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An Investigation Of The Relationship Between Cognitive Developmental Stage And Quantitative Skills In College Students

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AN INVESTIGATION OF THE RELATIONSHIP BETWEEN COGNITIVE DEVELOPMENTAL STAGE AND QUANTITATIVE SKILLS IN COLLEGE STUDENTS

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Abstract of Dissertation

PROBLEM: The purpose of this study was to investigate the relationship between Piagetian cognitive developmental stage and quantitative skill levels, as measured by placement tests, in college students taking introductory level mathematics courses.

PROCEDURE: Data were collected from students enrolled in self-paced remedial/developmental courses in pre-algebra, elementary algebra, intermediate algebra and regular courses in statistics and elementary functions at the University of the Pacific in the Fall 1982 semester. The Descriptive Test of Mathematical Skills (DTMS) was used to place students. Demographic data collected were sex, age and number of high school mathematics and science courses taken. Bond's Logical Operations Test (BLOT) was used to classify the cognitive stage of the students. The Kurtz/Karplus group test of Piagetian stage was also given to students in the remedial/developmental class. At the end of the semester, the BLOT and DTMS were re-administered to students in the remedial/developmental class. A matched pair design was used to analyze DTMS and BLOT gains for students given special problem-solving instruction. Gains in DTMS scores by cognitive level were tested using a covariate analysis. Contingencies between sex, cognitive level, placement level and number of high school science and mathematics courses were investigated.

FINDINGS: Significant relationships existed between mathematics placement level and Piagetian stage with students placed at higher levels having higher mean cognitive assessment scores. Gains in mathematical skills in the remedial/developmental course were related to cognitive stage. No gender differences were found in mean DTMS or BLOT scores. Gender differences favoring males were found in number of mathematics courses taken and the Kurtz/Karplus test scores. The experimental problem-solving instruction was successful in raising gains in DTMS scores but not BLOT scores. There was a 30% exact agreement between the two cognitive assessment instruments used.

CONCLUSIONS: College instructors should recognize that in lower placement levels in mathematics all students may not be formal operational and thus, the traditional lecture format may not be appropriate for these classes. Activities

which encourage the development of formal thought should be added to remedial/developmental courses. More research is needed on group assessment instruments which categorize college students as concrete or formal operational.

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CHAPTER I

INTRODUCTION

Instructors in college level mathematics and science classes assume of their students basic quantitative skills as well as the ability to use abstract patterns of thought. The quantitative skill levels required are minimally those of elementary or intermediate algebra while the abstract thinking abilities include the capacity to use the variable concept, to formulate and test hypotheses, to use symbols, to solve problems and to use probabilistic and proportional reasoning. These requisite thinking abilities correspond to the formal stage of cognitive development according to the theory of the Swiss psychologist, Jean Piaget (Beilen, 1971; Arons, 1979). A student who enters college without quantitative skills and/or without having developed abstract thinking abilities may have considerable difficulty in meeting the demands of college courses, particularly in the areas of mathematics and science.

Many students entering colleges and universities in the 1980's are required to meet minimal competency in quantitative skills through either placement testing programs or general education quantitative graduation requirements (McCurdy, 1982). A significant number of students are placed in remedial/developmental programs operated by mathematics departments or independent learning centers. Because of resource allocations and pedagogical concerns, these remedial/developmental courses are often self-paced modularized courses.

The University of the Pacific at Stockton, California, developed a mathematics placement testing system and associated remedial/developmental program beginning in 1976. Both placement testing and remediation are handled through a Mathematics Resource Center which is administratively a part of the Mathematics Department. The Mathematics Resource Center is staffed by two professionals and 20-25 student tutors. All placement testing in mathematics and associated remedial/developmental classes is done through the Mathematics Resource Center. A general education quantitative graduation requirement is being added in 1983 which will require competency at the intermediate algebra level of all entering freshmen. Based on three years of placement data, it seems that a large proportion of entering students will require remedial/developmental work. Appendix A contains data on placement testing for the period 1979 to September, 1983 at the University of the Pacific.

The ability of some students to profit from remedial/developmental work at the college level is possibly related to their attainment of the formal operational cognitive stage (Barrow and Shonberger, 1981). Alternatively, the lack of quantitative skills may be a symptom of a lack of cognitive development at the level which facilitates the abstract reasoning necessary in mathematics courses (Lawson, 1980). Several studies support the assertion that not all college age students have attained the formal operational stage (McKennon & Renner, 1971; Kolody, 1975; Cowan, 1978; Kuhn, 1979).

Although some college level instructors have shown correlations between cognitive stage and achievement in standard science and mathematics courses (Cantu & Herron, 1978; Walker, 1979; Barrow & Shonberger, 1981), little research has been done specifically dealing with cognitive development in modularized remedial/developmental mathematics courses at the college level. A careful examination of the course background and cognitive stage of students related to placement and attained skills in the remedial/developmental mathematics course seems desirable. Pencil and paper group assessments of cognitive level are in the developmental stage as yet, and more research is necessary on the usefulness of such tests for college instructors. Concern has been expressed nationally about the problem-solving skills of students in general (National Council of Teachers of Mathematics, 1980). It is also important to begin an investigation of the ways in which abstract problem-solving skills can be enhanced for students in the context of remedial/developmental classes at the college level.

The Purposes of the Study

This study was designed to determine whether a relationship exists between lack of quantitative skills and failure to attain the formal stage of cognitive development. The effect of cognitive stage on skill gains in a self-paced remedial/developmental class was investigated. Relationships between sex, high school course background, placement level and cognitive level were also investigated. The effectiveness of special problem-solving instruction given in the context of a self-paced modularized remedial class and the usefulness of two types of group cognitive stage assessment instruments were also investigated in this study.

Statement of the Problem

The major question under examination was whether there is a relationship between lack of quantitative skills, as measured by standardized placement tests, and non-attainment of the formal stage of cognitive development as measured by pencil and paper group assessment instruments. Another focus of the study was the effect of cognitive stage on pre-to-post mathematics skill gain in a self-paced remedial/developmental class. Whether sex is a factor in the attainment of the formal level of cognitive development or in the acquisition of quantitative skills was also a question of interest. The type of high school background in mathematics and science potentially may have a bearing on the previous issue and was investigated (Fox, Fennama & Sherman, 1977). This study also attempted to experiment with course materials which might be effective in enhancing both quantitative skills and abstract thinking. The efficacy of such materials was tested in a remedial/ developmental mathematics class at the University of the Pacific.

Research Hypotheses

This study analyzed the relationship between attained quantitative skills as measured by mathematics placement levels and attained stages of cognitive development in college students. The Hypotheses were as follows:

1. Students who are placed in higher levels of college mathematics on the basis of quantitative skills obtain higher mean scores on Piagetian cognitive stage tests. Students placed at lower levels in mathematics obtain lower mean scores on such tests.

2. Students placed in remedial mathematics classes show higher skill gains if they are formally operational regardless of the level of placement.

3. Sex is unrelated to the following factors: attainment of the formal cognitive stage; quantitative skill level; and the number of science and mathematics classes taken at the high school level.

4. There is a positive correlation between the number of science and mathematics courses taken at the high school level and scores on cognitive assessment tests.

5. The ability to deal with proportional relations is enhanced by remedial/developmental mathematics instruction for students placed in these classes.

6. A subset of students given special problem-solving instruction as part of their self-paced remedial/developmental mathematics class show:

   a. greater skill gains than students with similar backgrounds but without such instruction

   b. ~~greater gains in cognitive assessment~~ scores than students with similar backgrounds

7. There is a positive correlation between the two different types of tests (objective and subjective) used to assess cognitive stage.

Definition of Terms

Remedial/developmental mathematics. Mathematics below the widely accepted college entry level of elementary functions or pre-calculus is referred to as remedial/developmental mathematics (Heine, 1982). This includes pre-algebra, elementary algebra, and intermediate algebra at the University of the Pacific. The term "remedial" may have some negative connotations implying previous exposure to the material and possible learning or retention problems. Although the term developmental historically referred to general instruction at any level, the term is gaining popularity as a way of implying that the material has not been previously covered. These two terms will be used interchangeably or together, as in the most common present usage.

Quantitative skill level. Quantitative skill level is defined as one of four placement levels: 1) pre-algebra, 2) elementary algebra, 3) intermediate algebra, 4) pre-calculus. Placement levels are determined at the University of the Pacific by scores obtained on four corresponding forms of the DTMS (Descriptive Test of Mathematical Skills) Test from the College Board. A student passing the pre-algebra or higher level test is eligible to enroll in Mathematics for Elementary Teachers for which such skills are a prerequisite. A student not passing the pre-algebra test is referred to the Mathematics Resource Center for remediation at the pre-algebra level. A student passing the elementary algebra test is eligible to enroll in two introductory level Statistics courses, otherwise remediation is required at that level. A student passing the intermediate algebra test is eligible for college level courses for which intermediate algebra is a prerequisite (Elementary Functions, Chemistry, Business Calculus, Finite Mathematics or Computer Programming). Students failing the intermediate algebra test are referred to the Mathematics Resource Center for intermediate algebra remedial/developmental instruction. Students passing the pre-calculus test enter Calculus; students not passing are referred to Elementary Functions which is not considered to be a remedial course. The DTMS test will be described in more detail in Chapter 3.

Cognitive level. Several theories of developmental psychology postulate the development of stages of cognition. Piaget defines four stages: sensory-motor; pre-operational; concrete operational; and formal operational (1964). These categories are described in Chapter 2. In this study, cognitive stage is determined by score on an objective group assessment test (Bond's Logical Operation Test, Bond, 1981). This test will be described and evaluated in Chapters 2 and 3.

Ability to deal with proportional relations. Ability to deal with proportional reasoning involves using two frames of reference simultaneously. This skill is measured by a subscale score on the BLOT (Bond's Logical Operation Test, Bond, 1981), called the INRC 4 Group subscale. The Identity, Negation, Reciprocal and Correlation operations form an abstract group of order four. These operations are part of the formal operational level of cognitive development and are related to the ability of persons to deal with reciprocal relations and proportional thinking (Cowan, 1978). This subscale will be described more completely in Chapter 3.

PSI instruction. PSI, (Personalized System of Instruction) or self-paced, modularized courses are characterized by individualized instructional materials, small group tutoring instead of lectures, self-pacing and mastery learning. The particular system used in the Mathematics Resource Center at the University of the Pacific is a modified Keller Plan PSI instructional system (Keller, 1968). PSI systems in general are reviewed in Chapter 2 while the University of the Pacific system is explained in Chapter 3.

Procedures

The data used in this study were gathered from students, most of whom had their quantitative skills assessed during the 1982 summer Freshman Orientation Program of the University of the Pacific. These students were then assigned to either the remedial/developmental program of the Mathematics Resource Center or to ordinary entry level mathematics courses such as Statistics, Elementary Functions or Calculus. Some students chose not to enroll in a mathematics course of any kind, and were not part of the population studied.

During the first two weeks of the Fall, 1982 semester, group pencil and paper tests of Piagetian Cognitive levels were given to all students in the remedial/developmental mathematics classes and during the same time period also given to two sections of Elementary Functions classes and one section of Statistics and Probability. The latter three classes served as comparison groups of students who had passed their respective placement tests.

All quantitative skills were assessed using the DTMS test from the Educational Testing Service of the College Board. Stage of cognitive development was assessed using an objective multiple choice test, the BLOT. For the remedial/developmental students, a second assessment was done using a subjective test modified after Kurtz and Karplus (Kurtz, 1979). Demographic data were gathered for all groups at the time of the BLOT cognitive assessment. Data included age, sex, and number and kind of mathematics and science courses taken in high school.

In the second month of the remedial/developmental mathematics courses, a special series of four workshops on analytic problem-solving was given to a group of 34 volunteer students from the class. The workshops met weekly during regularly scheduled class hours. Instructional material included problems from _Problem Solving and Comprehension_ by Whimbey and Lochhead (1980), and selected types of verbal or "word problems" from assorted mathematics texts. Workshop format was group discussion with emphasis on structured approaches to problems. Individual approaches were shared and evaluated. Concrete aids to problems, such as charts and tables, were used. The workshop material and procedures are explained in more detail in Chapter 3.

At the end of the semester, a blind match was made to find controls for the experimental subjects. Controls were matched on the basis of sex, quantitative skill level, high school mathematics background and BLOT cognitive assessment score. Gains in quantitative test scores were compared using a matched-pair design analysis. At the end of the remedial/developmental courses, quantitative skills were again tested using the DTMS test. The BLOT Piagetian cognitive assessment test was also re-administered.

Statistical Analysis

The research hypotheses of this study were examined using the data collected during the Fall 1982 semester at the University of the Pacific. All statistical analyses were done using the SPSS (Statistical Package for the Social Sciences) computer program on a Burroughs B6700 computer at the University of the Pacific. The level of statistical significance was set at .05 for all tests.

Descriptive statistics were provided for all variables of the study. An analysis of variance was run on BLOT scores using placement level as the independent variable.

A covariate analysis on DTMS post scores was done using pre-score as the covariate and cognitive stage as the independent variable for all student data from the remedial/developmental classes. A dependent *t*-test was run on the matched-pair data obtained from the problem-solving workshop experiment. Chi square contingency tests were run involving sex, high school mathematics, science course background, mathematics placement level and comparison of agreement of cognitive stage. Classification between objective and subjective Piagetian assessment instruments was done using correlation techniques. Analysis of variance on pre-test quantitative scores and BLOT scores using demographic categories as independent variables was done. Pre-to-post gains in ability to deal with proportional reasoning were tested using INRC 4 Group BLOT subscale scores with cognitive stage as the independent variable.

Limitations and Assumptions

1. Because of the nature of PSI instruction, it is impossible to assume consistent quality of mathematical instruction across the 27 different sections of remedial/developmental mathematics in progress at the Mathematics Resource Center during the Fall 1982 semester. Twenty-one different proctors and one other supervisor interacted with the 202 remedial/developmental students initially involved in this study. Although this variation between proctors affects the implementation of the treatment, it should also imply generalization of results to other PSI remedial/developmental mathematics courses.

2. Testing conditions were not constant for the comparison groups or the remedial/developmental students. Although quantitative skill assessment and cognitive BLOT assessments were timed, standardized tests, there was variability in other testing conditions such as noise level, lighting, time of day and other aspects of the testing environment. This study assumes then, that these cognitive assessment scores and quantitative testing scores adequately

reflect the true level of the variables which the tests attempted to measure.

3. Positive changes in cognitive ability as measured by BLOT score, over the course of one semester for the remedial/developmental students, can not be ascribed solely to the experience in the Mathematics Resource Center. Other experiences, such as adjustment to college or other course experiences which might affect cognitive growth, were not monitored or controlled by this study.

4. The experimental problem-solving workshop's effectiveness may be related to the particular instructor involved.

5. Students who are in the ESL (English as a Second Language) program or who were identified as having significant language difficulties were dropped from the samples. This was done because the subjective Piagetian assessment test required proficiency in English.

6. Another limitation of the study was the difficulty of matching experimental subjects with controls in the problem-solving portion of the study. Students were matched on the basis of sex and high school mathematics background as well as placement test score and objective cognitive assessment score. Of the 34 students in the experimental group, matches were found for only 15 subjects.

7. Students in the experimental problem-solving instructional group may be subject to the Hawthorne effect.

8. It is assumed that students placed in remedial/

developmental mathematics at the University of the Pacific are representative of students in similar institutions. It is also assumed that proctors working in the PSI class are typical for such programs.

Significance of the Study

The large number of college students requiring remedial/ developmental mathematics instruction suggests a careful study of factors which may affect the ability of a student to benefit from such instruction. Piaget's theory of cognitive developmental stages may provide one such explanatory factor for lack of quantitative skills. Such findings would enable instructors to more accurately assess the abilities of their students to benefit from instruction in basic skills. Supplementary material that contributes to the development of abstract thinking at the formal stage may be necessary in such courses in order for students to maximize their skill gains.

This study also investigated means by which differentiation between concrete and formal levels of cognitive development can be assessed by instructors dealing with groups of students in PSI remedial/developmental mathematics classes at the college level. Demographic factors such as sex or course background which might affect skill gain or cognitive level were also investigated. The efficacy of problem-solving instruction in increasing quantitative gains or affecting cognitive assessment score was studied. Results of this study could be used by any learning center attempting to improve the problem-solving abilities of its students or the effectiveness of its remedial mathematics instruction.

Summary

Increased need for remedial/developmental mathematics at the college level has prompted interest in questions pertaining to the reasons for skill deficiencies and the most effective means for carrying out remediation programs. The Piagetian construct of formal thought is relevant to many aspects of successful learning at the college level, particularly in science and mathematics. Self-paced or PSI instruction is the most frequently used teaching methodology in remedial mathematics courses taught through learning centers. There is a need to investigate whether cognitive stage is a variable which might affect quantitative skill gains in these types of courses.

This study was conducted to determine whether contingencies exist between Piagetian cognitive stages and quantitative skill levels as measured by placement test scores. Gains made in self-paced remedial/developmental mathematics classes at the college level were analyzed relative to cognitive stage. Relationships between cognitive stage, quantitative skill level, sex, and high school mathematics and science background were also investigated. Experimental problem-solving materials to improve quantitative skill gains and possibly improve cognitive assessment scores were designed and tested. Agreement of cognitive level classification between objective and subjective group Piagetian assessment instruments given to the remedial/developmental mathematics classes was also done.

CHAPTER II

REVIEW OF THE LITERATURE

The major focus of this study was the examination of the relationship between cognitive stage and quantitative skills in college students placed in remedial/developmental or introductory mathematics classes. This chapter reviews literature related to trends in remediation at the college level and Piagetian theory. Literature pertaining to PSI instruction and formal thought as well as to studies of the relationship between formal thought and performance at the college level are surveyed. Additionally, relevant studies on gender differences in cognitive development and quantitative skill level are reviewed.

Manual searches of ERIC (Educational Resource Information Center), Dissertation Abstracts International and the Educational Index were done. A computer search was conducted at the University of the Pacific through the DIALOG database which accesses the ERIC database and the current index to over 700 journals in education. Professional journals in chemistry, physics, and mathematics were also searched and two national conferences dealing specifically with remedial/developmental mathematics at the college level and Piagetian research in higher education were attended.

Historical Background

In 1965, The Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America published a report entitled, A General Curriculum in Mathematics for Colleges. The lowest level college mathematics course which they suggested was an Elementary Functions course which combined algebra, trigonometry and analytic geometry (CUPM, 1965). The CUPM recommended that remedial mathematics not be taught at the college level as there was at that time a shortage of college mathematics teachers and the outcome of remedial instruction was doubtful (CUPM, 1965).

Even that year, however, approximately 20 percent of all students enrolled in four year colleges were taking courses below the level recommended by CUPM (Hudspeth, 1978). Five years later, a subsequent CUPM report reversed the 1965 position and in 1971, the committee recommended the establishment of a basic course in mathematics at the college level which included arithmetic (CUPM, 1971).

What social changes caused the CUPM committee of the American Mathematical Association to reverse its position concerning the appropriateness of college level remedial mathematics? The turbulent sixties, which witnessed so many social changes offer a partial explanation. Minority students and lower socioeconomic groups, as well as women, were demanding more educational opportunities at the college level. Access to non-traditional fields such as medicine, science and engineering created more demand by these groups for courses teaching the requisite quantitative skills. Educators were attempting to respond to student demands for these relevant courses (Hudspeth, 1978; Grant, 1977). The 1970's saw the beginnings of a Back-to-Basics movement as mathematics educators attempted to meet these needs.

Open admissions or special programs to admit minority students or women contributed to the need for remedial mathematics. Under open admissions, the burden of responsibility was shifted from the student to the college. The college was expected to provide skills and support (Schultz, 1971). Large scale programs providing remedial mathematics at the college level were thus initially designed to meet the needs of special populations entering college. It must be recognized, however, that historically, most colleges and universities had "bonehead" courses in English and mathematics for underprepared freshmen. Declining mathematical skills in the general population as indicated by SAT scores (Jones, 1981) imply that remedial mathematics is not only necessary for open admission students, but also necessary for a large proportion of college freshmen who have a deficient mathematical education (Hudspeth, 1978).

Other aspects of the trend toward providing remedial mathematics in college are the declining pool of college age students and renewal of graduation requirements in mathematics. In 1981, the Committee on Improving Remediation Efforts in the Colleges (Mathematical Association of America) indicated that colleges are adding programs in remediation because they anticipate more vigorous recruitment of students and they want to be sure that their curricula provide access to degree programs for all potential students. These schools are also developing a mathematics requirement for graduation and mathematics departments are being charged to develop courses that will prepare the students to meet this requirement (Bumcrot, et. al., 1981).

Therefore, in the late 1970's, remedial education in mathematics for underprepared college freshmen or returning adults became a major academic enterprise (Hechinger, 1979). For all of the above arguments, most college and university mathematics faculties believe that some remedial education in mathematics is necessary, at least, and probably desirable.

Piagetian Theory

Introduction

During approximately the same period of time that needs for college level mathematics remediation were growing, the cognitive developmental theories of the Swiss psychologist, Jean Piaget, were being popularized in the United States. A major source of research information on Piagetian Theory at the college level is The Journal of Research in Science Teaching. In 1964, this journal published a series of papers including lectures by Piaget himself on his theory of cognitive development (Piaget, 1964). Piaget's lectures had originally been given at conferences held at the University of California at Berkeley and at Cornell University. These conferences were sponsored by the National Science Foundation and indicated the early interest of university level scientists in Piaget's theory and its implications for science teaching. Therefore, much of the early college level research involving Piaget's theory has been done by physicists, chemists, and biologists, and their results do not appear in standard educational research journals, but rather in discipline-based journals.

Developmental Stages

Piaget describes knowledge as the creation of internal structure based on experiences or actions. Action, rather than perception, is the primary source of knowledge (Sinclair, 1971). The formal theory that Piaget developed is all expressed in terms of transformations. To know is equated with the ability to act, to modify, transform, create or negate.

Piagetian developmental theory is an outgrowth of Piaget's particular epistimological viewpoint. Piaget proposed stages of development which were based on the type of actions which were either possible or characteristic of a particular age period. Piaget proposed four levels of cognitive development: sensory-motor (approximately 0-2 years); pre-operational (approximately 2-7 years); concrete operational (approximately 7-11 years) and formal operational (approximately 12-15 years) (Piaget, 1969). These stages of cognitive development differ qualitatively in the kinds of intellectual tasks possible. Since this study concentrates on college students, functional behavior at the concrete, early formal (transitional) or formal operational level will be emphasized. The first two stages will be briefly described in order to provide an overview of the total theory.

Sensory-motor and pre-operational stages. The sensory-motor period is generally thought of as preverbal period during which the ability to symbolize is perhaps the most important development. Object permanence also develops at the beginning of this period. During the first few months of life, an object is seen as having no permanence. If the object disappears from the perceptual field, it no longer exists to the child (Piaget, 1964). Later, a child will try to find the object, indicating the development of the object concept and spatial organization. In addition, innate schemes of action such as sucking or kicking are directed toward objects indicating will and some sense of basic causality is developing. Imitation and goal directed behaviors emerge as the child experiences and integrates reality.

In the development of language, sensory-motor symbols are private and have meaning for the child only in terms of what he or she has done with the symbols (Cowan, 1978). There is a high degree of egocentrism present. Reasoning is done without the aid of language, probably by means of mental images (Elkind, 1977). The abilities being developed in this stage are necessary for movement to the next stage which is called the pre-operational stage.

At the pre-operational stage the child further develops the ability to deal with symbols and objects. Language skills are developed so that the child can describe his or her behavior both verbally or in thought using words which have shared meanings with other individuals. The child is concerned with causality and thus mythologies and magical belief systems are constructed.

Although this period produces tremendous social and intellectual growth, it is most often described in terms of what actions the child is not capable of performing. The individual in this stage of development has difficulty with class inclusion and hierarchical set relations. Seriation is difficult unless an abundance of clues are provided (Cowan, 1978). This stage is also characterized by confusion between an object and the word for the object. Often, only one dimension of a situation can be manipulated. Although egocentrism is lessening, there is still a tendency to base word meanings on private experience.

The first elements of quantitative thought emerge around the age of two when children begin asking for "some" or "more" or "all". Many children learn to rote count during this period, initially repeating the words with no sense of one-to-one correspondence between word and object. Number is generally not conserved by transformations during this period. Space, time and quantity are also not conserved.

Concrete operations. The concrete operational stage (approximately 7-11 years) introduces the first use of operations--that is, mental transformations which are reversible. This stage is generally characterized by the achievement of the skills which were lacking at the pre-operational stage. Stable hierarchies of classes and relations can be constructed and quantity and number are conserved.

Sanders (1978) characterizes the concrete thinker as basing his understanding upon reference to familiar actions, objects and observable properties. Concrete thinkers are capable of conservation, classification and seriation (Cowan, 1978), but use formal reasoning in a partial or unsystematic way. They are not aware of inconsistencies or contradictions in their own thinking. Piaget (1964) suggested that the concrete thinker possessed all the fundamental operations of elementary class logic, elementary mathematics, elementary geometry and even elementary physics.

Concrete operational does not mean that all transformations must be done concretely with material objects. At this stage, an individual is capable of mental operations. The concreteness refers to the applicability of the transformations to real situations whether the situation is present or not. What is lacking is the ability to formulate hypotheses, reason contrary-to-fact, isolate and control variables, think proportionally, use two reference systems simultaneously and operate in more than two dimensions.

According to Cowan (1978), the concrete operational stage is generally broken down into two substages labeled early and late concrete. The early concrete stage is marked by logical grouping, conservation and reversible mental operations in two spatial or temporal dimensions while the late concrete stage is characterized by use of spatial co-ordinates, perspective and use of arbitrary measurement units (Cowan, 1978). In a sense, the late concrete stage presents a move towards more than two dimensional thought and is preparation for the next stage which is the formal operational stage.

Formal operational stage. Formal operational thinking is characterized by the ability to reason with concepts, relationships, abstract properties and theories. Formal thinkers can use symbols to express ideas, are capable of probabilistic reasoning and can use variables to investigate relationships (Sanders, 1978). Formal thought allows students to reason about contrary-to-fact propositions (Elkind, 1977) and enables them to state and interpret relationships in mathematical form (Karplus, 1977).

At this stage, Piaget maintains, all of the translations characteristic of formal symbolic thought are available although they may not be expressed in the symbolic abstract form. Piaget found group properties and lattice structure in the sixteen binary operations possible on propositions using implications, conjunction, disjunction and negation. He closely tied the formal operational stage to formal logic and mathematical structure. Piaget is saying that now the young adult can understand all types of logical operations possible in a propositional calculus. That is, the forms of an argument can be followed regardless of the content of the argument (Cowan, 1978).

As an example, if the proposition, "If it is May in Stockton, California, the temperature will reach 90° F. at least one day of the month." is given, a formal operational thinker should be able to decide what evidence will negate this claim. The answer is: May in Stockton, California in which the temperature is below 90° F. on every day will negate this proposition. If this proposition is analyzed symbolically, it stands as a P -> Q statement and negation is $\sim(P \rightarrow Q)$ which is P and $\sim Q$. The form of the statement, regardless of the content allows negation. Even though the formal symbolic proof is not done, there is an intrinsic logical structure present in the formal thinker which allows that person to deal with negation of implications independent of the context. This type of logical structure is necessary in dealing with probability, combinatorics and hypothesis formulation and testing. These thought processes create a more flexible, comprehensive type of problem-solving ability. Thus, the transition from concrete operational thought to formal operational thought is necessary for more advanced mathematical and philosophical thought.

The formal operational stage is also divided into two substages called early and late formal (Cowan, 1978). The major differences in the substages seem to be the consistency and ease with which the formal operations are used. Early formal thinkers may experiment more whereas late formal thinkers have a systematic strategy from the start (Cowan, 1978). Early formal may be thought of as a transitional stage between late concrete and late formal.

The formal thinker, then, is able to reflect or think systematically about her or his own thought. Valid conclusions are drawn from the form of an argument or strategy, regardless of the premise or content. Proportional thinking, in which two frames of reference are simultaneously changed, is possible at this stage. The formal thinker can imagine the full range of possibilities in a real or hypothetical situation and thus is able to deal with probability, combinatorics, isolation of variables and hypothesis formulation and testing.

The Piagetian Theory of Cognitive Development

In considering Piaget's stage theory of cognitive development, some attention must be paid to the ordering of stages and the mechanisms through which stage development occurs. The learning theory of Piaget, thus is tied inseparably to his epistomological position.

Beilen (1970), in a review of Piagetian research, makes the case that there is general agreement that the age of acquisition of logical operations differs as a function of cultural experience. However, none of the cross cultural or subcultural studies show an acquisition order that is different from the stage order reported by Piaget.

Piaget viewed the learner as an interactive generator-transformer who acts upon objects or thoughts to construct knowledge. He stated (1964) that students can only benefit from teaching experiences if they are at a stage where the information can be understood; that is, only if the student is at the appropriate cognitive stage. Concrete operational students would thus be at a disadvantage in an algebra class which dealt formally with mathematical concepts.

There are four conditions through which stage progression is accomplished: 1) maturation, 2) experience with the effects of the physical environment which change the structure of intelligence, 3) social transmission in the broadest sense (education, conversation, etc.) and 4) equilibration (Piaget, 1964). The first three conditions are self-explanatory and understood by most persons. Equilibration, or self-regulation, is a special term used by Piaget to explain the dynamic balance between external information and the internal logical structure already present in the individual. External information may be assimilated into an existing structure or the individual may have to accommodate his or her internal logical structure to new external information. Assimilation is the term Piaget used to describe the process of actively transforming that which is incorporated into existing mental structures. Accommodation implies transforming mental structures on the basis of new internal or external information. Thus, a dynamic balance is created between internal operational structures and external experience.

Creating disequilibration, then, is a way of stimulating cognitive development. A situation is created where the student's present cognitive functions are unable to account for or explain an external situation or a new concept. The student must then struggle to either assimilate or accommodate in order to restore equilibration. Active accommodation results in cognitive growth.

Disequilibration is often referred to as cognitive dissonance. Cowan (1978) implies that the most important role of teachers is to serve as disequilibrators for their students. By appropriate use of questions, material, discovery learning and problem posing, teachers can help to provide an optimal amount of match and mismatch between the student's cognitive structure and the external environment (Hunt in Cowan, 1978).

A second educational implication of Piaget's stage development theory is that reasons for answers are sometimes more important than the answer itself. It is the reason that often reveals intellectual structure and the quality of understanding present in the student. In considering implications for education, it is important to remember, however, that Piagetian theory is a theory of staged cognitive development and not an instructional technology.

Assessment of Cognitive Level

Piaget's stage theory of cognitive development has generated much research to confirm or disprove his theory. David Elkind (1962), working with children, adolescents and adults, was one of the first researchers in the United States to verify Piaget's theories. However, in order to conduct large scale research programs, it was necessary to carefully consider the techniques by which cognitive stage was determined.

Clinical Interview

The method used by Piaget to assess levels of cognitive development in subjects is the "methode clinque" or clinical interview. Cowan (1978) described the clinical method as an unstandardized set of probes and manipulations of material to explore the child's version of a task and his or her responses to the experimentor's questions. In The Growth of Logical Thinking, Inhelder and Piaget (1958) developed 15 experimental situations derived from chemistry and physics which could be used to test for formal thinking. These experiments generally dealt with the action and effect of various variables. An example of one of the experiments is the pendulum with variable weights and variable string lengths. Piaget and Inhelder maintained that the correct solution of these tasks required the formal operational group of logical operations.

Exact methods of administering and assessing the Piaget/ Inhelder tasks are not explicitly stated, and, therefore, there is "method variance" in individual assessments (Kuhn, 1979). Lawson (1980) hypothesized that the validity of the Piagetian tasks for adults was questionable because of their content bias toward science. Bond (1981) felt that the difficulty of administering clinical interviews prohibits their use on a wide scale, since the thorough assessment of one subject would require many hours for both the investigator and the subject.

Researchers in many different areas are thus interested in the development of a psychometric written test that could be used with adolescents and adults to determine cognitive stage. The development of such an instrument would permit large-scale assessment projects as well as eliminate the method variance effects inherent in the clinical interview technique. Tests of both subjective and objective written group assessment types have been proposed. A literature review by Patterson and Milakofsky (1978) showed 17 group paper and pencil Piagetian tests. The first such test was developed by Longeot in France in 1962. Most of these tests were designed to measure concrete or formal operational thought. Of serious concern is the fact that few of the tests reviewed by Patterson and Milakofsky (1978) had reliability and validity studies associated with their use.

Patterson and Milakofsky list criteria for choosing an instrument for measuring Piagetian formal thought including: 1) provides comprehensive coverage of both concrete and formal thought; 2) requires minimal language and reading skills; 3) can be administered to a range of ages; 4) can be quickly and objectively scored; and 5) has been standardized and adequately studied for reliability and validity. The following sections give examples of the types of subjective and objective tests that have been developed with associated reliability and validity measures.

Subjective Group Tests

The most commonly used type of group assessment test is that which tries to capture the essence of the Piagetian tasks in pencil and paper exercises. Subjects are expected to write explanations of their answers and grading is subjective. These tests are often used in dissertations (Phillips, 1980) or for small research projects (Reif, 1982). A body of written tasks has emerged similar to the body of Piagetian tasks used in the clinical interview. An example of such a test is one used at the University of the Pacific in pilot research on the relation between cognitive development and grades in beginning computer programming classes (Christianson, 1982). This test was modified from one used by Kurtz and Karplus (1979) in similar research. They, in turn, had taken some of their questions from other researchers. A description of the ten questions used at the University of the Pacific is given below with the original source of the question given in parenthesis.

Item 1: Proportional Reasoning (Kurtz and Karplus, 1979) Students are told that in a particular photograph a mother is 8 cm high and her daughter is 6 cm high. Students are asked to predict the mother's height if the picture is enlarged so that the daughter is 15 cm high. They are asked to explain their answers.

Item 2: Permutations (adapted from Longeot, 1965) Students are given a hypothetical situation in which four stores (a barber shop, a discount store, a grocery store, and a coffee shop) are to be arranged side by side on the ground floor of a shopping center. The students are asked to list all possible ways that the stores can be arranged.

Item 3: Proportional Reasoning (Kurtz, 1979) Students are told that they are investigating the running abilities of a horse and a dog. Each time the horse takes a step, the dog also takes a step. The stride of the horse is measured and found to be 12 feet long. This horse can run a particular course in 30 seconds.

If the dog has a four foot stride, the student is asked how long it will take the dog to complete the same course? The students are asked to explain their answer.

Item 4: Propositional Logic (after Wason and Johnson-Laird, 1972) Students are asked to test the truth or falsity of the following rule: if a card has a vowel on one side, then it has an even number on the other side. Students are shown successive pictures of cards displaying E, 4, K, 7 and in each case asked, "Would you need to know what is on the other side of this card to test the rule? Explain your answers."

Item 5: Probabilistic Reasoning (adapted from Lawson, 1977) Three blue chips and seven red chips are placed in a container on the left, while two blue chips and four red chips are placed in a container on the right. Students are asked which container they would choose to have the best chance of drawing a blue chip on the first try. An explanation of their choice is requested.

Item 6: Correlational Reasoning (adapted from Lawson, 1978) Shown a picture with six birds having long beaks and short tails, two birds having short beaks and short tails, two birds having long beaks and long tails, and six birds with short beaks and long tails, students are asked if they think there is a relationship between the length of beak and the length of tail. Students are asked the strength of the relation and to explain their answers.

Item 7: Combinations (Lawson, 1976) Students are told that biologists are dissecting crab stomachs to find out if they are eating red, yellow, blue or green algae or other food. They are to list all possible combinations of varieties of algae which might be found in the stomach of the crab (assuming order is not important).

Item 8: Propositional Reasoning (Lawson, Karplus, Adi, 1978) Students are asked to test the truth or falsity of the following hypothesis: If a rat has lipid in its blood, then it will be fat. Students are asked:

1. Given blood samples with lipids, would you

need to know if they came from fat or thin rats?

2. Given blood samples with no lipids, would you need to know if they came from fat or thin rats?

3. Given several fat rats, would you need to know if there are lipids in these rats' blood?

4. Given several thin rats, would you need to know if there are lipids in these rats' blood?

Item 9: Separation of Variables (Lawson, private communication to Kurtz; Kurtz, 1979) Students are shown four pictures: 1) a healthy plant that received a tall glass of water and light plant food, 2) an unhealthy plant that received a tall glass of water, dark plant food, and leaf lotion, 3) a healthy plant that received a small glass of water, light plant food, and leaf lotion, and 4) an unhealthy plant that received a small glass of water and dark plant food. Told that another plant is receiving a small glass of water, light plant food and no leaf lotion, students are asked to predict how the plant is doing and explain the basis of their prediction.

Item 10: Deductive Logic (Karplus and Karplus, 1970) Shown a picture of four islands, named Bean, Bird, Fish, and Snail, students are given the following clues:

Clue 1: There is a way to fly between Bean Island and Bird Island.

Clue 2: There is no way to fly between Bird Island and Snail Island.

The students are asked: Is there a way to fly between Bird Island and Fish Island? (Yes, no, not enough information. Why?)

Clue 3: There is a way to fly between Bean Island and Fish Island.

Is there a way to fly between Fish Island and Snail Island? (Yes, no, not enough information. Why?)

An examination of the items in this test reveals a high degree of language skills needed to read the questions as well as a moderate bias toward scientific content. The grading protocols for this test are subjective and scores depend somewhat on the subject's ability to explain his/her answers. Appendix B contains the grading protocols used in this study.

Some of the questions on this test have been individually validated (Lawson, 1977; Longeot, 1965) but such tests as a whole are often not checked for reliability or validity. It is clear, however, that tests of these types generally do sample tasks which are characteristic of formal operational thought. In situations where such tests have been evaluated (Phillips, 1980) good test re-test correlations have been found ($r = .75$) and construct validity, compared with individual assessment techniques, has been high. Subjectively evaluated tests of this type offer a first alternative to the clinical interview that offers some practicability with larger groups of subjects.

Objective Group Tests

Because of the fact that subjective tests of cognitive development require subjective judgements to be made in grading and take a great deal of care and time to grade, interest has grown in the development of objective tests which would be suitable for large scale group assessment research projects. A search of the literature revealed three such tests which are capable of discriminating between concrete and formal levels and which have related reliability and validity studies. These tests will be reviewed individually.

The equilibrium in balance test. Adi and Pulos (1980) used the Equilibrium in Balance Test (EBT) in a multiple choice format to assess formal operational performance of college students in relation to other variables such as field dependence. This test is designed to measure performance on applications of proportional reasoning using a Piagetian task, the balance beam, in an objective format. The test requires balancing a beam by either changing weight, changing distance or finally, altering weight and distance in a compensating manner. Reliability data is available for this test and it has been used in other research projects (Barrow & Shonberger, 1981). The test is not highly verbal, but it only examines one area of formal thought and thus is somewhat limited in its applications.

The inventory of Piaget's developmental tasks. Milakofsky and Patterson (1979) report data of The Inventory of Piaget's Developmental Tasks (IPDT) authored by Hans Furth, B. Ross and J. Youniss of Catholic University, Washington, D. C. This test uses some of the usual Piagetian tasks in a picture format with multiple choice responses. Validity for the IPDT was determined using individually administered Piagetian tasks. Test-retest reliability at a one month interval for two groups of college students was .67 and .95 (Patterson & Milakofsky, 1978). The IPDT may suffer from a ceiling effect because it has too few tasks measuring formal thought. It has been used in research studies with Navaho Indian children (Patterson & Milakofsky, 1978) and in research involving college level chemistry classes (Milakofsky & Patterson, 1979).

Bond's logical operation test. Another well researched objective group test is Bond's Logical Operation Test (BLOT) developed by Trevor Bond of James Cook University, Townsville, Queensland, Australia. The BLOT was purposely constructed to distinguish between adolescents thinking at the formal stage and those thinking at less sophisticated levels (Bond, 1981). The test consists of 35 multiple choice items which can be administered and interpreted for large groups of students. Individual items on the BLOT do not use the Piagetian tasks but rather the logical model of the formal stage. Areas covered include such items as conjunction, disjunction, complete negation, equivalence, incompatibility, correlation, etc. The test is verbal but is not biased toward scientific content.

The BLOT has test-retest reliability of .91 over a six week interval and an 86.6% agreement of classification of the subjects on individually administered Piagetian tasks. The test was developed using a sample of 899 secondary pupils

(Bond, 1981). Bond is currently using it to measure formal operational ability in research projects in Queensland and New South Wales (Bond, in press).

The existence of tests such as the EBT, IPDT, and BLOT seem to indicate that objective psychometric tests of formal thought can be developed. These tests are of great value in research projects in which a large group of subjects must be assessed.

PSI Instructional Techniques and Piagetian Theory

The most common instructional technology in use in remedial mathematics classes at the college level is PSI or self-paced systems of instruction.

When students enter college planning to study mathematics, placement becomes a substantial responsibility of the university (Zwerling, 1979). By placement, one means assigning students to the optimal point in an instructional sequence on the basis of knowledge (Stronck, 1978). As placement tests are given, students requiring remedial mathematics instruction before beginning college level work in mathematics and science are identified. Generally, no assessment of cognitive development is made.

The traditional lecture format does not work well with remedial students (Zwerling, 1979). Commonly, a learning center is established and the most preferred mode of instruction is self-paced or personalized (PSI; Personalized

System of Instruction; Keller Plan). The initial motivation for using a learning center approach to remedial education is to shift instructional responsibility to the student (Musser & Thompson, 1977); however, individualized instruction is often necessary for groups of students who are all at different stages of mathematical competence.

Educational responses to individual student differences have been a recurring theme in American education (Talmage, 1975). There have been many approaches to individualized instruction which have been documented in the annals of psychological research. The problem seems to have been that most programs were developed on a small experimental scale and were not accepted in the mass educational markets (Nash, 1975). A fairly recent exception to this pattern is the Personalized System of Instruction developed by Keller, Sherman, Azzi and Bori (Keller, 1968).

PSI instruction is characterized by the self-paced feature which permits a student to move through material at a speed commensurate with his or her ability; the mastery requirement which lets students go ahead to new material only after demonstrating mastery of previous material; the use of student proctors which permits repeated testing, immediate scoring and feedback and tutoring; the use of lectures as a motivating device only rather than as the only source of critical information; and finally by the stress upon the written word through the use of study guides (Keller, 1968). PSI instruction has been widely adopted as an instructional technology in learning center environments.

Introduced in 1964 by Fred S. Keller and J. Gilmore Sherman, PSI is probably the best known of the behavioral instruction systems. Behavioral instruction systems are based on the principles of operant conditioning as developed in laboratories and applied research (Johnson & Ruskin, 1977). The learning theories of B. F. Skinner are clearly a basis for most behavioral instruction systems. Some persons feel that "Skinnerian" behavior control is anathema to liberal education and personal development. Yet motivation for initiating PSI instructional techniques is oftentimes a humane consideration for the needs of individual students.

Reasons given for the initiation of a PSI course in statistics at the University of Wyoming included low success rates in the traditional course (50-60%), resentful student attitudes toward the course, and concern about actual student learning and retention (Anderson & Cook, 1979). Thus a mixture of behavioristic theory and humanistic concern for students combine in the development of PSI courses to serve the needs of special groups of students at the college level. In 1973, The Carnegie Commission and The Fund for the Improvement of Post-Secondary Education provided funds for the establishment of the Center for Personalized Instruction at Georgetown University with Keller and Sherman on the staff (Johnson & Ruskin, 1977).

When traditional lecture classes are compared with classes using PSI instruction, PSI generally proves to be a superior mode of instruction. Eight of nine college level studies evaluated by Hassett and Thompson (1978) favored PSI instruction (final exam scores were used to compare traditional lecture and PSI classes). Attitude improvements due to PSI instruction also have been noted (Hassett & Thompson, 1978). Since a necessary component of remedial mathematics education is a system responsive to the heterogeneous preparation of students, individualized instruction would seem to be needed (Gaonkar, Douglas & Krishnan, 1977).

Instruction in mathematics has been considered to be one of the courses which might lead to intellectual development in the Piagetian sense (Sanders, 1978). PSI instruction in mathematics for quantitatively deficient students seems especially suited to the development of formal thought. Lovell (1971) expressed the opinion that knowledge of and attitude toward the subject material are likely to facilitate formal thought. The previously stated effectiveness of PSI instruction in terms of student attitude and achievement infer a more positive effect on cognitive development for PSI instruction as compared to traditional lecture format.

Piaget (1964) emphasized the activity of the learner as the crucial element in learning. He believed that without this activity there is no pedagogy that significantly transforms the student. He suggested that logico-mathematical discovery experiences are necessary for cognitive growth. Penrose (1978) implies that Piaget's assumption that to understand is to invent means that, among other things, each student is to work at his or her own pace. Piagetian theory emphasizes individual experiences which produce disequilibration leading to cognitive development (Cowan, 1978). Learning and growth are thus possibly only when there is active accommodation by the learner. The self-paced individualized learning experiences in a PSI course provide such activities through proctor/student interaction.

The immediate feedback provided by proctors in a PSI environment is important to the process of self-regulation. Piaget (1964) uses the term self-regulation as a fundamental factor in development in the sense of cybernetic processes which have feedback. Proctor feedback is an essential part of self-paced learning and hence may lead to disequilibration and accommodation on the part of students involved in the process. PSI instruction in remedial mathematics for students who lack quantitative skills thus provides the individual learning experiences which are a part of cognitive development in Piagetian theory.

Research Findings

Gender Differences

Formal thought. The literature reviewed for this study presented conflicting evidence for gender differences on attainment of formal thought. Studies by Elkind (1962) and Karplus, Formisano and Paulsen (1977) found differences indicating a higher proportion of males at the formal level of cognitive development among adolescents. The Karplus et al. study was international in scope (7 countries) and focused on 13-15 year old students tested on proportional reasoning and control of variables. Tuddenham (1971) also found males performing at the formal level significantly more often in his study.

McKennon and Renner (1971) tested 131 members of the freshman class at an Oklahoma university using volume conservation, separation of variables, exclusion of irrelevant variables and elimination of contradiction. Males scored significantly higher than females on their test. Graybill (1975) postulated that girls may become formal at a later age than boys. Other researchers (Karplus, et al., 1977) suggested that males may have had more of the experiences of the type that foster formal thinking.

On the other hand, other studies, (Sayre & Ball, 1975; Brekke & Williams, 1979; Phillips, 1979) found no differences in attainment of formal thought between males and females. Sayre and Ball sampled 205 high school students in Colorado. They found performance of girls on five formal operational Piagetian tasks not significantly different from boys. The tasks included proportional thought, combinatorial logic and deductive reasoning, syllogisms and a balance beam problem. Phillips (1980), in a study involving ninth grade students in economics classes in California found no difference by sex in cognitive development in her experimental group, but she did find gender differences in her control group.

Tomlinson-Keasy (1972) in a longitudinal study of formal thought in females from age 11 to 54, found, in her sample of college coeds, 67% at the formal operational level. This contrasts favorably with a study by Kolody (1975) estimating that the proportion of college students at the formal level is 50%.

Thus, the literature reviewed reports conflicting results concerning gender differences in attainment of the formal level of cognitive development. Few studies have been done with large samples using group assessment techniques at the college level. Controlling for science and mathematical background and avoiding the use of tests with content bias towards science should enable this study to provide needed information concerning gender differences in cognitive development, at least, among college students.

Quantitative skills. The controversy and evidence for gender differences in quantitative skills is as evident as the conflicting studies reviewed in the area of cognitive development at the formal level. Historically, there has been an assumption that women are inferior to men in quantitative skills as a result of either socialization patterns or genetic inferiority (Luchins, 1981). A study in which 100 psychology text books published between 1875-1975 were reviewed revealed that 78.8% of the texts reported males as being better with numbers or computations while 91% of the texts cited superior male spatial abilities (a factor linked to quantitative skills) (Luchins, 1981).

A recent study which advances the argument that differences in mathematical achievement are biological in origin was the "Study of Mathematically Precocious Youth" conducted at John Hopkin's University (Benbow & Stanley, 1980). This study found that the top scores in the SAT-M test (Scholastic Aptitude Mathematics Test) were always earned by males and that males outnumbered females 2-1 in having SAT-M scores over 500. The researchers administered the SAT-M to a volunteer sample of talented seventh and eighth students. They concluded that sex differences in achievement result from superior male ability which may be related to greater male ability in spatial tasks.

The "Women in Mathematics Survey", which was part of the 1978 National Assessment found no significant differences in quantitative skills favoring males except on a problem-solving subtest for 12th graders (Armstrong, 1981).

For 13 year olds, females outperformed males on computation and spatial visualization and were evenly matched with males on problem-solving ability. Fennama and Sherman (1978), who controlled for differential course taking, found that sex related differences varied from school to school, making it highly unlikely that sex differences alone could account for the variability in quantitative skills.

Research regarding the effect of socialization on female mathematical skills indicates that there are many factors tending to limit the participation of women in scientific and mathematical classes. Brophy and Goode (1970), for example, found that girls receive less praise for correct answers than boys do. Teachers also sex stereotype academic fields, making more contact with girls in reading and with boys in mathematics (Leinhart, Seewald & Engel, 1979). Social scientist Patricia Lind Casserly is quoted as finding enough examples of teachers and counselors discouraging females from mathematical pursuits that she calls it "misplaced nurturance" (Tagliamonte, 1981).

The debate concerning the reasons for observed gender differences in quantitative skills continues. Most earlier studies that found differences (Macoby & Jacklin, 1974; Aiken, 1976) failed to control for course taking (Fennama & Sherman, 1976). Sex differences in quantitative skills, favoring males, may then be the result of comparing groups with different academic backgrounds.

This study attempts to determine if gender differences exist in quantitative skill levels of college students tested with a standardized achievement test. High school background in mathematics will be controlled so that differences which may occur cannot be attributed to differential course taking. No attempt will be made to ascribe any differences which may be found to either biological or sociological causes, since control of sociological factors is not within the scope of this study. Given the present conflicting evidence of differential abilities by sex in the quantitative area, this study should provide information toward the resolution of this question.

Formal Thought and Academic Achievement

Originally, Piaget proposed that formal thought was achieved at age twelve by most individuals (Cowan, 1978). Subsequently, researchers as well as Piaget himself, recognized that early experiments had been done on a privileged group of seventh and eighth graders from Geneva, Switzerland and there was considerable variability in the age at which formal thinking was attained (Cowan, 1978).

David Elkind, whose works supported Piaget's stage theory in the United States, tested 240 college students on volume conservation. He found that only 58% had abstract concepts of volume (Elkind, 1962). In 1976, Haley and Good summarized studies done on college students with respect to determination of the proportion exhibiting formal thought.

The studies reviewed found percentages of college students determined to be fully formal ranging from 11-61%. Summaries of research with high school students showed an average of 44.5% fully formal (Haley and Good, 1976). Chiapetta (1976), in his review of cognitive developmental studies relevant to science instruction at the secondary and college level, found percentages of students at the concrete level ranging from 77-83% for junior high students, 22-85.8% for high school students and 0-52% for college students.

Lawson and Renner (1975) found that only 52% of high school students who were enrolled in science classes such as biology, chemistry and physics were fully formal. Chiapetta (1976) reported that a large percentage of students rated as formal operational functioned at the concrete level when tested on their understanding of physical science subject matter. These students could substitute correctly into mathematical formulas but they could not give examples to show their understanding of scientific concepts and principles. Most estimates, however, of the number of college students not attaining formal reasoning approximate the upper range as 50% (McKennon & Renner, 1971; Kolody, 1975; Sayre & Ball, 1975; Kuhn, 1979).

Piaget (1972) hypothesized that people may only acquire formal operational skills in areas of interest and experience. An experiment by Pulos and Linn (1979) confirmed that rural and urban students differ significantly on tasks measuring formal thought when the tasks contained material familiar to only one of the groups. This study would seem to imply that students would have difficulty exhibiting formal thought in areas such as science and mathematics if they have little background in these subjects.

Herron (1976) claimed that much of science is abstract and requires proportional and combinatorial logic for understanding. According to Herron, there is no alternative to the development of formal thought for successful performance in science and mathematics. Lawson and Nordland (1976) recommended that teachers recognize and appreciate individual differences in reasoning ability in order to better articulate subject matter with level of cognitive development. Karplus (1977) has observed large differences in student ability to understand science concepts with some students being capable while others demonstrate peculiar and inappropriate reasoning styles.

Cantu and Herron (1978) point out the difference in logical concepts that abstract thought requires. They maintain that difficulties that students have in dealing with the abstract are associated with the students' intellectual development in the Piagetian sense. Kuhn (1979) stated that the problem of acquiring formal thought has profound and far reaching implications for all education and especially for adolescent and adult education.

It has been shown that when the basis for awarding grades in a college level course demanded higher order cognitive process (formal thought), there was a high correlation with Piagetian tests of attainment of formal reasoning and grades (Lawson, 1980). Similar research has shown significant positive correlation of Piagetian tests of formal reasoning with achievement in high school and college courses in genetics, physics, chemistry, biology and mathematics (Walker, 1979; Baumen, 1976; Barrow & Shonberger, 1981; Lawson, 1980). Correlations have also been shown with general science concept attainment (Cantu & Herron, 1978). Jordan and Jenson (1979) reviewed a number of correlational studies between arithmetic achievement and cognitive stages which indicated moderate positive relations. Piagetian formal thought has also been associated with success in learning computer programming at the college level (Kurtz, 1979; Christianson, 1981; Zbyszynski, 1981).

Sayre (in Herron, 1976) found an association between performance on IQ tests and formal thought. Brekke and Williams (1979) found a significant correlation ($r = .50$) between formal reasoning and a measure of spatial reasoning. Adi and Pulos (1980) found significant correlations in college students between formal thought and field independence ($r = .54$). Linguistic sophistication was also significantly correlated with formal thought, although at a more moderate level ($r = .32$). Sayre and Ball (1975) found that students defined as formal operational received higher scholastic grades than students defined as non-formal.

Other studies have attempted to identify factors differentiating between successful and unsuccessful students in remedial/developmental mathematics courses. Only entering quantitative skill level was identified as a factor in a study by Barcus and Kleinstein (1981) in which sex, age and major were found to be non-predictive. Cognitive level was not assessed. Only two other research projects were found which link success in a remedial/developmental college mathematics class and cognitive stage. Barrow & Shonberger showed positive results in a traditional lecture setting with relatively small samples. Ricketts (1982) is in the midst of a research project at DePauw University using PSI instruction and a subjective Piagetian test. Final results are, as yet, unavailable from this study.

The studies which were reviewed above emphasize the significance of attainment of formal thinking for students in college level science and mathematics classes. Although such studies have indicated relationships between attainment of higher cognitive levels and achievement, few college level teachers are aware of the cognitive developmental stage of their students (Renner, et al., 1976). Further studies are required to confirm contingencies between achievement and cognitive stage. This study contributes new information on whether cognitive level produces differential skill gains in a PSI remedial/developmental mathematics class at the college level.

Problem Solving

The National Council of Teachers of Mathematics has drawn attention to the lack of problem-solving skills in students in mathematics classes (1980). According to Whimbey (1979), college students need problem-solving skills as well as quantitative skills, not only in mathematics courses but in many other college disciplines as well. In the past, the theory of mental discipline suggested that students learn to think logically and solve problems by studying Latin or geometry or other highly structured content areas. These ideas were discredited by Thorndike's research. Currently, a variety of cognitive process instruction programs have been started at colleges around the country, mainly based on Piagetian developmental theory, to directly teach problem-solving skills (Lochhead, 1979).

Piagetian theory implies that existing knowledge plays an important part in how problems are perceived and hence how solutions are attempted. Arons (1979) states that assumed problem-solving capacities include the reasoning patterns characterizing the Piagetian stage of formal operations.

The process of problem-solving in mathematics has been reflected upon by Polya (1945). In his famous book, How to Solve It, he suggests that dividing the problem-solving experience into stages facilitates solution. These stages serve as a way of organizing discussions of problem-solving strategies. Cognitive process instruction seeks to identify strategies used by successful problem-solvers.

Goldberg (1981) found cooperative small group problem-solving to be an effective way for students to learn problem-solving from their peers. Whimbey (1979) experimented with adding problem-solving instruction to a non-credit algebra course offered at Bowling Green University for remedial/developmental students. He speaks about "non-analytical" students learning the thinking patterns needed for successful problem-solving. His experiment showed significant pre-to-post gains in ability to solve word problems. Students involved in the study reported that the problem-solving instruction taught them to work mathematics problems in steps and also to read their textbooks with greater attention to meaning.

Teaching problem-solving at the college level is an area of recent research interest. The most relevant approach for remedial/developmental mathematics students seems to be the cognitive process instruction, based on Piagetian theory, of Whimbey and Lochhead (1978).

Summary

The literature reviewed here dealt with seven interrelated topics relevant to this study of formal cognitive development and its impact on quantitative skills at the college level. First, a review of historical trends in remedial/developmental mathematics at the college level was presented. This material provides an explanation for the current renewed interest in post-secondary remedial mathematics. Next, a brief exposition of Piagetian cognitive developmental theory was presented. The concrete and formal stages were emphasized as these stages are most likely to occur in a college age population. Types of assessment techniques were described and related to Piagetian theory. Studies suggesting that sex differences occur in both attainment of the formal cognitive level and quantitative skills were reviewed. The relationship between achievement and cognitive stage was discussed for both regular classes and remedial/developmental mathematics classes at the college level. Finally, problem-solving instruction was briefly discussed.

The following summary presents the main points of the review.

1. A large number of students, for various reasons, will be placed in remedial/developmental mathematics at the college level.

2. Research indicates that the formal cognitive stage facilitates achievement in mathematics and science at both the college and secondary level.

3. Self-paced or PSI classes are commonly used to teach remedial/developmental classes at the college level and seem to offer some positive benefits for improving quantitative skills and perhaps, also cognitive development.

4. There is no agreement on whether or not gender differences exist in cognitive development.

5. There is no agreement on whether or not gender differences in quantitative skills are actual, or, artifacts that result from socialization or differential course backgrounds.

6. There have been recent attempts to teach problem-solving at the college level based on Piagetian theory.

CHAPTER III

METHODOLOGY

This study investigated the relationships between formal cognitive development and quantitative skill at the introductory college level. Particular emphasis was placed on gain analysis for students enrolled in remedial/developmental mathematics classes. Gender differences in cognitive assessment scores, quantitative skills and number of high school mathematics and science courses taken were also investigated. An experiment designed to improve quantitative and cognitive scores by teaching problem-solving was performed. Correlation between two different types of cognitive assessment instruments was done. This chapter describes the methodology and procedures used to collect data relevant to this study.

1. The population of the study and the sample from which data were collected are defined.

2. The instruments used to test quantitative skills and cognitive level are discussed and evaluated as to appropriateness in measuring the variables under investigation.

3. The information concerning the questionnaire used to obtain demographic information is also presented.

4. The course organization in the Mathematics Resource Center at the University of the Pacific is described.

5. The special problem solving workshop techniques and materials are described.

6. The research hypotheses are stated.

7. The statistical analysis relevant to each procedure is given.

8. A summary of the research procedures is given.

Population and Sample

The target population is the group of students electing to take introductory mathematics courses at the college level. This group usually consists of young adults from age 17-22, but may include older returning students. The accessible population was comprised of students studying introductory level mathematics at the University of the Pacific in the Fall 1982 semester. The University of the Pacific (UOP) is a small, private university offering both liberal arts and professional degree programs. It is located at Stockton in the central valley of California. The Fall 1982 enrollment was 3,911 students on the Stockton campus.

The Mathematics Department at UOP is relatively large with 15 faculty members. However, most departmental courses are service courses for engineering, science and business majors. In the Fall 1982 semester, 1,574 students were enrolled in mathematics courses through the Mathematics Department. Six hundred ninety-one of these students (44%) were enrolled in introductory mathematics classes requiring satisfactory placement scores for admission.

The Mathematics Resource Center is administratively a part of the Mathematics Department. The primary function of the Resource Center is to administer placement tests, provide remedial/developmental instruction and maintain records for placement purposes. Supervision of mathematics and computer science tutoring and administration of one open computer laboratory are also part of the staff duties. Computerized records of placement test scores are used for registration checks and basic skills competency checks. The Mathematics Resource Center provides tutors for individuals from the Stockton community and also does some statistics and computer consulting work. The Mathematics Resource Center has been in operation since 1976 and was previously called The Mathematics Learning Center.

The sample consisted of students enrolled in Introduction to College Algebra, a two unit remedial/developmental course taught in the Mathematics Resource Center. This course consisted of 27 laboratory periods of approximately eight students per period ($\underline{N}$ = 202). Students signed into this course based on a placement examination given during summer freshman orientation or immediately prior to the start of Fall classes. The class was voluntary but students in certain majors such as business, science and education are strongly urged to take the course by their faculty advisors if their placement scores indicate that remediation is necessary. Students may not enroll in regular mathematics classes unless they pass the appropriate placement test or complete Introduction to College Algebra.

Introduction to College Algebra is an individualized self-paced class. Course material is taught at three skill levels: 1) pre-algebra, 2) elementary algebra and 3) intermediate algebra. Students are assigned to a level based on their placement test scores with some consideration given to background. Sixty percent of the students enrolled in remedial/developmental class were freshmen in the Fall 1982 semester. The age range of the students in the sample was from 17 to 58 with the majority of the students in the 18 to 20 year age range. The median age was 18.6 with 6% of the students above age 22. Because the bulk of the students were in the normal freshman age range, relationships in cognitive level and quantitative skills by age or class level were not investigated.

A comparison sample of students who passed the mathematics placement test at the elementary algebra level and enrolled in Introduction to Probability and Statistics was tested (N = 40). Twenty-five percent of this group were freshmen. A comparison group of students who passed the mathematics placement test at the intermediate algebra level and enrolled in Elementary Functions was also tested (N = 43). Sixty percent of this group were freshmen. The age range of the students in the comparison group was

18-33. The median age was 18.9 with 4% of the students above age 22.

The selection of a group of students from the remedial/ developmental class to participate in the problem-solving experiment posed special ethical problems. The sessions involved four hours of class time which the students would have to give up so that the extra materials could be covered. It was decided to use volunteers. A total of 34 students volunteered. Some students initiated interest on their own whereas others were referred to the problem-solving sessions by their proctors because the students were having difficulty with verbally stated problems. Therefore, the sample was not random. It was felt, however, that a matched-pair design would somewhat compensate for this problem. In order not to bias student instructor interaction, the matches were made anonymously using a numerical code instead of names. Matches were done at the end of the semester before post test scores and final grades were known. Students in the experimental group were paired with other students in the remedial/ developmental mathematics course on the basis of sex, high school mathematics background, placement test score and initial score on the cognitive stage assessment instrument. Students were matched exactly on sex and high school mathematics background. Placement scores were matched to within three points in raw score. Cognitive assessment scores were matched exactly on classification of developmental level and within three points on raw score. Predictably, not all students in the experimental group could be satisfactorily matched. A total of 15 matched pairs were available for this study.

Students who were in the ESL (English as a Second Language) program or who were identified as having significant language difficulties were not included in the sample. This was done because the measures of cognitive stage were pencil and paper assessments requiring proficiency in English.

Instrumentation

The choice of testing instruments for this study involved two decisions. Both quantitative skill level and cognitive stage would have to be reliably assessed. It was decided to choose the standardized placement test used at the University of the Pacific to measure quantitative skill. The objective Piagetian test chosen was Bond's Logical Operation test. The subjective Piagetian test chosen was modified after Kurtz (1979). These tests are described in detail in the following section.

Quantitative Skills

The University of the Pacific Mathematics Department has chosen the Descriptive Test of Mathematical Skills (DTMS) as its placement test. The DTMS is a product of the College

Entrance Examination Board of the Educational Testing Service. There are actually four tests in multiple choice format: a 35 item Arithmetic Skills test, a 35 item Elementary Algebra test, a 30 item Intermediate Algebra test, and a 30 item Functions and Graphs Pre-Calculus test. These tests correspond to four placement levels: 1) mathematics for elementary teachers 2) elementary statistics 3) chemistry, business calculus and elementary functions 4) calculus. Students who do not obtain the necessary scores for entrance into the regular mathematics courses are placed in the remedial/developmental class. The exception is that students not passing the calculus placment test usually take Elementary Functions which is not considered to be remedial at the college level (Heine, 1982).

The primary purpose of the DTMS is to assist colleges in the proper placement of admitted students into the sequence of mathematics courses offered by the institution (Bridgeman, 1980). Because subscale scores are available, the tests can also be used for individual diagnostic testing as well as large scale placement. Each DTMS is given in a 30 minute period and the tests are currently administered during freshman orientation and prior to registration in the spring and fall. Tests are computer graded and given by the staff of the Mathematics Resource Center. Test scores are considered valid for placement purposes for an 18-month interval.

All of the levels of the DTMS have test-retest reliability coefficients between .84 and .91 (College Entrance Board, 1979). A validation study was completed by the Educational Testing Service in 1980 (Bridgeman, 1980). The content validity study was done using a sample of 36 two- and four-year colleges. Gain analyses indicated that scores on the DTMS increased significantly over the course of one semester. This implies the test is accurately aimed at course content. Concurrent validity was assessed by a correlation of course grade with DTMS tests administered at the end of the course. Validity coefficients at the various colleges sampled ranged from .42 to .78.

Predictive validity was assessed by giving the DTMS at the beginning of the semester and correlating scores with end of semester grades. Correlation coefficients were in the .25 to .77 range. DTMS scores were better predictors of success than scholastic aptitude mathematical test scores. An unpublished study from UOP also indicated that placement test scores were better predictors of final grades than either course background or scholastic aptitude mathematical test scores (Christianson, 1977). This same study indicated a drop in failure rates in mathematics classes at the University of the Pacific from approximately 40% to 10% after placement testing was instituted. These results imply the effectiveness of the placement test in reliably assessing quantitative skills.

A content analysis by college faculty rated the DTMS as providing generally good coverage of the key concepts of mathematics courses (Bridgeman, 1980). An indirect relationship between student perception of course difficulty and DTMS scores was also established by Bridgeman's study. Statistical data for the four levels of the DTMS are summarized in Table 1, page 66. Based on these data, the DTMS is deemed to be a reliable and valid testing instrument for placing students into introductory level mathematics courses.

Cognitive Level

Two tests of cognitive assessment were used in this study. Bond's Logical Operation Test is an objective test based on Piagetian theory of formal logical operations with very little scientific subject matter content. The second test was subjective and a modification of a test constructed by Karplus and Kurtz. This test has a higher level of scientific content. Both tests were given to students in the remedial/developmental class in order to do a correlational study of scores on the two different types of cognitive assessment group tests.

Bond's logical operations test. The primary instrument chosen to assess cognitive stage for the purposes of this study was Bond's Logical Operation Test (BLOT). This test is a 35 item multiple choice instrument which has a 30 minute time limit. The test was constructed to reliably and

Table 1

Comparative Test Data for the Four Levels of the Descriptive Test of Mathematical Skills

	Scaled Score	Mean	Standard Deviation	KR20 Reliability	Standard Error	% Completing 75% of Test
Arithmetic Skills	101-125	115.4	5.9	.87	2.1	97
Elementary Algebra	201-225	209.6	6.9	.91	2.3	95
Intermediate Algebra	301-325	309.8	6.3	.86	2.5	93
Functions and Graphs	401-425	408.8	5.9	.84	2.5	98

Data are based on the 1978 morning administration of the DTMS by the College Entrance Examination Board of the Educational Testing Service.

validly distinguish between subjects at the formal stage of cognitive development from those who operate at less sophisticated levels (Bond, 1980). The test does not use Piagetian experiments but is based on the formal logic operations expected at the formal cognitive stage.

Test-retest reliability was done by Bond over an interval in excess of six weeks. The reliability coefficient was $r = .91$ ($p < .001$) for a sample of 91 subjects (Bond, 1980). Validity was established by selecting a random sub-sample of 30 students and administering three standard Piagetian tasks in a clinical interview situation. Agreement of ranking by the BLOT and the Piagetian tasks was .93 ($p < .0005$), using a Spearman rank order correlation coefficient corrected for ties. Agreement of classification using concrete and formal categories was 90% (Bond, 1980).

A factor analytic study, involving a sample of 1,201 subjects ranging from grades 7 through post-secondary, has been done using the items comprising the BLOT. The analysis identified only one dominant factor, called by Bond the formal cognitive stage, within the set of BLOT items (Bond, 1980).

The BLOT has been used in a major research project by Bond which indicated that students in the academic subject stream had higher BLOT scores than those in the commercial industrial track. At present, the BLOT is being used as a measure of formal operational ability in several research projects in Queensland and New South Wales, Australia. This author is unaware of the use of this test in the United States prior to this study. This test was selected for this study because of its reported reliability and validity, its construction according to Piagetian logico-mathematical theory and its apparent avoidance of a scientific bias. Table 2 shows the item content, difficulty and discrimination indices of the items of the BLOT.

There are three different subscale scores on the BLOT: Concrete, INRC 4 Group, and the 16 Formal Logical Operations. The INRC 4 Group consists of the identity, negation, reciprocal and correlation operations. These operations can be thought of as a mathematical group of order four. Only the INRC 4 Group subscale was chosen to investigate the effect of the remedial/developmental mathematics course on the BLOT test scores. Generally, this cluster of operations is thought to be related to proportional thinking, probability, correlation coordination of two systems of reference and multiplicative compensation (Cowan, 1978). There are 15 items comprising this subscale. Three of the 11 instructional units in the remedial/developmental Pre-Algebra and Elementary Algebra class cover some aspect of proportional thought. These units include algebraic fractions, ratios and proportions and conversion from one system of measurement to another. It was thought that mathematics instruction might positively affect this subscale score.

Table 2

Item Content, Difficulty and Descriminability of Bond's Logical Operations Test (Manual for the BLOT; Bond, 1980)

Item No.	Logical Operation	Difficulty	Correlation
1	Mechanical Equilibrium	.86	.35
2	Mechanical Equilibrium	.87	.29
3	Implication	.60	.47
4	Incompatibility	.76	.49
5	Multiplicative Compensation	.90	.31
6	Correlation	.91	.42
7	Correlation	.79	.46
8	Correlation	.73	.54
9	Conjunction	.74	.55
10	Disjunction	.82	.48
11	Conjunctive Negation	.67	.50
12	Affirmation of p	.94	.42
13	Reciprocal Exclusion	.59	.41
14	Probability	.88	.38
15	Reciprocal Implication	.58	.46
16	INRC 4 Group & Proportionality	.85	.39
17	INRC	.70	.62
18	INRC	.78	.59
19	INRC	.75	.49
20	INRC	.79	.47

Table 2 continued

Item No.	Logical Operation	Difficulty	Correlation
21	INRC	.45	.32
22	INRC	.80	.44
23	INRC	.69	.43
24	Co-ord'n of 2 Systems of Reference	.80	.44
25	Complete Negation	.74	.41
26	Complete Affirmation	.65	.54
27	Negation of p	.86	.52
28	Non-implication	.57	.53
29	Affirmation of q	.76	.55
30	Equivalence	.61	.44
31	Negation of q	.73	.51
32	Negation of reciprocal implication	.60	.50
33	Probability	.78	.51
34	Co-ord'n of 2 Systems of Reference	.81	.49
35	Co-ord'n of 2 Systems of Reference	.75	.51

Kurtz/Karplus subjective test. A subjective test of cognitive stage was also given to all students in the remedial/developmental mathematics class for comparison to BLOT socres. The test items were given in Chapter 2 and were a subset of a test constructed by Kurtz and Karplus (Kurtz, 1979). This test required students to answer ten problems of a mathematical, logical or scientific nature. The student was also required to write a short explanation of his/her answers. An answer was graded as either correct (+1) or wrong (0) depending on both the answer and the reason. All ten items require skills usually thought of as requiring formal operational thought.

The author graded all subjective tests. Since grading is subjective, a reliability study was done on the grading protocols. A random sample of ten papers was graded by one other person using the same set of grading instructions. Correlation of scores with the author's graded scores was $r = .96$ which is statistically significant ($p < .005$). Thus the scoring of this test in this study is judged to be sufficiently objective.

Validity and reliability of individual questions have been established in previous studies (Longeot, 1961; Lawson, 1977; Phillips, 1980) which used these items. The subjective test was given in an unlimited time format. Students taking the test usually took less than one hour.

Agreement of classification using the two different types of measures of cognitive stage, objective and subjective, was examined as part of this study. The two tests were given to all remedial/developmental students in two separate administrations during the first two weeks of the Introduction to College Algebra course.

Demographic Data

Students sampled for this study filled out a questionnaire used in obtaining demographic data relevant to the research. This sheet was completed at the time that the BLOT test was given. The student gave his/her name, birth date and sex. A check list of high school mathematics and science courses was completed using standard course titles (Algebra I, Algebra II, Geometry, Trigonometry or higher mathematics, Biology, Chemistry or Physics).

For the purposes of this study, age was recorded to the nearest month and converted to a two place decimal. A numerical code was created for high school mathematics background based on completed courses. A similar code was created for science background.

Instructional Procedures

Students were enrolled in the remedial/developmental course in 17 different time schedules of three hours per week. Each time period was usually broken up into two proctor groups. Proctors were chosen through a formal application procedure which included three training periods and three hours of direct class observation in the semester preceding employment. All proctors have completed pre-calculus mathematics although they are not necessarily mathematics majors. A total of 27 groups were formed in the Fall of 1982. Twenty-one different student proctors were employed and two staff members, one of whom was the author, supervised the course. During any one class period, 8-24 students would be present with 1-3 student proctors and 1-2 supervisors.

Regular Course Procedures

Student placement in course materials was based on DTMS scores and mathematical background. Teaching materials used included Developing Mathematical Skills by Whimbey/Lochhead for pre-algebra, Basic Algebra by Gilligan/Nenno for elementary algebra and Intermediate Algebra by Gilligan/Nenno for intermediate algebra. Standard types of PSI study guides, written by the author, were provided (Christianson, 1982). A suggested pacing schedule was provided and students were given points towards their final grade for meeting the scheduled deadlines. Two midterms and a final examination, structured as modules, were given. A total of 14 units of material, including the two midterm examinations and the final were to be completed for each of the three levels of the remedial/developmental course. Students who did not complete all units were given incomplete grades and required to finish the material in the spring semester. All students were tested for cognitive stage using the BLOT and the ten question subjective test during the first two weeks of school. The post-DTMS and BLOT was given as part of the course final examination. Students receiving incompletes ($\underline{N}$ = 70) were given DTMS and BLOT tests during the final examination period as part of the requirements for obtaining an incomplete.

Problem Solving Component

The 34 students who volunteered for this portion of the study were enrolled randomly through nine different time periods. Therefore, students were either given the treatment individually or, as was the usual case, in small groups of 2-6 students. The treatment consisted of four weekly one hour periods during which special material was covered in a lecture/discussion format. The material covered is outlined below.

Session one consisted of eight problems which were taken from the text Analytic Problem Solving by Whimbey/ Lochhead. The problems emphasized reading technically difficult material and using diagrams, charts and tables to represent relations given in verbal statements. Students worked on a problem and then solutions were discussed. Possible different approaches were suggested by the members of the group. A homework assignment of twelve problems considered to be of a similar type was given. The eight problems used in the group sessions are presented in Appendix C.

These problems were chosen because they begin with some logic statements of the if/then type, introduce order relations, gradually lead to quantitative problems and solutions are facilitated by diagrams and tables. Students in the group quickly discovered the importance of careful reading and the usefulness of diagramming relationships.

Session two consisted of a discussion of the homework assignment plus an introduction to mathematical vocabulary. The various ways of saying add, subtract, multiply and divide were discussed. The students were given a vocabulary list and then introduced to the idea of writing expressions such as "the difference of a number and 8" as N - 8. Several examples of verbal expressions were translated into symbolic form. Sixty-one practice translations were given as homework. During this session, a simple word problem was introduced. The problem was:

> A woman leaves an estate of $84,000 to be divided between a hospital and her daughter. The daughter's share is twice as large as the hospital's share. How much was the daughter's share? How much was the hospital's share?

The problem was solved by the group as a translation problem using a diagram to represent the relations given in the problem. Five similar problems were given as homework.

In the third session, students were given groups of word problems with common structures. The objective was to discover a pattern of solution. Categories of problems included basic sum problems, ratio problems, proportion problems, percent problems and value problems (Lewis & Smyth, 1982). One or two problems of each type were solved by the students in class with three or four more problems of each type assigned as homework. Use of diagrams and charts was stressed.

The last session consisted of covering problems involving value, distance, interest, and mixtures using the table format. Again, students proposed solutions and general patterns were suggested. The text used in the intermediate algebra level emphasizes table solutions of these types of problems. Homework problems were assigned but not discussed as this was the last group session.

All of the students in the groups were encouraged to participate in sharing problem solving techniques. False starts were not corrected until inconsistent results were obtained. Students were encouraged to develop checking techniques and to use diagrams and tables whenever appropriate. Careful reading was stressed along with patterned problem attacks. Twelve males and 22 females participated in the problem-solving sessions. The drop rate was 12% in this group which was slightly higher than the class drop rate of 10%. Matched controls were found for three males and 12 females.

Null Hypotheses and Statistical Analysis

All data collected for this study were processed using the SPSS (Statistical Package for the Social Sciences) program on the University of the Pacific B6700 Burroughs computer. A significance level of .05 was chosen. Tests were either one or two-tailed depending on theoretical considerations or indications of direction from previous studies reviewed in Chapter 2.

For some students, because of illness or administrative errors, not all measurements were obtained. For the purposes of this study, missing observations were eliminated from analysis whenever they occurred. There was a 10% deop rate in the remedial/developmental class which also affected the total sample available for analysis.

Null Hypothesis 1. There is no correlation between placement level and cognitive stage.

Hypothesis 1 addresses the relationship between cognitive developmental stage and quantitative skills as measured by the DTMS placement test. An analysis of variance was performed using BLOT scores recoded to Piagetian stages of late concrete, early formal, and late formal as the independent variable and DTMS scores as the dependent variable. A chi square contingency test was also run on coded placement level and coded Piagetian level. Data were used from both the comparison group of students and the students enrolled in the remedial/developmental mathematics class.

Null Hypothesis 2. There is no difference in skill gains, as measured by DTMS scores, between students at different cognitive stages as determined by BLOT scores.

Hypothesis 2 is concerned with the relationship of cognitive stage to the skill gain in a remedial/developmental mathematics course taught in PSI format. A covariate analysis using post DTMS score as the dependent variable, pre-DTMS score as the covariate and with cognitive level as the independent variable was done. A two-way analysis with interaction was also done to investigate possible interaction of remediation level within the course with post DTMS score and cognitive stage using pre-DTMS scores as a covariate.

Null Hypothesis 3. Thre is no difference in quantitative skills as measured by Pre-DTMS scores, by sex.

Null Hypothesis 4. There is no difference in cognitive stage as measured by pre-BLOT scores, by sex.

Null Hypothesis 5. There is no contingency between number of mathematics courses taken in high school and sex.

Null Hypothesis 6. There is no contingency between high school science background and sex.

Hypotheses 3, 4, 5 and 6 deal with the relationship between sex, quantitative skill level, cognitive development and participation in high school science and mathematics courses. Because of contradictory results of other studies dealing with sex differences, tests of the null hypothesis were two-tailed. Chi square contingency tests were run on categorical data, while an independent *t*-test on difference of means by sex in BLOT and DTMS scores was done.

Null Hypothesis 7. There is no difference between mean pre-to-post INRC 4 Group subscale scores on the BLOT test.

Hypothesis 7 deals with the assumed null effect of the remedial mathematics course on changing INRC 4 Group subscale scores over a one semester period. A paired difference test on pre-to-post BLOT INRC 4 Group subscale scores was performed. Theoretical implications suggested a one-tailed test of the null hypothesis was appropriate.

Null Hypothesis 8. There is no contingency between number of mathematics courses and cognitive stage.

Null Hypothesis 9. There is no contingency between number of science courses and cognitive stage.

Hypotheses 8 and 9 pertain to the relationship between cognitive stage and participation in high school science and mathematics courses. A chi square contingency test was done. An analysis of variance of the number of courses with cognitive level as the independent variable was performed to give additional input into the question.

Null Hypothesis 10. There is no difference between experimental group post DTMS scores and matched control scores.

Hypothesis 10 deals with whether or not the experimental problem-solving instruction was effective in raising post DTMS scores. A paired-difference test was run using the pairs generated through the blind matching technique. Because it was expected that instruction would affect DTMS scores positively, the test was run one-tailed.

Null Hypothesis 11. There is no difference in post BLOT scores between the experimental group and the matched control scores.

Hypothesis 11 deals with whether or not the experimental problem-solving instruction was effective in raising BLOT scores. A paired difference test was run using the matched pairs. A one-tailed test of the null hypothesis was run because positive effects were expected.

Null Hypothesis 12. There is no correlation between objective and subjective cognitive assessment scores.

Hypothesis 12 deals with whether or not subjective and objective scores of Piagetian cognitive stage will be correlated. A Pearson product-moment correlation coefficient was calculated. Since both tests purport to measure the same trait, a positive correlation would be expected and the test was run as a one-tailed test of the null hypothesis.

A chi square contingency test on agreement of classification was also performed by coding both scores to late concrete, early formal and late formal.

Summary

This study investigated the relationship of cognitive developmental stage to quantitative skill level in college students. A sample of students placed in remedial/developmental mathematics was tested for cognitive developmental level using the BLOT multiple choice test of Piagetian stages. In addition, a subjective test which also assessed Piagetian cognitive stage was administered. A comparison group of students placed in regular introductory level mathematics courses was tested using the BLOT. Demographic data from both groups were collected including age, sex, and number of mathematics and science courses taken in high school.

The DTMS and BLOT were re-administered to the students in the remedial/developmental class at the end of the semester. The comparison group was not re-tested because of access problems. An analysis of skill gains with respect to cognitive stage was done. Contingencies between sex, cognitive stage, placement level and number of high school science and mathematics courses were investigated.

A subgroup of students in the remedial/developmental group received special instruction in problem-solving skills. A matched-pair design was used to investigate the efficacy of the problem-solving instruction in improving DTMS scores. The effect of the problem-solving instruction on cognitive level as measured by BLOT scores was also assessed.

The effect of the remedial/developmental course on the INRC 4 Group subscale score of the BLOT was determined by using a pre-to-post dependent *t*-test. The INRC 4 Group measured facility with proportional relations. Finally, a Pearson product-moment correlational measure was computed for the two types of Piagetian assessment instruments used in this study.

CHAPTER IV

ANALYSIS OF RESULTS

The purpose of this study was to investigate the relationship between Piagetian cognitive developmental stage and quantitative skills of students enrolled in introductory and remedial/developmental mathematics classes at the college level. Students sampled were enrolled at the University of the Pacific in the Fall 1982 semester. Subjects were tested for cognitive stage using Bond's Logical Operation Test (BLOT), an objective group assessment instrument. Quantitative skills were assessed using the Descriptive Test of Mathematical Skills (DTMS). Demographic information on sex, high school science background and high school mathematics background was also recorded for each subject. In addition, a second subjective assessment of cognitive stage, the Kurtz/Karplus test, was given to the students enrolled in the remedial/developmental mathematics class.

A group of remedial students was given special experimental problem-solving instruction. A matched-pair design was used to assess the effectiveness of this instruction in raising DTMS and BLOT scores. At the end of the semester, DTMS and BLOT tests were re-administered in the remedial/developmental mathematics class. Changes in BLOT INRC 4 Group subscale scores over one semester were determined. Twelve hypotheses were tested at a .05 level of significance. The results of these analyses are presented in this chapter.

Cognitive Development and Quantitative Skills

The major focus of this study was the investigation of the relationship between cognitive stage and quantitative skill level. Cognitive stage was determined by score on Bond's Logical Operation Test (BLOT), a 35-item multiple choice test. Scores on this test classified subjects as late concrete (0-26), early formal (27-30), or late formal (31-35). Quantitative skill level was measured by scores on the Descriptive Test of Mathematical Skills (DTMS). Scores on the Kurtz/Karplus subjective Piagetian assessment test classified subjects as late concrete (0-3), early formal (4-6) or late formal (7-10).

Hypothesis 1. There is no correlation between mathematics placement level and cognitive stage.

A total of 249 students were classified, according to BLOT scores as concrete or late concrete (N = 29), early formal (N = 80) and late formal (N = 140). Students were assigned to one of four placement levels on the basis of DTMS test scores: pre-algebra (N = 3), elementary algebra (N = 81), intermediate algebra (N = 94) and Statistics or Elementary Functions classes (N = 83).

Table 3 shows the number of students at each cognitive stage, as classified by BLOT scores, in each placement group. Contingencies were found between placement level and cognitive developmental stage ($p < .05$). No contingencies were found at the .05 level between placement level, as determined

Table 3

Number of Students in Each Developmental Stage in Each Mathematics Placement Level as Determined by BLOT Scores

Mathematics Course Placement Level

Piagetian Stage	Pre-Algebra	Elementary Algebra	Intermed. Algebra	Regular Course	Total
Late Concrete	1	16	4	8	29
	*(.4%)	(6.4%)	(1.6%)	(3.2%)	(11.6%)
	+(.6%)	(9.6%)	(2.4%)		
Early Formal	1	21	31	27	80
	(.4%)	(8.4%)	(12.4%)	(10.8%)	(32.1%)
	(.6%)	(12.6%)	(18.7%)		
Late Formal	1	39	52	48	140
	(.4%)	(15.7%)	(20.9%)	(19.3%)	(56.2%)
	(.6%)	(23.5%)	(31.3%)		
Total	3	76	87	83	249
	(1.2%)	(30.5%)	(34.9%)	(33.3%)	(100%)
	(1.8%)	(47.7%)	(52.4%)		

* % of total group
+ % of remedial/developmental group

X^2 = 12.638 p = .0492 contingency coefficient = .21978 by the subjective Piagetian test given in the remedial/developmental mathematics class. Table 4 shows the number of students at each cognitive stage as classifed by the Kurtz/Karplus subjective Piagetian test. Table 5 shows the descriptive statistics for the BLOT scores for each placement level.

An analysis of variance on pre-DTMS scores by Piagetian developmental level indicated no significant differences. If our analysis is restricted to the remedial/developmental class, however, there is a significant difference by BLOT cognitive stage in pre-DTMS score ($F = 4.08$, $p < .02$). Late concrete subjects have significantly lower mean pre-DTMS scores in the remedial/developmental class.

Hypothesis 2. There is no difference in skill gains, as measured by DTMS scores, between students at different cognitive stages as determined by BLOT scores.

Hypothesis 2 was concerned with the effect of cognitive stage on mathematical skill gain, as measured by pre-to-post DTMS gain, in the self-paced remedial/developmental mathematics course. It was initially determined using a two-way analysis of covariance with interaction given post DTMS score as the dependent variable and developmental stage and placement group as the two independent variables and pre-DTMS scores as the covariate, that there was no interaction between the level of remediation and cognitive level. Table 6

Table 4

Number of Students at Each Developmental Stage in Each Placement Level as Determined by Kurtz/Karplus Scores

Piagetian Stage	Pre-Algebra	Elementary Algebra	Intermediate Algebra	Total
Concrete	2 (1.4%)	37 (25.3%)	27 (18.5%)	66 (45.2%)
Early Formal	0 (0.0%)	27 (18.5%)	33 (22.6%)	60 (41.1%)
Late Formal	0 (0.0%)	5 (3.4%)	15 (10.3%)	20 (13.7%)
Total	2 (1.4%)	69 (47.3%)	75 (51.4%)	146 (100%)

$X^2 = 9.401$ $p = .0518$ contingency coefficient = .2460

Table 5

Bond's Logical Operation Test and Kurtz/Karplus Test Mean Scores for Each Placement Level

	BLOT Test			Kurz/Karplus Test		
	N	$\bar{x}$	S	N	$\bar{x}$	S
Placement Level						
Pre-Algebra	3	26.00	9.64	2	1.50	2.12
Elementary Algebra	76	29.58	4.24	69	3.38	1.98
Intermediate Algebra	87	30.76	2.98	75	4.51	2.21
Elementary Functions and Statistics	83	30.48	3.57	0*	NA	NA
Total	249	30.25	3.74	146	3.93	2.18

*The Kurtz/Karplus Test was only given in the remedial/developmental class.

summarizes the results of this analysis. Thus data from the three remediation levels were combined and a covariate analysis was done on post DTMS scores using pre-DTMS scores as the covariate and cognitive stage as the independent variable ($N = 152$). The adjusted means differed significantly ($F = 6.28$, $p < .01$). Table 7 shows the ANCOVA results and also a multiple classification analysis showing adjusted means.

A similar analysis was done for the Kurtz/Karplus subjective test. Results were significant ($F = 7.46$, $p < .01$). Table 8 shows the ANCOVA results and the multiple classification analysis with adjusted means.

Percentage gains are reported since the different levels of the DTMS have different numbers of items. The standard error of measurement is approximately 3 percentage points for pre-algebra, 6.6 percentage points for elementary algebra and 8 percentage points for intermediate algebra. For the three DTMS tests used in this study, the standard error of measure overall is approximately 6 percentage points or about two points in raw score. The mean gain is 27.6 percentage points for all students included in the study. Late concrete students gained only 20.5 percentage points while early formal subjects gained 31.4 and late formal subjects gained 26.3 percentage points. The mean gain for all students was about five standard errors of measurement.

Table 6

Two-Way Analysis of Covariance of Final DTMS Score Between Remediation Level and Initial Piagetian Developmental Stage with Initial DTMS Score as the Covariate

	SS	df	F	p
Initial DTMS Score (Covariate)	2478.053	1	13.014	.001
Main Effect	2076.966	4	2.727	.04
Developmental Stage	1375.132	2	3.611	.03
Remediation Level	905.686	2	2.378	.09
Two-way Interaction	343.188	2	.091	.41
Explained	4898.207	7	3.675	.001
Residual	27800.189	146		
Total	4314.481	153		

Table 7

Analysis of Covariance of Final DTMS Scores Between Initial Developmental Stage as Determined by BLOT Scores With Initial DTMS Score as the Covariate

	SS	df	F	p
Initial DTMS Score (Covariate)	1767.086	1	9.125	.003
Main Effect Cognitive Stage	1882.247	2	4.86	.009
Explained	3649.333	3	6.281	.000
Residual	29049.064	150		
Total	32698.396	153		

Multiple Classification Analysis

Grand mean 69.38	Unadjusted Dev.	Adjusted Dev.	Adjusted Means
Concrete	-9.56	-7.62	59.82
Early Formal	2.48	2.10	71.76
Late Formal	.42	.26	69.80

Table 8

Analysis of Covariance Between Initial Developmental Stage as Determined by Kurtz/Karplus Scores and Final DTMS Scores With Initial DTMS Scores as the Covariate

	SS	N	F	p
Initial DTMS Score (Covariate)	2778.215	1	15.143	.000
Main Effect Cognitive Stage	1325.279	2	3.612	.030
Explained	4103.494	3	7.456	.000
Residual	24216.911	132		
Total	28320.404	135		

Multiple Classification Analysis

Grand mean 69.18	Unadjusted Dev.	Adjusted Dev.	Means
Concrete	-3.23	-2.59	66.59
Early Formal	1.64	1.55	70.73
Late Formal	5.48	3.63	72.81

Gender Differences

Hypothesis 3, 4, 5 and 6 dealt with the relationship between sex, quantitative skill level, cognitive stage and participation in high school science and mathematics courses. As was indicated in the literature reviewed, there is conflicting information regarding gender differences, especially in the area of quantitative skill and cognitive development. Two hundred fifty-seven pre-DTMS placement test scores were available for analysis. There were 101 males in the group and 156 females. These subjects were either in the remedial/developmental class or enrolled in Elementary Functions or Statistics classes.

> Hypothesis 3. There is no difference in quantitative skills as measured by mean pre-DTMS scores between male and female college students.

Hypothesis 3 was tenable using an independent *t*-test with pooled variance. The mean male DTMS score was 47.6% while the female mean score was 47.8%. There was no significant difference in variability of scores within the two groups. This analysis included scores from both the comparison group and the students in remedial/developmental mathematics courses. If the comparison group is tested separately from the remedial/developmental group, the same pattern of no differences by sex is observed in DTMS scores. Further, no contingencies were found between sex and placement level.

Hypothesis 4. There is no difference in cognitive scores as measured by pre-BLOT scores, between male and female college students.

No differences were found in BLOT cognitive developmental score by sex (N = 257) using an independent t-test with pooled variance. The mean score for males was 30.2 while for females, the mean was also 30.2. When the analysis of differences in pre-BLOT scores by sex was restricted to the remedial/developmental group, no difference was found. Again, the same pattern of no differences was present when the comparison group was analyzed separately from the remedial/developmental group. Chi square contingency tests showed no reglationship between sex and cognitive developmental level for either of the two tests of cognitive stage.

There is a difference by sex in mean raw scores on the Kurtz/Karplus subjective test ($t = 2.78$, $p < .01$). The male mean was 4.56 while the female mean score was 3.55. The Chi square contingency test using Kurtz/Karplus test determined stages was not significant at the .05 level. Table 9 summarizes the data for DTMS scores, BLOT scores and Kurtz/Karplus scores by sex.

Hypothesis 5. There is no relationship between number of mathematics courses taken in high school and sex.

Hypothesis 6. There is no relationship between number of science courses completed in high school and sex.

Table 9

Descriptive Data for BLOT Scores, DTMS Scores and Kurtz/Karplus Scores by Sex

	N	Mean	Standard Deviation	t value	2-tailed p value
Pre-BLOT					
Males	99	30.24	3.81	.02	.982
Females	150	30.25	3.70		
Pre-DTMS					
Males	101	47.63	18.69	.06	.955
Females	156	47.77	18.71		
Kurtz/Karplus (restricted to remedial/developmental group)					
Males	55	4.56	2.04	2.78	.006
Females	91	3.55	2.19		
Pre-BLOT (restricted to remedial/developmental group)					
Males	61	30.13	3.98	0.00	.997
Females	105	30.13	3.74		
Post-BLOT (restricted to remedial/developmental group)					
Males	61	32.29	2.74	2.89	.004
Females	105	30.83	3.34		

Table 10 summarizes the course background by sex of the subjects sampled in this study. Science and mathematics high school background was also investigated for differences in number of courses taken by sex. It was found that there was no difference by sex in number of science courses taken in high school. However, there was a contingency between sex and number of high school mathematics courses taken ($p < .05$). Differences favored males.

In the comparison group alone, there was also a significant difference in number of mathematics courses between males and females ($t = 2.11$, $p < .05$) with males taking 3.52 courses on average and females taking 3.15 courses. A difference in variability in number of mathematics courses also existed in the comparison group with females showing significantly higher variability ($F = 2.62$, $p < .01$) in number of courses. Differences in variability in number of mathematics courses by sex did not exist in the remedial/developmental group.

Cognitive Level and Science and Mathematics Participation

Besides focusing on whether there was a difference in number of science and mathematics courses taken in high school between males and females, this study also related cognitive stage to high school course background. Mathematics and science areas were chosen because many of the Piagetian tasks are oriented toward science, mathematics

Table 10

Number of Mathematics and Science Courses Taken in High School by Males and Females

High School Mathematics in Years	t = 2.88	p = .004				
	Male		Female		Total	
0	1	(1.0%)*	2	(1.3%)	3	(1.2%)
1	4	(3.9%)	11	(7.0%)	15	(5.8%)
2	22	(21.4%)	50	(32.0%)	72	(27.8%)
3	35	(34.0%)	55	(35.3%)	90	(34.8%)
4	41	(39.8%)	38	(24.4%)	79	(30.5%)
Totals	103	(100%)	156	(100%)	259	(100%)
High School Science in Years	t = .84	p = .403				
0+	7	(6.8%)	19	(12.2%)	26	(10.0%)
1	31	(30.0%)	58	(37.2%)	89	(34.4%)
2	51	(49.5%)	56	(35.9%)	107	(41.3%)
3	14	(13.6%)	23	(14.7%)	37	(14.3%)
Totals	103	(100%)	156	(100%)	259	(100%)

* % of column total

+ Note: Introduction to Physical Science or General Science was not counted in this study because of the variability of the content of such courses.

and problem-solving.

Hypothesis 8. There is no relationship between number of mathematics courses taken in high school and cognitive stage.

Hypothesis 9. There is no relationship between number of science courses taken in high school and cognitive stage.

Table 11 summarizes data on participation in high school science and mathematics courses by cognitive stage as reported by subjects sampled for this study. Hypotheses 8 and 9 pertain to the relationship between pre-BLOT cognitive level and participation in high school science and mathematics courses. Chi square contingency tests were done with cognitive level and number of mathematics courses (N = 246) and number of science courses (N = 225). Results were not statistically significant at the .05 level for either analysis. The expected contingency between number of mathematics courses and placement level exists ($p < .01$). A similar contingency exists between number of high school science courses and placement level ($p < .01$). Subjects placed at higher levels in the mathematics sequence tend to have taken more mathematics and more science courses.

Experimental Problem Solving Instruction and INRC Subscale Scores

As a part of this study, a matched-pairs experiment was

Table 11

Number of High School Science and Mathematics Courses Taken by Subjects at Different Piagetian Cognitive Stage

High School Mathematics in Years	Concrete	Early Formal	Late Formal	Total
	Piagetian Cognitive Stage			
	$X^2 = 9.400$	$p = .3097$		
0	0 (0%)	0 (0%)	3 (1.2%)	3 (1.2%)
1	4 (1.6%)	4 (1.6%)	7 (2.8%)	15 (6.0%)
2	10 (4.0%)	20 (8.0%)	35 (14.1%)	65 (26.1%)
3	10 (4.0%)	32 (12.8%)	47 (18.8%)	89 (35.7%)
4	5 (2.0%)	24 (9.6%)	48 (19.3%)	77 (30.9%)
Totals	29 (11.6%)	80 (32.1%)	140 56.2%)	249 (100%)
High School Science in Years	$X^2 = 4.876$	$p = .5598$		
0	3 (1.2%)	8 (3.2%)	13 (5.2%)	24 (9.6%)
1	14 (5.6%)	29 (11.6%)	42 (16.9%)	85 (34.1%)
2	10 (4.0%)	33 (13.2%)	62 (24.9%)	105 (42.2%)
3	2 (.8%)	10 (4.0%)	23 (9.2%)	35 (14.1%)
Totals	29 (11.6%)	80 (32.1%)	140 (56.2%)	249 (100%)

done to test the effectiveness of special instruction in verbal problem-solving in raising DTMS and BLOT scores. The effect of the remedial/developmental class itself on raising a subscale score of the BLOT was also tested.

Hypothesis 7. There is no difference between pre-to-post INRC 4 Group mean subscale scores on the BLOT test.

Hypothesis 7 deals with the possible effect of the remedial mathematics course on changing INRC 4 Group Subscale scores over a one semester period. The INRC score purports to measure the ability to deal with proportion, negation, reciprocal and correlational operations. These operations are possible at the formal stage of cognitive development according to Piaget. It was hypothesized that experience in a mathematics course which covers material involving ratios, proportions, algebraic fractions and unit conversions should improve the INRC subscale score on the BLOT test.

Pre-to-post INRC subscale scores were available for 154 subjects. A paired difference test indicated a statistically significant improvement in scores ($\underline{t}$ = 4.09, $\underline{p}$ < .01). The mean difference in scores, however, was only .60 with a .29 standard error. Although this difference is statistically significant because of the large sample size, it indicates less than one point improvement in raw score on the average. Therefore, the practical significance of this change is questionable.

Hypothesis 10. There is no difference between experimental group post DTMS scores and matched control scores.

Hypothesis 10 deals with whether or not the experimental problem-solving instruction was effective in raising post DTMS scores. A paired difference test was done using post DTMS scores which were available from 15 matched pairs. Significant differences were found at the .05 level favoring students who had received the experimental instruction. The mean difference was 6.73 percentage points. Table 12 shows the matched-pair data for the 15 pairs included in the analysis.

Hypothesis 11. There is no difference in post BLOT scores between the experimental group and the matched control group.

Hypothesis 11 deals with whether or not the experimental problem-solving instruction was effective in raising BLOT scores. A paired-difference test was run using post BLOT scores which were avilable from 14 matched pairs. No significant difference was found indicating that the problem-solving instruction did not affect post BLOT scores.

Correlation of Assessment Instruments

Two different cognitive assessment tests were given to the students enrolled in the remedial/developmental mathe-

Table 12

Data on Matched Pairs Used in Problem-Solving Group

Pair No.	Sex	Cognitive Stage	Years Math	Remedial Level	Difference in Post-DTMS (Exp-Control)
1	f	Formal	4	Int. Alg.	+20
2	f	Formal	4	Int. Alg.	+14
3	f	Formal	2	Int. Alg.	+30
4	f	Concrete	1	El. Alg.	-11
5	f	Formal	1	El. Alg.	+14
6	m	Formal	2	El. Alg.	+23
7	f	E. Formal	2	Int. Alg.	+ 7
8	f	Formal	3	Int. Alg.	- 7
9	m	Formal	2	El. Alg.	+ 5
10	m	E. Formal	4	El. Alg.	+11
11	f	E. Formal	2	El. Alg.	- 5
12	f	E. Formal	3	El. Alg.	+ 9
13	f	Formal	2	El. Alg.	- 3
14	f	Formal	3	Int. Alg.	-14
15	f	E. Formal	3	Int. Alg.	+ 7

$\underline{\bar{x}}_d = 6.67$ $\underline{S}_d = 12.75$ $\underline{t} = 2.206$ $\underline{t}_{(.05,14)} = 1.761$

matics course. The BLOT is a new multiple choice assessment instrument which was to be compared with a more traditional subjective group pencil and paper test modeled after the Piagetian tasks (Kurtz, 1979).

Hypothesis 12. There is no correlation between objective and subjective cognitive assessment scores.

Hypothesis 12 deals with whether or not raw scores on the subjective and objective group tests of formal cognitive development were correlated. A total of 137 subjects from the remedial/developmental mathematics classes took both tests. The Pearson product-moment correlation coefficient was $\underline{r} = .48$. This positive correlation is statistically significant ($\underline{p} < .001$) but of rather low magnitude. Table 13 summarizes the classification relations between the two tests.

A contingency test on agreement of classification of the two tests was done. Both tests classify students as late concrete, early formal and late formal on the basis of raw scores. There was 29.9% exact agreement of classification and classifications were significantly contingent ($\underline{p} = .0053$). If early formal and late formal categories are combined the exact agreement is 53%. There were six subjects (4.4%) classified as late formal by the subjective test who were classified as early formal by the objective test. There were 63 subjects (46%) classified as late formal by the objective test who were classified at lower levels by

Table 13

Cognitive Level Classifications Categorized by the Kurtz/Karplus Subjective Piagetian Test and Bond's Logical Operations Objective Test

Kurtz/Karplus Test	Bond's Logical Operations Test			
	Concrete	Early Formal	Late Formal	Total
Concrete	13 (9.5%)	23 (16.8%)	24 (17.5%)	60 (43.8%)
Early Formal	4 (2.9%)	14 (10.2%)	39 (28.5%)	57 (41.6%)
Late Formal	0 (0.0%)	6 (4.4%)	14 (10.2%)	20 (14.6%)
Total	17 (12.4%)	43 (31.4%)	77 (56.2%)	137 (100%)

the subjective test. There were four subjects (2.9%) classified as early formal by the subjective test who were classified as late concrete by the objective test. There were 23 subjects (16.8%) classified as early formal by the objective test who were classified as late concrete by the subjective test. Seventeen percent of students classified as late formal by the BLOT test were classified as concrete by the Kurtz/Karplus test. Thus there seems to be many exceptions to the agreement of the two tests.

The objective group assessment instrument categorizes 56.2% of the subjects as late formal, 31.4% as early formal and 12.4% as late concrete. The subjective group assessment test categorizes 14.6% of the subjects as formal, 41.6% of the students as early formal and 43.8% as late concrete.

Ancillary Findings

At the same time that the 12 major null hypotheses of this study were tested, additional tests were run to assist in the interpretation of major results. These findings are presented here.

The relationship that exists between mathematics placement level and cognitive stage can be examined inversely. An analysis of variance on BLOT scores with placement level as the independent variable was significant ($F = 3.48$, $p < .05$). Table 5 on page 88 shows the mean BLOT scores for each placement level and also the mean Kurtz/Karplus scores for each placement level.

Although the analysis of pre-DTMS scores by Piagetian developmental level showed no significant differences, if the early formal and late formal group are combined and compared to the late concrete group using analysis of variance on DTMS scores by BLOT stage, there is a significant difference at the .05 level favoring the formal group ($F = 3.74$, $p < .05$). As was previously stated, if we restrict our analysis to the remedial/developmental class, there is a significant difference by BLOT cognitive stage in pre-DTMS score ($F = 4.08$, $p < .02$). Late concrete subjects have significantly lower mean pre-DTMS scores in the remedial/developmental class.

In examining gender differences in course background, it was decided to combine number of high school science and mathematics courses. There is a significant difference in mean number of courses taken with males having taken a higher mean number of classes ($t = 3.64$, $p < .01$). In examining gender differences in cognitive assessment scores, although no differences were found by sex in the BLOT scores initially, at the second administration of the test at the end of the semester in the remedial/developmental class a significant contingency between sex and cognitive stage was found ($X^2 = 8.23$, $p < .02$). Males were, proportinately, at higher levels. Males made significantly higher gains over one semester in BLOT scores ($t = 2.94$, $p < .01$).

When concrete operational students as a group were compared with students classified as early or late formal using an independent t-test with pooled variance, a significant difference in mean number of mathematics courses is found with the formal operational group having taken more classes ($p < .01$). The formal group had a mean of 2.98 courses while the concrete group had a mean of 2.55 courses.

When number of high school science and mathematics courses were combined and treated as a single independent variable, an analysis of variance run on BLOT scores was significant ($F = 3.21$, $p < .01$). Table 14 shows the ANOVA results. A similar analysis on Kurtz/Karplus subjective test scores showed no significant differences.

An analysis of variance done on pre-DTMS scores by combined number of high school mathematics and science courses was also significant ($F = 3.91$, $p < .01$). Table 15 summarizes this information.

There is a contingency between high school science and high school mathematics participation ($p < .01$). Subjects reporting few mathematics courses also reported few science courses. Table 16 shows participation in science and mathematics jointly for the subjects in this study.

Although the mean gain on the BLOT INRC 4 Group subscale score was quite small (.60 in raw score), an examination of mean gain by cognitive level reveals a more practical significance for concrete operational subjects. It should

Table 14

Analysis of Variance of Pre-BLOT Score with Combined Number of High School Mathematics and Science Courses as the Independent Variable

	SS	df	MS	F	p
Between Groups	295.497	7	42.214	3.214	.0028
Within Groups	3165.065	241	13.133		
Total	3460.562	248			

Number of Years of Science and Mathematics	$\bar{X}$	N	s
0	32.5	2	.71
1	24.5	4	7.89
2	29.9	19	4.34
3	29.3	42	4.11
4	30.9	50	3.25
5	29.8	64	4.01
6	30.8	44	2.51
7	31.8	24	2.51
Group as a whole	30.2	249	3.74

Table 15

Analysis of Variance of Pre-DTMS Score with Combined Number of High School Mathematics and Science Courses as the Independent Variable

	SS	df	MS	F	p
Between Groups	8839.746	7	1262.821	3.914	.0005
Within Groups	80336.519	249	322.637		
Total	89176.265	256			

Number of Years of Science and Mathematics	$\bar{x}$	N	s
0	54.3	3	16.3
1	31.2	4	13.9
2	35.1	21	14.9
3	45.6	46	17.1
4	44.3	50	17.8
5	49.9	64	18.3
6	52.2	45	17.9
7	57.4	24	21.7
Group as a whole	47.7	257	18.7

Table 16

Participation in High School Science Courses Related to Participation in High School Mathematics Courses

Years of High School Mathematics	Years of High School Science: 0	1	2	3	Total
0	2 (0.8%)	1 (0.4%)	0 (0.0%)	0 (0.0%)	3 (1.2%)
1	3 (1.2%)	10 (3.9%)	2 (0.8%)	0 (0.0%)	15 (5.8%)
2	11 (4.2%)	37 (14.3%)	22 (8.5%)	2 (0.8%)	72 (27.8%)
3	7 (2.7%)	26 (10.0%)	47 (18.1%)	10 (3.9%)	90 (34.7%)
4	3 (1.2%)	15 (5.8%)	36 (13.9%)	25 (9.7%)	79 (30.5%)
Total	26 (10.0%)	89 (34.4%)	107 (41.3%)	37 (14.3%)	259 (100%)

be noted that initially, INRC scores were quite high. This would be expected according to Piagetian theory which suggests that college students should be, in the majority, formal operational. The mean pre-score was 13.09 out of a total of 15 possible points. Thus most of the students sampled in this study seemed to score highly on this subscale of the BLOT.

When, however, pre-INRC scores are broken down by cognitive stage using an analysis of variance, the results are significant ($p < .01$). The concrete operational group had lower INRC subscores. The mean for the concrete group was 10.4 while the early formal group had a mean score of 12.9 and the late formal group had a mean score of 13.97. Piagetian theory would infer the successively increasing mean scores. The concrete group made a statistically significant mean gain of 2.44 points in raw score over one semester. This brings the group mean closer to that of the pre-INRC early formal mean score.

When INRC 4 Group subscores are broken down by placement level, there is a statistically significant difference by level ($p < .05$). The pre-algebra mean score is 10.67, while the elementary algebra group mean is 12.92 and the intermediate algebra group mean is 13.30. It should be noted that 55.5% of the remedial/developmental students taking the Kurtz/Karplus subjective test missed the first question which dealt with a proportional calculation.

In determining the effect of the remedial/developmental class on changing INRC subscale scores, the overall changes in BLOT assessment of cognitive stage classification were also examined. Table 17 shows the pre- and post BLOT cognitive levels for students in the remedial/developmental class. As expected there is a significant relationship ($p < .001$). The percentage of students remaining at the same stage over the course of one semester was 64.7% ($N = 99$); 7.2% of the students moved to a lower stage ($N = 11$) and 28.1% of the students moved to a higher stage ($N = 43$).

The difference between pre- and post raw BLOT scores was also examined. An analysis of variance of differences with cognitive stage as the independent variable was significant ($F = 37.00$, $p < .001$). Students at the concrete stage made the highest mean gain of 5.44 points. Students at the early formal stage gained 2.07 points and students at the late formal stage gained .18 points on the average. The small mean gain of the late formal group would be expected.

Summary

This study was conducted to determine whether Piagetian cognitive stage was related to quantitative skills for students in introductory level and remedial/developmental mathematics classes at the college level. Additionally, gender differences and relationships to high school science and mathematics course background were investigated. Two

Table 17

Developmental Stage of Students in the Remedial/Developmental Class at the Beginning and End of the Semester

Beginning of the Semester	End of the Semester			
	Late Concrete	Early Formal	Late Formal	Total
Late Concrete	4 (2.6%)	10 (6.5%)	4 (2.6%)	18 (11.8%)
Early Formal	2 (1.3%)	19 (12.4%)	29 (19%)	50 (32.7%)
Late Formal	1 (.7%)	8 (5.2%)	76 (49.7%)	85 (55.6%)
Total	7 (4.6%)	37 (24.2%)	109 (71.2%)	153 (100%)

types of group cognitive assessment instruments were used and scores from both tests were correlated.

Significant but modest relationships were found between cognitive stage and mathematics placement level with higher cognitive levels associated with higher placement levels. Students classified as concrete operational had somewhat lower mean DTMS scores than students classified as early or late formal.

It was found that within the remedial/developmental class, post DTMS score was related to cognitive stage after controlling for entering DTMS score through analysis of covariance. There was no significant effect of remediation level nor any interaction between remediation level and the effects of cognitive stage on post DTMS score after controlling for entering DTMS score using a two-way analysis of covariance with interaction.

No significant gender differences were found in DTMS scores, pre-BLOT scores or number of science courses taken. There was, however, a significant difference in mean scores on the subjective Piagetian test with males having higher mean scores. There was also a significant difference, favoring males, in number of high school mathematics courses taken. Males also had significantly higher mean gains on the BLOT test over a one semester time period so that post BLOT scores showed a significant difference by sex with males having a higher mean score.

There was a statistically significant pre-to-post difference in INRC 4 Group subscale scores for students enrolled in the remedial/developmental mathematics course with concrete students showing significantly higher gains. These gains had little practical significance for students at the formal level but for students in the concrete group, the gains were large enough to move them into the early formal category, on the average.

The experimental problem-solving instruction did significantly raise post DTMS scores using a matched pair analysis. No significant differences were found between post BLOT scores.

No contingencies were found between cognitive stage as classified by BLOT score and participation in high school science and mathematics classes separately. However, when number of mathematics and science courses are combined, there is a significant difference by BLOT cognitive stage. Expected relationships existed between placement level and number of mathematics and science courses taken.

The correlation between the BLOT objective and the subjective Piagetian assessment scores was positive but low. There was an only 30% exact agreement of classification between the two tests. Over the course of one semester, 28.1% of the students sampled from the remedial/developmental course moved to a higher cognitive stage according to BLOT scores with concrete students making significantly higher gains. The implications of these results are discussed in Chapter 5.

CHAPTER V

DISCUSSION OF RESULTS

Since a large number of college students require remedial/developmental mathematics instruction, factors which affect placement and instruction need to be identified and studied. One factor which has been linked to achievement in college level mathematics and science courses is Piagetian cognitive developmental stage.

This study investigated the relationship between cognitive development and quantitative skill levels as measured by the DTMS standardized placement test. The relationship between gains in a self-paced remedial/developmental mathematics class and entering cognitive assessment score were studied. Possible gender differences in scores on cognitive assessment tests and mathematics placement test scores were examined as well as differences by sex in number of high school science and mathematics courses taken. The effect of remedial/developmental mathematics instruction over a one semester time period on changing INRC 4 Group BLOT subscale scores (which measure ability to deal with the identity, negation, reciprocal and correlational operations) was examined. The results of experimental problem-solving instruction in affecting cognitive assessment and quantitative skill scores were determined. Correlational studies of two different cognitive assessment instruments were done. The students sampled in this study were enrolled in introductory and remedial/developmental mathematics classes at the University of the Pacific in the Fall of 1982.

Cognitive Development and Quantitative Skill

The major focus of this study was the relationship between cognitive developmental stage, as assessed by the BLOT and Kurtz/Karplus tests, and quantitative skills, as measured by the DTMS placement tests. Significant contingencies were found indicating that students placed at higher levels in the mathematics course sequence were more likely to be formal operational and score at a higher level on the BLOT test. A similar result was found for the Kurtz/Karplus subjective group test. The concrete operational students show a lower DTMS score than formal students at all placement levels. The determination that a significant relationship does exist between placement level and cognitive stage suggests that students placed at lower levels of the college mathematics curriculum need instructional approaches that do not necessarily assume formal operational abilities. The traditional lecture/demonstration method is possibly inappropriate, in some areas, for concrete operational students. Such students require more experiential learning which takes individual rates of accommodation into consideration. These students may have difficulty with verbal problems, the functional concept, and other topics generally covered at this level in mathematics courses.

Some experiences aimed at stimulating cognitive development should be a part of remedial/developmental courses. These experiences might include cognitive process instruction similar to the experimental problem-solving instruction used in this study or material taught using Piagetian learning cycle structure.

Experiences based upon a pilot study for this project indicated that concrete operational students in the remedial/developmental course had difficulty with concepts such as the distributive property, negative numbers and ratios and proportions. In contrast, a student at the pre-algebra remedial/developmental quantitative level who was later determined to be formal operational was remembered to have had no difficulty in dealing with such concepts. This student, who was part of the experimental problem-solving instruction group, was taught to solve distance and mixture problems in the context of group instruction before she had formally studied variables in the context of the class. These experiences indicate the importance of the attainment of the formal stage for students in mathematics classes.

Null hypothesis 2 was rejected indicating that students at higher cognitive levels made higher pre-to-post gains in DTMS scores regardless of the level of remediation. The mean overall gain is 27.6% which is approximately five standard errors of measurement for the DTMS test. Early formal students make the largest gains while concrete operational students make the smallest gains. Thus, ability to profit from remedial/developmental instruction is related to cognitive developmental stage. The significant covariate effect of pre-DTMS score on post DTMS score may indicate that late formal subjects do not make the highest gains because of regression toward the mean. Regression toward the mean could also partially explain the higher gains of the concrete operational students.

Students who are identified as concrete operational at the beginning of an instructional period may profit from special support and perhaps, special experiences which might lead to movement towards the formal operational stage. Careful performance feedback which assists in accommodation plus self-pacing may be helpful. Extra experience in problem-solving, including possibly control of variable experimentation might be desirable. Mathematics instruction itself does have some positive effect on cognitive assessment scores as indicated by the gain of concrete operational students in pre-to-post BLOT INRC 4 Group subscale scores. Providing instruction in content such as ratio and proportion provides experiences which may lead to the formal stage of cognitive development.

Gender Differences

No evidence was found in this study to indicate that differences in cognitive stage as measured by BLOT scores or differences in quantitative skill as measured by DTMS score initially existed between male and female college students in introductory matheamtics classes below the calculus level at the University of the Pacific. No differences in variability between the two groups were found indicating rather uniform performance between sexes on each of these tests. No differences were found generally or when course background was controlled. The results of this study are consistent with the most recent National Assessment of Educational Progress (Armstrong, 1981) but not with the conclusions of other studies such as Benbow and Stanley (1980) which dealt with talented seventh and eighth grade students.

There was a difference by sex in the subjective Kurtz/Karplus cognitive assessment scores facoring males. This may reflect somewhat the scientific orientation of the test which may bias it for women who, in this study, took significantly fewer mathematics and science courses combined than did males. Alternately, the Kurtz/Karplus test may more accurately reflect differences in mathematics and science background than the BLOT test which is constructed to test Piagetian logical structure in the formal sense.

There was a difference by sex favoring males in gain in BLOT score over one semester in the remedial/developmental class. At the second administration of the BLOT, male mean scores were significantly higher than female mean scores.

No explanation for this result is readily apparent unless a sex differentiated effect of the remedial instruction is postulated. Differences in experiences which promote cognitive growth, separate from the remedial/developmental class and not controlled in this study, may also explain this difference.

There was a significant difference between males and females in mean number of mathematics courses taken in high school. This difference indicated that males, in this sample, took a higher number of mathematics courses in high school on the average. At the same time, there was no significant difference in DTMS placement score by sex. This would seem to suggest that although on the average women students have less exposure to high school mathematics classes, they retain a basic core of knowledge sufficiently well so that on basic skill placement tests these differences in background are not clearly evident.

There was no difference by sex in number of high school science courses taken. There were no contingencies between cognitive stage and science or mathematics course background either. However, if concrete students are separated as a group and compared to early and late formal students as a single group, the concrete students had taken significantly fewer mathematics courses. When number of high school mathematics and science courses are combined and analyzed by sex, there is a significant difference in mean number of courses with males having taken a larger mean number of courses.

Problem Solving and INRC Subscale Score

Consistent with Piagetian theory, concrete operational, early formal and late formal groups had significantly different pre-scores on the INRC 4 Group subscale of the BLOT. Pre-to-post gains were significant for the whole group of students, but there is little practical significance in the size of the gain. Students classified as concrete operational at the beginning of the remedial/developmental mathematics class made significantly higher gains by the end of the course. The mean soore of the concrete operational students was almost exactly the same at the end of the course as the mean that the early formal group had scored at the beginning of the course. This suggests that the PSI remedial/developmental class may have positively affected the INRC subscale score of the BLOT for concrete students so that their exiting score closely approximates that of the early formal student.

INRC 4 Group subscale scores also differed significantly by placement level with lower levels of placement having lower mean scores. This result seemed to substantiate the view that there is a definite relation between this subscale and quantitative skills.

The experimental problem-solving instruction was effective in significantly raising post DTMS scores for subjects when matched with controls who had not had such instruction. It should be recalled, however, that the sample tested consisted of volunteers and this may have affected the results of the experiment. No effect was noted on BLOT cognitive assessment scores. Several students who participated in the problem-solving group instruction commented favorably on it in open comment portions of the course evaluation. This instruction may be an effective way of improving mathematical skills for remedial/developmental students. Further experimentation with these instructional techniques and materials is necessary.

There was a significant gain in BLOT score over the course of one semester for students in the remedial/developmental class for whom pre- and post test scores were available. Concrete operational subjects made the highest gains and late formal subjects made the lowest gains as Piagetian theory would suggest. The number of students who moved to a higher stage according to BLOT scores was 28.1% or 43 students. These gains cannot be directly attributed to experiences in the remedial/developmental class as the many other variables which could affect cognitive development for college students in this study were not controlled.

Correlation of Assessment Instruments

There was significant correlation between the objective BLOT and the Kurtz/Karplus subjective cognitive assessment test. Exact agreement was 29.9% of the 137 assessments available for correlation. The BLOT objective test classified 56.2% of the subjects as fully formal which seems to agree well with previous recent studies (Kuhn, 1979). The subjective instrument classified only 14.6% of the students as fully formal. The subjective test classified almost four times as many students at the concrete level as the objective BLOT. Certainly the objective test does not examine reasons for correct answers as Piagetian theory suggests it should, but, on the other hand, the subjective test may be too difficult and contain too many items with a science orientation.

Only four of the students characterized as concrete by the BLOT test were classified at a higher stage by the Kurtz/Karplus test. On the other hand, 47 students classified as concrete by the Kurtz/Karplus test were at a higher level on the BLOT. The BLOT then seems to be more conservative in placing students at the concrete operational stage. The Kurtz/Karplus classified 20 students at the late formal stage. Six of these subjects were classified as early formal by the BLOT and none as concrete. Thus, the Kurtz/Karplus seems to be more conservative in identifying students at the upper stage of late formal. Most non-agreement of classification between the two tests occurred at the early formal stage which, if it is a transitional stage, would be expected.

The BLOT test which was given at the beginning and end of the Fall 1982 semester to students in the remedial/developmental class showed fairly stable classification results. A total of 64.7% of the students remained at the same stage over this time period.

Recommendations for Further Study

It seems clear from this study that there is a relationship between quantitative skill level and cognitive abilities. The nature of the relationship is, however, not clear. Do students do poorly in mathematics and thus avoid the subject area as a result of not achieving the formal operational stage or do students not attain the formal operational stage because of their inability to deal with mathematical concepts? The cause/effect relationship also may be affected by other intermediate variables such as IQ or socialization. Further study, perhaps at the secondary level, should be undertaken to investigate this relationship. A sample of students at the calculus level should be tested and data compared with results of this study to ascertain whether the relationship between cognitive stage and quantitative skill continues into a higher college level mathematics entry course.

Gender differences still remain a questionable area. The finding that women, as a group, have less course background in mathematics than males and yet do not differ significantly on standardized placement tests is surprising. Repetition of this study with a larger sample of students at another university is suggested to determine if this result can be replicated. The finding that males made significantly higher gains in BLOT scores over a one semester period in the remedial/developmental class needs to be replicated and if such results consistently occur, investigated.

The self-paced PSI remedial/developmental mathematics class did increase the INRC 4 Group subscale score of students initially classified as concrete operational. A study analyzing other subscales of the BLOT test is suggested along with some attempt to find the topics of instruction which have most effect on this subscale.

The experimental problem-solving instruction was effective in improving DTMS test score in a matched pair design involving volunteers. A study in which random samples are used should be done. The technique should be tested by other instructors to insure that the methods can be generalized to other teaching situations.

More studies involving use of the BLOT Piagetian cognitive assessment test should be done. It appears to be an effective, practicable test for assessing cognitive developmental stage at the concrete, early and late formal levels. The BLOT was correlated with placement level and skill gain in the remedial/developmental mathematics course. It is also constructed according to Piagetian logic theory and does not have a content bias toward science. Students identified as concrete by this test would probably be placed at that stage by other Piagetian tests such as the Kurtz/Karplus test. It is felt by the author that if the purpose of cognitive assessment is to identify students at the concrete level, the BLOT is a more dependable test in the sense that students identified by this test as concrete are more certainly at that level. Also, more studies need to be done in which several different group assessment instruments are administered to the same set of subjects so that other studies of agreement of classification can be done.

This study was carried out in a self-paced PSI setting. An interesting experiment related to the effectiveness of the PSI technology would be to repeat this study in both lecture and PSI settings and compare pre-to-post gains in both types of courses.

Summary

This chapter summarized the results of the analyses conducted in this study. The major focus of the study was to investigate relationships between cognitive developmental stage and quantitative skill levels as reflected by placement in introductory mathematics courses. Significant relationships were found between BLOT determined stages and placement levels for the students enrolled at the University of the Pacific in the Fall of 1982. Students placed at lower levels in the mathematics sequence tended to have lower Piagetian assessment scores.

Students placed in remedial/developmental classes were found to have made higher gains over the course of one semester if they were at the formal rather than the concrete stage. A strong covariate effect of initial placement score was found for both types of cognitive assessment tests. After correcting for this effect, significant differences in gains by cognitive level were found for both the BLOT test and the Kurtz/Karplus test.

No gender differences in placement test scores or BLOT scores were found. Differences in mean Kurtz/Karplus cognitive assessment scores existed favoring males. Males also reported a significantly higher mean number of mathematics courses taken in high school. No differences in mean number of science courses were found but when number of science courses and mathematics courses were combined, males had a significantly higher mean sum.

The expected positive relationship between number of science and mathematics courses and mathematics placement level was found. Participation in science and mathematics courses in high school was also positively correlated. Only the BLOT cognitive stages showed any significant relationship to science and mathematics background. Subjects at the formal stage of cognitive development, according to BLOT scores, had higher mean number of science and mathematics courses than subjects at the concrete stage.

INRC 4 Group subscale scores of the BLOT showed a positive relation with placement level. Concrete students showed a large enough gain over one semester of remedial/developmental mathematics instruction to move their mean scores into the early formal range. The experimental problem-solving instruction was successful in significantly raising DTMS scores when a matched pair analysis was done. A total of 28.1% of the students tested in the remedial/developmental class showed movement to a higher cognitive stage over the course of a one semester period.

The two types of cognitive assessment tests showed 29.9% exact agreement on cognitive stage classification. Scores were significantly, although moderately, correlated. The Kurtz/Karplus test places more students at the concrete and early formal stage than the BLOT test. It is the author's opinion that students identified as concrete operational by the BLOT are more reliably at this level than students identified as concrete operational by the Kurtz/Karplus test.

Recommendations for further study include: replication of the problem-solving instruction experiment involving other instructors, exploration of the reasons for and order of the relationship between cognitive stage and quantitative skills, further examination of gender differences in quantitative skills and cognitive assessment scores, comparison of PSI remedial/developmental classes and regular lecture classes with respect to the variables involved in this study and examination of which mathematical topics affect cognitive assessment scores. Further studies involving Bond's Logical Operation Test are suggested as well as more studies comparing group cognitive assessment tests.

Recommendation for implementation in remedial/developmental classes in mathematics include consideration of cognitive stage when planning instruction. Students assigned to pre-algebra or elementary algebra remediation levels should not be assumed to be formal operational. Special problem-solving instruction (cognitive process instruction) should be part of all remedial/developmental mathematics courses.

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APPENDIX A

The following data represent placement testing results from 1979 to September, 1982, at the University of the Pacific. The placement test used was The Descriptive Test of Mathematical Skills (DTMS) of the College Entrance Examination Board.

Year	Total Tested	Number Requiring Remedial Work in Pre-Alg.	Number Requiring Remedial Work in El. Alg.	Number Requiring Remedial Work in Int. Alg.	Number Placed in Elem. Functions
1979	1,028	10	52	231	62
1980	871	8	53	258	70
1981	777	4	65	200	59
1982*	868	+	117	121	96

*Data for Summer, Fall freshman orientation plus transfer orientation only.

+Pre-algebra was eliminated as a level for placement testing on an experimental basis as part of a pilot basic skills study during the 1982 orientation. Nine students were remediated at this level during the 1982-83 school year.

APPENDIX B

The grading protocols below were used in grading the Kurtz/Karplus subjective test of Piagetian stage.

1. Proportional Reasoning: The correct answer is 20. Students may write an equation such as 6/8 = 15/x or make some multiplicative statement such as "the daughter is 2 1/2 times larger so the mother will be 2 1/2 times larger". The answer 17 is an additive answer.

2. Permutations: Students should show the 24 unique permutations of four items. A pattern in writing the permutations down should be apparent but is not necessary for a correct solution. Students missing one or two of the 24 permutations but exhibiting a pattern should be given credit for the problem.

3. Proportional Reasoning: The correct answer is 72 seconds or 1 minute and 12 seconds. The reason given should imply proportional thinking such as "the horse's stride is 2.4 times as long as the dog's stride so the dog will take 2.4 times as long to cover the course".

4. Propositional Logic: The E and 7 card need to be turned over. The E card needs to be turned over to see whether an even number is on the other side. The 7 card needs to be turned over since if it has a vowel on the other side, the rule is false. The 4 does not have to be turned over, it

could have a vowel or consonant and the rule would still be true. The k need not be turned over since the rule does not involve consonants. Only the correct two must be indicated in order to receive credit for this problem.

5. Probability: The correct answer is the container on the right. The reason given should include a statement of the proportion of blue balls on the right (1/3) being larger than the proportion of blue balls on the left (3/10).

6. Correlation: The correct answer to Part a) is yes; to part b) a strong relationship. The explanation should include a statement that most birds with short tails have long beaks (6/8) and most birds with long tails have short beaks (6/8).

7. Combinations: The student should list each color separately, then in unique pairs and triples and finally all four colors together. There are 16 combinations in all including the combination none. Give credit for 15 if they forgot the combination none.

8. Propositional Reasoning: Part a) is a statement of the implication and should be marked. Part d) is the negative of the implication and must be valid if the implication is valid. No other responses should be marked.

9. Separation of Variables: The correct answer is yes. The reason must mention the fact that it is the dark plant

food that causes a plant to do poorly.

10. Deductive Logic: All three parts of this problem must be correct to get credit for the problem. A. Not enough information. B. Yes, by flying from Fish to Bean to Bird Island. C. No, because if there was, you could fly from Bean to Bird which contradicts clue 1.

APPENDIX C

The problems below were used in the first session of the experimental problem-solving instruction. These problems are taken from Whimby and Lochhead's book, Analytic Problem Solving.

1. If the circle below is taller than the square and the cross is shorter than the square, put a K in the circle. However, if this is not the case, put a T in the second tallest figure.

2. If the word 'sentence' contains less than nine letters and more than three vowels, circle the first vowel. Otherwise, circle the consonant which is farthest to the right in the word.

3. Tom is heavier than Fred but lighter than Marty. If from this information, you can determine which of the three men is the heaviest, circle his name. Otherwise, write indeterminable in this space.

4. If Bob and Fred are both taller than Tom, while Hal is shorter than Bob but taller than Fred, which man is the shortest and which one is next to the shortest, or can this not be determined from the information given?

5. Cathy knows French and German, Sandra knows Swedish and Russian, Cindy knows Spanish and French, Paula knows German

and Swedish. If French is easier than German, Russian is harder than Swedish, German is easier than Swedish, and Spanish is easier than French, which girl knows the most difficult languages?

6. Paul, Sam and Tom differ in height. Their last names are Smith, Jones and Calvin, but not necessarily in that order. Paul is taller than Tom but shorter than Sam. Smith is the tallest of the three and Calvin is the shortest. What are Paul's and Tom's last names?

7. Three fathers--Pete, John and Nick--have between them a total of 15 children of which nine are boys. Pete has three girls and John has the same number of boys. John has one more child than Pete, who has four children. Nick has four more boys than girls and the same number of girls as Pete has boys. How many boys each do Nick and Pete have?

8. Lester has twelve times as many marbles as Kathy. John has half as many as Judy. Judy has half as many as Lester. Kathy has six marbles. How many marbles each do Lester and John have?