1973

A Comparison Between Certain Piagetian Logical Thinking Tasks And The Subtraction Ability Of First-Grade, Second-Grade, And Third-Grade Children.

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A COMPARISON BETWEEN CERTAIN PIAGETIAN LOGICAL THINKING TASKS AND THE SUBTRACTION ABILITY OF FIRST, SECOND, AND THIRD GRADE CHILDREN

A Dissertation
Presented to
the Faculty of the School of Education
The University of the Pacific

In Partial Fulfillment
of the Requirement for the Degree
Doctor of Education

by
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April 1973

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Dated May 23, 1973

ABSTRACT OF DISSERTATION

PROBLEM: Educators who plan the child's sequence of learning experiences in mathematics must know how the child's logical development compares to his ability to do mathematical operations.

PURPOSE: This investigation was conducted to explore the comparison between first, second, and third grade children's ability to subtract and their ability to think logically as measured by conservation of numerical quantities, seriation, and quantitative class inclusion tasks.

PROCEDURES: A group of ninety public school children were randomly selected on the basis of a computation test of subtraction. After the children were selected, they were given a subtraction test based on the use of manipulative materials. Depending on the difficulty levels of the tests passed, children were given one or more of the following Piagetian tasks: conservation of numerical quantities, seriation, quantitative class inclusion. A chi-square test of significance was used to test differences between children's operative and figurative knowledge of subtraction and their ability to do the logical thinking tasks.

FINDINGS: Children's operative and figurative knowledge of the subtraction facts one through nine was not significantly affected by the presence or absence of the ability to conserve numerical quantities. The ability to do the subtraction facts ten through eighteen either on an operative or a figurative level of understanding was not significantly affected by the presence or absence of conservation of numerical quantities or seriation abilities. Conservation of numerical quantities was found to be a highly significant factor for children who had an operative understanding of subtraction of two- and three-digit numbers above twenty that do not require regrouping. However, an operative or a figurative knowledge of subtraction at this level is not significantly affected by abilities to do problems of seriation or quantitative class inclusion.

CONCLUSIONS: The results of this investigation suggest that children can acquire learned patterns of thought which allow them to channel their thinking in such a way as to avoid the use of some of the logical abilities tested. Children are able to substitute previously learned

methods of solution or use learned techniques of counting to solve the subtraction facts one through eighteen. If children have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping, they must have established the ability to conserve numerical quantities. Based on the results of this study it appears that the logical abilities of seriation and quantitative class inclusion are not necessary in solving subtraction problems both on the operative and figurative level with two- and three-digit numbers above twenty that do not require regrouping.

RECOMMENDATIONS FOR FURTHER RESEARCH: Three further investigations are recommended: (1) Initiate a similar investigation with addition. (2) Determine the comparison between subtraction problems not studied in this investigation and logical development. (3) Find the logical development necessary to understand place value.

ACKNOWLEDGMENTS

I wish to thank Professor Jerry King for helping me to set goals and meet tentative deadlines for the completion of this study. His encouragement and advice were greatly appreciated.

Appreciation must also be expressed to Professor William Theimer for his insight into the many problems faced during the writing of this study. I also wish to express my gratitude to the other members of my dissertation committee: Professors John Schippers, Douglas Smith, and William Topp for their willingness to help whenever asked.

Further, my thanks goes to the principals, teachers, and children of Lodi Unified School District who cooperated to make this study possible. And special thanks to CTB/McGraw-Hill for assistance in constructing the computation test of subtraction.

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CHAPTER I

INTRODUCTION TO THE STUDY

> The implications of Piaget's theories for mathematics education have not yet been realized. Studies by competent researchers involving American children are badly needed. New curricular materials based on sound psychological evidence should be written. And, in teacher education, more work involving Piaget's theories and their implications would serve as landmarks in improving instruction in the elementary school.[1]

Three difficulties occur when Piagetian theories are applied to the educational curriculum.[2] First, it is not clear how much knowledge a child must acquire through "rote" learning before he can think constructively about number relations and classifications. Second, Piaget's methods are best suited for a one-to-one teacher-student relationship; in American schools this type of relationship is rare. Third, the connection between language and the acquisition of knowledge is poorly understood.

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[1]Paul Rosenbloom, "Implications of Piaget for Mathematics Curriculum," Improving Mathematics Education for Elementary School Teachers--a Conference Report. Edited by W. Robert Houston (Michigan State University, 1967), sponsored by the Science and Mathematics Teaching Center and the National Science Foundation, p. 49.

[2]Harry Beilin, "The Training and Acquisition of Logical Operations," Piagetian Cognitive-Development Research and Mathematical Education (Washington, D.C.: National Council of Teachers of Mathematics, Inc., 1971), pp. 115-116.

Recent trends in American education towards individualized instruction should provide an avenue through which teachers can use small group techniques which make use of some of Piaget's methods. The role of "rote" memory and language in the acquisition of knowledge may be clouded for some time to come until further research can be completed.

~~Acceleration of learning or schooling is an~~ important question that has a direct bearing on the implications that can be drawn from Piaget's theory. Bruner, in his book The Process of Education, indicated that he could teach anything in an intellectually honest way to any child at any age if he did so in the correct manner.[3] Piaget does not agree with Bruner on this point. In a recent speech at a New York university in March of 1967, Piaget stated:

> A few years ago Bruner made a claim which has always astounded me; namely that you can teach anything in an intellectually honest way to any child at any age if you go about it in the right way. Well, I don't know if he still believes that . . . it's probably possible to accelerate but maximum acceleration is not desirable. There seems to be an optimum time. What this optimum time is will surely depend on each individual and on the subject matter.[4]

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[3]Jerome S. Bruner, The Process of Education, Vintage Books (New York: Alfred A. Knopf, Inc. and Random House, Inc., 1960), p. 33.

[4]Frank Jennings, "Jean Piaget, Notes on Learning," Saturday Review, May 20, 1967, p. 82.

Engelmann[5] claimed that Piagetian logical developmental tasks could be taught to young children without taking into account the natural developmental sequence that Piaget described. Engelmann taught seven kindergarten children, without manipulative materials, verbal rules that could be applied to problems of logical structure of conservation and specific gravity. These children were able to correctly answer questions pertaining to the conservation of volume and specific gravity that children ordinarily could not answer until their teens. However, further investigation showed that children made responses that characterized their stage of development when a situation did not lend itself to the application of a rule. Almy summarized the effect of schooling on the development of thinking as follows: ". . . schooling may affect the acquisition of information but it is not likely to be crucial in changing basic ways of organizing and assimilating facts."[6]

Statement of the Problem

The development of logical thinking can be measured, for Piaget has identified a series of stages through which

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[5]Siegfried E. Engelmann, "Does the Piagetian Approach Imply Instruction?" *Measurement and Piaget*, ed. by Donald Ross Green, Marguerite P. Ford, and George B. Flamer (New York: McGraw-Hill Book Co., 1971), pp. 119-147.

[6]Millie Almy and Associates, *Logical Thinking in Second Grade* (Columbia University: Teachers College Press, 1970), p. 10.

a child must progress in the development of his powers of logical thought. Adults who plan the child's sequence of learning often disregard these unique patterns of thinking. Experimental evidence is needed concerning the comparison between the development of logical thinking, as defined by Piaget, and the primary child's ability to understand the concepts underlying arithmetical algorithms.

Arithmetical algorithms of subtraction are especially appropriate for an investigation into the comparison between the development of logical thinking and the primary child's ability to understand the concepts that underlie an arithmetical algorithm. Subtraction algorithms are appropriate because the algorithms may be arranged from problems that are relatively easy to compute to those which can be solved only with some difficulty. Curriculum planners have made use of this in planning the mathematics curriculum for primary children. However, the grade placement of a particular group of problems that represent a certain level of difficulty for a child has never been established by comparing the inferred difficulty with the child's powers of logical thought.

In present primary arithmetic programs nearly every child is forced to attempt the same problems in subtraction; it is only when the child repeatedly fails that the teacher is aware that the child is not capable of understanding the problems. Some children manage to mechanically perform a subtraction algorithm, thus the teacher does not realize that the child lacks understanding. Experimental evidence of a significant comparison between logical thinking and subtraction would have importance to the teacher as well as the curriculum planner.

Rationale

The rationale of this study is based on the following assumptions from Piaget's theory of logical development[7] and from instructional ideas about subtraction:

1. Children pass through four stages of development: sensori-motor stage, pre-operational stage, concrete operations stage, and formal operations stage.

2. Each stage of a child's logical development is marked by a characteristic manner of thought.

3. Children display two types of arithmetical knowledge: *figurative* and *operative*.[8]

4. Subtraction as normally presented in arithmetic textbooks can be ordered in terms of the level of learning difficulty as follows:

   Level 1. The subtraction facts with numbers one through nine.
   Level 2. The subtraction facts with numbers ten through eighteen.
   Level 3. Subtraction of two- and three-digit numbers above twenty that do not require regrouping.
   Level 4. Subtraction of two-and three-digit numbers above twenty that need to be regrouped.

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[7] John L. Phillips, Jr., *The Origins of Intellect: Piaget's Theory* (San Francisco: W. H. Freeman and Company, 1969), pp. 16-90.

[8] Kenneth Lovell, *The Growth of Understanding in Mathematics: Kindergarten through Grade Three* (New York: Holt, Rinehart and Winston, Inc., 1971), p. 11.

Children in first, second, and third grade range in age from six to nine years. According to Piaget,[9] children who are below the age of seven are usually in an intermediate stage of development between the sensori-motor stage and the concrete operations stage. This stage is called the pre-operational stage and is marked by the child's inability to conserve numerical quantities. From about the age of seven to eleven, children are in the concrete operations stage. During this period children are able to conserve numerical quantities and deal with certain problems of classification and seriation.

Because of what is known about the logical development of children from the ages of six to nine and the implied connection between certain Piagetian logical developmental tasks and subtraction, the following tasks have been selected for use in this study:

1. Conservation of numerical quantity.
2. Quantitative class inclusion.
3. Seriation.

Piaget found that children display two types of arithmetical knowledge which he terms _figurative_ and _operative_.[10] The term _figurative_ refers to the child's

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[9] Hermine Sinclair, "The Training and Acquisition of Logical Operations," _Piagetian Cognitive-Development Research and Mathematical Education_ (Washington, D.C.: National Council of Teachers of Mathematics, Inc., 1971), pp. 1-9.

[10] Lovell, _Growth of Understanding_, p. 11.

knowledge of the symbolic manipulation and its end result. The child is only aware of the perceptual images and not the reality that brought about the situation. When the child has an understanding of the reality behind the symbols, he has an operative knowledge of the arithmetical process.

A child who has an operative knowledge of subtraction would be able to use concrete materials; such as, Diense Blocks, Cuisenaire Rods, Unifix Cubes, etc. to perform a subtraction operation that is presented without its algorithmic representation. It can be assumed that a child is operative on a particular level of subtraction difficulty if his performance on the concrete level matches his performance on the symbolic level.

Definition of Terms

The following definition of terms will be used in this study:

1. Subtraction: the inverse operation of addition.[11]

2. Operative knowledge of subtraction: a child will be said to have an operative knowledge of subtraction if he passes both the computational and manipulative tests given at a particular level of subtraction.

3. Figurative knowledge of subtraction: a child will be said to have a figurative knowledge of subtraction if he passes the computational test for a particular level of subtraction difficulty but fails the manipulative test of subtraction for that level.

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[11] Jack E. Forbes and Robert E. Eicholz, Mathematics for Elementary Teachers (Reading, Mass.: Addison-Wesley Publishing Co., 1971), p. 116.

Statement of Hypotheses

A child must reach a certain level of logical thought before he can have an operational knowledge of subtraction. To establish this assumption about subtraction, the following hypotheses will be tested:

1. A significantly greater number of children who have an operative knowledge of the subtraction facts one through nine will be able to give a greater number of operational responses on conservation of numerical quantities tasks than children with a figurative knowledge of the same subtraction facts.

2. A significantly greater number of children who have an operative knowledge of the subtraction facts ten through eighteen will give a greater number of operational responses on conservation of numerical quantities tasks than children with a figurative knowledge of the same subtraction facts.

3. A significantly greater number of children who have an operative knowledge of the subtraction facts ten through eighteen will give a greater number of operational responses on seriation tasks than children with a figurative knowledge of the same subtraction facts.

4. A significantly greater number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping will give a greater number of operational responses on conservation of numerical quantities tasks than children with a figurative knowledge of the same subtraction facts.

5. A significantly greater number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping will give a greater number of operational responses on seriation tasks than children with a figurative knowledge of the same subtraction facts.

6. A significantly greater number of children who have an *operative* knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping will give a greater number of operational responses on quantitative class inclusion tasks than children with a *figurative* knowledge of the same subtraction facts.

Limitations of the Study

This study was limited to first, second, and third grade children attending Garfield, Erma Reese, and Vinewood Elementary Schools. These schools are located within the City of Lodi and children attending these schools come from a representative cross-section of the community.

Summary

The need for research on the implications of Piaget's theory of intellectual development for the teaching of subtraction is apparent. To establish a useful comparison between the child's stage of intellectual development and his subtraction ability would improve instruction in the elementary school. The remaining four chapters will be organized as follows:

1. *Chapter II*: A review of the literature of Piaget's theory of intellectual development and other relevant research on how children learn subtraction.

2. *Chapter III*: The procedures and methods of collecting the research data will be described.

3. *Chapter IV*: The data that were collected will be presented and the findings will be revealed.

4. *Chapter V*: Conclusions of the study and recommendations for further research will be discussed.

CHAPTER II

REVIEW OF THE LITERATURE

The studies reviewed in this chapter will be organized into five sections. First, Piaget's theory of intellectual development will be briefly reviewed. Second, the child's conception of number will be discussed as it pertains to three operational structures: conservation of numerical quantities, seriation, and quantitative class inclusion. Third, other research findings that pertain to the child's development of conservation of numerical quantities, seriation, and quantitative class inclusion will be examined. Fourth, the child's conception of number as it relates to his developing ability to use arithmetical ideas in the classroom will be examined to determine the positive trends that might be revealed. Fifth, how the learning of subtraction is related to the child's conception of number will be reviewed.

Piaget's Theory of Intellectual Development

The nature of intelligence

Piaget states that knowledge does not originate from within the child but is a result of an interaction between the child and his environment.

> Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy, or image, of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed. An operation is thus the essence of knowledge; it is an interiorized action which modifies the object of knowledge.[1]

The act of knowing has two different aspects depending upon the physical circumstances.

> The aspects of knowing which deal essentially with fixed states we shall call figurative aspects of the _figurative function._ Examples of this function which deals with static configurations independent of transformations are perception, imitation, and mental imagery . . . . The aspects which focus on transformations I shall refer to as the _operative function._ In this we shall include physical actions which transform objects in one way or another, and we shall include operations, that is interiorized actions which have become reversible and are coordinated with other operations in a structure.[2]

The process by which the child coordinates the operations within a structure is called a _construction._[3] A _construction_ is both a coordination of the child's actions and an interrelation between objects. An early example of these _constructions_ occurs in the first year of

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[1]Jean Piaget, "Cognitive Development in Children: The Piaget Papers," in _Piaget Rediscovered: A Report of the Conference on Cognitive Studies and Curriculum Development_, ed. by Richard E. Ripple and Verne N. Rockcastle (Cornell University: School of Education, 1964), p. 8.

[2]_Ibid._, p. 21.

[3]Jean Piaget, "Piaget's Theory," in _Carmichael's Manual of Child Psychology_, ed. by Paul H. Mussen (New York: John Wiley and Sons Inc., 1970), p. 704.

the child's life. Between the age of nine to twelve months the child discovers that an object which no longer can be seen does not just dissolve but has a permanence. When permanence is first established in the sensorimotor stage, the child learns that if an object disappears at a certain point in his visual field, the object will again reappear at that same point. In this way the child learns ~~to look for an object at the point where it first~~ disappears even though at a later time it disappears at a completely different point.

<u>Stages in the development of intelligence</u>

Piaget[4] identifies four stages of intellectual development: sensorimotor period, preoperational period, concrete operations period, and the formal operations period. Children pass through these stages of cognitive development in a continuous growth pattern.[5] The chronological age which is associated with each stage of development represents the age at which at least three-fourths of the population has acquired a particular concept.

The period beginning with birth and lasting to about the middle of the second year is identified as the sensorimotor stage of development. Near the end of the

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[4]Richard W. Copeland, <u>How Children Learn Mathematics</u> (New York: The Macmillan Company, 1970), pp. 10-11.

[5]<u>Ibid.</u>, p. 7.

sensorimotor period children acquire the notion that objects have permanency and can be retrieved even if the object is out of the perceptual field.

The second stage, preoperational period, is an intermediate stage of development between the sensorimotor period and the concrete operations period. Children below the age of seven are usually in the preoperational stage. During this period, children are not able to conserve numerical quantities. The child has a tendency to center his attention on one detail of an arrangement and exclude others. His thinking goes from point to point with little connection between ideas.

The third stage, concrete operations period, lasts from about the age of seven years to eleven years. This stage is very significant because it marks the period when children can engage in logical thought on a concrete level. In Piaget's terminology "concrete" refers to real objects and the term "operation" refers to the child's ability to internalize actions which are reversible (the doing and undoing of a process).

During the concrete operations stage, the child is able to accomplish a number of grouplike structures of transformation.[6] The grouping structures that the child

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[6] Hermine Sinclair, "Piaget's Theory of Development: The Main Stages," Piagetian Cognitive-Development Research and Mathematical Education (Washington, D.C.: National Council of Teachers of Mathematics, Inc., 1971), pp. 7-8.

is able to think logically about include conservation, classification, and seriation.

At about the age of eleven or twelve, the child enters the fourth stage of development, the formal operations period. The child is able to reason logically using symbols that are not based on concrete objects. He is able to use a hypothetic-deductive procedure of thinking that is not tied to existing reality.

The Child's Conception of Number

According to Piaget the child's conception of number is bound directly to the child's development of intelligence.

> Our hypothesis is that the construction of number goes hand-in-hand with the development of logic, and that a pre-numerical period corresponds to the pre-logical level.[7]

There are three main operational structures which Piaget identifies in the concrete operations period of intellectual development which coincide with operational structures in the child's conception of number. They are conservation of numerical quantities, seriation, and class inclusion. Conservation is thought by Piaget to be very fundamental. He states, " . . . conservation is a necessary condition for all rational activity . . . ."[8]

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[7] Jean Piaget, The Child's Conception of Number (New York: W. W. Norton and Company, Inc., 1965), p. viii.

[8] Ibid., p. 3.

Conservation may be defined as the ability to understand that a particular dimension of an object will remain unchanged even though other irrelevant aspects of that object may undergo change. Piaget indicates that there are three stages in the development of the understanding of conservation. Each of the stages may be clearly identified by the characteristic responses made by the child. At the first stage there is an absence of conservation. The child thinks that quantity varies with irrelevant aspects of the object, i.e., a change in the arrangement of a set of markers also changes the quantity of markers present. The second stage is a transitional stage. A child may at first assert conservation when perceptual changes are not great but then revert to non-conservation when perceptual relationships come into conflict. Conservation occurs immediately in the third stage. The child will maintain his conviction regardless of perceptual conflict.

Piaget identifies three types of perceived quantity that relate to the three stages of conservation: gross quantities, intensive quantities, and extensive quantities.

> At the level of the first stage, quantity is therefore no more than asymmetrical relations between qualities, i.e., comparisons . . . . As soon as this intensive quantification exists, the child can grasp, before any other measurement, the proportionality of differences, and therefore the notion of extensive quantity. This discovery, which alone makes possible

the development of number, thus results from the child's progress in logic during these stages.[9]

An adequate concept of number must also include an understanding of seriation and classification. Seriation first appears at the sensorimotor level. Children can perceptually seriate a number of objects if differences in elements of the series are great. Operational seriation differs from perceptual seriation; to be operational the child must be able to perform four tasks:

> A child understands ordinal number when he is able to do four things. First, a child must be able to arrange in a sequence a set of objects which differ in some aspect. Piaget calls this action seriation. Second, he must be able to construct a one-to-one correspondence between two sequences of objects in which the elements of the sequences correspond because they have the same relative positions in the sequences. Such a one-to-one correspondence is called a serial correspondence. Third, he must be able to conserve a serial correspondence when it is no longer perceptible. Fourth, a child must be able to conserve an ordinal correspondence between two sequences of objects. The conservation of an ordinal correspondence is accomplished when a child can find an object in an unordered set (but a set which is capable of being ordered) which corresponds to a given object in an ordered set. The act of conserving an ordinal correspondence requires a child to arrange a sequence of objects and construct a serial correspondence between two sequences, either mentally or physically.[10]

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[9] Ibid., p. 5.

[10] Arthur F. Coxford, Jr., "The Effects of Instruction on the Stage Placement of Children in Piaget's Seriation Experiments," Current Research in Elementary School Mathematics, ed. by Robert B. Ashlock and Wayne L. Herman, Jr. (New York: The Macmillan Company, 1970), p. 113.

Operational seriation appears around the age of seven or eight. At about the same time, classification based on inclusion appears.[11] Seriation is somewhat more closely allied with perception and classification with language.[12] However, language alone is not sufficient to explain the conceptual system of class inclusion.

The understanding of quantitative class inclusion depends upon the prior conception of such words as "all" and "some." An understanding of quantitative class inclusion is demonstrated by the ability to answer questions in the following form: "Suppose one class A to be included in another class B, without being equal to the whole of B, are there more A's than B's or more B's than A's?"[13] Children tend to fail questions of inclusion because they cannot think of the whole in relation to its parts. When a child tries to answer a question about the relation of the A's to the B's he may compare the A's to themselves. Inhelder gives an example of how this might occur.

> "Ducks are birds; it's the same thing," says the child, "so there are the same number of both."

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[11] Bärbel Inhelder and Jean Piaget, The Early Growth of Logic in the Child (New York: Harper and Row, 1964), p. 249.

[12] Ralph Scott and Ludwig Sattel, "Perception and Language: A German Replication of the Piaget-Inhelder Position," Journal of Genetic Psychology, CXX (1972), 203.

[13] Inhelder and Piaget, Growth of Logic, p. 100.

> Everything seems to show that a young child can compare A and A' only while neglecting B. Or else he can only compare A and B while neglecting the complementarity of A and A' . . . some years later--the child finally understands that B > A. And he expresses his logical reasoning in such statements as: "There must be more birds than ducks. All those which aren't ducks are birds, and they have to be counted along with them."[14]

Research Related to Piaget's Findings on Number: Conservation of Numerical Quantities, Seriation, and Quantitative Class Inclusion

Conservation of numerical quantities

Elkind[15] undertook a replication study of the development of children's quantitative thinking because Piaget's studies "have been devoid of statistical methods and systematic design."[16] Elkind wished to substantiate those observations which Piaget made about the ages of children and the order of stages in which they perceive quantity. The following is what Piaget observed:

> Children at the first stage (usually age 4) succeeded only when a comparison of gross quantity was the minimum requirement for success. At the second stage (usually age 5) children succeeded when a comparison of gross or intensive quantity

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[14] Bärbel Inhelder, "Some Aspects of Piaget's Genetic Approach to Cognition," Cognitive Development in Children (Chicago: University of Chicago Press, 1970), p. 31.

[15] David Elkind, "The Development of Quantitative Thinking: A Systematic Replication of Piaget's Studies," Journal of Genetic Psychology, 98 (1961), 37-46.

[16] Ibid., p. 37.

> was the minimum requirement for success. Third stage children (usually age 6-7) succeeded when a comparison of gross, intensive, or extensive quantity was the minimum requirement for a successful result.[17]

The manner in which these observations were tested is summarized as follows:

> Eighty school and pre-school children were divided into three Age Groups (4, 5, 6-7) and tested on three Types of Material for three Types of Quantity in a systematic replication of Piaget's investigation of the development of quantitative thinking. Analysis of variance showed that success in comparing quantities varied significantly with Age, Type of Quantity, Type of Material, and two of the interactions. Correlations for Types of Material were positive, high, and significant. Correlations of comparison scores and W.I.S.C. scores were positive, generally low, and sometimes significant.[18]

There was a very close agreement with Piaget's findings that quantity is perceived in ordered stages that relate to the age of the child. Each statistical test was significant beyond the .01 level with the exception of a number of sub-test correlations on the W.I.S.C. intelligence test which compared the children's quantity scores to scores on the intelligence test.

Elkind found that the judgments that children could make about quantity varied with their logical development. Judgments that involved gross quantity could be made easier than ones involving intensive quantity, and it was less difficult to make judgments of intensive quantity than extensive quantity. Success in making quantity

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[17] *Ibid.*, p. 38.

[18] *Ibid.*, p. 45.

judgments also varied with age. Younger children could make judgments of gross quantity but could not make judgments of extensive quantity until they were older. Elkind found exceptions to the linear relation between age and the type of quantity the child could judge. A child who could make judgments of extensive quantity with one type of material might be able to only make judgments of intensive quantity with other materials--this phenomena is called horizontal decalage. A child may exhibit a characteristic cognitive structure but not be able to perform all of the tasks within that structure. Flavell states: "In brief, the existence of horizontal decalages seems to point up a certain heterogeneity where only homogeneity might have been suspected."[19]

The correlations that Elkind found between children's quantity scores and their I.Q. as measured by the W.I.S.C. intelligence test were low. Although the correlations were positive, there is doubt as to the role intelligence may play in conservation tasks. The role of I.Q. was explored further in a study by Feigenbaum.

Feigenbaum's[20] major interest was in the relation between the child's I.Q. and his understanding of

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[19] John H. Flavell, The Developmental Psychology of Jean Piaget (Newark: D. Van Nostrand, Inc., 1963), p. 23.

[20] Kenneth D. Feigenbaum, "Task Complexity and I.Q. as Variables in Piaget's Problems of Conservation," Child Development, 34 (1963), 423-432.

conservation of discontinuous quantities. He tested three hypotheses concerning Piaget's contention that each stage of logical development determines the method and mode of thinking that a child will employ.

To test his first hypothesis that age is not the sole determiner of a child's development of conservation of discontinuous quantities, fifty-four nursery school and elementary school children were used. The children were divided into two age groups: forty-five to sixty-four months and sixty-five to eighty-seven months. Children were given tests of correspondence and conservation of discontinuous quantities. Children in the oldest age group did significantly better than children in the younger age group. The level of significance was at the .01 level. Although there was statistical significance for the difference in age groups, it was noted that some of the children in the younger age group were able to solve most of the problems.

The second hypothesis tested the children's level of success in relation to intelligence as measured by the Stanford-Binet Test of intelligence. Children in the experimental group were divided into two groups according to intelligence: children with I.Q. scores above 119 and children with I.Q. scores below 119. The results of the chi-square test of significance for the conservation tasks yielded a .05 level of significance. Inspection of the data revealed that children with higher I.Q.s performed at a level superior to children who were older but had lower I.Q.s.

An analysis of the modes of responses made by children tended to agree with Piaget's findings, however, there were some notable exceptions. One of the exceptions was the use of counting. Piaget did not report this mode of response in his investigations. Feigenbaum found counting to be a mode of response that was significant at the .01 level for children with a mean age of about sixty-eight months. Since the investigator did not mention how many children used counting or the procedure used to determine the level of significance, it would be difficult to do more than note that children make operational responses other than those mentioned by Piaget.

The third hypothesis dealt with materials of various sizes and shapes used in the conservation and correspondence tests. Two groups of children were used to test this last hypothesis: one group contained fifteen children and the other group contained twenty-one children. Performance differences of the two groups were equated as to age, I.Q. and conceptual ability. Most of the differences noted in the two groups were not significant.

A major finding of this study would indicate that age is not a definite barrier in the acquisition of the concept of conservation. The data indicates that intelligence has a significant effect upon the age at which a child acquires the concept of conservation. Ability to conserve appears to be both a function of age and I.Q. Feigenbaum indicates that counting was used by children as one of the modes of responses to the correspondence and conservation problems; these responses could have been due to the testing procedures. Children were prompted on tests of correspondence and asked to count the number of beads; this may have had a carry-over effect to the conservation test. In the study done by Dodwell, which is reviewed next, further evidence is given that the stages of development as identified by Piaget are subject to variation.

Dodwell[21] initiated an investigation of the child's understanding of number to assess the generalities of behavior Piaget described for children between the ages of five and eight years. Two hundred fifty public school children ranging in age from five years and six months to eight years and ten months took part in the investigation. Children were tested with similar test materials as were used by Piaget. The tests which were given are as follows:

1. Relation of perceived size to number (conservation of numerical quantities).
2. Provoked correspondence.
3. Unprovoked correspondence.
4. Seriation.
5. Cardination and ordination.

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[21] P. C. Dodwell, "Children's Understanding of Number and Related Concepts," Canadian Journal of Psychology, 16 (1960), 191-195.

The results of the investigation showed that children could be classified into three groups according to their answers. These groups are the same as the ones identified by Piaget: global comparisons, intuitive judgments, and concrete operations. Although children could be grouped by their responses, there was considerable variation in the number of children giving operational responses for the various ages. There did not seem to be a "typical" age for the attainment of a concrete operational activity. Children's performance on conservation of numerical quantities was somewhat varied: 60 per cent of the children at five years and ten months gave operational responses, and 50 per cent of the children at six years, five months gave operational responses. Eighty per cent of the children from ages seven years and six months to eight years and six months gave operational responses. The children's responses on the seriation test showed on the average more operational responses than was shown on the cardination and ordination test.

Dodwell's results on conservation of numerical quantities problems show that there is considerable variation in abilities to conserve for younger children, but there is greater stability for older children, those in the higher grades. Variations found in the ability to conserve as related to age would suggest that I.Q. or other factors such as familiarity of the material used in testing have an effect on the child's responses. Although

Dodwell was not able to assess the effect of I.Q. on the child's performance of conservation problems, it would seem that Feigenbaum's findings on the relation of I.Q. to success have some bearing.

Seriation

Coxford[22] had two purposes for his study:

> (a) to replicate Piaget's experiments in seriation, ~~serial correspondence, and ordinal correspondence,~~ and (b) to ascertain the effect of instruction on advancing a child from one stage to the next.[23]

Sixty children were chosen for the experiment. Their ages ranged from three years, six months to seven years, five months. All sixty children were given a pretest to determine their ability to seriate. Children were classified according to their responses by stages:

Stage 1: All parts of the test were done incorrectly.

Stage 2: Some of the items on the test were done correctly, but mistakes were made on various subtests.

Stage 3: The entire test was done correctly.

After the pretest was given, twenty-four children were selected for instruction on seriation and another group of twenty-four was selected as a control group.

The material which was used in both the pretest and posttest was ten cardboard balloons (varying uniformly

[22]Arthur F. Coxford, Jr., "The Effects of Instruction on the Stage Placement of Children in Piaget's Seriation Experiments," in Current Research in Elementary School Mathematics, ed. by Robert B. Ashlock and Wayne L. Herman, Jr. (New York: Macmillan Company, 1970), pp. 113-120.

[23]Ibid., p. 114.

in size from small to large) and ten complementary cardboard sticks. Results of the pretest were as follows: the mean chronological age for children in Stage 1 was 56.2 months, Stage 2 was 69.8 months, and Stage 3 was 77.6 months. These findings tended to agree with Piaget's predictions of age as related to stage of development. Two exceptions were noted; one very intelligent child of 46 months tested at Stage 3 and another child of 83 months tested at Stage 1.

Of the group of twenty-four children that were selected for instruction, twelve children were in Stage 1 while the remainder were in Stage 2. Both groups received objects made of cardboard that were similar to the balloons and sticks used in the pretest. Children were given games to play with the materials designed for each of four sessions that would help them to overcome particular difficulties noted on the pretest. At the end of the teaching session, the posttest was given and scores of the experimental group were compared to scores made by children in the control group. The greatest gain was recorded for children in Stage 2 of the experimental group. Six of the twelve children were able to make Stage 3 responses after instruction. This gain was significant at the .05 level of significance.

Coxford's research indicates that a child's experience with seriable objects can help the child to become operational sooner than he might without these experiences. Experience, however, is not the only factor--the child must be at a transitional stage for the experience to be effective.

Churchill[24] conducted a small investigation involving a group of five year old children. Sixteen children were selected for the investigation. Churchill was primarily interested in the effects of instruction on the growth of numerical ideas in young children. Children in the investigation were given three series of tests at the beginning of the experimental period and again at the end. Two tests were given to assess the child's understanding of a one-to-one correspondence (qualitative and numerical correspondence) and the third test was given to assess the child's ability to perform tasks that involved numerical seriation as well as qualitative seriation. Eight children in the experimental group met twice a week for four weeks. Each session lasted one-half hour. Instruction consisted of using familiar objects that were placed in groups and series; children were helped to solve the problems during the instructional periods by using some form of counting.

Differences in scores between the experimental and control groups with respect to change in performance on the Piagetian tests from the pretest to the posttest

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[24] Eileen M. Churchill, "The Number Concepts of the Young Child: Part I," Researches and Studies, Leeds University, XVIII (1958), 34-49.

were significant at the .01 level. Evidence of this investigation suggests that experiences of the kind provided in the experimental group meetings contributed to the earlier formation of basic concepts. The use of counting by the experimental group on the posttest was a characteristic feature in the child's thinking.

> Actually a rigorous analysis of the whole series showed that the use of counting in a *numerical correspondence* was one of the characteristic features which marked out the more advanced children from the others. The children who used counting tended to do so throughout the series, though not always as their initial reaction. In contrast, none of the children who made no use of enumeration showed more than "stage-one" responses.[25]

Evidence from Churchill's study would suggest that instruction can contribute significantly to the early formation of logical concepts. In this regard, both Coxford and Churchill are in agreement. Children's use of counting was also noted by Feigenbaum in his work with children on correspondence and conservation problems. Both Feigenbaum and Churchill found counting to be a mode of thinking which was used by children to deal with problems of logical thinking. Counting is used by children to solve simple problems in addition and subtraction. Whether Feigenbaum's and Churchill's findings about counting have relevance for the learning of mathematics has yet to be established.

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[25] Eileen M. Churchill, "The Number Concepts of the Young Child: Part 2," *Researches and Studies, Leeds University*, XVIII (1958), 34.

Quantitative Class Inclusion

Dodwell[26] investigated Piaget's contention that the development of number occurs side by side with the development of class inclusion. Some understanding of class inclusion is a necessary condition for dealing in a consistent fashion with number. Dodwell selected sixty public school children between the ages of five years, two months and eight years, eight months.

The material used in the tests for class inclusion were all familiar objects to the children (rakes, shovels, dolls, and cars). Children's responses were easily classified into three categories: responses that were clearly operational, responses that were non-operational, and responses that were indefinite.

Dodwell gave tests of provoked and unprovoked correspondence; they were used to measure the child's concept of "cardinal number." In the test for provoked correspondence, eggs and eggcups were used to provoke an obvious perceptual correspondence between the individual members of the two sets. Two sets of poker chips were used in the test for unprovoked correspondence to set up a perceptual correspondence which was then disrupted by the experimenter (one set was pushed into a bunch).

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[26] P. C. Dodwell, "Relation between the Understanding of the Logic of Classes and of Cardinal Number in Children," Canadian Journal of Psychology, XVI (1962), 152-159.

The results of the study showed that the correlations between composition of classes and number were all low. This would indicate a very small tendency for children to answer correctly questions of correspondence ("cardinal number") and class inclusion. There was a significant tendency for children who answered one part of the class inclusion test correctly to answer other parts correctly. Dodwell concludes:

> Although there is no clear relation between the development of the two types of concept, either in terms of priority of appearance or concomitance, they both develop within the same age range. It can be argued, as was done in the case of the number concept test (Dodwell, 1960), that invariability is largely due to learning specific responses for particular types of situations and material, and generalization of such responses to novel situations is imperfect.[27]

Logically there should be a relation between "cardinal number" and class inclusion. The fact that Dodwell did not find one suggests that children may be able to consistently deal with number at an elementary level with or without being fully operational on class inclusion problems.

The Child's Conception of Number as it Relates to his Developing Ability to Use Arithmetic Ideas in the Classroom

Hood[28] wished to trace the characteristic stages

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[27] *Ibid.*, p. 159.

[28] H. Blair Hood, "An Experimental Study of Piaget's Theory of the Development of Number in Children," *British Journal of Psychology*, LIII (1962), 273-286.

of development identified by Piaget. A secondary consideration was to relate these findings to children's performance in arithmetic. The experiment was conducted in England with 126 children between the ages of four years, nine months and eight years, seven months. Each child in the study was given the Terman-Merrill scale "L" to assess his mental age; comparisons could then be made between mental and chronological ages at which a child attained a concept. Eight different number concept tests were given to children; these tests consisted of various tests of correspondence, seriation, and class inclusion.

Teachers in the study were asked to rank their pupils according to arithmetic ability. Five categories were established and ranked as follows:

<u>Rank</u> <u>1</u>. Children with no number ability and who were unable to pick out five or more objects from a group.

<u>Rank</u> <u>2</u>. Children who could pick out five or more objects from a group.

<u>Rank</u> <u>3</u>. Children who could do simple addition and subtraction with or without the use of counters.

<u>Rank</u> <u>4</u>. Children who could do simple problems stated in writing or verbal form with apparent understanding.

<u>Rank</u> <u>5</u>. Children who could do all of the problems done by all of the children in the lower ranks and beyond.

Teachers found it difficult to rank children according to the categories mentioned; however, after the children were ranked, the data revealed the following trend.

Sixty-two per cent of the children ranked in the fifth category gave operational responses to Piagetian number concepts. Twelve to 13 per cent of the children in ranks 2 through 4 gave operational responses while none of the children ranked in the first category gave operational responses.

Although Piaget never intended to have his experiments used to measure how much arithmetic a child should be able to do, it has been demonstrated by Hood that there is a positive comparison between number concepts and arithmetic ability. Children with greater ability in arithmetic tend to have a higher per cent of operational responses. While children with less ability in arithmetic tend to have a lower per cent of operational responses. Children may, as Hood observes, be taught methods of problem solving.

Hood observes:

> A child may be trained, not only in mechanical processes, but on problems work, to act as if he understood number. Methods of solving a problem may be skillfully taught, . . . and the presence or absence of the concepts themselves does not constitute for him an element in the problem.[29]

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[29] *Ibid.*, p. 279.

How the Learning of Subtraction is Related to the Child's Conception of Number

LeBlanc[30] studied the performances of first grade children in problem solving and the relation of this performance to four levels of conservation of numerousness. The children in the sample were also divided into I.Q. groups so that it was possible to assess the relationship of three factors: levels of conservation of numerousness, levels of intelligence, and levels of problem solving difficulty.

The subjects were 400 first grade children, all of whom were given four tests: a Kuhlman Anderson Group I.Q. Test, a pretest of conservation of numerousness, a problem solving test in subtraction, and a subtraction facts test. Children included in the study were divided into three groups based on their I.Q. scores. The ranges of I.Q. scores were 78-100, 101-113, and 114-140. Approximately one-third of the total cumulative frequency was within each I.Q. range. The four categories in which children were placed on the pretest of conservation of numerousness were as follows: level 1, all four items on the test were answered correctly; level 2, all of the items on two tests were answered correctly; level 3, all of the

[30] John Francis LeBlanc, "The Performance of First Grade Children in Four Levels of Conservation of Numerousness and Three I.Q. Groups when Solving Arithmetic Subtraction Problems," (unpublished Ph.D. dissertation, University of Wisconsin, 1968), pp. 1-189.

items on one test were answered correctly; level 4, none of the items on the test were answered correctly. The subtraction problem solving test contained eighteen problems. These problems were divided into two groups of nine problems each. The first group of problems involved no transformations (real or implied physical action that transforms the object); the other group of problems involved transformations. Each group of nine problems was subdivided so that there were three problems with manipulative aids, three problems with pictorial aids, and three problems with no aids. Each problem was read to the child and the experimenter displayed the appropriate material if the problem involved a manipulative or pictorial aid. The subtraction facts test, like the problem solving test, contained subtraction combinations with numerals less than nine.

An analysis of variance was used to test the statistical significance of the data gathered. Statistical analysis revealed that the relation between conservation of numerousness and problem solving was significant at the .01 level. LeBlanc states:

> The most significant outcome of this study is the relationship of conservation of numerousness as measured by the pretest to children's performance in a problem solving test. Although all children received training based on the same curriculum, the performances of the children categorized into four levels of conservation of numerousness were significantly different. The children who did well on the conservation test did well on the problem solving test. Likewise, the children

> who did poorly on the conservation test did poorly on the problem solving test.[31]

Children in the various I.Q. groups did not do significantly better than any other I.Q. group on the subtraction problems. A correlation between problem solving tests and the number facts test was found statistically significant but the correlation was low (.39). LeBlanc concludes:

> . . . the relationship between children's knowledge of number facts and their performance of the problem solving test is questionable. Surely, knowledge of basic facts is not sufficient for success in problem solving.[32]

Children also performed significantly better on some types of problems than others. Children performed better on problems where there were aids and a transformation and poorer on problems where there were no aids or transformations.

The results of LeBlanc's study have a number of implications for the present study of subtraction. While conservation of numerousness is a significant factor in a child's problem solving ability, it remains to be established that conservation is a significant factor in computation of the subtraction facts. Children's performance with problems that involved aids was better than their performance without aids. This would seem to support Piaget's contention that children in the concrete operational periods of development can think logically

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[31] *Ibid.*, p. 154.

[32] *Ibid.*, p. 159.

with real materials but have difficulty when they are asked to think logically in the absence of materials. Since LeBlanc did not give the ages of the children who were operational or non-operational on the conservation test, comparison to the ages predicted by Piaget cannot be made.

Summary

Research reviewed in this chapter was organized into five sections. First, Piaget's theory of intellectual development was reviewed. Next, the discussion of the child's conception of number was limited to conservation of numerical quantities, seriation, and quantitative class inclusion. Piaget leaves little doubt that there is a direct connection between the child's logical development and his development of the conception of number. In the third section, other research studies were reviewed. In general these studies indicate that, while there is a close connection between the child's age and his stage of development, it is subject to individual variations. Intelligence of the child seems to be one of the factors that is responsible for some of the variations noted. The last two sections address the child's problem of logical development as it relates to his ability in arithmetic and to his ability to do subtraction. Although very few studies have been done on subtraction, evidence available indicates that a search for statistically significant relationships between subtraction and conservation of numerical quantities, seriation, and quantitative class inclusion may be fruitful. In the chapter that follows, the methods and procedures used in this investigation will be discussed.

CHAPTER III

EXPERIMENTAL PROCEDURES

The procedures which were used in this investigation will be discussed under the following six headings: population and sample, procedures, hypotheses, measures of ability to subtract, measures of logical thinking, and statistical design.

Population and Sample

Population

Lodi is the second largest community in San Joaquin County and has a population of about 30,000 people. Because of rapid growth of the city's population in recent years, Lodi has one old residential area and one large new area. Families that live in the older area tend to have less wealth than families that live in the newer residential areas.

The majority of the residents are middle-class Whites. Since there are very few industries in Lodi, many of the residents commute to nearby areas such as Stockton or Sacramento. Agricultural interests in and near Lodi provide employment for a number of agricultural workers.

Sample

A total of ninety children were selected from Garfield, Erma Reese, and Vinewood Elementary Schools. Garfield School is located in the older residential area of town while Erma Reese and Vinewood Schools are located in the newer residential areas. Garfield School has grades kindergarten through the third grade. About 230 first, second, and third grade children attend Garfield School. According to the 1972 statistics compiled by the San Joaquin County Schools Department, 59 per cent of the children attending Garfield School come from homes with low incomes. Erma Reese School has grades kindergarten through the sixth grade. There are about 240 children in the first, second, and third grade; 8 per cent of the children in this attendance area come from homes with low incomes. Vinewood School also has grades kindergarten through the sixth grade; 4 per cent of the children at Vinewood School come from families with low incomes.

Thirty children were randomly selected from each school on the basis of a computation test in subtraction. The method which was used in the selection process and the testing procedures will be discussed in the next section.

Procedures

Selection of subjects

Children in the first, second, and third grades at

Garfield, Erma Reese, and Vinewood Schools were given a paper and pencil test in subtraction computation. The computation test consisted of four tests with seven problems on each test. Children able to pass the first two tests were given the next two tests. The following is the order of difficulty of each test:

<u>Test 1</u>. The subtraction facts with numbers one through nine.
<u>Test 2</u>. The subtraction facts with numbers ten through eighteen.
<u>Test 3</u>. Subtraction of two- and three-digit numbers above twenty that do not require regrouping.
<u>Test 4</u>. Subtraction of two- and three-digit numbers above twenty that need to be regrouped.

Testing was done by the investigator with the cooperation of the classroom teachers. The following tests were given at each grade level:

<u>First grade</u>: tests 1 and 2.

<u>Second grade</u>: tests 1, 2, 3, and 4.

<u>Third grade</u>: tests 1, 2, 3, and 4.

Children in the first grade who passed both tests 1 and 2 were given tests 3 and 4. To pass a test the child was required to correctly compute five out of the seven items on each test. Copies of the four computational tests may be found in Appendix A.

Upon completion of the testing, children were ranked according to the test of highest numerical value they passed. Children passing test 4 were eliminated from the sample, because their computational ability was beyond the limits of this investigation. A list of random numbers was used to select ten children from each of the remaining three groups. A total of thirty children were selected from lists compiled at each school for a total of ninety children.

Testing of subjects on the dependent variables

Three different subtraction tests that required the use of manipulative materials were constructed. Each of the tests were comparable to one of the computation tests given in the preceding section. Samples of the script used for the manipulative tests may be found in Appendix B.

Children were given the manipulative test which corresponded to the computational test with the highest rank which was passed. The minimum number of problems the child was expected to perform correctly was two problems out of three. Since the last two problems on each of the tests involved a physical transformation (some of the blocks were hidden from the child's view), these problems were considered to have greater importance when the test was scored.

Following each of the subtraction tests each child was given a Piagetian logical thinking task. These tasks were given as follows:

1. Conservation of numerical quantities tasks were given to children in the subtraction level 1 group.

2. Conservation of numerical quantities and seriation tasks were given to children in the subtraction level 2 group.

3. Conservation of numerical quantities, seriation, and quantitative class inclusion tasks were given to children in the subtraction level 3 group.

Samples of the scripts used for each of the above Piagetian tasks may be found in Appendix C.

Training of tester

The individual tests for subtraction and logical thinking were given by the investigator and one volunteer who had two years of teaching experience with elementary and secondary students. Prior to the administration of the individual subtraction tests that required the use of manipulative materials and again prior to the administration of the Piagetian tasks, practice sessions were held. Each script was carefully followed.

Approximately half of the children in the sample were tested by the principal investigator. The other half were tested by the volunteer tester. At the end of each testing session, the responses made by each child were reviewed and scored cooperatively.

Controls over testing and scoring

Each of the testing sessions was recorded. In cases where there was some doubt about the administration or scoring of a particular test, the tapes were reviewed by the principal investigator.

Hypotheses

The hypotheses stated in Chapter I are stated here in the form of null hypotheses.

HO_1: There is no significant difference between the number of children who have an operative knowledge of the subtraction facts one through nine and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks.

HO_2: There is no significant difference between the number of children who have an operative knowledge of the subtraction facts ten through eighteen and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks.

HO_3: There is no significant difference between the number of children who have an operative knowledge of the subtraction facts ten through eighteen and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to seriation tasks.

HO_4: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks.

HO_5: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to seriation tasks.

HO_6: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to quantitative class inclusion tasks.

Measures of Ability to Subtract

Computational tests of subtraction

Problems for the computation tests of subtraction were provided by CTB/McGraw-Hill located in Monterey, California. The problems were selected from eight different standardized tests which were administered to more than 200,000 students. The items were chosen on the basis of item analysis for their ability to discriminate grade levels. Information by the publisher concerning each test item may be found in Appendix D. The per cent of children in the national sample who passed each item and the grade level of the children tested are given. A letter from the Director of Test Development gives further information about the computational items supplied for this investigation.

Forty test items were provided by CTB/McGraw-Hill; ten items for each of the four tests. Although each of the ten items had four answer choices as provided by the publisher, these item responses were not used in the experimental testing. Children were required to respond by recalling the correct response in each testing. Three of the ten items for each of the four tests were randomly selected for use with the subtraction test that requires the use of manipulative materials. Since a manipulative test was not constructed using the subtraction problems from the fourth test, these three items were discarded. The twenty-eight subtraction problems that remained were given to sixty-one first, second, and third grade children not included in the study. Two weeks later the test was given again and a test-retest reliability coefficient for each of the grades was calculated. The coefficients found for each grade were as follows:

<u>Grade 1</u>. The Pearson <u>r</u> correlation coefficient for twenty children in the sample was .72.

<u>Grade 2</u>. The Pearson <u>r</u> correlation coefficient for seventeen children in the sample was .56.

<u>Grade 3</u>. The Pearson <u>r</u> correlation coefficient for twenty-four children in the sample was .75.

Since test scores were used to establish the level of subtraction difficulty the child could compute, the reliability coefficients were generally acceptable. The fact that the correlation coefficient for the second grade class was much lower than the first and third grade coefficients could not be explained. Data used to calculate each correlation is displayed in Appendix E.

Manipulative tests of subtraction

The subtraction problems selected for use with manipulative materials were presented in the following order.

1. The child was given a set of blocks to count. A problem was then stated in which the child was required to show the number of blocks in the set that belonged to a remainder set.

2. The child was given a set of blocks to count. A screen was then placed in front of the blocks. After some of the blocks were placed in front of the screen, the child was instructed to find the number of blocks that remained behind the screen. Extra blocks were made available to the child for the solution of the problem.

3. Two equal sets of blocks were presented to the child. After each set of blocks was counted, some of the blocks from one of the sets were placed behind a screen. The child was then instructed to find the number of blocks that were placed behind the screen.

Between eight to ten children were randomly selected from each of three groups: children who could only compute problems on test 1, 2, or 3. (All of the children selected were also part of the reliability study described in the previous section.) Each child was then given the appropriate manipulative test of subtraction that corresponded to his placement as determined by the computation test. Children who were unsuccessful on the computation test were unsuccessful on the manipulative test. Children who were relatively successful on the computation test were not always successful on the manipulative test; however, children who were highly successful on the computation test were highly successful on the manipulative test.

Measures of Logical Thinking

Conservation of numerical quantities

To test conservation of numerical quantities, two conservation tasks were used. These tasks are similar to the conservation tasks used by Millie Almy[1] in her work with second grade children. Out of a group of 629 second grade children participating in the study, Almy found 366 second grade children who were operational on the two conservation tasks of numerical quantities.

In the first conservation task eleven yellow blocks and fourteen blue blocks were placed in front of the child. After the yellow and blue blocks were arranged in two parallel rows, the child was asked if there were as many yellow blocks as blue blocks. He was then given some yellow blocks and asked to make the yellow row the same as the blue row. After the child achieved this task, the arrangement of the blocks in the blue row was changed. The

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[1] Almy and Associates, Logical Thinking, p. 117.

blue blocks were pushed together. Again, the child was asked if the two sets of blocks were equivalent. Next the row of yellow blocks was spread out and the child was asked the same question.

In the second task sixteen yellow blocks were placed in a row in front of the child. He was then instructed to count the blocks. After the blocks were counted, the spaces between the blocks were increased so that the row was longer. The child was then asked to tell how many blocks there were in the row (he was not allowed to recount the blocks). Next, the blocks were pushed into a pile. The child was asked to tell without counting how many blocks were in the pile. A copy of the script used for the conservation of numerical quantities tasks and the other Piagetian tasks that follow are found in Appendix C.

Seriation

Instead of using three dimensional objects as were used by Piaget, a set of cards with pictures of farmers and shovels were used. Almy[2] also used pictures in her research with second grade children. Her results indicate that this task was extremely difficult for second graders. Only 5 per cent of the children were operational on the ordination, seriation, and reordering task and 15 per cent of the children were operational on both the ordination and seriation tasks.[3]

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[2] *Ibid.*, p. 37. [3] *Ibid.*, p. 154.

Almy's results on the seriation task with second graders agrees favorably with the results obtained in a similar experiment done by E. H. Hood:

> Using drawings of ten boys differing in size and and a complementary set of ten hoops, (the test) required the child to put each set into serial order and then to make the two sets correspond. She found this to be one of the most difficult of the tasks related to the concept of number. At age seven only 6 per cent of her normal subjects had reached the level of operational thought that enabled them to solve this problem of seriation. Of the eight-year-olds, 34 per cent, and of the nine-year-olds, 75 per cent, were at this level.[4]

In this investigation, two sets of ten cards were used. One set of cards had ten men differing in size by three-eights of an inch in height. The second set of cards contained a complementary set of shovels. Three types of problems were posed with the cards: serial order, ordination, and reordering.

Serial order.--The two sets of cards were shown to the child. From the set of cards with men on them, the two smallest and the two largest men were placed two feet apart. The child was asked to arrange the remaining cards in their proper order. If the child ordered the set properly, he proceeded to the second task; however, if the child made an error, the error was corrected before the child was allowed to proceed to the second part. In the second part, the child was asked to find the shovel that went with each

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[4] *Ibid.*, p. 37.

man after the largest and smallest shovel had been placed at the foot of each corresponding man.

Ordination.--In this task, each man was matched with a complementary shovel; then the row containing the shovels was displaced three cards to the left. The child was then asked to find the shovels that belonged to three specified men.

Reordering.--The men were placed in a row and the shovels were scrambled. The evaluator then asked the child to find a shovel that belonged to a designated man.

Quantitative Class Inclusion

When Piaget[5] first conducted his experiments with class inclusion he found that children were not able to understand the logical relationship of the parts to the whole until about the age of seven. In subsequent studies done by Inhelder and Piaget[6] it was found that the majority of children are not truly operational until after the age of seven. This finding is substantiated by Almy[7] who found that children do not begin to gain a good grasp of class inclusion until they are beyond the age of seven.

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[5] Jean Piaget, The Child's Conception of Number, The Norton Library (New York: W. W. Norton and Company, Inc., 1952), pp. 161-184.

[6] Bärbel Inhelder and Jean Piaget, The Early Growth of Logic in the Child (New York: Harper and Row, Publishers, 1964), pp. 100-149.

[7] Almy and Associates, Logical Thinking, p. 163.

In this study two class inclusion tasks were required of the student. The tasks selected were similar to those used by Almy.[8] Copies of the scripts used for the class inclusion tasks may be found in Appendix C.

Plastic spacemen were used in the first task. A box containing five blue and three white spacemen was placed in front of the child; the following questions were asked:

1. What material are the spacemen made of?
2. Are there more blue spacemen, more white spacemen or are they the same?
3. Are there more blue spacemen, more plastic spacemen or are they the same?
4. How can you tell?

In the second task seventeen plastic Unifix Blocks were used, twelve of the blocks were blue and five of the blocks were yellow. The blocks were placed in front of the child and the following questions were asked:

1. Are the blue blocks made of plastic? Are the yellow blocks made of plastic?
2. Are there more blue blocks, more plastic blocks or are they the same?
3. How can you tell?

Scoring of Test Data

Subtraction tests

The computation tests of subtraction were scored on the basis of right minus wrong. A child was required to correctly compute at least five of the seven computation

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[8] *Ibid.*, pp. 30-34.

problems on a test to have a passing score for that test.

Each of the manipulative tests of subtraction contained three items. A correct numerical solution for each of the items was given one point; a second point was given to the child for each item on which he gave verbal responses that indicated his method of solution was rational and not just a guess. Total point scores of five or six were passing scores. If a child received a score of four points, his score was passing if he received two points on each of the last two problems.

Logical thinking tasks

A major consideration in scoring the Piagetian tasks was to identify those children who clearly gave characteristic operational responses to each of the tasks. Children who gave operational responses for some but not all of the tasks were not considered to be fully operational.

Conservation of numerical quantities.--Each of the two conservation tasks contained three parts. In the first two parts of each test one point was given for a correct answer. A characteristic operational response on the last part of each test was given one point. To be considered operational on the conservation tasks, the child had to receive a total of six points.

Seriation.--The seriation task consisted of three parts: serial order, ordination, and reordering. The first two parts contained three items that were scored zero if there were errors and one if done correctly. The reordering task was scored zero if the child made an error and two points for a correct answer and a characteristic operational response. A child was considered to be operational on the ordination task if he received a score of three points and operational on both ordination and reordering if his total score was five.

Quantitative class inclusion.--Each of the class inclusion tasks contained two parts. A child was given one point for a correct response and no points for an incorrect or non-operational response. To be considered operational on the class inclusion tasks, the child had to receive a total of four points.

Statistical Design

Data collected will be entered on the contingency tables shown below. Tables 1 through 6 will be used to test the hypotheses stated earlier. A chi-square test of statistical significance will be used to test each hypothesis. Significance at the .05 level will be accepted.

TABLE 1.--Contingency table showing the number of children who are judged to be operational or non-operational on conservation of numerical quantities tasks and who have either an operative or a figurative understanding of level 1 subtraction.

Ability to Subtract	Ability to Conserve Oper.	Non-oper.	Total
Operative			
Figurative			
Total			

TABLE 2.--Contingency table showing the number of children who are judged to be operational or non-operational on conservation of numerical quantities tasks and who have either an operative or a figurative understanding of level 2 subtraction.

Ability to Subtract	Ability to Conserve Oper.	Non-oper.	Total
Operative			
Figurative			
Total			

TABLE 3.--Contingency table showing the number of children who are judged to be operational or non-operational on seriation tasks and who have either an operative or a figurative understanding of level 2 subtraction.

Ability to Subtract	Ability to Seriate Oper.	Non-oper.	Total
Operative			
Figurative			
Total			

TABLE 4.--Contingency table showing the number of children who are judged to be operational or non-operational on conservation of numerical quantities tasks and who have either an operative or a figurative understanding of level 3 subtraction.

Ability to Subtract	Ability to Conserve Oper.	Non-oper.	Total
Operative			
Figurative			
Total			

TABLE 5.--Contingency table showing the number of children who are judged to be operational or non-operational on seriation tasks and who have either an operative or a figurative understanding of level 3 subtraction.

Ability to Subtract	Ability to Seriate Oper.	Non-oper.	Total
Operative			
Figurative			
Total			

TABLE 6.--Contingency table showing the number of children who are judged to be operational or non-operational on quantitative class inclusion tasks and who have either an operative or a figurative understanding of level 3 subtraction.

Subtraction Ability	Class Inclusion Ability Oper.	Non-oper.	Total
Operative			
Figurative			
Total			

Summary

In this chapter the experimental procedures and methods for gathering the data have been discussed. The sample consisted of ninety first, second, and third grade children in three different Lodi elementary schools. Children in the sample were given subtraction tests and logical thinking tasks for the purpose of comparing their ability to subtract with their ability to perform certain logical thinking tasks.

The research data collected will be presented in Chapter IV. A statistical analysis of this data will be performed by using a chi-square test of significance.

CHAPTER IV

RESULTS OF THE STUDY

Introduction

Data gathered to test the comparison between children's logical development and their ability to subtract will be presented in the order that the hypotheses were stated in Chapter I. For the purpose of clarity the hypotheses will be considered in groups according to the level of subtraction difficulty tested. Hypothesis number one will be presented, then hypotheses numbers two and three, followed by hypotheses numbers four, five, and six.

Results

HO_1: There is no significant difference between the number of children who have an operative knowledge of the subtraction facts one through nine and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks.

The first hypothesis was tested with thirty children who passed the level 1 computation test of subtraction. Each child was given two additional tests: a subtraction test using manipulative materials and a conservation of numerical quantities task. Data gathered from these sources are tabulated in Table 7.

TABLE 7.--Results of the manipulative test in subtraction and conservation of numerical quantities tasks for children passing the level 1 computation test of subtraction.

Subject	Age in Months	Level 1 Manipulative Test of Subtraction: Operative Scores	Level 1 Manipulative Test of Subtraction: Figurative Scores	Conservation Scores
01	87		4	4
02	79		2	6
03	79	5		6
04	82	6		6
05	85		2	1
06	79	6		3
07	79	6		6
08	83	6		3
09	76	6		6
10	78	6		0
11	77	6		6
12	80	6		6
13	87	6		6
14	90	6		3
15	79	5		3
16	111	6		1
17	79	6		1
18	78		2	6
19	87	5		3
20	79		3	1
21	79	6		6
22	85	6		6
23	82	6		6
24	78	6		4
25	86	6		1
26	78	5		6
27	80	5		3
28	82		3	3
29	86		2	3
30	77		2	3

Table 7 reveals that the mean age of the children in the sample is eighty-two months. Twenty-two children out of thirty were judged to have an <u>operative</u> knowledge of the subtraction facts one through nine. Only eight children out of thirty have a <u>figurative</u> knowledge of these same subtraction facts. A score of six on the conservation test indicated the child was operational; all other scores are considered to be non-operational scores. Thirteen children out of thirty received operational scores; seventeen children received non-operational scores. Data used to calculate the chi-square statistical test of significance is displayed in Table 8.

TABLE 8.--Contingency table of the number of children who are judged to be operational or non-operational on conservation of numerical quantities tasks and who have either an <u>operative</u> or <u>figurative</u> understanding of level 1 subtraction.

Ability to Subtract	Ability to Conserve Oper.	Non-oper.	Total
Operative	11 (9.5)	11 (12.5)	22
Figurative	2 (3.5)	6 (4.5)	8
Total	13	17	30

$\chi^2 = 0.69$

The critical value for rejection of the null hypothesis at the .05 level of significance is a value greater than or equal to 3.84 for one degree of freedom. Since the value calculated is 0.69, differences between the number of children who have an operative knowledge of subtraction facts one through nine and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks are non-significant.

HO_2: There is no significant difference between the number of children who have an operative knowledge of the subtraction facts ten through eighteen and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks.

HO_3: There is no significant difference between the number of children who have an operative knowledge of the subtraction facts ten through eighteen and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to seriation tasks.

To test hypotheses two and three, a sample of thirty children were selected who passed the level 2 computational test (the subtraction facts ten through eighteen). These children were given an equivalent subtraction test using manipulative materials. Following this test, children were given two Piagetian tasks: conservation of numerical quantities and seriation. The data for each child tested is recorded in Table 9.

TABLE 9.--Results of the level 2 manipulative test in subtraction and two Piagetian logical development tasks for children who passed the level 2 computation test of subtraction.

Subject	Age in Months	Level 2 Manipulative Test of Subtraction Operative Scores	Level 2 Manipulative Test of Subtraction Figurative Scores	Conservation Scores	Seriation Scores
01	87	4		6	3
02	94	6		6	2
03	100	6		3	3
04	105	6		3	2
05	94		2	6	3
06	96		2	6	2
07	96		0	6	2
08	91		0	0	2
09	90		2	2	1
10	91		4	3	1
11	89	6		6	3
12	89	6		6	3
13	103	6		3	1
14	76		4	6	2
15	88		3	6	0
16	89		2	6	1
17	71		2	3	3
18	94		2	3	1
19	85		0	3	2
20	83		4	3	2
21	93	4		6	3
22	88		4	6	3
23	83		3	6	3
24	77		4	6	2
25	84		3	6	1
26	83		2	3	3
27	74		4	3	3
28	76		2	3	2
29	89		3	3	1
30	80		2	3	0

The cumulative data from Table 9 reveals that the mean age of the children in the sample is eighty-eight months. Eight children out of thirty were judged to have an operative knowledge of the subtraction facts ten through eighteen; twenty-two out of thirty have a figurative knowledge of these same facts. Children receiving a score of six on the conservation tasks had operational scores. ~~Scores less than six were considered non-operational.~~ A score of five on the seriation tasks was an operational score for the entire series of tasks; a score of three on the seriation tasks was an operational score for all of the seriation tasks up to and including ordination (conservation of a serial correspondence). Scores less than three were considered to be non-operational scores. The table above reveals that fifteen children have operational scores on the conservation of numerical quantities tasks, and fifteen children have non-operational scores. Eleven children had operational scores on conserving a serial correspondence, and nineteen children were considered to be non-operational. None of the children in the sample had operational scores for the ordinal correspondence task which would have given them a total score of five. Data used to calculate the chi-square statistical test of significance is displayed in Table 10.

TABLE 10.--Contingency table of the number of children who are judged to be operational or non-operational on conservation of numerical quantities tasks and who have either an operative or a figurative understanding of level 2 subtraction.

Ability to Subtract	Ability to Conserve Oper.	Non-oper.	Total
Operative	5 (4)	3 (4)	8
Figurative	10 (11)	12 (11)	22
Total	15	15	30

$\chi^2 = 0.17$

The critical value for rejection of the null hypothesis at the .05 level of significance is a value greater than or equal to 3.84 for one degree of freedom. Since the value calculated is 0.17, differences between the number of children who have an operative knowledge of the subtraction facts ten through eighteen and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks are non-significant.

Data used to calculate the chi-square test of significance for hypothesis three is displayed in Table 11.

TABLE 11.--Contingency table of the number of children who are judged to be operational or non-operational on seriation tasks and who have either an operative or a figurative understanding of level 2 subtraction.

Ability to Subtract	Ability to Seriate Oper.	Non-oper.	Total
Operative	5 (2.9)	3 (5.1)	8
Figurative	6 (8.1)	16 (13.9)	22
Total	11	19	30

$\chi^2 = 1.89$

The value calculated for the chi-square test of statistical significance for hypothesis three is less than 3.84; therefore, differences between the number of children who have an operative knowledge of the subtraction facts ten through eighteen and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to seriation tasks are non-significant.

HO_4: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks.

HO_5: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to seriation tasks.

HO_6: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to quantitative class inclusion tasks.

To test hypotheses four, five, and six, one group of thirty children were selected who passed the level 3 computation test (subtraction using two- and three-digit numbers above twenty that do not require regrouping). These children were given four additional tests: a manipulative test of subtraction, conservation of numerical quantities tasks, seriation tasks, and quantitative class inclusion tasks. The data gathered on each child is recorded in Table 12.

TABLE 12.--Results of the level 3 manipulative test in subtraction and three Piagetian logical development tasks for children who passed the level 3 computation test of subtraction.

Subject	Age in Months	Level 3 Manipulative Test of Subtraction Operative Scores	Figurative Scores	Conservation	Seriation	Class Inclusion
01	105	6		6	3	4
02	90	6		6	3	4
03	102	6		6	3	2
04	98	4		6	5	4
05	89	6		6	2	4
06	111	6		6	3	4
07	99	6		6	3	2
08	98		0	1	3	0
09	109	6		6	3	0
10	93		1	3	1	0
11	108	6		6	3	4
12	99	6		6	3	4
13	100	6		6	3	0
14	104	6		6	3	0
15	100	6		6	2	0
16	101	6		6	1	0
17	107	5		6	1	0
18	103	4		6	2	0
19	94		4	4	2	4
20	114		2	6	2	0
21	100	6		6	3	0
22	110	6		6	5	4
23	103	6		6	5	0
24	109	5		6	3	4
25	90	6		6	3	0
26	112	4		3	2	0
27	102		4	3	2	4
28	104		2	3	3	4
29	80		2	4	2	0
30	80		2	6	2	0

Data from Table 12 reveals that the mean age of the children in the sample is 100 months. Twenty-two children out of thirty were judged to have an operative knowledge of subtraction of two- and three-digit numbers greater than twenty; eight children out of thirty have a figurative knowledge of these same subtraction problems. Children receiving a score of six on the conservation tasks had operational scores; less than six was considered non-operational. A score of five on the seriation tasks was an operational score for the entire series of tasks; a score of three on the seriation tasks was an operational score for all of the seriation tasks up to and including ordination (conservation of a serial correspondence). Scores less than three were considered to be non-operational scores. Three children received a score of five on the seriation tasks. Eighteen children out of thirty were operational on the ordination task; twelve children out of thirty were non-operational. A score of four on the quantitative class inclusion tasks was considered to be an operational score; scores less than four are non-operational. Twelve children out of thirty received operational scores, and eighteen children out of thirty received non-operational scores. Data used to calculate the chi-square test of significance for hypothesis four is displayed in Table 13.

TABLE 13.--Contingency table showing the number of children who are judged to be operational or non-operational on conservation of numerical quantities tasks and who have either an <u>operative</u> or a <u>figurative</u> understanding of level 3 subtraction.

Ability to Subtract	Ability to Conserve Oper.	Ability to Conserve Non-oper.	Total
Operative	21 (16.9)	1 (5.1)	22
Figurative	2 (6.1)	6 (1.9)	8
Total	23	7	30

$\chi^2 = 12.25$, p   .05

The critical value for the rejection of the null hypothesis at the .05 level of significance is a value greater than or equal to 3.84 for one degree of freedom. A chi-square value as high as 12.25 is significant beyond the .001 level. Differences found between the number of children who have an <u>operative</u> knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a <u>figurative</u> knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks are significant beyond the .001 level of significance.

Table 14 displays the data used to statistically test hypothesis five.

TABLE 14.--Contingency table showing the number of children who are judged to be operational or non-operational on seriation tasks and who have either an *operative* or a *figurative* understanding of level 3 subtraction.

Ability to Subtract	Ability to Seriate Oper.	Non-oper.	Total
Operative	16 (13.2)	6 (8.8)	22
Figurative	2 (4.8)	6 (3.2)	8
Total	18	12	30

$\chi^2 = 3.76$

The chi-square value calculated for hypothesis five is 3.76. This value indicates that differences between the number of children who have an *operative* knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a *figurative* knowledge of the same subtraction facts in their ability to give operational responses to seriation tasks are non-significant.

In Table 15 the data used to statistically test hypothesis six is displayed.

TABLE 15.--Contingency table showing the number of children who are judged to be operational or non-operational on quantitative class inclusion tasks and who have either an operative or a figurative understanding of level 3 subtraction.

Subtraction Ability	Class Inclusion Ability Oper.	Non-oper.	Total
Operative	9 (8.8)	13 (13.2)	22
Figurative	3 (3.2)	5 (4.8)	8
Total	12	18	30

$\chi^2 = 0.64$

Results of the statistical test for hypothesis six reveal a 0.64 chi-square value. This indicates that the differences found between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to quantitative class inclusion tasks are non-significant.

Summary

Six hypotheses were tested. The results of these hypotheses are as follows:

HO_1: There is no significant difference between the number of children who have an operative knowledge of the subtraction facts one through nine and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks. (Non-signficant at the .05 level.)

HO_2: There is no significant difference between the number of children who have an operative knowledge of the subtraction facts ten through eighteen and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks. (Non-significant at the .05 level.)

HO_3: There is no significant difference between the number of children who have an operative knowledge of subtraction facts ten through eighteen and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to seriation tasks. (Non-significant at the .05 level.)

HO_4: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to conservation of numerical quantities tasks. (Significant at the .001 level.)

HO_5: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to seriation tasks. (Non-significant at the .05 level.)

HO_6: There is no significant difference between the number of children who have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and the number of children who have a figurative knowledge of the same subtraction facts in their ability to give operational responses to quantitative class inclusion tasks. (Non-significant at the .05 level.)

The conclusions that can be drawn from these hypotheses will be discussed in the next chapter. Following the conclusions, the implications of this study for teaching of subtraction will be discussed, and specific recommendations will be given for further research.

CHAPTER V

CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

The conclusions that can be drawn from the data presented in Chapter IV will be discussed in the order that the hypotheses were presented. Following the conclusions, the implications and recommendations for further research will be given.

Conclusions

Hypothesis One

The statistical test for hypothesis one was non-significant. This result indicates that there are no differences in conservation abilities between children who have a computational knowledge of subtraction of numbers one through nine and children who have an *operative* knowledge of these facts. Non-significance should not be interpreted to mean that conservation of numerical quantities is irrelevant to the child's ability to subtract. Instead, the data suggest that children do not necessarily need to rely on the logical ability tested to solve subtraction problems. It was noted during both the group computation test and the individual test of subtraction based on manipulative materials that children relied on counting techniques to find answers to problems.

In LeBlanc's[1] study of the performance of first grade children in the solution of subtraction word problems, he found a significant relation between the ability to solve subtraction word problems and the ability to conserve numerical quantities. A correlation was done between problem solving and the number facts found in each word problem. Since the correlation was very low (r = .39), one would expect the statistical relation between subtraction and conservation of numerical quantities to be non-significant. This investigation provides the empirical evidence that a non-significant relation does exist between the subtraction facts one through nine and conservation of numerical quantities.

Hypothesis Two

A chi-square test of statistical significance proved to be non-significant for hypothesis two. The evidence indicates that there are no differences in ability to conserve numerical quantities between children with operative and figurative knowledge of the subtraction facts ten through eighteen. This was an unexpected

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[1] John Francis LeBlanc, "The Performance of First Grade Children in Four Levels of Conservation of Numerousness and Three I.Q. Groups when Solving Arithmetic Subtraction Problems," (unpublished Ph.D. dissertation, University of Wisconsin, 1968), pp. 159-160.

outcome. However, an interpretation based on the facts indicates that children can learn to solve the more difficult subtraction facts without using the ability to conserve.

An ability that children used most often was counting. On both the computation test and the subtraction test based on manipulative materials, in this study children frequently used their fingers or made tally marks on their papers. Piaget observed that the ability to count need not be based on logical processes, and the ability to count rationally only requires that the child be able to make a one-to-one correspondence.

Successful subtraction strategies were also displayed by children who had learned techniques for solving the number facts ten through eighteen. On one of the problems presented in the manipulative test four blocks were taken from a group of thirteen blocks. Several children found the amount that was left by saying, "You take the three from the four. This makes ten. Then you take the one from the ten and this makes nine. That's the way I know 13 - 4 = 9."

Hood[2] made comparisons between children's logical development and their arithmetic ability. The children in

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[2] H. Blair Hood, "An Experimental Study of Piaget's Theory of the Development of Number in Children," British Journal of Psychology, LIII, 1962, p. 279.

Hood's study also demonstrated the ability to solve computational as well as word problems by using learned techniques without the need to think in channels that relied on the concepts of conservation.

Hypothesis Three

Hypothesis three was found to be statistically non-significant. There was no significant difference between children's operative and figurative knowledge of the subtraction facts ten through eighteen and their ability to seriate (conserve a serial correspondence). Again, children appeared to be able to channel their methods of solution so that seriation was not a necessary ability. Some of the counting techniques used by children to solve subtraction problems seemed to make use of seriation abilities; however, close inspection of these counting techniques proved this assumption to be false.

In one of the manipulative tests of subtraction thirteen blocks were placed behind a screen. Four blocks were taken from behind the screen and placed in front of the child. The child was then asked to determine the number of blocks that remained behind the screen. Several children after seeing the four blocks counted on their fingers, touching each finger saying, "Thirteen, twelve, eleven, . . . , five." After the child counted the fifth finger, he then determined the number of fingers touched.

Hypothesis Four

Perhaps the most significant result of this study is the outcome of hypothesis four. A chi-square test of significance showed that differences tested are significant beyond the .001 level. This result would strongly suggest that children's ability to understand the operative aspect of subtraction (subtraction of two- and three-digit numbers greater than twenty that do not require regrouping) is related to their ability to conserve numerical quantities. Children who lack the ability to conserve numerical quantities are able to compute subtraction problems at this level; however, their ability is not based on logic. Piaget[3] identifies conservation as the most fundamental of all logical processes and as a necessary condition for all rational activity. Piaget's prediction was found to be quite accurate in the case of subtraction of two- and three-digit numbers greater than twenty that do not require regrouping. At this level of subtraction difficulty, children need to use their conservation abilities to solve subtraction problems based on real material of the type used in this investigation.

From the comparison of the results of hypotheses one through four it can be conjectured that in subtracting

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[3]Jean Piaget, The Child's Conception of Number (New York: W. W. Norton and Company, Inc., 1965), p. 3.

the facts one through eighteen the child can treat real quantities as though they were composed of finite units which do not necessarily have ordinal properties. These units may be distributed along a linear continuum so that they may be treated in easily perceived groups. Thus, the child may carry out the subtraction operation by taking one unit at a time or a small group of units away from a collection of real objects. This approach to subtraction is not successful when applied to concrete situations where the quantities involved are greater than twenty and the materials are not easily broken into individual units which can be distributed in a linear fashion. Lack of success at this point is due to at least two factors: the magnitude of the quantities involved and the ability to think of the quantities involved as being transformed into another arrangement.

Hypothesis Five

The statistical test for hypothesis five was non-significant. The results indicate that there are no differences in the seriation ability of children who have a computational knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and children who have an operative knowledge of subtraction on the same level of difficulty. As in the earlier findings the non-significant results indicate that the child can use abilities other than seriation (conservation of a serial correspondence) to find answers to the subtraction problems. Evidence gathered from the individual tests of subtraction based on manipulative materials substantiates this conclusion. The preferred method of solution used by the children in this experiment was to treat the subtraction of two large numbers as a series of smaller subtraction problems. The hundreds were subtracted first, then the tens, followed by the ones. Each of the differences were then combined or stated as separate differences. Since regrouping was not involved in any of the problems, children were able to find the answer by simply stating the separate differences beginning with the hundreds.

Hypothesis Six

In hypothesis six the differences found between children with an *operative* and *figurative* knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping and quantitative class inclusion are non-significant. This outcome is in agreement with the results of hypothesis five on seriation; however, the ability of the child to conceptually combine the Diense blocks (hundreds, tens, and units blocks) should logically entail the inclusion of the separate groups of blocks into an integrated whole. In the discussion of the childrens' methods of solution in the preceding section, it was found that children can avoid the problem of inclusion by not combining the subordinate parts to create an integrated whole.

Out of the thirty subjects tested only two children were noticeably troubled by the part to whole relation. During one of the manipulative tests of subtraction, the investigator took a hundreds block and seven units blocks from a group of Diense blocks. The child was asked to determine the number of blocks that were missing. One child looked at the hundreds stack that remained on the table and said, "You took one hundred because there is one missing." Then he looked at the units blocks and said, "No, you took seven. (Pause) No, you took 701 blocks . . . . No, you took 107 blocks." A second child who did not get the correct answer gave a series of identical responses; except, the child ended by saying, "No, you took 107 blocks . . . . No, you took 1007 blocks."

Summary

Children's operative and figurative knowledge of the subtraction facts one through nine was not significantly affected by the presence or absence of the ability to conserve numerical quantities. The ability to do the subtraction facts ten through eighteen either on an operative or a figurative level of understanding was not significantly affected by the presence or absence of conservation of numerical quantities or seriation abilities.

Conservation of numerical quantities was found to be a highly significant factor for children who had an operative understanding of two- and three-digit numbers above twenty that do not require regrouping. However, an operative or a figurative knowledge of subtraction at this level is not significantly affected by the ability to do problems of seriation or quantitative class inclusion.

An analysis of data accumulated from the subtraction tests indicates that children can learn patterns of thought which allow them to avoid using the concrete operational structures tested. Children are able to substitute previously learned methods of solution or use learned techniques of counting to solve the subtraction facts one through eighteen. If children have an operative knowledge of subtraction using two- and three-digit numbers above twenty that do not require regrouping, they must have established the ability to conserve numerical quantities. It is not necessary to have the ability to conserve at this level of subtraction difficulty to solve purely computational problems. Based on the results of this study, it appears that logical abilities of seriation and quantitative class inclusion can be avoided in solving subtraction problems both on the operative and figurative level with two- and three-digit numbers above twenty that do not require regrouping.

Implications and Recommendations for Further Research

Implications

It has been commonly recognized by teachers that children can be taught quickly by rote techniques to perform simple arithmetical operations. Findings in this study indicate that young children can perform simple subtraction operations that involve the subtraction facts one through nine with a high degree of "understanding," but their degree of "understanding" diminishes greatly with the subtraction facts ten through eighteen. (The term "understanding" is being used in the place of operative knowledge.) The initial success that teachers have with rote techniques of instruction should not be used as evidence to diminish the importance of the use of concrete materials. A one-sided textbook approach does not provide the child with the opportunity to discover important mathematical relations or an opportunity to use logical structures such as conservation of numerical quantities.

When instruction is based on symbolic material from textbooks and the primary means for evaluating children's progress is through computation tests, the major factor which seems to determine "learning" is the ability to apply patterns of behavior that give correct answers. In contrast instruction which makes use of symbolic material as well as manipulative material allows for a fuller use of the spectrum of logical abilities that a child may possess.

A major implication of this study is that conservation of numerical quantities tasks can be used as a readiness test for subtraction that involves two- and three-digit numbers above twenty that do not require regrouping. The child's failure to pass conservation of numerical quantities tasks similar to those used in this investigation would signal caution to the teacher. Teachers should allow these children to have more time working with simple subtraction situations. However, an operational performance on conservation of numerical quantities tasks would imply that the child has the potential to perform at an *operative* level of understanding with two- and three-digit numbers above twenty that do not require regrouping. Both a computational test and a manipulative test of subtraction should be used to verify this readiness.

Children who show by their performances on the conservation and subtraction tests that they are ready for more advanced work should not be pushed into working with two- and three-digit numbers above twenty that involve regrouping. Each child should be allowed time to develop symbolic models which can be tried in real situations and revised to accommodate the new facts.

In order to develop a broad conceptual understanding of subtraction children need to be given experiences with a large variety of manipulative materials. Reliance on one type of material, such as markers, would tend to reinforce only one model of subtraction. Other types of materials should be used to help children understand the relational aspects of subtraction.

Recommendations for further research

During the course of this investigation three research questions surfaced which are related to the learning of arithmetic. They are as follows:

1. A logical precursor to this study might have been an investigation of addition for children in grades one, two, and three. An assumption made about subtraction is that it is the inverse operation of addition. Theoretically, if a child has gained reversibility of thought, he should be able to subtract as well as he adds. Would a similar study of addition with first, second, and third grade children produce statistically similar results?

2. What logical thinking abilities are significant for the learning of subtraction of two- and three-digit numbers that require regrouping? An answer to this question would help to complete a better understanding of how logical thinking is related to subtraction. One of the problems inherent in such an investigation is the ability of the researcher to find ways of determining the child's <u>operative</u> knowledge of subtraction.

3. Children have difficulty dealing with place value. Most of the mathematical operations that a child performs beyond the elementary ones require some knowledge of place value. How is the child's logical development related to his understanding of place value?

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APPENDIX A

COMPUTATION TESTS OF SUBTRACTION*

Test 1

(1) $\begin{array}{r} 3 \\ \underline{-1} \end{array}$ (2) $\begin{array}{r} 6 \\ \underline{-3} \end{array}$

(3) $\begin{array}{r} 8 \\ \underline{-0} \end{array}$ (4) $\begin{array}{r} 7 \\ \underline{-7} \end{array}$

(5) $\begin{array}{r} 9 \\ \underline{-5} \end{array}$ (6) $\begin{array}{r} 3 \\ \underline{-2} \end{array}$

(7) $\begin{array}{r} 7 \\ \underline{-3} \end{array}$

Test 2

(1) $\begin{array}{r} 11 \\ \underline{-8} \end{array}$ (2) $\begin{array}{r} 12 \\ \underline{-5} \end{array}$

(3) $\begin{array}{r} 13 \\ \underline{-9} \end{array}$ (4) $\begin{array}{r} 10 \\ \underline{-2} \end{array}$

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(5) $\begin{array}{r} 15 \\ \underline{-7} \end{array}$

(6) $\begin{array}{r} 14 \\ \underline{-8} \end{array}$

(7) $\begin{array}{r} 13 \\ \underline{-6} \end{array}$

Test 3

(1) $\begin{array}{r} 33 \\ \underline{-31} \end{array}$

(2) $\begin{array}{r} 38 \\ \underline{-25} \end{array}$

(3) $\begin{array}{r} 66 \\ \underline{-40} \end{array}$

(4) $\begin{array}{r} 457 \\ \underline{-106} \end{array}$

(5) $\begin{array}{r} 939 \\ \underline{-18} \end{array}$

(6) $\begin{array}{r} 76 \\ \underline{-32} \end{array}$

(7) $\begin{array}{r} 490 \\ \underline{-130} \end{array}$

Test 4

(1) $\begin{array}{r} 843 \\ \underline{-184} \end{array}$

(2) $\begin{array}{r} 58 \\ \underline{-29} \end{array}$

(3) $\begin{array}{r} 67 \\ \underline{-38} \end{array}$

(4) $\begin{array}{r} 673 \\ \underline{-537} \end{array}$

(5) $\begin{array}{r} 75 \\ \underline{-49} \end{array}$

(6) $\begin{array}{r} 756 \\ \underline{-148} \end{array}$

(7) $\begin{array}{r} 616 \\ \underline{-507} \end{array}$

APPENDIX B

MANIPULATIVE TESTS OF SUBTRACTION

Test 1 - Part 1

The materials used in parts 1, 2, and 3 of Test 1 are as follows:

1. Square inch blocks all of the same color.

2. A poster board screen.

Orientation

Nine blocks are placed on the table.

____A. "How many blocks are on the table?"

Testing

____1. "Pretend you are going to paint all of these blocks white. If you painted eight blocks and then ran out of paint, how many blocks would there be left to paint?"

Answer.______.

____2. "How do you know?"

____a. Global response.
____b. Counts using fingers or objects to find answer.
____c. Child cites a number fact that relates to the situation.
____d. Other.______________________________.

Test 1 - Part 2

Orientation

Six blocks are placed on the table in front of a screen. Extra blocks are placed near the child. He is told, "You may wish to use these later."

____A. "How many blocks are in front of the screen?"

Testing

____1. (Place the screen in front of the blocks and then reach behind the screen and bring four blocks around to the front of the screen.) "How many blocks are behind the screen?"

Answer.______.

____2. "How do you know?"

____a. Global response.
____b. Counts using fingers or objects to find answer.
____c. Child cites a number fact that relates to the situation.
____d. Other.________________________________.

____3. "Would you like to see how many are behind the screen?" (Remove the screen.) "What do you think?"

____a. Child rationalizes his response.
____b. Child verifies his conclusion.
____c. Other.________________________________.

Test 1 - Part 3

Orientation

Two sets of seven blocks are placed on the table.

____A. "How many blocks are there in each of these piles?"

____B. "Is there the same number of blocks in each pile?"

Testing

_____ 1. "I am going to take some of the blocks from one of these piles and I don't want you to see how many I take. Close your eyes." (Take five blocks from one of the piles and place them behind the screen.) "Open your eyes. How many blocks did I take from this pile?"

Answer._____.

_____ 2. "How do you know?"

_____ a. Global response.
_____ b. ~~Counts using fingers or objects to find~~ answer.
_____ c. Child cites a number fact that relates to the situation.
_____ d. Other.________________________________.

_____ 3. "Would you like to see how many blocks I took?" (Remove the screen.) "What do you think?"

_____ a. Child rationalizes his response.
_____ b. Child verifies his conclusion.
_____ c. Other.________________________________.

Test 2 - Part 1

The same materials used in Test 1 will again be used in Test 2.

Orientation

Eleven blocks are placed on the table.

_____ A. "How many blocks are on the table?"

Testing

_____ 1. "Pretend you are going to paint all of these blocks white. If you painted three blocks and then ran out of paint, how many blocks would there be left to paint?"

Answer._____.

_____ 2. "How do you know?"

____a. Global response.
____b. Counts using fingers or objects to find answer.
____c. Child cites a number fact that relates to the situation.
____d. Other.________________________________.

Test 2 - Part 2

Orientation

Thirteen blocks are placed on the table in front of a screen. Extra blocks are placed near the child. He is told; "You may wish to use these later."

____A. "How many blocks are in front of the screen?"

Testing

____1. (Place the screen in front of the blocks and then reach behind the screen and bring four blocks around to the front of the screen.) "How many blocks are behind the screen?"

Answer.____.

____2. "How do you know?"

____a. Global response.
____b. Counts using fingers or objects to find answer.
____c. Child cites a number fact that relates to the situation.
____d. Other.________________________________.

____3. "Would you like to see how many are behind the screen?" (Remove the screen.) "What do you think?"

____a. Child rationalizes his response.
____b. Child verifies his conclusion.
____c. Other.________________________________.

Test 2 - Part 3

Orientation

Two sets of sixteen blocks are placed on the table.

____A. "How many blocks are there in each of these piles?"

____B. "Is there the same number of blocks in each of these piles?"

Testing

____1. "I am going to take some of the blocks from one of these piles and I don't want you to see how many I take. Close your eyes. (Take nine blocks from one of the piles and place them behind the screen.) "Open your eyes. How many blocks did I take from this pile?"

Answer.____.

____2. "How do you know?"

- ____a. Global response.
- ____b. Counts using fingers or objects to find answer.
- ____c. Child cites a number fact that relates to the situation.
- ____d. Other.________________________________.

____3. "Would you like to see how many blocks I took?" (Remove the screen.) "What do you think?"

- ____a. Child rationalizes his response.
- ____b. Child verifies his conclusion.
- ____c. Other.________________________________.

Test 3 - Part 1

The following materials will be used for parts 1, 2, and 3 of Test 3.

1. A small plastic bucket which contains the following amounts of Diense base ten blocks:

a. Twenty units blocks.
b. Twenty "longs" (blocks containing an equivalent of ten units blocks).
c. Twenty "flats" (blocks containing an equivalent of one hundred units blocks).

2. A poster board screen.

3. Two spinners with digits marked in the following order: 1, 3, 6, 2, 5, 8, 1, 7, 4, and 9. One of the spinners is used to indicate the number of tens and the other spinner is used to indicate the number of units.

<u>General introduction to materials</u>

____A. (The bucket of blocks are poured onto the table.) "Will you help sort these blocks." (Sorting is completed.) "We are going to play a game with these blocks; can you tell me how many unit blocks it takes to make a 'long'?"

____B. "How many 'longs' does it take to make a 'flat'?"

____C. "If the units blocks are called one, what is the value of a 'long'? 'flat'?"

____D. (Next, a game is played with the blocks.) "These spinners are used to play a game called '500.' We take turns spinning this pair of spinners. On your turn you will receive the amount of blocks indicated on the spinner dials. You must change the wood you collect into hundreds ('flats'). If a mistake is made in changing smaller pieces for larger pieces or if an exchange is not made when one could be made, you will only be allowed to spin the 'units' spinner on your next turn. The game ends when someone reaches the value of 500. The winner of the game receives a score equal to the number of pieces of wood (after exchanges are made) which are in the excess of five 'flats.'" (This game is played a number of times until the child is familiar with the blocks.)

<u>Orientation</u>

____A. (The bucket of Diense blocks are poured onto the table.) "Help me sort these blocks into piles that go together." (After the task is completed.) "These blocks will be left here in case you want to use them later."

____B. (Three "flats," 9 "longs," and 2 units are placed in front of the child.) "If each unit block has the value of one, what is the value of this pile of wood?" (If the child makes an error, the error is corrected.)

Testing

____1. "Pretend that these blocks are made of ice and they have been left in a hot place so that 201 of the units cubes melt. How many cubes would be left?"

Answer.____.

____2. "How do you know?"

____a. Global response.
____b. Counts using fingers or objects to find answer.
____c. Child cites a relevant chain of number facts.
____d. Other.________________________________.

Test 3 - Part 2

Orientation

____A. (Four "flats," 9 "longs," and 8 units are placed in front of the child. A screen is placed immediately behind the blocks.) "How many blocks are in front of the screen?"

Testing

____1. (Place the screen in front of the blocks and then reach behind the screen and bring 2 "flats," and 3 units around to the front of the screen.) "How many blocks are behind the screen?"

Answer.____.

____2. "How do you know?"

____a. Global response.
____b. Counts using fingers or objects to find answer.
____c. Child cites a chain of number facts that relates to the situation.
____d. Other.________________________________.

____3. "Would you like to see how many blocks I took?" (Remove the screen.) "What do you think?"

____a. Child rationalizes his response.
____b. Child verifies his conclusion.
____c. Other.________________________________.

Test 3 - Part 3

Orientation

____A. (Two sets of blocks each containing 8 "flats," 2 "longs," and 8 units are placed in front of the child.) "How many blocks are there in each of the two piles?"

____B. "Is there the same number of blocks in each pile?"

Testing

____1. "I am going to take some of the blocks from one of these piles and I don't want you to see how many I take. Close your eyes." (Take 1 "long" and 7 units from one of the piles and place them behind the screen.) "Open your eyes. How many blocks did I take from this pile?"

Answer.____.

____2. "How do you know?"

____a. Global response.
____b. Counts using fingers or objects to find answer.
____c. Child cites a relevant chain of number facts.
____d. Other.________________________________.

____3. "Would you like to see how many blocks I took? (Remove the screen.) "What do you think?"

____a. Child rationalizes his response.
____b. Child verifies his conclusion.
____c. Other.________________________________.

APPENDIX C

PIAGETIAN LOGICAL THINKING TASKS

Conservation of Numerical Quantities Without Counting

Orientation

____A. (Eleven yellow blocks and fourteen blue blocks are placed in two horizontal rows.) "Are there as many yellow ones as blue ones?"

____B. "Make it so there are as many yellow ones as blue ones."

____C. (Take a yellow one.) "Are there as many yellow ones as blue ones?" (Return the yellow one.) "What about now?" (Continue taking different amounts of blue and yellow blocks and returning them. If the child does not understand that the two rows are equivalent, do not continue testing.)

Testing

____1. (Push the blue blocks into a pile.) "Are there as many yellow ones as blue ones now?" "Are there more yellow ones, more blue ones, or are they the same?"

____More yellow ____More blue ____Same

____2. (Spread out the row of yellow blocks.) "Now, are there more yellow ones, more blue ones, or are they the same?"

____More yellow ____More blue ____Same

____3. "Why do you think so?"________________________________.

Conservation of Numerical Quantities

With Counting

Orientation

____A. (Place thirteen yellow blocks in front of the child.) "Would you count the number of blocks I have?" (If the child is unable to correctly count the blocks after a few tries, do not continue testing.)

Number____.

Testing

____1. (Spread the blocks out into a long row.) "Without counting can you tell me how many blocks there are now?"

____a. Does not know, must count to find out.
____b. Knows how many without counting.
____c. Other.________________________________.

____2. (Collect the blocks into a bunch.) "Without counting can you tell me how many blocks there are now?"

____a. Does not know, must count to find out.
____b. Knows how many without counting.
____c. Other.________________________________.

____3. "How do you know?"________________________________.

Seriation

Orientation

____A. "Here are some pictures of men." (Each card is coded with a letter of the alphabet. When the cards are placed in proper order from the biggest to the smallest they spell UZDPIAGETS.) "This is the biggest man (U)." (Place the card next to the child.) "This is the smallest man (T)." (Place the card about two feet to the right of the first card.) "Here is the next biggest man (Z) and it goes here, and this is the next smallest man (T) and it goes here."

Testing

_____1. "Arrange the rest of the cards from the smaller one to the bigger one. Tell me when you have finished." (When the child is finished, record the series below.)

U Z ___ ___ ___ ___ ___ ___ T S

Orientation

_____B. (If the men have not been ordered correctly, make correction by saying, "That's almost right, but this one is a little bigger than this one," etc.) "Here are some shovels for the men." (The proper order for these cards spell ERYTHMOFAC when matched with the proper man.) "The biggest shovel (E) belongs with the biggest man." (The card is placed under the man.) "The smallest shovel (C) belongs here with the smallest man."

Testing

_____2. "Arrange the rest of the shovels so they go with the man that is the right size. Tell me when you have finished." (When the child is finished, record the series below.)

E ___ ___ ___ ___ ___ ___ ___ ___ C

Orientation

_____C. (If the order is not correct, say: "That is almost right; a few of them are mixed up.") "Watch what I do next." (Push the row of men closer together. Move the shovels to the left so that card (C) is to the left of man (T).)

Testing

_____3. (Point to each man.) "Which shovel belongs to this man?"

_____a. Man (E) _____b. Man (P) _____c. Man (G)

____4. (Leave the men in the correct order. Mix up the shovels.) "Can you find the shovel that goes with this man?" (Point to man (I).)

Child's method of solution:

Reorders through card I____
Reorders entire series_____
Visual estimate____________
Random choice______________
Other______________________

Quantitative Class Inclusion

Task 1

A box containing five blue plastic spacemen and three white plastic spacemen is placed in front of the child.

Orientation

____A. "What material are these spacemen made of?" (Make sure that the child understands the question.)

____B. "Sort the plastic spacemen into two groups which belong together."

____C. "Put all of the plastic spacemen into one group."

Testing

____1. "Are there more blue spacemen, more white spacemen, or are they the same?"

____More blue spacemen.____More white spacemen.
____Same.

____2. "Are there more blue spacemen, more plastic spacemen, or are they the same?"

____More plastic spacemen. ____More blue spacemen.
____Same.

____3. "How can you tell?"

____.

Task 2

In this task seventeen plastic Unifix blocks are used: twelve blue blocks and five yellow blocks.

Orientation

____A. "Can you sort these blocks into two piles that belong together?"

____B. "What are the Unifix blocks made of?" (Make sure the child understands the blocks are made of plastic.)

____C. "Put all of the plastic blocks into one group."

Testing

____1. "Are there more blue blocks, more plastic blocks, or are they the same?"

____More plastic blocks. ____More blue blocks. ____Same.

____2. "How can you tell?"

____.

APPENDIX D

COMPUTATION TEST OF SUBTRACTION

Publisher's Data for Test 1

Item	Problem	Per cent Giving Correct Responses	Grade Level	Item	Problem	Per cent Giving Correct Responses	Grade Level
1	3	59	1.6	6	6	40	1.6
	−1	87	2.6		−4	78	2.6
2	6	58	1.6	7	9	41	1.6
	−3	90	2.6		−5	77	2.6
3	9	55	1.6	8	3	43	1.6
	−8	88	2.6		−2	70	2.6
4	8	64	1.6	9	7	46	1.6
	−0	88	2.6		−3	80	2.6
5	7	56	1.6	10	7	35	1.6
	−7	86	2.6		−5	67	2.6

Publisher's Data for Test 2

Item	Problem	Per cent Giving Correct Responses	Grade Level	Item	Problem	Per cent Giving Correct Response	Grade Level
1	11 -8	74 83	2.6 3.6	6	15 -7	63 86	2.6 3.6
2	12 -5	70 86	2.6 3.6	7	14 -8	59 85	2.6 3.6
3	13 -9	79 91	2.6 3.6	8	13 -4	63 87	2.6 3.6
4	11 -3	69 86	2.6 3.6	9	13 -6	74 90	2.6 3.6
5	10 -2	88 93	2.6 3.6	10	16 -9	49 81	2.6 3.6

Publisher's Data for Test 3

Item	Problem	Per Cent Giving Correct Responses	Grade Level	Item	Problem	Per Cent Giving Correct Responses	Grade Level
1	33	42	2.6	6	939	46	2.6
	-31	78	3.6		-18	77	3.6
2	38	54	2.6	7	76	64	2.6
	-25	84	3.6		-32	86	3.6
3	66	61	2.6	8	756	54	2.6
	-40	87	3.6		-148	82	3.6
4	392	45	2.6	9	647	48	2.6
	-201	79	3.6		-195	84	3.6
5	457	56	2.6	10	616	60	2.6
	-106	81	3.6		-507	87	3.6

Publisher's Data for Test 4

Item	Problem	Per Cent Giving Correct Responses	Grade Level	Item	Problem	Per Cent Giving Correct Responses	Grade Level
1	843	23	2.6	6	673	21	2.6
	-184	44	3.6		-537	52	3.6
2	58	45	2.6	7	75	75	4.6
	-29	74	3.6		-49	83	5.6
3	67	18	2.6	8	756	75	4.6
	-38	48	3.6		-148	87	5.6
4	645	27	2.6	9	647	71	4.6
	-536	49	3.6		-159	80	5.6
5	45	13	1.6	10	616	18	2.6
	-37	25	2.6		-507	43	3.6

CTB/McGraw-Hill

Del Monte Research Park, Monterey, California 93940 • Telephone 408/373-2932

January 31, 1973

Marvin L. Sohns
2125 W. Walnut St.
Lodi, CA 95240

Dear Mr. Sohns:

Enclosed is a carbon copy of the arithmetic subtraction test items that I sent to you earlier with permission to use in a research project. Beside each item is recorded in red ink the grade levels at which the item was presented to a national standardization sample. Beside that figure is the percentage of students at that grade that correctly answered that item.

For example, for item #1 in Test 1: in the national sample, 59 percent of those children in the sixth month of grade 1 correctly answered the item, and 87 percent of those in the sixth month of grade 2 also answered it correctly.

These test items were selected from eight different tests: from two levels and two forms each of the California Achievement Tests - 1970 edition and the Comprehensive Tests of Basic Skills. The first form of each of these series was administered to more than 200,000 students - between 12,000 and 20,000 at each grade. The second form was administered to smaller numbers - about 1,000-3,000 for each grade to provide data for equating the two forms. The percentages then for these items taken from CAT, form B, and CTBS, form R, are "estimated" national difficulties and are not the actual percentage of students in the "equating study." For the smaller group in the equating study we did not have a "perfect" representative sample of the nation; but by using the statistical technique of equating, we feel we succeeded very well in presenting data for those forms that would characterize a representative national sample.

Sincerely yours,

William E. Kline

William E. Kline
Director, Test Development

WEK:tk

Encl.

APPENDIX E

TEST RETEST DATA USED TO CALCULATE

PEARSON r CORRELATION COEFFICIENTS

Subject	Grade	Test Score	Retest Score	Subject	Grade	Test Score	Retest Score
01	1	12	13	32	2	13	13
02	1	11	9	33	2	14	20
03	1	10	13	34	2	14	15
04	1	10	12	35	2	19	22
05	1	9	9	36	2	13	13
06	1	8	13	37	2	15	18
07	1	8	8	38	3	21	27
08	1	8	7	39	3	21	21
09	1	8	8	40	3	21	21
10	1	8	7	41	3	25	27
11	1	7	12	42	3	23	23
12	1	7	7	43	3	23	20
13	1	7	5	44	3	24	25
14	1	6	6	45	3	22	23
15	1	6	8	46	3	27	28
16	1	4	7	47	3	21	21
17	1	2	2	48	3	19	27
18	1	9	9	49	3	21	24
19	1	7	6	50	3	20	20
20	1	8	11	51	3	20	20
21	2	15	12	52	3	20	21
22	2	14	17	53	3	14	15
23	2	12	19	54	3	20	25
24	2	11	14	55	3	18	21
25	2	15	21	56	3	24	28
26	2	10	14	57	3	26	26
27	2	10	10	58	3	28	27
28	2	19	15	59	3	27	28
29	2	18	19	60	3	26	27
30	2	16	17	61	3	26	25
31	2	12	15				

Marvin Lee Sohns

Born in Porterville, California, January 25, 1935

Graduated from Porterville Union High School, 1952

A.A., Porterville Junior College, 1954

B.A., University of the Pacific, 1959

M.A., University of the Pacific, 1967

Other course work at University of California, Berkeley

Credentials:

General Elementary Teaching Credential
General Secondary Teaching Credential
General Administrative Credential

Professional Experience:

Teacher, seventh and eighth grades, 1958-1967
Teacher, fourth grade, 1967-1969
Teacher, ESEA-I Mathematics, 1969-1971
Graduate Assistant, University of the Pacific, 1971-1972
Consultant, ESEA-I Reading and Mathematics, 1971-1973

Professional Societies:

Phi Delta Kappa
American Educational Research Association
National Council of Teachers of Mathematics
California Mathematics Council
Elementary School Science Association
California Teacher Association
National Education Association