additionally, the NCES (2016) reports that 28% of secondary mathematics teachers are between 30 and 39 years of age, while only 21% of the participants in this study were between 30 and 39 years of age. However, both the NCES age distribution and the age distribution in this study have an approximate normal distribution.

Like the secondary mathematics teacher statistics reported by the NCES, which reports that 54.6% of secondary mathematics teachers currently have 10 or more years teaching experience, more than half (63.92%) of the participants in this study reported having 10 or more years of teaching experience. The NCES (2016) reported that a majority (60.67%) of secondary mathematics teachers have a master's degree, similarly (61.39%) of the participants in this study reported a master's degree as their highest level of educational achievement. Seventy-five percent of the participants teach in California; the remaining 25% of the survey respondents teach in 15 other states with no more than four teachers per state.

The third section of the online survey asked teachers two questions about the student population that they serve. A summary of the student characterization data collected is displayed in Table 5.

Table 5

Student Demographics as Reported by Teachers

Select Demographics	n	Percentage
Best Estimate of Students Classified as English Language Learners		
10% or below	55	54.46%
Between 11% and 50%	38	37.62%
51% and above	8	7.92%
Best Estimate of Proportion of Students Receiving Free and Reduced Lunch		
Below 15%	13	12.87%
Between 16% and 29%	19	12.87%
Between 30% and 49%	18	18.81%
Between 50% and 69%	21	20.79%
Above 70%	30	29.70%

Almost half of the participants (45.54%) reported that English language learners comprised 11% or more of their student population. Under 10% (7.92%) of teachers reported that more than half of their students were English language learners. Over half of the teacher participants, however, indicated that they serve student populations where over half of the students receive free or reduced lunch.

Participant Responses Descriptive Statistics

The third section of the online survey also captured teachers' perceptions of site-based variables relative to their learning communities, as well as their personal perceptions relative to CMT instruction. Summaries of the teacher perception survey data collected using a 4-point Likert-type scale are displayed in Tables 6 through 12. Additionally, a weighted average was computed for each survey item using the formula below, with responses being numbered (1) strongly disagree to (4) strongly agree.

$$\frac{\sum_{i=1}^{4} i * (number\ of\ students\ with\ response\ i)}{total\ number\ or\ student\ responses\ (N)} \tag{5}$$

Curriculum.

Table 6

Curriculum

Survey Item	Strongly Disagree	Disagree	Agree	Strongly Agree	Weighted Average
My district adopted primary curriculum and supplemental resources include challenging math tasks aligned to the CCSSM that afford students an opportunity to engage in problem solving.	8.82% n = 9	20.59% n = 21	46.08% n = 47	24.51% n = 25	2.86
My district provides a selection of challenging math tasks that can be solved in multiple ways.	8.82% n = 9	31.37% n = 32	47.06% n = 48	12.75% n = 13	2.64
My district introduces educators to challenging math tasks, which may come from online resources, that allow students to use multiple representations (e.g., tables, graphs, and equations) when solving.	4.90% n = 5	30.39% n = 31	51.96% n = 53	12.75% n = 13	2.73

Note. N = 102.

A majority, 71%, of teachers agreed or strongly agreed that their primary curriculum and supplemental resources included Challenging Math Tasks (CMTs) aligned to the CCSSM. However, a smaller percentage (60%) of teachers agreed or strongly agreed that the district provided a selection of CMTs that could be solved in multiple ways, with a higher percent of teachers (65%) indicating that their district introduces educators to CMTs, which may come from online resources, that allow students to use multiple representations (e.g., tables, graphs, and equations) when solving. It may be that districts are introducing teachers to CMTs, but not supplying teachers with a selection of such tasks. Over a quarter of the teachers strongly disagreed or disagreed that their districts were instrumental in providing teachers with CMTs through their primary curriculum (29%). An even larger percentage (40%) of teachers disagreed or strongly disagreed that their district provides a selection of CMTs that can be solved in multiple ways.

Mathematical practices.

Table 7

Mathematical Practices

Survey Item	Strongly Disagree	Disagree	Agree	Strongly Agree	Weighted Average
Challenging math tasks give students an opportunity to make sense of problems and persevere in solving them.	0.00% n = 0	.98% n = 1	45.10% n = 46	53.92% n = 55	3.53
Challenging math tasks are an important tool which allow students an opportunity to engage in the (Mathematical Practice Standards) MPS.	.98% n = 1	0.00% n = 0	43.14% n = 44	55.88% n = 57	3.54
Procedural exercises do not allow students sufficient practice with the MPS, but challenging math tasks do.	3.92% n = 4	25.49% n = 26	48.04% n = 49	22.55% n = 23	2.89

Note. N = 102.

Teachers almost unanimously agreed or strongly agreed that CMTs give students an opportunity to make sense of problems and persevere in solving them (99%) and that CMTs are an important tool which allow students an opportunity to engage in the MPS (99%). Teachers clearly see CMTs as important for students to practice the habits of mind associated with effective problem solving, the MPS. Twenty-nine percent of teachers, however disagreed or strongly disagreed with the third indicator for the mathematical practices construct, which stated that procedural exercises do not allow students sufficient practice with the MPS, but that CMTs do. This seems to indicate that some teachers see both CMT and more procedural math assignments as mediums by which educators offer students an opportunity to hone math practices.

Assessment and teacher evaluation.

Table 8

Assessment and Teacher Evaluation

Survey Item	Strongly Disagree	Disagree	Agree	Strongly Agree	Weighted Average
State SBAC assessments contain challenging math tasks which encourages me to use CMTs.	4.90% n = 5	25.49% n = 26	57.84% n = 59	11.76% n = 12	2.76
Teacher evaluation encourages me to use challenging math tasks.	5.88% n = 6	33.33% n = 34	52.94% n = 54	7.84% n = 8	2.63
District level testing encourages me to use challenging math tasks.	14.71% n = 15	40.20% n = 41	38.24% n = 39	6.86% n = 7	2.37

A majority of the teachers agreed or strongly agreed that state SBAC assessments contain CMTs that encourage them to be used (69.6%). However, more than a quarter (30%) of the teachers indicated that they strongly disagreed or disagreed with the statement that state SBAC assessments contain CMTs that encourage them to be used. Even with the impetus of state testing to make instructional changes, like using CMTs, some teachers do not seem to feel that CMTs on state tests necessitates the use of CMTs in classrooms. Teachers indicated that they agreed or strongly agreed (61%) that teacher evaluation encourages them to use CMTs, while a lesser percent (45.1%) agreed or strongly agreed that district-level testing encourages them to use CMTs. This seems to indicate that, in particular, district-level testing interferes with teacher use of CMTs at this time. Fifty-five percent of teachers strongly disagreed or disagreed that district-level testing encourages them to use CMTs.

Professional development.

Table 9

Professional Development

Survey Item	Strongly Disagree	Disagree	Agree	Strongly Agree	Weighted Average
District professional development has addressed the importance of maintaining the cognitive demand as students work challenging math tasks.	6.86% n =7	23.53% n =24	51.96% n =53	17.65% n =18	2.80
My district has provided professional development opportunities where I am able to work challenging math tasks alongside my colleagues.	10.78% n =11	27.45% n =28	42.16% n =43	19.61% n =20	2.71
District professional development has addressed implementing (facilitating) challenging math tasks with students (e.g., supporting productive struggle, classroom management, collaborative groupings, and academic discourse).	9.80% n =10	26.47% n =27	50.98% n =52	12.75% n =13	2.67

Note. N = 102.

Since PD implementations that focus on mathematical tasks is not uncommon (Boston, 2013; Foster & Noyce, 2004), this portion of the survey helped to quantify the prevalence of district-initiated math task PD. A common PD model in mathematics education is to have teachers work problems together. Yet, about 36% of the survey participants strongly disagreed or disagreed with the statement that their district provided them with PD opportunities where they were able to work CMTs alongside their colleagues. Inviting teachers to work CMTs alongside one another is among the easier forms of math PD to employ and would be an appropriate mitigation for this result.

District guidance with facilitation of CMTs was also addressed in the survey. When asked if district PD has addressed implementing (facilitating) CMTs with students (e.g., supporting productive struggle, classroom management, collaborative groupings, and academic discourse), 38% strongly disagreed or disagreed. In addition, 30% strongly disagreed or disagreed that district PD addressed the importance of maintaining cognitive demand as students work CMTs.

Familiarity and preparation (implementing CMTs).

Table 10

Familiarity and Preparation (Implementing CMTs)

Survey Item	Strongly Disagree	Disagree	Agree	Strongly Agree	Weighted Average
I encourage my students to use varied approaches and strategies to solve math problems.	0.00% 0	1.96% 2	45.10% 46	52.94% 54	3.51
I purposefully select math tasks that build on and extend student learning from our previous work. *(n = 101)	0.00% 0	3.96% 4	56.44% 57	39.60% 40	3.36
I give my students math tasks on a regular basis that require a high level of cognitive demand.	0.00% 0	17.65% 18	55.88% 57	26.47% 27	3.09

Note. N = 102*.

The vast majority of teacher participants in this survey responded that they employ CMTs. About 82% of teachers reported that they give their students math tasks on a regular basis that require a high level of cognitive demand, while about 96% of teachers agreed or strongly agreed that they purposefully select math tasks that build on and extend student learning based on previous learning. Ninety-eight percent of teachers agreed or strongly agreed that they encourage their students to use varied approaches and strategies to solve math problems. So, despite potential district PD, math task PD shortcomings, or outside influences, such as evaluation and testing, a large majority of teachers indicate that they employ CMTs.

Collaboration.

Table 11

Collaboration

	Strongly Disagree	Disagree	Agree	Strongly Agree	Weighted Average
When planning for instruction, I regularly discuss challenging math tasks with colleagues.	8.82% 9	28.43% 29	43.14% 41	19.61% 20	2.74
I regularly discuss the scaffolding of challenging math tasks with colleagues.	4.90% 5	30.39% 27	51.96% 47	12.75% 11	2.73
My district prioritizes collaboration time to score students' work on challenging math tasks.	26.47% 27	39.22% 40	24.51% 25	9.80% 10	2.18

A majority (63%) of teachers reported that when planning for instruction they regularly discuss CMTs with colleagues. Nearly the same percentage (65%) of teachers reported that they regularly discuss the scaffolding of CMTs with colleagues. A much smaller percentage of teachers (34%) reported that their district prioritizes collaboration time to score students' work on CMTs. This is not surprising given that colleague-to-colleague interaction is the most immediate form of collaboration as instructional issues tend to emerge day-to-day in real time. Many educators espouse the strength of teacher-to-teacher collaboration (Baum & Krulwich, 2017). Some declare its superiority over district provided PD or district organized collaboration as an avenue to instructional improvement. Baum and Krulwich (2017) remarked that "within the [mathematics teachers] team is where these teachers' real professional development happens" (p. 63). However, for mathematics teachers who do not collaborate regularly, district prioritization to collaborate seems more critical. In this survey, about 37% of the respondents strongly disagreed or disagreed with the statement that when planning for instruction they regularly discuss CMTs with colleagues. Thirty-five percent of teachers also indicated that they strongly disagreed or disagreed with the statement that they regularly discuss the scaffolding of CMTs with colleagues.

Student characterization.

Table 12

Student Characterization

	Strongly Disagree	Disagree	Agree	Strongly Agree	Weighted Average
Student behavior/disposition effects the regularity with which you use challenging math tasks.	2.94% 3	25.49% 26	50.98% 52	20.59% 21	2.89
Students' prior academic preparation effects the usefulness of challenging math tasks. *(n = 101)	1.98% 2	22.77% 23	43.56% 44	31.68% 32	3.05
Students' disposition towards engaging in productive struggle effects the regularity with which you use challenging math tasks.	1.96% 2	26.47% 27	50.98% 52	20.59% 21	2.90

Note. N = 102*.

Students' behavior, prior academic preparation, and disposition towards engaging in productive struggle all seem to affect the regularity with which a majority of teachers choose to use CMTs. About 72% of teachers either agreed or strongly agreed with the statement that student behavior/disposition effects the regularity with which they use CMTs. An even larger percentage of teachers (75%) reported that they agreed or strongly agreed that students' prior academic preparation effects the usefulness of CMTs. This is referred to as the *basics first approach* to mathematics, which tends to moderate who experiences more authentic mathematical problem solving. About 72% of the survey participants indicated that they agreed or strongly agreed that students' disposition towards engaging in productive struggle effects the regularity with which they use CMTs. Being aware of the factors that may bias instructional decisions may lead to more regular use of CMTs with all students.

CFA Model Fit

CFA was used to test how well measured variables (three per factor) represented seven factors believed to predict teachers' perceptions of CMTs. The seven factors: curriculum, math practices, assessment and evaluation, PD, doing math tasks, collaboration, and student characterization, were allowed to covary. Unique error variances were added to each of the measured variables with unit loading identification. Additionally, one path coming from each of the seven latent variables were modified so that one path regression weight was set to 1. Model fit was examined with the comparative fit index (CFI), Tucker-Lewis index (TLI), and root mean-square error of approximation (RMSEA) where values of .95 and above indicate good fit for CFI and TLI, while values of .05 or less indicated good model fit for the RMSEA. Model comparisons were evaluated using change in λ^2. The initial model is pictured below in Figure 5.

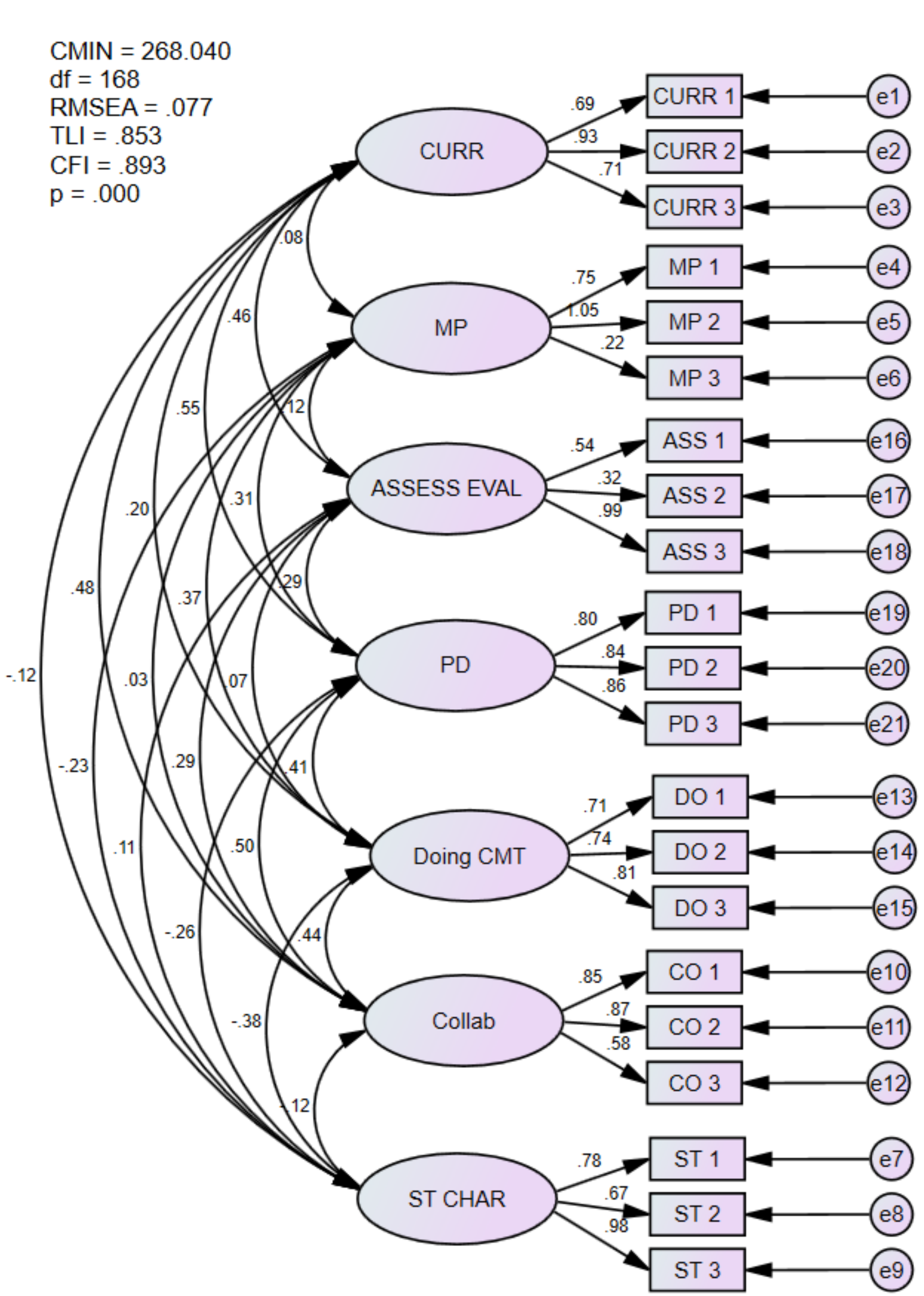

Figure 5. Initial AMOS model.

The fit of the initial model was poor (CFI = .893; TLI = .853; RMSEA = .077; 90% RMSEA Confidence Interval [CI] = .059 to .094). Factor loadings were .5 or higher for five of the seven subtests. The smaller loadings were for the third indicator in the mathematics practices subtests (.22) and the second indicator in the assessment and evaluation subtests (.32); these two loadings were not statistically significant. Eliminating these two indicator variables with low factor loadings resulted in a significant decrease in χ^2 ($\Delta\chi^2 = 71.517$, $\Delta df = 37$, $p < .001$); accordingly, the model modification was retained.

The modified model with indicator variables all greater than .6 with rounding to the 10th place still did not have good model fit (CFI = .927; TLI = .894; RMSEA = .070; 90% RMSEA CI = .049 to .090). Next, the modification indices indicated a possible model improvement by creating a path from latent PD variable to CURR 3, which constituted a cross-loading. This alone did not result in good fit (CFI = .952; TLI = .937; RMSEA = .058; 90% RMSEA CI = .031 to .079). Examination of the standardized residual covariance matrix revealed multiple covariance residuals with an absolute value approaching 2. Keith (2015) indicated that "one rule of thumb suggests examining standardized residual covariances (commonly referred to as standardized residuals) greater in absolute magnitude than 2.0" (p. 350). The standardized residual covariance matrix is shown below in Table 13. The higher residuals indicate that the student characterization indicators seemed to be causing an issue with the models fit (see Table 14). Eliminating student characterization as a factor in the model resulted in significant model improvement and good model fit (CFI = .972; TLI = .962; RMSEA = .050; 90% RMSEA CI = .000 to .080). Figure 6 shows this final model.

Table 13

Standardized Residual Covariance Matrix

	1	2	3	4	5	6	7	8	9	10
ST_Q3	.000									
ST_Q2	.003	.000								
ST_Q1	.013	-.204	.000							
CO_Q3	-.889	-.827	**-1.791**	.000						
CO_Q2	.192	-.255	.863	.328	.000					
CO_Q1	.023	.013	-.030	-.485	.026	.000				
DO_Q3	-.293	.232	.430	.451	-.017	.595	.000			
DO_Q2	.694	.598	1.012	-.208	-.153	-.025	-.078	.000		
DO_Q1	-.585	-.885	-.158	**-1.574**	-.371	-.031	-.297	.565	.000	
PD_Q3	-.223	-.147	-.638	1.295	.176	.163	.373	.127	.213	.000
PD_Q2	.111	-.337	.420	.751	-.493	-.320	.288	-.047	-.846	.091
PD_Q1	.595	-.541	.370	1.241	-.085	.352	-.181	-.213	-.178	-.265
AS_Q3	-.074	.675	.632	.284	-.141	.138	.552	-.468	-.024	.310
AS_Q1	-.295	**1.724**	.172	-.450	-.382	.435	-.621	-.278	.646	-.936
MP_Q2	.198	-.676	.610	.097	.189	.363	.726	-.584	.013	-.032
MP_Q1	.576	-.321	1.203	-.711	.607	1.397	.665	-.003	.620	.198
CU_Q3	-.475	-.655	-.953	.316	**-1.557**	-.399	.513	-.824	-.599	.154
CU_Q2	.044	.924	-.521	.292	-.321	.204	.447	-.541	-.220	.264
CU_Q1	**1.577**	1.300	.961	.117	-.281	.196	.254	-.203	**-2.176**	-.189

	11	12	13	14	14	16	17	18	19
PD_Q2	0								
PD_Q1	0.166	0							
AS_Q3	0.153	1.376	0						
AS_Q1	-1.442	-0.669	0	0					
MP_Q2	0.461	0.38	-0.539	1.188	0				
MP_Q1	1.208	**1.507**	-0.423	1.036	0	0			
CU_Q3	-0.549	0.522	-0.095	-0.863	0.688	1.079	0		
CU_Q2	-0.706	0.445	0.334	-0.718	0.297	0.8	0.008	0	
CU_Q1	-0.15	-0.01	0.52	-0.022	1.102	1.431	0.694	0.016	0

Table 14

Summary of Fit Statistics

Model	χ^2	df	$\Delta\chi^2$	Δdf	p	RMSEA	TLI	CFI	AIC
1. Initial	268.040	168				.077	.853	.893	436.040
2. Indicators with low factor loadings removed	196.523	131	71.517	37	<.001	.070	.894	.927	352.523
3. Cross loading of PD to CURR 3	172.652	130	23.871	1	<.001	.058	.937	.952	292.652
4. Student characterization factor and indicators removed	112.860	88	59.792	42	<.05 (0.0367)	.053	.953	.966	204.860

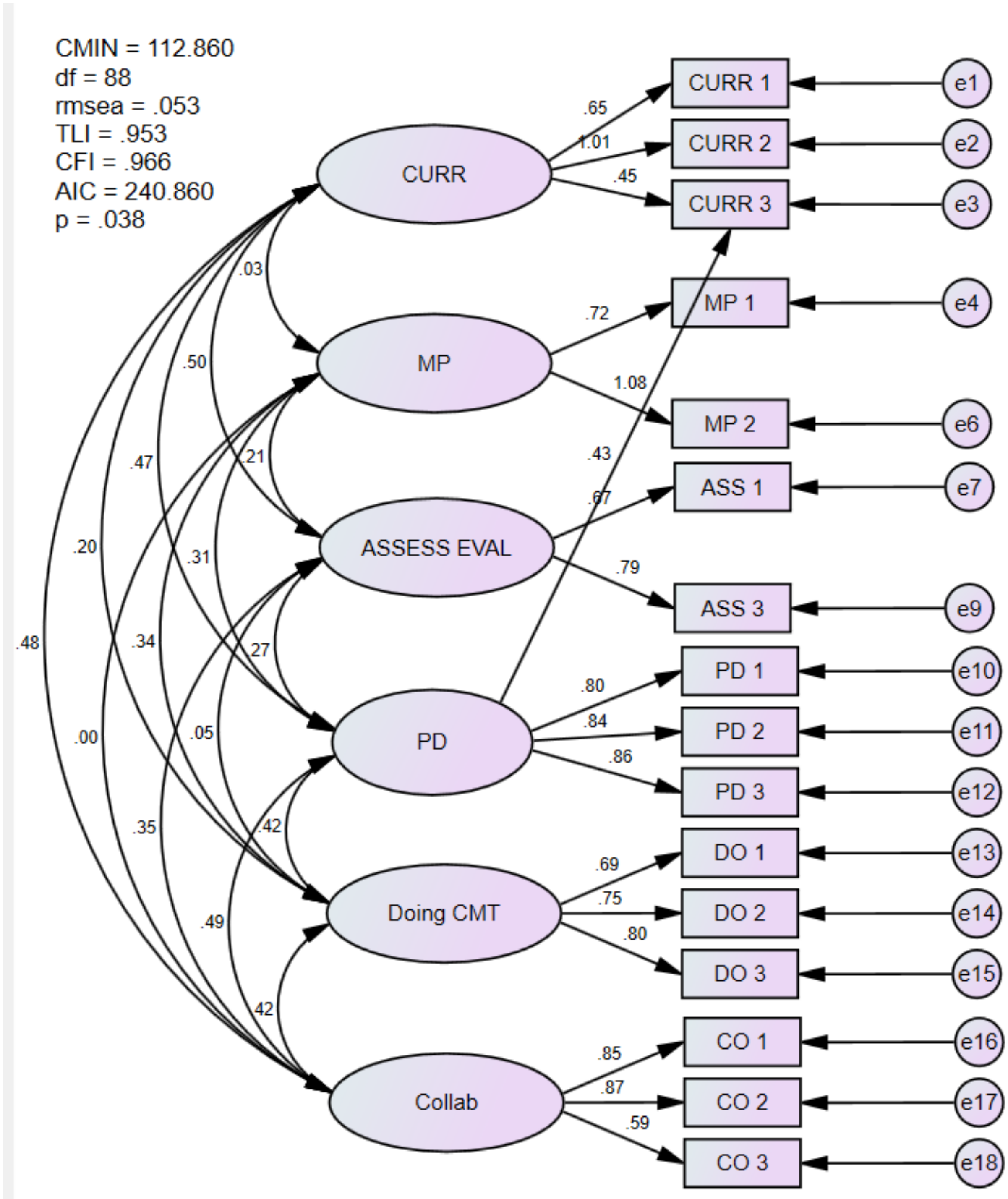

Figure 6. Final model without the student characterization factor and with cross-loading.

From a theoretical perspective, it made sense to make CURR 2 an indicator variable for both the curriculum variable and the PD variable as it indeed could be considered both a measure of curriculum as well as of PD. To recap, the CURR 3 indicator stated that the district introduces educators to CMTs, which may come from online resources, that allow students to use multiple representations (e.g., tables, graphs, and equations) when solving. Online resources or other supplementals are often provided to teachers as a companion to one's primary curriculum. The initial intent of this question was to capture teachers' perception of the availability of CMTs that may be introduced to teachers by the district, but that are not a part of their primary curricula. However, the act of districts *introducing* CMTs constitutes a form of PD. Additionally, elimination of the latent variable, student characterization, made theoretical sense. All of the other latent variables could be more easily attended to by districts by attending to teachers PCK. Students' disposition, prior preparation, and behavior effect teachers' CMT use, but student characteristics cannot be attended to easily by the district. Whereas, access to CMT curriculum, CMT PD, assessment and evaluation relative to CMTs, addressing the importance of the MPS, teacher collaboration around CMTs, and ensuring that teachers actually do CMTs, are all more realizable from a district vantage point.

Convergent validity for the final model. The convergent validity rules of thumb as specified to be sufficient by Hair et al. (2010), were all met with the exception of two factor loadings that were below .5. However, the rules of thumb for AVE and CR were all met sufficiently with all AVE measures being greater than .5 and all CR measures being .7 or higher, as shown in Table 15.

Table 15

Convergent Validity (AVE and CR)

Latent Variable	AVE	CR
Curriculum	.548	.767
Mathematical Practices	.842	.911
Assessment and Teacher Evaluation	.541	.701
Professional Development	.568	.832
Implementing CMTs	.559	.791
Collaboration	.610	.820
Student Characterization	Eliminated from model	

Divergent validity for the final model. According to Hair et al. (2010), "AVE estimates for two factors should be greater than the square of the correlation between the two factors to provide evidence of discriminate validity" (p. 673). This condition was met as evident in Table 16. All of the AVE estimates were above .5 and all of the squares of the standardized interconstruct correlations were less than .5.

Table 16

Standardized Interconstruct Correlations and Squares

Factors	Standardized Interconstruct Correlation	Square of the Standardized Interconstruct Correlation
CU <--> MP	0.031	0.000961
CU <--> AS	0.497	0.247009
CU <--> DO	0.197	0.038890
CU <--> CO	0.484	0.234256
MP <--> AS	0.206	0.042436
MP <--> DO	0.339	0.114921
MP <--> CO	-0.002	0.000004
AS <--> DO	0.050	0.002500
AS <--> CO	0.351	0.123201
DO <--> CO	0.422	0.178084
AS <--> PD	0.271	0.073441
CU <--> PD	0.471	0.221841
MP <--> PD	0.305	0.093025
DO <--> PD	0.417	0.173889
CO <--> PD	0.492	0.242064

Summary

This study examined the degree of fit of a proposed model with a septenary factor structure that was hypothesized to model teachers' perceptions of CMTs. Student characterization was eliminated from the final model's factor structure to achieve good model fit.

Chapter 5: General Discussion and Conclusions

"Challenging tasks are complex and absorbing mathematical problems with multiple solution pathways, whereby the whole class works on the same problem" (Russo & Hopkins, 2017, p. 31). Effective teachers design mathematical experiences for students that include exposure to Challenging Math Tasks (CMTs); "effective teachers don't leave these things to chance" (Hattie et al., 2017, p. 83). With the Common Core mathematics standards in place, this research sought to investigate teacher's perceptions of CMTs at this time, through the lens of the Math Task Framework (MTF), keeping in mind that instruction is not monolithic, but ever changing. In addition, this work confirmed, using Confirmatory Factor Analysis (CFA), a hypothesized relationship between teachers' perceptions of CMTs and their educational learning communities' site-based characteristics (i.e., curriculum, assessment and evaluation, PD, and collaboration), and teachers' own Challenging Math Task (CMT) Pedogical Content Knowledge (PCK).

Task Selection

MPS. A student's mathematical habits of mind do not necessarily take hold on their own, and need to be facilitated by teacher actions that promote a predilection to using the Standards for Mathematical Pratice (SMP) (Schoenfeld, 1992). Historically, a skills-only approach proved mathematically disadvantageous; "the back-to-basics movement was a failure" (Schoenfeld, 1992, p. 3). In this study, teachers nearly unanimously (99%) indicated that CMTs are important tools that allow students an opportunity to engage in the MPS, with 82% of teachers reporting that they give their students math tasks on a regular basis that require a high level of cognitive demand. Ninety-six percent of teachers agreed or strongly agreed that they purposefully select math tasks that build on and extend student learning based on previous learning. This study supports the notion that mathematical reform in the United States in secondary classrooms is taking hold in terms of a shared collective consciousness that teachers should play an active role in teaching the SMP through the use of CMTs. Teachers' view of the MPS was a factor found to explain variability in teachers' perceptions of CMTs.

Curriculum. The incorporation of CMTs has been a more recent approach to teaching problem solving and critical thinking. Recent research by McDuffie et al. (2017) found that middle school mathematics teachers' perceptions of the Common Core State Standards of Mathematics (CCSSM) was affected by the "extent to which digital/electronic curriculum resources are used" (p. 19). Additionally, they found that middle school mathematics teachers were significantly supplementing, perhaps indicating that the primary curricula proved insufficient in meeting the rigorous content exposure called for under the CCSSM. McDuffie et al. (2017) indicated that 39.3% of middle school mathematics teachers were using supplementary, inquiry-based activities at least weekly.

Little was found in the research regarding high school teachers' perceptions of their primary curricula. This study contributes to our understanding of high school mathematics teachers' perceptions of their primary curriculums' selection of CMTs aligned to the CCSSM. Over a quarter of the high school math teachers (29%) surveyed indicated that their district-adopted primary curriculum and supplemental resources did not include challenging math tasks aligned to the CCSSM. For these high school mathematics teachers, more effort and skill are required at the selection or design phase of the MTF. Not surprisingly, CMTs which have multiple solution pathways or address multiple representations were reportedly less accessible through district provided resources. Forty percent of teachers disagreed or strongly disagreed that their district provides a selection of CMTs that can be solved in multiple ways. If this, 40%

of teachers were to seek out supplemental materials weekly, this conceivable supplementation would be in line with the statistical findings of McDuffie et al. (2017) who reported that 39.3% of middle school mathematics teachers were supplementing inquiry-based activities at least weekly.

Testing. Davis, Choppin, McDuffie, and Drake (2017) found in their survey research that 66.8% of Middle School Mathematics Teachers (MSMTs) felt that state assessments helped them “decide what CCSSM mathematics content and practices to emphasize” (p. 244). In this study, a larger majority (69.6%) of high school mathematics teachers indicated that state Smater Balance Assessment Consortium (SBAC) assessments contain CMTs which encourages them to use CMTs. Both State CCSSM assessment and district level assessment were found to help explain the variability in teachers’ perceptions of CMTs. It would seem that in terms of state testing, Schoenfeld (2015) was correct that WYTIWYG (what you test is what you get). Assessment was a factor which was found to help explain the variability in teachers’ perceptions of CMTs. State assessment explained 44.89% of the variability in teachers’ responses, while district level assessment explained 62.41% of the variability in teachers’ responses regarding what encourages them to use CMTs. However, 55% of teachers disagreed or strongly disagreed that district-level testing encourages them to use CMTs. This may indicate that district-level testing remains more procedural in nature. Directors of curriculum and assessment and administrators may want to examine site-based assessment practices.

As a final note, it may be of interest to educators that the indicator variable designed to measure the effect of teacher evaluation on their use of CMTs was eliminated from the model. The original hypothesis that teacher evaluation effects teachers’ perception of CMTs was not supported in this study. It could be that teacher evaluations do not focus extensively enough on implementing CMTs to provide teachers encouragement for the enactment of CMTs. Perhaps teacher evaluation foces more, in general, on a teacher's ability to control the classroom.

Setup and Collaboration

Davis et al. (2014) identified the frequency of teachers' collaborative instructional planning as a factor which effected MSMTs' perception of the CCSSM. In this study, teacher collaboration was also a variable that explains the variability in High School Mathematics Teachers' (HSMTs) perceptions of CMTs. Of the three indicator variables considered to be representative of the collaboration latent variable, all of the indicator variables were retained in the model. Teachers reported (62.75%) that they agreed or strongly agreed that when planning instruction they regularly discuss CMTs with colleagues. An even higher percentage of teachers (64.71%) reported that they regularly discuss the scaffolding of CMTs with colleagues. This may indicate that teachers find determining appropriate scaffolding as something to be negotiated at the setup phase of the MTF, which is of paramount importance in terms of successful implementation of CMTs. Research suggests that when teachers do not over-scaffold a task, students may have an opportunity to learn to plan a course of attack, instead of being given one (Sullivan & Mornane, 2014). CMT teacher collaboration, where teachers discuss CMTs and discuss scaffolding, is presumed to lead to increased math task prelogical knowledge over time, as well as a more favorable perception of CMTs. Thus, teacher collaboration was found to be a factor that explains teachers' perceptions of CMTs.

Implementation

Professional development. It has been espoused that the success of "the CCSS hinges on the success of professional development" (Marrongelle et al., 2013, p. 203) and Davis et al. (2014) found that "the degree to which professional development is supported" (p. 19) is a factor that effects teachers' perceptions of the CCSSM. This study further established that teacher CMT professional development is a factor that effects teachers' perceptions of CMTs under the CCSSM. In terms of the third phase of the MTF, implementation, 64% of educators indicated that district PD has addressed implementing CMTs with students (i.e., supporting productive struggle, classroom management, collaborative groupings, and academic discourse). Allowing students to struggle and persevere appropriately requires instructional math task tacit and math task PD can prove beneficial in ensuring all students experience CMTs under the CCSSM. Boston and Smith (2009) showed that through a CMT PD intervention, teachers could acquire optimize math task knowledge.

Student characterization. Among the factors investigated to effect teachers' perception of CMTs under the CCSSM, student characterization was not found to be a factor and was eliminated from the model. However, student characterization remains a gatekeeper as to whom will be granted exposure to CMTs. About 72% of teachers either agreed or strongly agreed with the statement that student behavior/disposition effects the regularity with which they use CMTs. An even larger percentage of teachers (75%) reported that they agreed or strongly agreed that students' prior academic preparation effects the usefulness of CMTs. While teachers' characterization of students was not shown to be a factor that effects their perception of CMTs under the CCSSM, access to CMTs is limited by student behavior and students' learning dispositions. Stein and Lane (1996) argued that "it was the opportunity to have such higher forms of thinking developed and supported" which produced "increases in student learning" (p. 55).

Doing CMTs. It was hypothesized that teachers' familiarity with CMTs as a result of implementing and using CMTs would be a factor which effected teachers' perception of CMTs under the CCSSM. Teachers' "doing CMTs" was found to be a factor which effected their perception of CMTs under the CCSSM. Eighty-two percent of teachers agreed or strongly agreed that "I give my students math tasks on a regular basis that require a high level of cognitive demand." This would seem to indicate that instructional reformation is under way. Davis et al. (2017) reported that 89% of teachers indicated that the CCSSM required them to "emphasize more solving of complex problems" (p. 244).

Limitations

The relatively small sample size (N = 102) is a limitation in this study, a larger sample size would have been more ideal. Keith (2015) explained that "a common rule of thumb for SEM studies is that researchers should strive for a 20:1 ratio of sample size to the number of parameters to be estimated (the N:q rule, Jackson, 2017)" (p. 530); yet, Keith (2015) further indicated that according to Kline (2011), "an N:q ratio of 10:1 may be acceptable" (p. 530). The results of this study may not be generalizable to the overall high school mathematics teaching population. It relied on samples of convenience when a more systematic collection strategy could have been devised. Triangulation of the data would have provided a richer context from which to analyze the results. Other methodologies, such as teacher interviews (qualitative) or, perhaps an ethnographic approach could have been pursued. Qualitative information is worthwhile in many research approaches. This study is more quantitative than qualitative, and thus lacks some of the richness that might be realized with a mixed methods approach.

Implications for Practice

With over a quarter of high school teachers (29%) indicating that their districted adopted and supplemental resources no not include CMTs aligned to the CCSSM, distric directors of curriculum should investigate accessibility at their site(s). Special attention should be paid to to identifying Common Core aligned CMTs with multiple correct solution pathways. Teachers in this study reported that CMTs with multiple pathway solutions are rarer than more general CMTs, with 40% of teachers disagreeing or strongly agreeing that their primary or supplemental curriculums supplied them with access to CMTs with multiple solution pathways. While this finding is not surprising, it is worth addressing at the site level.

Since this study identified teacher collaboration and teacher professional development to be factors which effect teachers perceptions of CMTs, educational leadership should strive to be intention about providing designated CMT collaboration and CMT professional development.

In terms of testing, district administration may want to make sure that district and state testing are more similar than different. In this study, 70% of teachers reported that SBAC state testing was encouraging their use of CMTs. However, a majority of teachers indicated that district level assessment was not encouraging them to use CMTs. Given this finding, district benchmark practices may need to be examined at the site level.

The opportunity to engage in the habbits of the mind that can be practiced through the use of CMTs is reported regularly afforeded to a relatively large majority of students (82%). Educational leaders should realize those classrooms where students do not have the regular opportunity to engage in CMTs are being inadvertently mathematically disadvantaged. While student characterization was not found to be a factor in teachers' perception of CMTs, student characterization does seem to have a effect on student access to CMTs. Over 70% of teachers agreed or strongly agreed that students' disposition, behavior, and prior academic preparation effects the regularity at which they use CMTs. Site administration may want to look at whose using CMTs. Is engagement in CMTs an experience all students are granted?

Suggestions for Further Research

In working towards bridging the achiement gap, further research might focus on the outcomes for learners of mathematics when teachers intentionally use CMTs with students who seem to display dispositional opposition to engaging with CMTs or whom are in need academic remediation. All students deserve an opportunity to grapple with mathematics. Would teachers and students be able to push past student dispositional opposition to working on CMTs, if professional development focused on intentionally teaching perserverance was enacted? It seems that in some instances, teachers have to preserve as much as students when enacting CMTs in the classroom. Would students' mathematical outcomes improve in classrooms identified as having behavior and/or lower levels of mathematical preparation through the encorporation of CMTs, despite teachers' feelings that CMTs are less useful for students who are less prepared academically?

While it seems that teachers seem to almost ananmously agree that CMTs are useful, 75% indicated that students' prior academic preparation effects the usefulness of CMTs. Could the reverse be true? The more lacking students' mathematical preparation, the more important the role of CMTs to the students mathematical understanding. Teacher beliefs matter because they effect student learning at the transactional level. Further research focused on teacher perception of CMTs for the academically underprepared and the potential oppotunity gap that may manifest, is an area for further study.

Conclusion

Being aware of the factors which effect teacher perception of CMTs under the CCSSM provides district leadership an opportunity to intervene on behalf of students and ensure that all students benefit from the practice of honing their individual habits of mind through exposure to

CMTs. Site-based variables are presumed to effect teachers' overall perceptions of CMTs. This study's results indicated that district testing at some sites contends in some way with the enactment of CMTs. Over half of the teachers surveyed (55%) disagreed or strongly disagreed with the statement that district-level testing encourages them to use CMTs. Despite challenges educators may encounter with the selection, setup, or implementation of CMTs under the CCSSM, teachers clearly see CMTs as important tools that allow students to practice the habits of mind associated with effective problem solving. Teachers nearly unanimously (99%) agreed that CMTs are important tools that allow students an opportunity to engage in the MPS. The CFA model with the latent factors of curriculum, mathematical practices, PD, collaboration, doing CMTs, and assessment had good model fit, supporting the hypothesized tenant that these precepts effect teachers' perceptions of CMTs.

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APPENDIX A: CONSENT FORM

Greetings,

My name is Mariya Sullivan and I am a doctoral student at the University of the Pacific. I am conducting a research study about high school mathematics teachers' perceptions of challenging math tasks and their instructional environments. I am asking if you would take about 5 minutes to complete a survey for this research project. Participation is completely voluntary and your answers will be anonymous. You will receive no direct benefits from participating in this research study. However, your responses may help us learn more about how to support mathematics teachers.

If you are interested, please click OK below and continue on to the survey.

If you have any questions, please do not hesitate to contact me ([redacted]).

Thank you for your time.

OK [Selecting the OK button takes participants to the electronic consent portion of the form.]

ELECTRONIC CONSENT: Please select your choice below. Clicking on the "Agree" button indicates that

- You have read the above information
- You voluntarily agree to participate
- You are 18 years of age or older

☐ Agree

☐ Disagree

APPENDIX B: SAMPLE CHALLENGING MATHEMATICS TASK

Performance Assessment Task
Hopewell Geometry Grade 10
This task challenges a student to use understanding of similar triangles to identifying similar triangles on a grid and from dimensions. A student must be able to use trig ratios to calculate an angle in a 3,4,5 right triangle. A student must be able to apply Pythagorean theorem to find missing dimensions in right triangles. A student must be able to construct arguments to prove that two triangles are similar.
Common Core State Standards Math - Content Standards
<u>**High School – Geometry – Similarity, Right Triangles, and Trigonometry**</u> **Understand similarity in terms of similarity transformations.** G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **Prove theorems involving similarity.** G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and prove relationships in geometric figures. **Define trigonometric ratios and solve problems involving right triangles.** G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Common Core State Standards –Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.5 Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the

insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

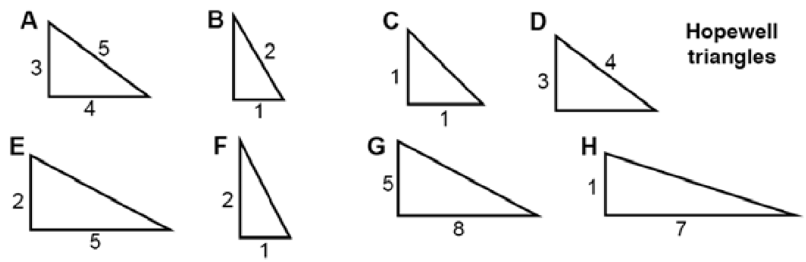
Grade Level	Year	Total Points	Core Points	% At Standard
10	2006	8	6	41%

Hopewell Geometry

This problem gives you the chance to:
• work with the Pythagorean Rule, angles and similarity in given triangles

The Hopewell people were Native Americans whose culture flourished in the central Ohio Valley about 2000 years ago.

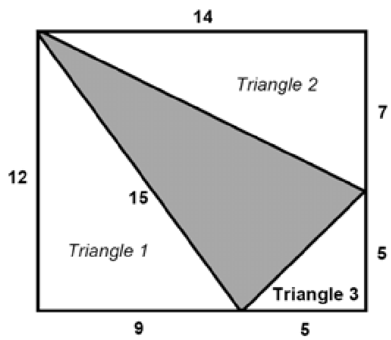
The Hopewell people constructed earthworks using right triangles, including those below.



1. What is the length of the hypotenuse of Triangle H?
   Give your answer correct to one decimal place. _______________
   Show your calculation.

2. What is the size of the smallest angle in Triangle A?
   Give your answer correct to one decimal place. _______________
   Show your calculation.

The diagram on the next page shows the layout of some Hopewell earthworks. The centers of the Newark Octagon, the Newark Square and the Great Circle were at the corners of the shaded triangle.



Not drawn to scale

The three right triangles surrounding the shaded triangle form a rectangle measuring 12 units by 14 units.

Each of these three right triangles is similar to one of the Hopewell triangles on the previous page.

For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? _______________

   Explain how you decided.

   ____

   ____

   ____

4. Is the shaded triangle a right triangle? _______________

   Explain how you decided, showing all your work.

   ____

   ____

   ____

8

Hopewell Geometry Rubric		
The core elements of performance required by this task are: • work with the Pythagorean Rule, angles and similarity in given triangles Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **7.1** (accept 7 or $5\sqrt{2}$) Shows correct work such as: $\sqrt{(1^2 + 7^2)}$	1 1	2
2. Gives correct answer: **36.8º to 36.9º** Shows correct work such as: $\sin^{-1}\frac{3}{5}$ or $\cos^{-1}\frac{4}{5}$ or $\tan^{-1}\frac{3}{4}$	1 1	2
3. Gives correct answer: Triangle **A** Gives correct explanation such as: Triangle 1 is an enlargement of Triangle A by a scale factor of 3.	1 1	2
4. Gives correct answer: **No**, **and** Gives a correct explanation such as by finding lengths of all three sides ($\sqrt{225}$, $\sqrt{50}$, $\sqrt{245}$) and showing they don't satisfy the Pythagorean Rule. $245 \neq 225 + 50$. *Other methods include:* • Using trigonometry to find the angles (71,6, 81.9, 25.5) • Triangle 3 is isosceles $\therefore$ it has two 45° angles. Triangles 1 and 2 are not isosceles $\therefore$ do not have 45° angles. Angle in shaded triangle = 180° - 45° - non 45° angle $\therefore \neq 90°$ *Partial credit* Gives a partially correct explanation.	2 (1)	2
Total Points		**8**