Interactive pace approach to learning in physics: method and materials

Mario Amézquita
University of the Pacific

Follow this and additional works at: https://scholarlycommons.pacific.edu/uop_etds

Recommended Citation
https://scholarlycommons.pacific.edu/uop_etds/1921

This Thesis is brought to you for free and open access by the Graduate School at Scholarly Commons. It has been accepted for inclusion in University of the Pacific Theses and Dissertations by an authorized administrator of Scholarly Commons. For more information, please contact mgibney@pacific.edu.
INTERACTIVE PACE APPROACH TO LEARNING IN PHYSICS:

METHOD AND MATERIALS

A Thesis

Presented to

The Graduate Faculty of the

University of the Pacific

In Partial Fulfillment

of the Requirements for the Degree of

Master of Science

by

Mario Amezquita

May 1976
Copyright

by

Mario Amézquita

Andres F. Rodriguez

1976
This thesis, written and submitted by

MARIO AMEZQUITA

is approved for recommendation to the Committee
on Graduate Studies, University of the Pacific.

Department Chairman or Dean:

[Signature]

Thesis Committee:

[Signature]  Chairman
[Signature]

[Signature]  [Name]

Dated  MAY 11, 1976
ACKNOWLEDGEMENTS

I would like to express my appreciation to all the faculty members of the Physics Department, University of the Pacific, for their assistance and guidance during the time of this research.

To Professor Andres F. Rodríguez, Chairman, particular recognition is due for this opportunity to study and investigate in the field of my choice at my own pace. Words cannot express my gratefulness for his continuous advice, assistance and support in the preparation of this paper.

As sincere as the accolades above are, they pale in comparison to the gratitude I feel for the many contributions and sacrifices made by my wife. Ligia's confidence in me and encouragement at the appropriate times were the paramount ingredients necessary in my completing this seemingly interminable quest. Sharing this experience with her has enriched the meaning of this achievement for me.

M.A.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Chapter 1: GENERAL CONSIDERATIONS</td>
<td></td>
</tr>
<tr>
<td>1.1 History of PSI</td>
<td>5</td>
</tr>
<tr>
<td>1.2 PSI Characteristics</td>
<td>9</td>
</tr>
<tr>
<td>1.3 Implementation</td>
<td>11</td>
</tr>
<tr>
<td>1.4 Reward-Motivated Learning</td>
<td>12</td>
</tr>
<tr>
<td>1.5 Final Annotations</td>
<td>14</td>
</tr>
<tr>
<td>1.6 Study Unit: PSI Philosophy</td>
<td>19</td>
</tr>
<tr>
<td>Chapter 2: STRUCTURING THE COURSE</td>
<td></td>
</tr>
<tr>
<td>2.1 System for Learning and PSI</td>
<td>22</td>
</tr>
<tr>
<td>2.2 Structure of Design</td>
<td>24</td>
</tr>
<tr>
<td>2.3 Purpose of the Course</td>
<td>27</td>
</tr>
<tr>
<td>2.4 Content of the Course</td>
<td>28</td>
</tr>
<tr>
<td>2.4.1 Structures in Physics</td>
<td>29</td>
</tr>
<tr>
<td>2.4.2 Spiral Approach</td>
<td>32</td>
</tr>
<tr>
<td>2.5 Activities and Resources</td>
<td>34</td>
</tr>
<tr>
<td>2.5.1 Discussion Group</td>
<td>35</td>
</tr>
<tr>
<td>2.5.2 Audiovisuals</td>
<td>36</td>
</tr>
<tr>
<td>2.6 Study Unit: Planning the Content of a PSI Course</td>
<td>41</td>
</tr>
<tr>
<td>Chapter 3: FORMULATING OBJECTIVES</td>
<td></td>
</tr>
<tr>
<td>3.1 Generalities</td>
<td>45</td>
</tr>
<tr>
<td>3.2 Characterizing Behavior Objectives</td>
<td>47</td>
</tr>
<tr>
<td>3.3 Structuring Objectives</td>
<td>51</td>
</tr>
<tr>
<td>3.4 Using Verbs</td>
<td>52</td>
</tr>
<tr>
<td>3.5 Classifying Objectives</td>
<td>57</td>
</tr>
<tr>
<td>3.6 Study Unit: Writing Instructional Objectives</td>
<td>60</td>
</tr>
<tr>
<td>Chapter 4: DESIGNING UNITS</td>
<td></td>
</tr>
<tr>
<td>4.1 Composing a Self-Contained Unit</td>
<td>65</td>
</tr>
<tr>
<td>4.2 Establishing Functions of Components</td>
<td>67</td>
</tr>
<tr>
<td>4.3 Structuring the Units</td>
<td>72</td>
</tr>
<tr>
<td>4.4 Study Unit: Designing a Study Unit</td>
<td>74</td>
</tr>
<tr>
<td>Chapter 5: ADDITIONAL MATERIALS</td>
<td></td>
</tr>
<tr>
<td>5.1 Autolectures</td>
<td>78</td>
</tr>
<tr>
<td>5.2 Film-Loops</td>
<td>80</td>
</tr>
<tr>
<td>5.3 Test Format</td>
<td>81</td>
</tr>
<tr>
<td>5.4 Grading Key</td>
<td>82</td>
</tr>
<tr>
<td>SUMMARY AND CONCLUSIONS</td>
<td>84</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>--------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>86</td>
</tr>
<tr>
<td>APPENDIXES</td>
<td></td>
</tr>
<tr>
<td>A. TAXONOMIES AND VERB LIST</td>
<td>89</td>
</tr>
<tr>
<td>B. CONTENTS OF PHYSICS 53 PSI COURSE</td>
<td>92</td>
</tr>
<tr>
<td>C. UNITS FOR MECHANICS OF POINT-LIKE BODIES</td>
<td>95</td>
</tr>
<tr>
<td>D. EXAMS AND GRADING KEYS</td>
<td>270</td>
</tr>
<tr>
<td>E. AUTOLECTURES</td>
<td>290</td>
</tr>
<tr>
<td>F. LIST OF FILM-LOOPS</td>
<td>339</td>
</tr>
</tbody>
</table>
INTRODUCTION

My dissatisfaction with the traditional courses in physics arose while I was teaching undergraduate introductory physics courses at the Universidad Javeriana, Bogotá, Colombia. At that time, I was looking for new trends to improve the physics teaching at the Universidad Javeriana. We, the teachers at the Javeriana, set up educational experiments to compare what we assumed to be different methodologies. One of our conclusions was that our supposed different methodologies were variations of the traditional format, but nothing else. In this search, we had the opportunity to participate in a Seminar on the Methodology of Teaching Physics at University Level directed by Professor Andres F. Rodriguez (then holding a Fulbright-Hayes Professorship) during the summer of 1973. Among the new trends of physics teaching examined, the PSI was the one that we felt was adequate to adapt to our educational environment in Colombia. During two additional meetings in 1974 we got more theoretical information about PSI.

Since then I have been deeply interested in the PSI system of instruction and its applications. One of the biggest problems reported by the users of PSI, but one which has not received the attention it deserves, is the written material required for a course. It seems that every time a professor wants to use PSI, he finds...
himself without the material for his course because the standard textbooks cannot be adapted to PSI format, and materials of this type are not available commercially. Then he has to spend several hours writing study guides without any rationale, yielding to a production of poor material in what J. G. Sherman calls "corruption of PSI". This situation led us to consider a study of production of PSI materials under a rational basis.

In private discussions with Professor Rodriguez I learned of his discontent with the individualism that this system can produce. It then appeared natural to try on an experimental basis a methodology that maintains the positive aspects of PSI but also uses a classroom strategy to emphasize the encouragement of student autonomy and involvement in active student group participation.

As the time that I started this work, the analysis of the purpose of the course led us to consider a slight deviation from the standard presentation of the canonical topics. We agreed with some learning theorists who argue for the critical importance that structure plays in instruction.

For the past one and one-half years, I attempted, along with Professor Rodriguez, to develop a methodology and written materials that are effective in teaching introductory physics. This thesis is a primary result of that effort.

The development of a "prototype" course took place within the already existing courses, Physics 53 and 55—courses offered at University of the Pacific, Stockton, CA. During the spring and fall terms of 1975 in Physics 55 and the spring term of 1976 in Ph53, the elements of a new methodology had been tested. Feedback obtained from
students during the academic year 1975-76 has been used to revise the study units expressly written for the course Physics 53. The third revision of Part I of the Mechanics Study Units is included as Appendix C of this work, and frequent explicit references are made to it in the chapters that follow.

**Goals and Outline**

This study has the following general objectives: first, to present in detail the techniques that the author has developed and used in designing the course and the final materials produced for an introductory physics course in Mechanics; second, to state and examine the important blended ingredients used to get a new strategy for teaching-learning in which an individualized pace together with a strong group interaction is used; third, to analyze a combination of structural and operational course where concept, structure formation, and problem solving are emphasized.

The first chapter of this study presents in rather broad outline the development and philosophy of the Personalized System of Instruction, which was the starting point. In the second chapter the ingredients for the new strategy are presented and discussed together with the importance of structures in Physics. In Chapter three the elements and procedures for formulating instructional objectives are presented, followed in Chapter four by a discussion of the design of a study unit. Chapter five presents our comments and limitations of the additional material developed to complement the course. Some implications are drawn in the conclusions' heading. There are six Appendices that form the material of this work, and without them the study could appear superfluous.
In order to develop the present study, I present, when it is necessary, some concurrent ideas related to teaching and learning—especially those that appear to apply to science courses at college level. The ideas interpreted and incorporated in the new strategy are those that are congenial with my own thinking and experience. This is not to say that the job is finished and the problems solved. This is a first step in searching for a great solution.
Chapter 1

GENERAL CONSIDERATIONS ON PSI

"The Personalized System of Instruction (PSI) is a well-defined system, based on the principles of the psychology of learning, for administering personalized instruction."

--PSI Newsletter [1]

1.1 History of PSI

The conception of this teaching methodology (PSI) appears to have occurred on the evening of March 29, 1963, when a group of four psychologists with a common belief that "better education can come from experimentation" got together to plan and discuss an introductory course.

Fred S. Keller (then at Columbia University), Carolina Martuscelli Bori, Rodolfo Azzi (then at the University of Sao Paulo, Brasil), and J. Gilmor Sherman (then Fulbright-Hayes Professor at the University of Sao Paulo) were collecting ideas, information, books, equipment, etc., to accomplish the task, commended to them, of founding a Department of Psychology at the University of Brasilia. This group believed in the reinforcement theory and the use of rewards to improve learning.

The first outline of the course was made by Fred S. Keller\(^\text{1}\) as

an imaginary course in a paper presented at the American Psychological Association, Philadelphia, August, 1963. It seems that the basic format described in that paper has been used almost without change in many courses in psychology [2]. The real application of this methodology was made by Keller in the winter term of 1963 at Columbia University in a course offered to three students. However, the plan went into effect at Brasilia in 1964 in the course called Introduction to the Analysis of Behavior [3]. The original aim was that one course lead to another according to each student’s progress, readiness, and desire, without time or administrative limitations, from the beginning to the end of his studies in psychology. In 1965 Keller and Sherman used two variations of the Brasilia plan at Arizona State University. The modifications involved: a) the use of student aides called proctors; and b) the use of final exam and letter grades as in a traditional lecture course. During the period 1965-67, the Basic Plan began to excite interest due to its presentation and discussion at conferences such as those of the American Psychological Association, the American Education Research Association, etc. Yet, the most significant fact was the publication by Fred S. Keller of an article titled "Good-bye, Teacher..." in 1968. This paper was read by many teachers, not all of them psychologists. Since then, many papers have been reported containing applications of PSI to courses in psychology as well as to other disciplines.

Probably the first PSI course in subject matter different from psychology was an introductory physics course given by Dr. Ben A. Green Jr. at MIT in the spring of 1969. With this paper Green gave hope to many physicists of changing their teaching strategy, making it more adaptable to their own needs and those of their students. Almost at the same time, Billy V. Koen, a nuclear engineer at the University of Texas at Austin, started PSI with a senior level nuclear engineering course.

Following these initial tryouts of the plan, its utilization has shown increasing diversity. Disciplines ranging from psychology, physics, engineering, mathematics, statistics, chemistry, biology, to astronomy, Spanish, library science, philosophy and photography have used PSI. Many of these extensions have occurred in the past three years, and the reports are preliminary and informal.

PSI is not limited to the United States; PSI has had an independent and parallel development in Latin American. Under the leadership of Professor Carolina Martuscelli Bori, PSI is used widely and is better established in the educational system of Brasil. Colombia, Venezuela and Chile have taken their first steps in that direction;

---


6 Andres F. Rodríguez & Mario Amézquita, Seminario sobre la Metodología de la Enseñanza de la Física a Nivel Universitario. Tomos I & III, (Bogotá, Colombia: Universidad Javeriana, 1974).
and the Monterrey Institute of Technology in Mexico has undertaken the largest trial of this kind in the world. Australia, Canada, France, Israel, Philippines and the United Kingdom have reported use of PSI.

There has been large-scale research such as the recently terminated PSI Project at the University of Texas at Austin; the CBI (self-paced) courses at Michigan State University; the plan Teaching For Excellence at the Monterrey Institute of Technology. The center for Personalized Instruction at Georgetown University acts as an information clearinghouse. There are many more reasons which seem to point out that one more attempt to improve the plan is not a vain effort. My modest contribution in that direction will be to try to systematize the design of written material for an introductory physics course. Finally, it is worthwhile to mention the earliest use of characteristics of the plan in classroom situations. Luella Colle\(^7\) reports the use of students in the teaching of other students in the Jesuit system in the sixteenth century. These student-helpers, called decurions, had the same role as that of the present proctors. Later on in the nineteenth century, young students were used in the "monitorial system" as well as in the "pupil-teacher system"\(^8\) to reduce the teacher's load and to increase the number of students taught. The go-at-your-own-pace has been a feature of the military training centers. The absence of lectures as well


\(8\) Ibid., pp. 607-609.
as the use of prepared written material are obvious features of correspondence school instruction. The prompt correction of answers to tests and the progression to a next higher step if the one taken before was mastered are features of programmed instruction. And even in its totality, it seems to have been anticipated by Mary Ward in 1912 at the San Francisco State Normal School as described by Washburn and Marland in their book.9

1.2 PSI Characteristics

The PSI10 system has five defining characteristics: (1) mastery learning, (2) self-pacing, (3) written-material, (4) proctors, and (5) noninformative lectures.

The mastery requirement means to learn well everything that is assigned, without implications if the assigned material is elementary or advanced, simple or complex, new or review. Mastery, excellence, and perfection are terms used synonymously to point out that subject-matter has been explored with care, whatever the level, and that the learner is able to perform in such a way that he manifests skillful work in the subject matter. Perfection is detected from performance. For that reason the student must be called upon to respond frequently. His errors should lead to a program of remediation, instead of punishment, and to a new trial. His success must be rewarded; the student

---


10 PSI stands for Personalized System of Instruction, although sometimes it means proctorial or programmed system of instruction, self-paced instruction, or Keller Plan. I will use it exactly for Personalized System of Instruction.
goes ahead with new material.

The go-at-your-own-pace feature is mandatory, given the mastery requirement, because no one can deny the individual differences in human beings. It must be allowed that each learner moves through the course at a speed commensurate with his ability, without a time limitation.

The last three features follow directly from the first two. If students go at different speeds, then the lock-step lectures are incompatible with self-pacing. The written material (textbooks, study units, or audiovisual media) is the remaining informative medium from teacher to students. This heavy reliance on the written word requires the material be presented to students more clearly than ever before, in a student-oriented style of writing, with objectives clearly specified and sequenced in small steps in a gradually increasing level of sophistication. When the student has demonstrated that he is ready to deal with new concepts, then he receives the new information. All this implies that the teacher must reexamine the course content and choose, not without some pain, only what is worth teaching, because now he knows that what is taught will be learned.

The proctor or tutor comes to relieve the teacher of his new burdens in repetitive testing of different units, immediate scoring, and unavoidable and limitless individual attention to students.

Finally as mentioned above, the lectures as informative devices are no longer required. The teacher can throw in a few lectures, but they must be as vehicles of motivation and inspiration.

In summary, all these characteristics can be derived from the concept of "Excellence Learning." If the teacher wants every
student to learn the material assigned (mastery), it is necessary to
give the slow student more time (self-pacing). If students go at dif-
ferent speeds, then the lectures are not necessary as sources of
critical information (no lectures) but rather as motivation vehicles.
The communication channels left are the textbook and study guides
(written-materials). Finally the teacher needs assistants (proctors)
to survive the excess of written work.

1.3 Implementation

The course material is divided into units. (Keller suggests
20 to 30, but this number depends on many factors). Three or four
units should be of the review type to avoid undue fragmentation and
to consolidate a core of knowledge.

Each unit may contain a reading assignment, objectives, study
questions, references, study problems, and any necessary introductory
or explanatory material.

For each unit one should prepare three or four equivalents
"readiness tests" to cover the same material of the unit (the solu-
tion time of each Keller's test was 10 minutes).

The student goes through the unit at his own pace at the time and
place that he prefers. When he feels that he has mastered the unit,
he asks a proctor for a "readiness test" for evaluation of the unit.
The student must make a grade of 100, but if he misses only a few
questions, the proctor can try to see if they were misunderstood and
can reword the question and even check if the right answers were
not a lucky guess. The proctor grades the test with pass or fail.
If the student passes he receives a new study unit and proceeds
to repeat the cycle. If the student does not successfully complete
the test, he receives advice about his mistakes, studies the unit
once more, or receives an additional set of study questions. He
can take the test again (new format) or many times without affecting
his grade. The only interest is that he ultimately demonstrates his
proficiency.

Lectures are infrequently scheduled for students who have com-
pleted a certain number of units and can understand the material.
Attendance is not compulsory, and the lecture-subject is not covered
on any examinations.

One of the components of the plan that has become increasingly
important is the proctor. He is an undergraduate student chosen
carefully for his mastery of the course material, interest, personal
qualities, and willingness to work. His success in proctoring de-
pends greatly on good preparation and guidance given by the instruc-
tor. Typically, there is one proctor to every ten students. The
assignment of students could be on a fixed basis or by free-choice
election.

A final examination can be added, if it is desired. It may
make a better seasoning for the course and may strengthen the final
product, integrating all the bits and pieces of information.

And, finally, using Keller's words, "watch carefully while
cooking."

1.4 Reward Motivated Learning

PSI is based on Skinner's\(^1\) theory of learning from the

\(^{11}\)B.F. Skinner, *The Technology of Teaching* (New York: Appleton-
perspective of "reinforcement theory" [5]. The elementary principle of behavior says that "an organism tends to repeat and enjoy behavior for which it is rewarded (i.e., positive reinforcement); it tends to avoid and dislike behavior for which it is punished (i.e., negative reinforcement)". In other words, the task of changing behavior, including learning behavior, is one of making appropriate reinforcements dependent upon specific samples of human conduct in clearly denoted environmental circumstances. From this it follows that a "learning situation" should have clear tasks identified, critical behaviors presented, and rewards assured.

The positive reinforcement for a student working under PSI comes in different ways. Keller [3] claims: passing at first attempt an easy first unit gives a reinforcement of satisfaction; small work units lead to greater density of reinforcement than do large ones; self-pacing allows the student to use time for study whenever he desires; getting ahead, unit by unit, permits him to gain a token reward with a partial A when a unit is passed; immediate grading of unit-tests strengthens a student's grasp of a concept; he has no penalty for failing a test one or more times; he has a chance to get an A up until the last day of class; study questions and sample problems focus student's attention on relevant material, reducing the possibilities for students to fail and increasing the frequency of success; the student experiences the satisfaction of getting a seat in a lecture. There are other more subtle reinforcers that come with study of a text--those little explosions of satisfaction when the answer to a question is found, when a new relationship is uncovered,
or when a new application of a principle suggests itself to the reader. 

The proctor is the greatest agent of reward. He is the source of several generalized reinforcers: attention, approval, and even affection, as well as the token reward of a grade. But the proctor also receives rewards from the system. Those appear as generalized reinforcers: attention, submission, acceptance from his students. He draws special satisfaction from the improvement of his students. He also improves in his readiness to deal effectively with concepts and to show his mastery of related skills: i.e., the increase in his knowledge of the course's subject-matter.

The instructor's reinforcements derive from several sources. Some of them come from a better appreciation of his course's content and a more intimate knowledge of his pupil's problems with it. Others come from the enthusiasm of his proctors, their growth in understanding, and their comprehension of the teacher's task. He also has rewards that come from his successful operation of the system as a whole.

Each person in the system gets his rewards from the behavior of others. The teacher gives work to the student; the student provides work for the proctor; the proctor and the student give the assistant his data; the assistant provides the instructor with proctor and student feedback; and the instructor starts the cycle all over again. Each one gets maximal satisfaction when the other one's work is well done. It is a system of mutual reinforcement.

1.5 Final annotations

About twelve years have passed since the first application of PSI. The system has spread everywhere. It has been used and is
being employed in many higher level institutions and is expanding to high schools. A review of evaluative research on PSI in science education had been made by Kulik\textsuperscript{12} and the following points are established:

(a) Self-pacing and student-proctor interaction seem to be the features most favored by the students.

(b) The system is attractive to most students. They report high satisfaction and enjoyment, as well as a preference for the unit format over typical course formats.

(c) Higher than average drop out rates and procrastination are two main objections to the system. It is possible that an adequate course design may control these drawbacks.

(d) Content learning (as measured by final examination) is adequate and in some cases reported students demonstrate somewhat better performance than they do in traditional lecture courses.

(e) More learning has been qualitatively reported by students who point out that more study-time and effort were necessary.

(f) The proctor, being an undergraduate student, benefits especially from the system.

(g) There is a possible cost-savings to institutions. The use of undergraduate assistants is one basis for the economy.

(h) There is a U-shape grade distribution as compared to bellshape distribution in traditional courses. Since the grades are

assigned in a manner having less parallel than in traditional courses, that distribution does not necessarily indicate that students learn more.

On the other hand, there has never been total success. Several failures and even disasters have occurred. There are, at least, three groups of problems: (1) inherent in the system, (2) created by modifications of the basic pattern, and (3) created by institutions' regulations.

(1) In this category the failures may appear because:

(i) the professor and the student need more time to work harder than ever

(ii) there is a substantial logistic and administration load created by the system and often by careless handling by the professor

(iii) the system turns out to be expensive for using graduate proctors or salaried undergraduate students

(iv) procrastination and withdrawals cause some students to dislike the system

(v) the materials needed in the course require production at time almost prohibitive.

(2) The system has been adopted with a surprisingly increasing speed for all kinds of people. But, do these people know enough about the psychology principles on which PSI is based to run correctly the system? Are they making a superficial copy and disdaining its real bases? The answers to these questions are becoming apparent for
the increasing number of those who are using SLI ("Something like it") instead of PSI. Other modifications include

(i) changing the requirement from mastery to another
criterion
(ii) adding contingencies
(iii) walking away from the system by the professor who
believes that when the program is done he has freed
himself
(iv) producing poor material.

(3) To this category belong all struggle situations between
the PSI-professor and deans, registrars, colleagues who do not under-
stand or like PSI grade distribution, the lack of lectures, the
absence of time structure, or other aspects of the system.

From everything said so far it is evident that PSI materials
have not received the attention that they deserve. But, it is also
undeniable that they are the third most important feature of the sys-
tem. PSI materials have to be extensive and must be written with
greatest care for they serve as the communication vehicle from profes-
sor to learner; they are also put on test immediately. Commercial
materials of this type for physics courses are not available, and
information exchange is not operating, as far as I know. Some study
guides are sporadically available, but it seems they are not adaptable
for a general course. As was mentioned above, one of the purposes
of the present work is to develop the most perfect possible material
needed for an introductory physics course.

The written material must meet requirements of being self-
contained, well structured, and readable. Moreover, material should
demonstrate good exposition, no excessive rigor, good organization and encouragement of student motivation. The design and elaboration of materials trying to meet those conditions will be presented and commented upon starting with Chapter 3.

Finally, we believe that learning physics requires not only content, motivation, and reward (reinforcement), but also the system of learning should involve the student in an active participation by requiring him to make decisions, to guess (sometimes), to conjecture over challenging problems by using intuition and imagination. This is a goal difficult to attain but not impossible. Our modest effort in that direction is presented in Chapter 2.
1.6 Study Unit: PSI PHILOSOPHY

Introduction

In this unit you will read about a new teaching strategy with a general purpose that the learner learn how to learn. The PSI (Personalized System of Instruction) is based on a learning theory in which every element is carefully chosen and interrelated to maximize student learning.

Objectives

To be able to:

1. Describe the Keller's five features of PSI method.
2. Explain how the reinforcement theory works in PSI.
3. List a minimum of two reasons why PSI is successful.
4. Identify additional characteristics of PSI from a given list of descriptive items.

Resource Materials


Suggested Procedure

Reading Keller's article is perhaps the most exciting way to become exposed to this system and to accomplish objectives one through four. It has a summary of the features which seem to distinguish the system.
Green's paper starts with Skinners' aspects of the PSI system and also in the context gives several reasons for the success of the PSI at Massachusetts Institute of Technology.
Study Unit: PSI Philosophy

(Note: In general, the number of each question is keyed to the corresponding objective's number.)

1. Express the essential five features of PSI.
2. Explain the role of reinforcement theory in the PSI System.
3. Give two reasons for the success of PSI courses.
4. Which of the following characteristics are in PSI System?
   (a) The teacher has all free-time when he has finished explaining the operation of the system to his students.
   (b) All students proceed through the course at the same rate.
   (c) Course material is divided in units.
   (d) Student helpers are used to individualize the instruction.
   (e) There is repeated testing without punishment.
   (f) Those who do not attend a conference miss test-material.
   (g) The classroom is used for study, group discussion, testing, proctoring.
   (h) Course objectives and procedures are explained in written study guides that are given to the student.
   (i) Grades are based on two tests and a final examination.
   (j) The instructor becomes a system manager.
Chapter 2

STRUCTURING THE COURSE

"To learn structure is to learn how things are related"

Jerome Bruner [6]

2.1 System for Learning and PSI

An educational system is developed and built around a nucleus, which is the purpose of the system. A modern point of view is to say that the purpose of education is to insure the attainment of specified knowledge, skills, and attitudes; thus, learning is the purpose around which the system is to grow. Instruction denotes functions or specific techniques introduced in the learning environment to promote more efficient and effective learning; processes are completed in order to facilitate mastery of specific learning tasks. The effectiveness of an instructional system, therefore, can be measured by assessing the degree to which it provides for the learner a system for learning.

In a learning-oriented system the typical classroom set-up does not work anymore, because all the persons in the class cannot learn in the same way. The rigid scheduling of class time and academic period terms would not exist either inasmuch as we know that people learn at different rates or speeds and therefore need different amounts
of time to master a particular learning task. The "passive" role of the student of receiving information from the teacher as the main source would also be ruled out. In the learning process the learner assumes an active role; he is always on stage, while the teacher would manage the learning. Furthermore, in the learning environment the audiovisuals and books are no longer aids to teaching or supplements to instruction. They might be for certain groups of learners, if they choose to use them in that way. But, on the other hand, if the capabilities of a certain medium or library resource indicate that it is the best component to facilitate learning, then they would be used or the learner could select them as the main component to bring about the desired learning.

If all these changes appear when we focus on the goal of education in learning, then what is the best process (operations and functions) in which the components would be engaged in order to accomplish the purpose of the system? There are several instructional methods which might work: Personalized System of Instruction, Computer Assisted Instruction, the Audio-Tutorial System, Individually Prescribed Instruction, and Programmed Instruction. These are some of the methods originated within the past two decades.

As we saw in Chapter 1, PSI is a method of instruction that combines the strengths of basic learning theory and individualized instruction: i.e., active responding, immediate feedback, small sequential steps, self-pacing, and mastery with a highly personalized relationship among students as well as between student and instructor. It has been noted that this instructional system, while
emphasizing five basic features, is by no means inflexible to the needs of different individual users in different disciplines at different institutions. For these reasons, we believe PSI is one of the best instructional techniques to achieve the goal of education—i.e., learning.

2.2 Structure of Design

The development of a system of learning is a decision-making operation. Decisions have to be made about what should be learned, how, by whom, when, and where; how learning should be evaluated and improved, and what resources should be involved in preparing for, providing for, and evaluating learning. The systems approach [7,8] to design and development offers a logical structure and the orderly use of strategies for making these curriculum decisions. In the decision-making structure, the objective of the system will determine whatever has to be designed and done to attain that objective. The design is then implemented and the output tested by criterion measures, developed on the basis of objective specifications. The test findings are interpreted in order to measure the extent to which objectives have been reached. If necessary, the system can be redesigned in order to insure the accomplishment of its objectives.

Sketching the Systems Approach

1. Analysis and Formulation of Objectives:
   a) System Purpose (what is the over-all purpose of the system?).
b) Specification of Objectives (what will the learner be able to do? how well? under what conditions?).

2. Construct Criterion Test. Develop a criterion test based on objectives and use it to test terminal proficiency.

3. Analysis and Formulation of Learning Task:
   a) Inventory of Learning Task (what has to be learned by the student so that he can behave in the way described by the objectives?).
   b) Input competence (will the learner bring to the learning situation some skills, information, attitudes, and so on, that are relevant to what he is supposed to learn?).

4. Design of the System.
   a) Function analysis (what has to be done and how)
      i) Selecting and organizing the content.
      ii) Selecting and organizing the learning experience.
      iii) Managing the learners.
      iv) Evaluating the learning and operating the system.
b) Components analysis (who or what has the potential to do it). It is selected on the bases of:
   i) potential to accomplish a particular function,
   ii) ability to integrate with other components,
   iii) relevancy to the learner,
   iv) practicality, and
   v) economy.

c) Distribution of functions among components (who or what will do exactly what).

d) Scheduling (when and where it will be done).

5. Implementation and Quality Control
   a) System training (preinstallation exercise)
   b) System testing (thinking-through process)
   c) System installation
   d) Evaluation (to determine if the learner behaves in the way initially described).

6. Change to improve (findings of the evaluation fed back into the system to determine changes if they are needed).

This structure was used to plan the introductory physics course for engineering and science students already mentioned. Also, it was systematically used to design each main component of the course. It is not worthwhile to discuss here the application of this structure to the over-all course. I shall make special comments about some elements incorporated into the design of our course and about what makes it different from standard PSI courses.
2.3 Purpose of the Course

In the learning system, a statement of purpose will establish the nucleus around which the system should grow. The statement tells us the reason for the system, its general environment, and the broad constraints under which the system is to operate. In this particular section the system is the Physics 53 PSI course taught at The University of the Pacific, Stockton, California. Let us state the purpose of the course as the students received it.

Instructional Goals for Physics 53 PSI. The purpose of this course is to provide you in the Sciences and Engineering with a solid background in Mechanics by giving you a unified view of physics so that you will be aware of how physics is related to many other sciences and to all fields of engineering. Mechanics is the cornerstone of the whole discipline of Physics, and this is why it is traditionally the first topic discussed in studying physics.

You will become thoroughly familiar with the fundamental principles of conservation, a basic handful of laws, and the fact that physical phenomena can be reduced to interaction between particles. It is expected that you get a clear understanding of these ideas. Secondly, you will develop the ability to manipulate those ideas and apply them to concrete situations.

As a result you should be able to demonstrate your competency by identifying and isolating a mechanical system from its environment, giving a physical interpretation of its behavior, and solving reasonable situations that involve the interaction of the moving body or the system with its environment by applying the conservation principles
and/or the laws of motion to find kinematics and/or the dynamics of unknown parameters. Although the course is calculus based, this is not a prerequisite; rather a concurrent course in the subject is required.

As we can see, this statement is general enough, but it is also explicit enough that the reasons and aims are stated. In addition, some limitation and even a criterion performance are given. However, specific objectives are not stated. These will be given for each study unit, and we will discuss their design in the next chapter.

2.4 Content of the Course

In agreement with our system model, this is the place to look for an answer to the question, what has to be done to enable the learner to master the task, or even more specifically, what is the learner supposed to learn? And in what sequence? As was already mentioned, the characterization of learning activities is a primary basis for selecting content. But other considerations are the availability of an item, its flexibility in saying different things, and learning ability which implies similarity, clarity, brevity and regularity.¹

After the content has been selected, it must be organized under the policies of sequencing, arrangement, and presentation. Our selection of content under these strategies came out of the traditional

content of Mechanics where the fundamental principles, needed to
describe and explain the motion we observe around us, are set up.
The type and amount of learning that this content requires, based
on the characterization of the learning activities, impose limitations
in that content itself. This restricted the content to Mechanics
of point-like bodies and of rigid bodies rotating around a fixed
axis.

The organization was guided by the aim of presenting physics
based on a small number of fundamental concepts rather than a col-
lection of areas of specific facts. In this sense we could say that
we tried to make a structural course. This topic will be commented
upon in the next subsection. It is worth mentioning now that our
structure was the conservation laws.

Finally, this content was integrated into specific learning
sequences of specific learning units. See a Table of Contents in
Appendix B. The design of these study units will be the topic of
Chapter 4.

2.4.1 Structures in Physics. In every area of knowledge studied by
man, certain patterns emerge that provide a mean for looking at the
subject in a coherent way. Those patterns and this way of viewing
is what Bruner [6] calls the structure of the field: "Grasping
the structure of a subject is understanding it in a way which per-
mits many other things to be related to it meaningfully." It is
worth pointing out that the opposite situation -- to "learn" a sub-
ject without understanding its structure -- is to learn an isolated
collection of facts. But this is not always obvious. It is true
that in a traditional course all the specific facts are covered, but only the most perceptive student is able to organize them to get a clear idea of the whole subject. The same happens in the standard textbooks when the material presented is too compartmentalized to reveal the essential unity of physics and its principles. The pattern exists there, but is so hidden that it is difficult to put it in evidence.

The standard student of general physics usually conjectures without understanding; it is formula guessing to apply to a problem or a calculation. It is not unusual to find the situation where students can evaluate the numerical value of a force using Newton's second law, but how many can describe it in their own words? Or they are able to describe separately linear motion and rotational motion, but how many can go by analogy from one kind of motion to another?

Therefore, for conjecturing, the student needs to know the fundamental principles of the subject: the structure. The role of this in Physics is to allow the student to see the subject organized in such a way that he can arrive at the intuition that provides an over all concept. If one knows the structure of the subject, one knows the fundamental relations that link the different parts. If one understands the structure, one can go on to understand other topics that have the same relation between ideas by analogy and induction. For instance, go from two to three dimensions in geometry, or go from kinetic translational energy to kinetic rotational energy. Knowing the structure supplies the induction. The majority of structures in all fields are founded in the invariance concept; this is, in fact, a humanistic concept.
In physics the invariance of energy, the invariance of momentum, the invariance of charge, the speed of light, and symmetries are the fundamental bases of any theory and therefore the structures of physics, because they give the relation among all the parts of the whole.

Another structure in theory is the condition of simplicity, where the simplest is preferred in theoretical formulations. This is also a humanistic concept. Science is always an act of creation by man, and in that way it must be taught and learned.

Some arguments in favor of learning structures are presented$^2$ [6].
1. Understanding fundamental ideas that link each part of the subject makes the subject more comprehensible, (i.e., general implies specific).
2. The learning by rote of structures is less easily forgotten than are specific facts (what is Lorentz' transformation for momentum and energy? what is the fundamental idea to derive it? which one do you answer?)
3. The understanding of fundamental principles promotes "Transfer of training," because from the general case one can go to the specific or to the analogous case.
4. Learning structures and principles places the learner nearer to "advanced" knowledge and to the frontiers of science.

---

2.4.2 Spiral Approach. The organization of the content of Physics 53 PSI is made using a spiral approach to present the relevance of the conservation laws, like the structure of the subject-matter. After learning the basic language on motion (Unit II), the learner is acquainted with the concepts of momentum and energy associated with motion (Unit III). They are presented in the simplest possible formulation, but they are placed in the proper level by emphasizing them as the most important physical variables used in quantitative description of physical phenomena. All at once they are employed to study one-dimensional collisions as well as the concept of potential in free fall motion.

When the study of motion is extended to two dimensions (Unit V), the same concepts of momentum, energy, and potential are used as fundamentals to link the parts, but here the treatment of these concepts is more profound and abstract than before. Their characteristics of conservation are enhanced with their applications to ballistic problems and collisions on a plane surface. The vectorial feature of momentum steps-up the abstraction of the concept a little more.

The presentations of Newton's Laws and the Concept of Force (Unit VI) are properly done using the concept of momentum vector. After the concept of force is established, its effects are immediately computed over space, yielding to its relation with kinetic energy.

Finally, momentum conservation and energy conservation are formulated more rigorously (Unit VIII and IX), and their efficacy to describe the interaction between any physical bodies is analyzed. Later, their use and importance are emphasized in the rigid bodies study.
This does not mean that the study of Laws of Motion has been carelessly taken. On the contrary, like momentum and energy, the use of Newton's Laws starts early (Unit III), but later (Unit VI) they are placed in their proper level and introduced in a spiral way; and their use and application are systematized (Unit VII) and their power for solving many kinds of problems illustrated.

The physical reasons for this approach can be summarized:

a) The structure of the subject (conservation laws) is always in evidence.

b) There is a wide variety of problems in which it is either not possible or not convenient to write an analytical expression for the force applied to the study-body (impulsive forces, scattering experiments, forces in atomic and subatomic levels, etc.).

c) In many-body problems the direct application of the laws of motion is not possible even if we are sure of the forces.

However, in all these situations, the conservation laws have universal application.

This approach of presenting each fundamental principle twice, once in a preliminary form in the early units and again in more detail in the last half of the sequence, is useful in the PSI format. In fact, in some institutions students do not have to finish all the units in lower division courses, but it is very important that students acquire a basic knowledge of the material to be used in upper division courses; and in this case the spiral approach is even more useful.
2.5 Activities and Resources

From the learning point of view, we should plan to have as many alternatives available as we need to cope with the great variations in which people learn; i.e., we should consider the specific characteristics of the learning group and the individual differences of the learners. Variation in the time needed to master a specific learning task is only one of the manifestations of individuals' differences. Some additional variations are interest, need, aptitude, achievement, warm-up period needed, ability to deal with abstractness or concreteness, interest span, sensory modes such as audio or visual perception, and many others involving imagination, creativity, motivation, and ability to solve problems.

To insure that the progress of any single student is not hindered, we considered study guides, textbook, readings, experiments in laboratory, weekly lecture, use of film-loops, autolectures recorded in audiotapes, group discussions, and group solving-problems. None of these activities are compulsory; they are alternatives available for the learner; he/she selects the one most appropriate for him/her to achieve his/her objectives. This means, we are no longer talking about the teacher and his teaching aids, but about the components of a system (the course) that can be considered and used by the learner on the basis of the ability to accomplish specific functions.

The human component includes the learner, the teacher, and the personnel engaged in support and service functions (tutors, proctors, teaching assistants, etc.). From component analysis, the teacher has new functions: his primary function includes providing for the motivation of the learner, for the planning and managing of
learning activities, and for examining and utilizing with the student the information the student has acquired. In other words, the teacher is best described as being the manager of learning. The learner becomes an active element of the system, a decision making subject, taking responsibility for his learning and forcing himself in a high degree of involvement. All these facts are not in complete agreement with PSI strategy.

It is worth mentioning that even though the course was individualized by PSI method, we do not overlook the characteristics of the group. After all man is a social human being striving for excellence within the group.

2.5.1 Discussion Group. This was based on and generally initiated from questions specifically related to the assigned reading corresponding to the average self-paced rate of students. Generally, the questions were related to assigned problems. Since all the homework problems require some level of understanding of the reading, the discussion quickly brought the current assignment into consideration. The answers to the questions were not given by the teacher; rather he restated or rephrased them and turned them back to the whole group so that they came out with a right answer.

It would be hard to overestimate the reticence and difficulties that many students experience in verbalizing their physical thinking—especially in front of an instructor. This seems to be the case, even though they may sincerely desire to participate. But, this reluctance disappears with time. They become willing to participate in class discussions and to contribute suggestions for improving the
quality of the discussions or the course in general, as time goes on. As the assistance to the discussion sessions in our study was rather small, the whole class approached a problem with the instructor, sometimes at the board, serving as scribe and moderator.

It is possible on the basis of these experiences, to state some general characteristics of this method.

1. It is a good alternative presented to those students highly motivated to act in groups, and even those "passive ones" who prefer to watch the show without participating still get benefits.

2. The method enables and encourages students to follow and verbalize their own physical reasoning, with immediate reinforcement from other students and the instructor. This enhances students' involvement and stimulates their self-reliant thinking (including the transfer of knowledge from one area to another.)

3. The method gives immediate feedback from students—feedback which is used to estimate their level of knowledge as the course progresses.

4. The instructor still serves as a model of a physicist: one who speculates, tries, perhaps fails, conjectures, and faces dilemmas; one who "provides exercises and occasions for the nurturing of a sense of conjecture and dilemma."³

2.5.2 Audiovisuals. The role of audiovisuals in the learning-oriented system has a new dimension compared with the traditional system. In the latter, the audiovisuals are considered as "aids" to teaching or

supplements to instruction. In the former system, they might be taken in that sense, but, more important, someone might select them as the main component to facilitate learning. This implies that the instructor should have enough audiovisual media to fit specific, predetermined functions. This is not a practical situation in most cases. Ours was not an exception.

The proper limitations led us to offer only two fronts: (1) film-loops, and (2) autolectures.

(1) Film-loops are motion pictures without sound. They show actual events and phenomena that would otherwise be impossible to observe in the classroom.

There are several features in the film loops that add a new dimension to instruction and facilitate carrying out some specific study activities [12]:

(a) They are short in time: approximately four minutes in length. This allows the student to watch one film two or three times without getting tired.

(b) The loop eliminates threading and rewinding. When the film comes to the end, it is automatically ready to start again.

(c) They present a specific concept, experiment, or event; some of them show step-by-step sequenced sophisticated techniques and experiments. Others show a particular procedure that can be reproduced from the film-step-by-step using real materials which are at hand. The student can view the first part, turn off the film, work on that part, turn on the film, view the second part, stop, turn off the film, work on it, and so on. Repetition of any segment or step of the procedure can easily be done.
(d) Single frame projection allows the student to "freeze" a particular scene to take measurements on the projected image, read dials, or obtain data in other ways and use these data to make graphs, tables, or other appropriate records. Or he may simply view problem-situations for analysis.

Some of the film-loops are self-contained, but others need additional commentaries and explanations to make them useful for individualized instruction. Then, it is necessary to use a film guide that provides the indispensable guidelines. In this way, film loops are ideally suited for individual and/or small group learning. Students are able to view the film repeatedly and progress at a rate compatible with their ability. A student can regulate the pace of the demonstration of the film to suit his own skill and see the demonstration without looking over the heads of other students. The use of film guides allows students to use film loops effectively with little supervision.

Our students used a stock of 60 film loops chosen from 200 of the BFA/Ealing Science Film-Loops Series. Their areas range from Demonstrations of Physics, Mechanics on Air Table, Mechanics on Air Track, Vector Kinematics, the series Project Physics Course with the topics: Concepts of Motion, Motion in the Heavens, the Triumph of Mechanics and the Nucleus; and other Individual Film-Loops. A detailed list of the films used by the students is given in Appendix F.

(2) Autolecture is a product, resultant from combining:

(a) cassette-tape prerecorded which includes a short lecture and programmed-instruction sequences,

(b) a sequenced set of transparencies to be used on an overhead
projector. The transparencies are synchronized on the overhead projector by the programmed instructions from the cassette recorder.

The major features of the autolecture are (a) it is a well organized, compact, "error-free" presentation, composed and kept up-to-date by the teacher himself, (b) the learner can use it as many times as he wants like a self-paced auto instructional medium, and (c) the teacher's time is better distributed. 4

The autolecture makes the student focus his attention more effectively than a regular lecture because he does not need to take notes. When he has finished watching the autolecture, he can take notes directly from the transparencies, or he can use an exact facsimile of the projected transparencies that are always available in the autolecture pack.

The uniqueness of the medium inevitable encourages the student's involvement. He has to look at definitions, diagrams, tables, problems, procedures, thinking about open questions, etc; he never just listens. But this involvement depends strongly on the clarity of sight, sound, and subject presentation.

The autolecture is short in time, the statements are brief, the critical points are clarified, and repetition by the tape narrator is avoided. Short questions and answers provide the immediate reinforcement necessary to help the student proceed confidently. Repetition can be achieved by the student through a replay of the appropriate tape segment. In demonstrations that require motion, an 8 mm projector

---

can be used parallel to the overhead projector; and the whole system is operated by the student.

We made a trial elaborating three autolectures. They dealt with (a) Basic Kinematics Definition: mathematical and geometrical interpretation, and applications to motion with constant acceleration and motion with acceleration proportional to time; (b) Vector Kinematics: addition of velocity vectors, ballistic problems, circular motion; and (c) Laws of Motion: theory and applications. These materials are presented in script form in Appendix E.

At first, we were sceptical about how attractive autolectures would be for the students, but very soon we found out that most of the students were using them as a complementary step in the study procedure before attempting to take a test. Others used them as a remedial procedure when they failed a Unit. When we ran out of autolectures, it was surprising to hear many claims about new autolectures on new topics. That apparent success has encouraged us to make a new but more formal trial of using this device in this personalized course in the near future.
2.6 Study Unit: PLANNING THE CONTENT OF A PSI COURSE

Introduction

In this unit you will read about the general considerations required to select and organize the content of a PSI Course. The procedure recommended is to follow a systems approach. You will find that selection of content and organization are decision-making operations. The process implied by this strategy is not linear. The questions raised here are many, and the commitment is a major one in terms of time and effort. The rewards are worth it.

Objectives

To be able to:

1. List three decisions that require considerable thought on the part of the instructor as he prepares his course content by a systems approach.

2. Recall various methods you might employ to select student objectives that should result from your course.

3. Order in a chronological sequence a list of planning activities.

4. Describe, in writing, Keller's suggestions for avoiding undue fragmentation of the course (the result of too many units) and for consolidating what the student has learned.

Resource Materials

Suggested Procedure

Microsystem Approach:

1. Decide what the students are going to learn in the course.
   
   This looks very trivial but it is not. The decision should be made in terms of what the students are going to do or say. The clear specification of learning objectives is crucial. Sequencing the objectives so that students can proceed systematically through the course involves some trial and error.

2. Decide how you will determine when the student has learned the material. This implies a clear statement on how he is going to do it. The instructor should provide explicit criteria by which the student's work is judged.

3. Decide what sources of information are pertinent to the objectives. Choose learning experiences that will facilitate learning by students. Experiments, readings, lectures, demonstrations, and field trips are some of them.

   The above remarks should enable you to satisfy Objective 1. Objective 2 is concerned with methods you might use to make the first decision—what should the student learn in the course. Remember the advice: think in terms of what the student is going to do or say (student competency). It is a simple matter to go through a book and identify all possible learning objectives. But which of those objectives define competencies (proficiencies, performances) that
you want every student to take away from your course? Concentrate on the **minimum learning objectives** (core). Take a look at your old final exams. Are these competencies the most important? You might even go so far as to enlist the advice of colleagues (shudder). Perform a **simulated task analysis**: imagine the oral examination setting, and develop a list of appropriate questions, problems, and tasks for each major course topic; sift out the core objectives by deleting the nonessential; consult other colleagues who teach the same course.

Your planning activities (Objective 3) might go like this:

1. Identify competencies desired (simulated task analysis, text, final exams, colleagues).
2. Determine criteria of evaluation (test items, practice exercises, study questions, assigned problems).
3. Write the minimum learning objectives based on (1) and (2).
4. Sequence the objectives and divide them into study units.
5. Specify the resource materials (and learning activities).

A simpler prescription has been offered by Dr. Keller. See page 78 of resource material [1]. The units of Mechanics of Point-like Bodies are examples of a course that was planned according to a systems approach.

Now, proceed to the Pretest.
Study Unit: Planning the Content of a PSI Course

1. Write three tasks an instructor has to deliberate over during the planning of a PSI course.

2. Name two procedures useful in determining just what the student is to learn (student competencies) in your course.

3. Order the following activities in the sequence you would perform them as you plan the content of a PSI course: (a) Perform a task analysis to identify the competencies students should be able to demonstrate when they have completed the course. (b) State explicitly the conditions and the criteria by which you intend to evaluate the students' performance; in other words, write test items or practice exercises that identify the competencies you want all of your students to attain. (c) Write the minimum learning objectives (core) in performance terms, e.g., what the student is to do or say. (d) Sequence the objectives: group them in study units (prepare a course outline). (e) Specify learning activities that will enable the student to satisfy the learning objectives, e.g., select resource materials, write practice tests.

4. What is Keller's recipe for consolidating what the student accomplishes in the separate study units?
Chapter 3

FORMULATING OBJECTIVES

"... if you are not sure where you are going, you are liable to end-up some place else--and not even know it."

Robert F. Mager [9]

3.1 Generalities

Everybody writes objectives all the time and is able to do it without any knowledge about taxonomies or indicator behaviors. We are always talking in terms of goals and purposes. These terms are often used by people to explain what they are intending to do or what they did.

We saw in our systems approach how important it is to establish clear objectives as the first step in designing any system. This chapter is intended to establish the general criteria of writing objectives for our units in the Mechanics course. This is our general objective; that is the purpose of this micro-system. Once the purpose or goal has been determined, specific objectives can be derived and described as specifically as it is possible and feasible.

From the preceding information, it is clear that a difference is established between goal (purpose) and specific objective. Educational goals or purposes or general objectives tend to be much broader
and less explicit than objectives. Yet they can both be considered to lie on a continuum of specificity. For instance, one goal for the physics course might be for the students to be able to understand Newton's Theory of Motion.

An objective stated in behavioral terms for the course might be:

To be able to state the forces acting on a study-body by drawing a free-body diagram and then to calculate the resultant acceleration of the body.

There are several approaches for delineating educational goals:

1. Instructor oriented. Emphasis is on the instructor teaching rather than on the student learning. The teacher uses a list of prescribed content to remind him what he must do to keep the students busy.

2. Activity oriented. Emphasis is on sequenced activities in which the learner will participate. The belief is that a variety of stimulating experiences will provide the learner with necessary input to enable and to motivate later responses.

3. Learning oriented (subjective). The instructor describes in general terms the concepts and skills the learner is to "master". It is not clear what "master" means. The peremptoriness is on the student and his development and focuses on the subjective development of the mind.

4. Behavior oriented (observable). Long-range terminal behavior has been analyzed and broken down into its short-range components. These specific behavioral objectives determine the content
of the curriculum and the learning activities to be included. They enable the teacher to identify the learner who does not reach the stated objectives, diagnose his difficulties, and prescribe remedial measures.

From this presentation a question is raised: what is the nature of an educational objective which provides sufficient direction for the effective structuring of curriculum materials, classroom teaching, and evaluation procedures? Before answering we recall that in the learning oriented system the educator must be able to describe in exact terms what he wants the learner to be able to do at the end of a learning experience. Then, every objective is to be defined in terms of behavior that ideally will result as a consequence of the learning experience. It is concluded that behaviorally stated objectives that equip the educator with an optimum structure is the answer. In consequence, the formulation of our objectives for Physics 53 PSI must be of the behavioral type. The objectives in our units are "content" objectives, in the cognitive domain, as opposed to "attitudinal objectives". In the next sections, we will describe the characteristics that an objective has to meet to be behavioral, its structuring, and the product as it applies to our course.

3.2 Characterizing Behavioral Objectives

An instructional objective is a statement that describes an intended outcome of instruction. It defines the goals of instruction. It is what you expect the learner to be able to do as a result of the instruction. As Mager says explicitly [9]: 
An objective is an intent communicated by a statement describing a proposed change in a learner—a statement of what the learner is to be like when he has successfully completed a learning experience. It is a description of a pattern of behavior (performance) we want the learner to be able to demonstrate.

Specific objectives help the teacher select appropriate learning activities. They must be stated meaningfully in the sense of communicating clearly to the reader the writer's instructional intent; and they must give both student and teacher standards for evaluating progress.

The following are offered as characteristics of well stated behavioral objectives:

1. The statement is directed to the student performance.
2. The statement specifies what the learner will be able to do or perform when he is demonstrating his mastery of the objective. (Is the skill called for the principal performance or is it an indicator behavior?)
3. The behavior is observable.
4. The behavior can be evaluated.
5. The essential characteristics of the derived behavior are explicitly stated.
6. The statement explicitly describes the conditions under which the behavior is to occur (givens or restrictions, or both).
7. The statement is written in appropriate vocabulary at an appropriate reading level.

In the real writing of objectives it is not necessary to include all those items. They help, but some of them are dispensable;
and one probably finds that some of them apply to some objectives, but not to others.

A simple procedure is indicated by Mager.¹ He suggests that one poses three questions to insure that one does communicate effectively:

1. Does the statement describe the performance (behavior, competency) by name, a name that will be accepted as evidence that the learner has achieved the objectives?

Present the ideas of Newton's theory and discuss its applications.

To present is not an instructional outcome but a teaching way.

An amendment might be:

Be able to understand the basic...

The question arises: what will the student do when he is asked to demonstrate his understanding of the basic...?

A better, corrected objective should be:

Be able to identify by name the external forces acting on a body.

2. Does the statement describe the conditions (givens, restrictions, or both) under which the learner must demonstrate his competence?

Distinguish between conservative and non-conservative forces.

In this case the student could name two different forces, or could name 20 different forces, or could give the definitions. The conditions are not stated. The objective could be improved by wording it in this way:

Given a list of 10 forces, distinguish between conservative and nonconservative forces.

Or better,

Classify in conservative and nonconservative forces from a list of 10 given forces.

In science the wording of the conditions might confuse the objective. The easy way to show the condition is by using sample problems or practice exercises. This practice was our favorite in developing our units in Mechanics.

(3) Does the statement describe how well the learner must perform to be acceptable (acceptable standards)?

Example of an objective without performance criterion:

Construct a free-body diagram of a body isolated from its environment.

An improved version might be:

Construct a free-body diagram of a body isolated from its environment, showing all the external forces acting on the body and naming correctly the source of each force.
A phrase is a criterion if it says anything about the excellence of performance that is expected of the student. A practice exercise is again a useful tool to provide the acceptable standard since the student can use it for self-evaluation. In our units, these come under the subsection Worked Problems.

### 3.3 Structuring Objectives

In writing instructional objectives we have to be careful about the structure of the objective. There are several variations of the declarative sentence form.

We have encountered in our work the following forms, with variations, of course. They are presented with some comments:

(a) The student will be able to add... . It is not directed to the student but to someone else. This format should be used if the objective is for teacher's use.

(b) You will be able to add... . It is directed to the student only. When it is repeated many times it becomes boring and sounds like baby talk.

(c) add ... . The imperative form sounds like a test item rather than an objective.

(d) To add ... . This infinitive form is a good one to use in directing the same objective to the student and the teacher.

(e) To be able to add ... . It is equivalent to the latter, but it has a drawback with respect to it. That is, for the question: "What is the objective?" the answer is, "The objective is to add ...". However, if the objective has a "given" stem, it is very awkward to use it with an infinitive: Given ..., to add ...; or, Given ..., to be able to add ...?
(f) Be able to add .... This form seems ideal. It directs the objective to the student. The inclusion of an introductory verbal phrase does not make the objective clumsy.

Nevertheless, to be able to... seems a better form when it is used as a common heading for a set of objectives starting with action verbs. This type was adopted to our units.

3.4 Using Verbs

Therefore, the essence of every objective is its verb. This is a matter of great discussion. In this section we will discuss some of the popular verbs that we believe are used and misused.

Understand: 1a: To grasp the meaning of, b: to grasp the reasonableness of, c: to have thorough or technical acquaintance with or expertness in the practice of; d: to be thoroughly familiar with the character and propensities of...²

Know: 1a (1): to perceive directly; have direct cognition of, (2): to have understanding of, (3): to recognize the nature of; b (1) to recognize as being the same as something previously known, (2): to be acquainted or familiar with...³

With these meanings for understand and know, the following question arises:

What do we mean when we say we want a student to know (or to understand) something? Obviously those words are open to a wide range of interpretations. The words refer to states of the individual.


³Ibid., p. 639.
These states are not directly observable. They are useful in describing broad goals from which a single objective may be a part of it. But here we are dealing with instructional objectives, not outlining curriculum goals.

For example:

To be able to know and understand the motion of a body.

In the first place, this objective is so broad that it is actually a goal for several units of work or even a term's work, and so it would be acceptable as a curriculum goal.

Let us try to be more specific and narrow the objective:

Be able to understand the rectilinear motion of a body.

This is more specific but still too broad. Let us write it in a way that when achieved will show us that the student understands this motion.

Be able to use the equations of rectilinear motion.

or,

Be able to solve problems of rectilinear motion such as the worked problem III.

or,

Be able to use correctly the equation of rectilinear motion to find unknown kinematics variables of motion of a body.

or,
Be able to find unknown kinematics variables of a body in motion with constant acceleration by using the equations already derived.

In conclusion, understand and know, used as performance terms, are simply not testable. They are not overt action.

Recognize. Like understand and know, this represents an underlying requirement of an objective.

To be able to recognize the conservation laws in problems.

It says nothing at all about ways for the student to demonstrate that he recognizes the conservation laws.

If we use the word identify, however, we have a legitimate objective.

Identify, can take care of a lot of requirements; but we have to be careful with it. Sometimes it is just too easy to use, and it is not always the best word. Or if we tell the student to identify something, he will wonder if we are going to give him a group of things from which to identify the something or if we are leaving him on his own to go find the thing he is to identify.

The verbs name, give, list, and state require recall. The main problem with these words is present in making a decision about testing. Usually the test item would be the same as the objective.

Describe and explain. Objectives that use these words usually cannot have a sample test because no two students will give the same description or explanation.
In general we can say there is no set form for writing objectives. But it is worth following the guidelines described above. Try not to use verbs of broad meaning or verbs without observable behavior. A list of possible verbs for use in stating cognitive behavioral objectives is suggested in Appendix A.3, as well as a list of words that one should avoid because they are open to many interpretations.

There are other considerations that we called style technicalities. Among those are the use of given and using. There is a trend to begin all objectives with the word given. It is desirable and necessary to state the condition; but a given stem is not the only way of expressing that condition. Some objectives do not require the expression of conditions. Furthermore, the word given does not necessarily have to be expressed at the beginning of the objective. Sometimes an objective will be clear and will sound better if the necessary given part is included as a dependent clause at the end of the sentence.

Given the masses, positions, and velocities of all particles in a system, be able to find the position and velocity of the center of mass and the total linear momentum.

Modified,

Be able to find the position and velocity of the center of mass and the total linear momentum when the masses, positions, and velocities of all the particles in a system are given.

Utilizing a using stem is probably the most obvious way to include conditions in the statement. But care must be taken not to write indiscriminately using instead of use and vice versa without
any reason. A *using* stem is a conditional part of the objective--not the main event.

Using a conversion factors table, be able to convert units from the MKS system to the Engineering System.

We prefer:

Be able to convert units from the MKS system to the Engineering system using a conversion factors table.

If the word *use* (instead of using) is written in an objective, the act of using is the main event rather than a simple condition.

Be able to use a conversion factors table to convert units from the MKS system to the Engineering system.

Whether *use* or *using* is chosen depends on the writer's intention--whether he wants the student to show that he is able to use a conversion factors table or whether he wants the student to show that he is able to get the correct answer.

Many educators believe it is against the "rules" to write objectives that have more than one sentence or ones that have listed parts. But there are no "rules". When the objective requires a lot of details and perhaps criteria for evaluation, it is a good practice to break it up in several sentences or to make a list of parts. This improves clarity, understanding, and intelligibility. To make continuous use of this practice is dangerous, because it is possible that in a long sentence one is really combining several objectives in one; and that is not the case. We recommend using several sentences
3.5 Classifying Objectives

If one is developing an instructional program, it is helpful to see what type of objective is produced; or if the set of objectives is covering all the intellectual abilities and skills that one desires, the student develops throughout the instruction. Therefore, one can get a tool for improving if the objectives are classified after one has finished writing the set corresponding to a Unit. But how can we classify them? What criterion can we use?

Of all books on learning behavior, Bloom's Taxonomy [10] is the most frequently quoted and used for working with instructional objectives. And that is the most familiar to me from my preceding work in evaluation. The Taxonomy of Educational Objectives: Handbook I, Cognitive Domain includes those objectives which deal with the recall or recognition of knowledge and the development of intellectual abilities and skills. Bloom's work attempts to classify levels of learning behavior in a classification system with a structure. He defines six levels ranging from simple to complex. These six major classes are described as Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation. See Appendix A.1 for a brief definition of each class and its subclasses. Bloom's Taxonomy does not define their categories in behavioral terms, but it has examples of objectives and test items that can be used to determine the kinds of behaviors required at each level. In Appendix A.3 we have a list of possible verbs that can be used to describe observable behaviors for each category.
We use these tools to classify our objectives rather than to design them. Just by classifying them, we become aware of the number of each kind of objective we have included. Then, in writing objectives for the areas that are underrepresented, we are likely to create new objectives that will promote critical thinking, problem solving, concept formations, and creativity. This approach is suggested by Julie S. Vargas.\(^4\)

Bloom's Taxonomy was structured to serve evaluation rather than curriculum design. Both are concerned with objectives, but the perspectives are different; whereas the evaluation has to categorize an objective already present in the course materials, the curriculum designer must pull his objective "out of the blue".

But if one is going to use Bloom's Taxonomy to design objectives, several applicability limits appear. The most important lies in the author's insistence on keeping the structure neutral. The designer needs a taxonomy where the structure when explored leads the designer to a comprehensive set of objectives, once a key objective has been defined. With such a taxonomy the designer would have a device for developing curricula that would provide a comprehensive referent for structuring learning materials—especially materials suitable for use in self-pacing instruction. Drumheller\(^5\) has modified the structure of the Bloom's Taxonomy eliminating the

---


hierarchical emphasis and emphasizing the logical relationship between the categories. The new structure provides guides for breaking complex behavioral objectives into teachable components: that is, identifying the terminal sub-objectives of a Unit once the terminal objectives have been identified. The chart in Appendix A.2 presents an overview of this modified taxonomy.

The aim of this Chapter is to present and discuss the methods and criteria that we adopted in writing the objectives of our course Physics 53 PSI. In this chapter I have also included some information related to my research on the subject of objectives: definitions, features, misuse of verbs, style technicalities, taxonomies of classification, and illustrations as applied to physics.

As final word on this matter, it is worth noting that there is nothing absolutely defined and infallible in writing objectives. There are certain guidelines that one must try to follow, but one can alter them at any time to meet specific needs. The important part is to choose the correct words to help oneself to communicate without misinterpretation.

"An objective will communicate your intent to the degree you have described what the learner will be DOING when demonstrating his achievement and how you will know when he is doing it."6

---

6 Mager, op. cit., p. 53.
3.6 Study Unit: WRITING INSTRUCTIONAL OBJECTIVES

Introduction

The objectives of a system are the main step in the design procedure because they will determine whatever has to be designed and done to attain the objectives themselves. Specific objectives of a course perform three functions: they set up the selection of appropriate learning activities, they communicate to others--particularly to the student--what he is expected to perform, and they give both student and teacher standards for evaluating progress. The student then knows exactly where he is going, can tell what progress he is making, and can tell when he gets there. He can direct his attention to the essential information that he needs; and he can make more efficient use of his time. These kinds of objectives are called behavioral objectives.

Objectives

To be able to:

1. Distinguish between objectives that do and do not communicate instructional intentions.

2. Differentiate between learning objectives that do and do not describe:
   (a) the kind of performance or learner competency to be demonstrated,
   (b) the conditions under which the competency is to be demonstrated, and
   (c) the minimum acceptable performance.

3. Write learning objectives that accomplish all of the following:
(a) describe the learner's competency (behavior performance) to be demonstrated as evidence of accomplishment of the objective;

(b) identify the conditions under which competency is to be demonstrated; and

(c) define acceptable standards competency.

You can use content material and a list of action verbs.

Resource Materials


4. Verbs List.

5. Mechanics of Point-Like Bodies Units, for example Unit VI Newton's Laws.

Suggested Procedure

The best starting point is to read Mager's book, (Resource Material 1). He says an instructional objective is an intent communicated by describing a proposed change in the student. It is a statement of what the student is to be like when he has successfully completed a learning experience. In other words, it is a description of a pattern of behavior (performance) that one wants the student to be able to demonstrate.

The first objective of this unit is concerned with communicating your instructional intent, whatever the objective; you should be detailed enough so that others understand your intent as you understand
it. The statement of objective should specify:

1. **What** the learner is expected to be able to do, by
   a. using a verb that denotes observable action,
   b. indicating the stimulus that is to evoke the behavior of the learner.

2. **Under what conditions** the learner must demonstrate his competency, by
   a. identifying the givens and/or restrictions,
   b. specifying resources (objects) to be used by the learner and persons with whom he should interact.

3. **How well** the behavior is expected to be performed by identifying
   a. accuracy or correctness of response,
   b. response length, speed, time, and so forth.

The best way to couch your objectives in "performance terms" is to use action verbs that describe what the learner will be doing, e.g., to write, to identify, to discriminate, to solve, to draw, to list, etc. Strive to avoid misinterpretation.

Often it is desirable to be more specific by stating the conditions that will be imposed on the learner when he is demonstrating his competency: what will be provided and what will be denied to the learner. Restrictions such as "given such and such" "using only...", and "without the aid..." might improve the ability of the objective to communicate.

Stating the minimum acceptable performance further specifies your learning goals. When appropriate, identify the standard or lower limit of acceptable performance, e.g., a time limit, a minimum number
of correct responses, percentage, or accuracy.

Summarizing,

(a) start a set of objectives with only one heading: "to be able to:"

(b) start each objective with an action verb

(c) express the condition by a given or a using, used as a single-word participle or in a dependent clause at the end of the sentence

(d) add a standard.

Features (c) and (d) are highly desirable but not absolutely indispensable. One method of insuring that your objectives are understood is to provide exercises or sample problems. Note the use of this in the Mechanics Units.

And, whatever you do, communicate!
PRETEST

Study Unit: Writing Instructional Objectives

1. Which of the following learning objectives communicate instructional intentions?
   (a) To develop an understanding of motion with constant acceleration
   (b) To appreciate the power of Newton's law
   (c) To describe with your own words Newton's three laws of motion
   (d) To be able to apply momentum conservation in collisions of point-like bodies
   (e) To calculate the period of a pendulum
   (f) To know how the frictional force works

2. Which of the learning objectives in the Newton's Laws Unit are written in performance terms?

3. Which of the above objectives specify the conditions under which the performance is to occur?

4. Which of the above objectives defines the standard of acceptable performance?

5. Do the three written objectives in this study unit contain the items that are listed in Objective 3? If no, rewrite them.

Answers
Chapter 4

DESIGNING UNITS

"What I hear, I forget; what I see, I remember; what I do, I understand."

Chinese Proverb

We already mentioned how important the materials are for any instructional method, especially for PSI courses, and how they are one of the biggest problems. But, we have already taken steps in search of a solution when we presented and discussed the formulation of objectives. We have suggested that one selects content, procedures, and methods appropriate to the objectives; that one stimulates students to interact with the subject matter in well-thought-out learning activities; and that one evaluates the student's performance according to the objectives or goals that were selected. In this chapter we are concerned with selecting the most efficient route for getting students to the goal.

4.1 Composing a Self-Contained Unit

The purpose of a self-contained unit is to enable the student to proceed through the textbook and other course materials mainly on his own so that he can do it in such a way as to achieve successfully the objectives of the unit. This function is more important than whatever form the unit might take.
Therefore, the unit of study must tell the student what the learning objectives are, how to accomplish them, and how to supply the materials or resources that have not yet been provided by other media. Whether this goal is or is not achieved can be determined by means of the test or exam for the unit. The self-contained feature means that the unit should present clear content in small, logically sequential, completed parts, requiring students to get actively involved so that each step during learning would be an intrinsically rewarding one.

Once the analysis of the purpose of the unit is done, it will lead to gathering of data from which through further analysis, a statement of objectives is developed. The description of terminal performances becomes a basis upon which to construct the (criterion) test. The test is the measuring instrument which is used to assess if the objectives have been achieved. It is the key to the quality control of the unit. The objectives must then be further analyzed in order to identify whatever the learner has to learn in order for him to behave in the way prescribed. This analysis provides the learning activities, which properly sequenced become a simple procedure.

Although the systems approach offers specific structure for instructional decision making, this structure is not rigid. It has a built-in flexibility that enables the designer to think ahead. We mentioned that it is not a one-directional structure, but one which allows not only for feedback but also for "feed-ahead" or "feed-forward". This characteristic enables us to do the following variation in the development of the components of a unit of study.

In general, the science teacher is more familiar with designing
tests and examinations than with constructing learning objectives. Then, we ask ourselves, "What questions, problems or tasks do we want the learner to be able to answer, solve, or perform, and what is the minimum acceptable performance?" In other words, we begin developing test items. The test items will identify the competencies completely and provide a guide to show us for what we must prepare the student to do.

So far, we already have the three essential parts of the unit: the Objectives, the Procedure (both in the Study Guide), and the Test. That is:

a) Objectives, Procedure, and Test must be congruent.

b) The Procedure must be appropriate to achieve the Objectives.

To these foundations we can add other sections or parts to supply text supplements for the assignment: resource materials, alternate procedures, or anything that seems to help the learner. Our units contain the Study Guide and Test Materials.

Our typical Study Guide is composed of Introduction, Learning Objectives, Suggested Procedure, Summary of Relations and Definitions, Worked Problems, Pretest, and Audiovisual Aids Section. Sometimes, we used an additional Supplementary Annotations Section, Tables, and additional Resource Materials.

Test Materials are composed of three test forms and their grading keys.

4.2 Establishing Functions of the Components

First, let us examine functions of the components for the Study Guide:
**Introduction.** This section tells the learner in a clear exposition about the subject of the unit and its links with the material of past or future units. It sets the scope, provides a concrete, familiar example of the physics contained in the unit, and shows the necessity of studying the material.

The introduction can be long or short, formal or chatty, depending on the material and the interest and style of the writer. It should be interesting—interesting enough to motivate the students.

**Objectives.** They state specifically the goals of the unit. The statement of objectives, in behavioral terms, tells the student what he will be able to do as a result of completing the unit, the competencies he will acquire and will be required to demonstrate. The list of objectives should direct the students to all of the subject matter the instructor intends to include in the unit test. It allows the student to determine for himself when he has met the mastery criterion.

**Procedure.** This section specifies the activities which will prove adequate for the students to accomplish the unit objectives. It directs the learner to the resource materials in the form of books, articles, audiovisuals, lectures, demonstrations, models, and other instructional materials that have maximum accessibility. It indicates the practice exercises, the problems to solve in order of increasing difficulty, and discussions identifying correct answers.

In some cases this may be as simple as stating the assignment to be read and stating that the student work out solutions for certain problems before attempting to take a test. Perhaps a few comments about what to skip, what to skim, and what to study in detail will be
sufficient. Other units may require detailed instructions. In either case the procedure section tells the students what to do, how to self-test his comprehension, how to decide whether to proceed or review, and how to decide when he has finished.

**Supplementary Annotations.** The point here is to comment on, supplement, or correct sections of the textbook. There may be outdated sections of the textbook that require the addition of current information to be complete. The professor may wish to change the stress or emphasis of some topics or to clarify the things that he might have said in the lecture. It is a chance for the instructor to present his own opinions and interests.

**Summary of Relations and Definitions.** This section may include a list of concepts, definitions, and/or mathematical relations that the student should be prepared to define or use for solving problems. This section provides a summary of the most important points presented in the unit. They are not supposed to be memorized; rather, they should focus the student's reading until he thoroughly "understands" them. Furthermore, the student could use the summary of relations and definitions during the exams in order that they may encourage him to "learn" physics through "thinking" rather than memorizing. Finally, it will serve as reference material which can be easily retrieved when it is needed by the students in subsequent units or in their future use of physics.

**Worked Problems.** This section also performs several functions. If there is at least a worked problem for each objective, then it serves to clarify the objective showing explicitly the conditions or restrictions of the objective and also the performance criteria.
for evaluation. This enables us to state the principal performance or skill that we want the student to demonstrate and eliminates any doubt about our intention in the objective.

On the other hand, the worked problems serve to guide the student in the proper line of approach to a problem and the "method of science." We still believe that a simple reading of the text cannot create an overnight understanding of physics. Physics is best understood through the experience of application of fundamental concepts to a variety of physical situations. Therefore, methods and techniques of problem-solving are emphasized, which, when mastered, should provide the student with the background and confidence required for more complex situations. Hence, this section of the study guide serves this purpose fully.

Audiovisual Aids Section. Even though the heading uses the word "aids" this section serves as a suggested procedure for those students who select them as the components to carry out specific learning. However, due to physical limitations, they are chosen by most of the students as an alternate procedure or a supplement. So that this section, functioning in either one of these two ways, tells the student about the content of film-loops, autolectures, transparencies, slides, and in certain cases asks questions that help the student to detect his progress and success.

Resource Materials. This is a list of the materials needed, or those that provide information for the learning experiences. The list may contain textbooks, journals, articles, equipment, and other media required.

Pretest. Matching each objective with a pretest item provides the student, as do the worked problems, with clear samples of what he
will be expected to do in order to show that he can achieve the objectives. Besides, it provides a private progress evaluation to help the learner realize his own problems and decide what to do about them; it also helps him to measure the increase in observable competencies.

The Pretest is intended to be representative of the regular Test Unit, but sometimes it can have more detailed questions than the actual test has, and then it can be considered as a Review test and Model test at the same time.

In considering the Test Materials, which are separated from the study guide but still form an integral part of the unit, we have:

Test. The unit tests are of crucial importance in determining the success or failure of any course. They are the device to measure the student's mastery requirement. Two aspects of the unit tests are dictated by prelogistical considerations. First, mastery learning requires that the tests evaluate the students' understanding of each major objective of the unit rather than "sample" what has been learned. Second, the questions must relate to the objectives specified in the unit rather than draw off some hidden program. The recommended totally "objective test" does not work in Physics because this requires problem solutions.

Test Grading Keys. Its function is to help in the proctor's duty of grading. It provides him with the solutions of test items; gives alternate solutions procedure; suggests key questions to allow him to deal appropriately with students who know the answer but misread the question, or those who do not know the material but appear to have attained mastery by a lucky guess; and gives informational
instructions that the proctor can use to guide the students to correct their own mistakes.

4.3 Structuring the Units

This is one of the first problems that a professor faces with a course of the type discussed in this paper: how many units will be needed for a particular course, how long and how difficult is each unit, and in what sequence should they be presented.

The number of units for a course depends on so many factors that it is better not to stress that point. However, a popular choice is to have at least as many units as there are weeks in the course. This provides the opportunity for the student to check in about once a week.

The unit size and test length depend on the nature of the topic under study. The tests have to be designed to quiz the student on every major unit objective and should not merely sample and thus estimate what has to be learned. This limits the unit size to a content that can be fully tested in the average time estimated. A design in which a unit covers about a week's work is a popular choice. Koen [13] suggests a process in which the material is first divided into chapter divisions or outlined from the course textbook. These are called "logical units" which undoubtedly result in too few units. Then, the units are changed and new ones are added to obtain units of essentially "equal difficulty." As a final step the unit structure is fine-tuned by estimating when reinforcement will be needed and creating "reinforcement units." This serves as a first tryout, and then the unit structure is improved on the basis of the feedback data of the unit test.

Given the above considerations, other factors suggest sequential
arrangement deviating from approximating "equal difficulty" units. It is advisable to have the first units slightly easier than the average unit of the course to provide an early reinforcement and to break the "set" that students have for traditional courses. After this, the reinforcing nature of the system will take over. Also, if the material is organized in a repeat spiral, the first half of the units should be simple to encourage the student to cover the most important material, especially that required for the following course.

Finally, review units should be included every five or six units to correlate important concepts and to provide practice in problem solving, if some units do not have it, and to integrate all parts in an overview of course material.
4.4 Study Unit: DESIGNING A STUDY UNIT

Introduction

The preceding unit dealt with formulating behavioral objectives that the professor intends students to reach at the end of a unit of study. In the learning system, that is the starting point, the selection of procedures, content, and methods must be congruent with the behavioral objectives. Assuming that you know what your destination is, this unit concerns selecting the most efficient route to get the students there.

Objectives

To be able to:

1. List the components of a study unit in the sequence they are presented to the student.

2. Write descriptions of the function of each component of a study unit and distinguish between properly and improperly written components.

3. List the components of a study unit in the order that they are developed by the instructor.

4. Describe a method of developing study units that is initiated by constructing test items.

Resource Materials


2. Sample Unit: for example, Momentum and its Conservation.
The first objective refers to the order of presentation of the component in a study unit. If you have read the study units of each Chapter of this work or the Mechanics Units you will surely recognize the order: Introduction, Objectives, Procedure (including directions, resource materials, and notes), and finally Pretest. The table below is patterned after material in Deterline and Lenn's manuals on Coordinated Instructional Systems—(Resource Material 1). You should examine this table until you can satisfy Objective 2.

### Study-Unit Components and Functions

<table>
<thead>
<tr>
<th>Component</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>Tells the learner what the unit is all about; identifies the topics, sets the scope; is brief, clear and interesting. Provides a concrete, familiar example of the physics contained in the unit.</td>
</tr>
<tr>
<td>Objectives</td>
<td>Tell the student what he will be able to do as a result of completing the unit: the competencies he will acquire and will be required to demonstrate.</td>
</tr>
<tr>
<td>Procedure</td>
<td>Facilitates learning by the student; directs the learner to the resource materials in the form of books, audiovisuals, lectures, demonstrations, models, and other instructional materials that have maximum accessibility; practice exercises and discussions identifying correct answers; indicates what to do, using which materials, located where, and why.</td>
</tr>
<tr>
<td>Pretest (or Practice Test, or Model Test)</td>
<td>Provides a private progress evaluation to help the learner realize his own problems and decide what to do about them; measures the increase in observable competencies; provides practice in the use of the skills acquired.</td>
</tr>
</tbody>
</table>

The order in which the components of a study unit are developed is not the same as the order of presentation to the student. It is
suggested that you start by deciding what the successful learner is able to do at the end of the study unit. If you follow the sequence below in developing study units, then you are more apt to lead the student to the desired competencies.

Order of Development - Study-Unit Components

(1) Test
(2) Objectives
(3) Procedure (learning activities, resource materials, practice exercises)
(4) Introduction

Designing the tests or exams is an easier method of identifying objectives than the method of constructing learning objectives first. The test items will identify the competencies completely and provide a guide to show you what it is that you must prepare the student to do. (This should give you a hint about Objective 4).
PRETEST

1. List the components of a study unit in the order they are presented to the learner.
2. What is the function of the Pretest or Practice Test?
3. Does the introduction in the unit "Momentum and its Conservation" fulfill the function of a properly written introduction?
4. List the components of a study unit in the order of development by the instructor.
5. Describe the use of test items in identifying your learning objectives.
Chapter 5

ADDITIONAL MATERIALS

"It is one thing to criticize the work of others, and quite another to show what you have done about it."

--- Popular Adage

In this chapter we will present brief additional comments on materials used in our course—materials which cannot be normally found, at least in its presentation, in the traditional course. Also, we will criticize the limitations of our materials. The additional materials considered are the autolectures format, the film-loop guides, the test format, and the test grading key.

5.1 Autolecture

The autolecture is composed of the lecture prerecorded in a cassette tape and the transparencies to be used on the overhead projector. The transparencies supply the image or visual information required by the audio in the cassette. In the autolecture we have elements attuned with two aspects of our learning system: self-paced instruction and student involvement.

We already mentioned that the autolecture has its own style and performance requirements. It is not a regular lecture or a substitute for it. Therefore, the usage of a personal conversational tone with an occasional bit of humor creates a familiar environment. In the audio the voice of the instructor provides timely information,
definitions, and parenthetical expressions with minimal effort for the learner. These helpful aids are often omitted from a student's written guide because of the inconvenience involved in looking up words and because such thoughts seldom fit well as part of a written text. The tone of voice places emphasis on important points and expresses authority not apparent in the written words. Postlethwait has gained a lot of experience in this field with his Audio-Tutorial Method [12] and has recommended some points to keep in mind while making a tape regarding the voice used, the content of the auto-lecture, and the mechanics of production. Our experience in this matter was rather short, and we slid into some pitfalls in the sense of doing quite formal presentations for following prior lectures notes. Moreover, extended didactic presentation if it is boring when live is worse in the audio tape. It can be narcotic. For instance, our Autolecture No. 2 is excessively extended.

The transparencies also need proper care. Use of color for coding key equations or portions of diagrams calls students' interest and attention. A large, uncluttered format places emphasis on structure, leaving many of the details for private study.

The format of our presentation consisted essentially of 1) a brief theoretical introduction of the subject matter corresponding to one or two units; 2) a complementary discussion of paragraphs, figures, diagrams, or problems of the textbook that in our judgment needed stress, emphasis, or addition of current information; 3) a set of worked-out problems presenting applications of physical concepts and problem solving procedures.
5.2 Film-Loop Guide

If the film-loop is going to be used as a main source of information on a certain topic, it is necessary that it be explicit enough that students can get the maximum from it. But, our system also requires that the student get involved in the action. The way to accomplish this, is by leading the student to interact with the situation shown in the film. Several films have that purpose incorporated when sufficient data is presented so that experimental results can be calculated with a high degree of precision. Others filmed with extreme slow motion photography permit observation, measure, and prediction of physical quantities. In consequence, a film-guide is required to make the student's work easy and beneficial.

In the study guide for the unit an audiovisual section was included. Its purpose was more informative than formative. It contains a list of the films which are related to or concerned with the subject matter of the unit. For each film a clear and concise explanation of the theme and comments about its relation with the matter in study are given. This elemental film-guide can be seen in each unit of Appendix C.

After a film is viewed for the first time, a student could receive a complementary aid, if he wanted, from a proctor. He could explain certain sequences, or guide the data taking procedure, or ask questions, or give clues for answering them.

However, to get the kind of student involvement described above, a better film guide should provide background information about the matter, bibliography, suggested procedure for data taking, hints, key questions, and follow-up activities. With this complementary tool,
the film-loop will fit better for self-paced personalized instruction.

This improvement could be in either of two ways: (a) a detailed written guide to be used at the time of the second running of the film, or (b) in a cassette tape. The student would be asked to synchronize the tape and the film so he can use them simultaneously. Sometimes, it would be useful to have the student view the film while listening to the tape and then have the student watch the film in silence. Either way, this would be better than our present film-guide.

These materials—film-loops, super 8 mm. projectors, cassette recorders, cassette tapes, overhead projectors, transparencies, 35 mm. slide projector with carousel adapter, blackboard, chairs, tables, textbooks and reference material—were conveniently placed in a PSI room staffed by one proctor or instructor at all times. This room was made available to students at any time during the normal working day, but they had to sign-in on a control sheet.

5.3 Test Format

It has already been mentioned how important it is to have congruency between objectives and test items. This is the main aspect of the Test, and beyond that there is no standard format. Some PSI courses have used essay questions and others only objective items. Several considerations have yielded to a "mixed" test format with a ten question quiz, two true-false questions, three multiple choice questions, two matching items, two complete-the-sentence questions and one short essay. This seems to work well in courses such as psychology, philosophy, and social studies.
We thought that the "mixed format" does not fit in a course such as ours where concepts, structures, and problem solving are emphasized. As we have between four and six objectives per unit, we chose to have at least one question per objective. The question could be a whole problem or a section of a problem. There are four problems on the average per unit test. These problems should encourage concept formation, thinking and transfer, while at the same time minimizing the role of guessing.

The average time for test completion was forty minutes, which is at odds with the PSI standard of 10 or 15 minutes in length. The critical thinking and decision making for solution of a problem require that the student takes more time than simply crossing out letters in a multiple choice exam. To this point, we recall that understanding the basic concepts and scientific arguments requires efficient study habits, many hours of problem solving, and discussion with other students and instructors. When a problem is presented to a student, it requires him to recall all his knowledge related to the problem and to transfer the information.

Three alternative and equivalent forms of each test were elaborated. It appears to us that any student who is unsuccessful after three attempts probably needs more careful guidance than a proctor can give. The professor should see the student to make a detailed analysis of his difficulties and to administer a special examination that suits the particular student's needs.

5.4 Grading Key

As it was mentioned in the section on functions of components of the unit in Chapter 4, the test grading key is in reality proctor
material. The key facilitates the proctor's job but is not limited to giving away answers; it helps the proctor to decide on the adequacy of students' answers and provides remedial instructions for those who have not met the criterion of mastery. Our format consists of a solution in the left column of the page, showing only the main steps in the procedure and leaving out the calculation details. In the right column there is a series of notes of what to verify, examine, and ask. Also, alternate solutions are suggested, as well as new motivational questions. Another feature is a number keyed to the objective that the section of the problem is supposed to evaluate. This helps the proctor to recommend exactly what the student should restudy before attempting the test once again.

This proctor material develops quite naturally and easily when it is written concurrently with the tests. If one delays writing proctor material, it becomes difficult to find page references useful to the proctor in guiding students who have made errors. A sample of our test and grading keys can be seen in Appendix D.

In conclusion, the value of thoughtfully conceived study and test materials should not be underestimated. Good materials that successfully guide the student to the importance of the content he is to learn and carefully prepared testing procedures are the cornerstones of our system and must not be short-changed.
SUMMARY AND CONCLUSIONS

In this work I have tried to present the development of an educational experiment in a Physics course. A study of the history, principles, and underlying psychological substructure involved in PSI is the starting point. Considerations about the reported poor quality of written material used in many PSI courses and careless "individualized" student treatment with a loss of group interaction led us to try to develop a new teaching-learning strategy.

A systems approach is used to make a clear statement of the purpose, and a logical and measured implementation. The result is a strategy which combines the PSI features of mastery, written materials, and self-pace with a group discussion method designed to encourage student autonomy and interactive group participation. That is why I called it the interactive pace approach. Through this course we emphasized concepts, structure formation, and problem solving. The content presentation and organization follow a spiral approach.

The work described in this thesis has been largely developmental, as explained before. For this reason, there are no "conclusions" of the kind associated with a traditional research experiment in the behavioral or physical sciences. However, we can qualitatively mention that in the interactive pace approach:

1. Emphasis is placed on student learning rather than on teaching.
2. Students can adapt the study pace to their ability to assimilate material.

3. Interaction between individuals in the group stimulates their self-reliance and command of the subject matter.

4. There is enough evidence to assume that students prefer this type of course to other approaches. They felt that they learned more by participating in group discussion and by being aware of at least some of the relationships between the concepts discussed.

I do not pretend that my work is the ultimate word in the teaching-learning process; on the contrary, more research on this methodology is necessary, but I believe that with this approach we have opened new roads in the educational process in Physics.
BIBLIOGRAPHY

1. P.S.I Newsletter Editorial. The PSI is available from The Center for Personalized Instruction, Georgetown University, Washington, D.C. 20007.


REFERENCES USED BUT NOT NUMBERED IN THE STUDY


18 Ruskin, Robert and Stephen B. Bono. eds Personalized Instruction in Higher Education. Proceedings of the First National Conference held by the Center for Personalized Instruction.


WORKS RELATED TO THE APPENDIX C.


12. CBP Modules. University of Nebraska. Lincoln; Nebraska. Used for the Study Units of the final section of each Chapter of this study.
KNOWLEDGE

1.0 KNOWLEDGE
Knowledge, as defined here, involves the recall of specifics and universals, the recall of methods and procedures, or the recall of a pattern, structure, or setting. For measurement purposes little more than bringing to mind the appropriate material. Although some alteration of the material may be required, this is a relatively minor part of the task. The knowledge objective emphasizes the processes of remembering. The process of relating is also involved in that a knowledge test situation requires the organization of a communication of a problem such that it will furnish the appropriate points and cues for the information and knowledge the individual possesses. To use an analogy, if one thinks of the mind as a file, a problem in a knowledge test situation is that of finding in the problem or task the appropriate points, cues, and clues which will most effectively bring out whatever knowledge is stored.

1.1 KNOWLEDGE OF SPECIFICS
The recall of specific and identifiable bits of information. The emphasis is on symbols with concrete referents. This material, which is at a very low level of abstraction, may be thought of as the elements from which more complex and abstract forms of knowledge are built.

1.2 KNOWLEDGE OF TERMINOLOGY
Knowledge of the terms used in a particular field, the general meaning of the terms, and the possible interpretations that can be given the terms. This enterprise not only emphasizes the knowledge of the symbols of a knowledge domain, but also the knowledge of the symbols of the knowledge domain.

1.3 KNOWLEDGE OF CONVENTIONS
Knowledge of the rules, styles, protocols, and formats used in a particular field. In particular, the focus is on the conventions for communicating in a specific domain.

1.4 KNOWLEDGE OF SPECIFIC FACTS
Knowledge of the specific data or exact magnitude of a phenomenon. This knowledge is used when making precise predictions or calculations.

1.5 KNOWLEDGE OF WAYS AND MEANS OF DEALING WITH SPECIFICS
Knowledge of the ways of organizing, studying, and critically examining the specific details of a problem. This includes the methods of investigation, the chronological sequence of events, and the disturbance in the field as well as the patterns of organization through which the mass of the fields themselves are generated and organized internally. The knowledge objective emphasizes the processes of remembering, organizing, and the role of the individual's knowledge of the nature of the concepts involved.

1.6 KNOWLEDGE OF TRENDS AND SEQUENCES
Knowledge of the process, directions, and movements of phenomena with respect to time.

1.7 KNOWLEDGE OF CLASSIFICATIONS AND CATEGORIES
Knowledge of the classes, sets, divisions, and groupings which are regarded as fundamental for a given subject field, purpose, argument, or problem.

1.8 KNOWLEDGE OF CRITERIA
Knowledge of the criteria by which facts, principles, opinions, and conclusions are tested or judged.

1.9 KNOWLEDGE OF METODOLOGY
Knowledge of the methods of inquiry, techniques, and procedures employed in a particular subject field as well as those employed in investigating particular problems and phenomena. This emphasizes the process of making a problem description and selecting a method of attack.

30 KNOWLEDGE OF THE UNIVERSALS AND ABSTRACTIONS IN A FIELD
Knowledge of the major schemes and patterns by which phenomena and ideas are organized. These are the structures, theories, and generalizations which dominate a subject field or which are particularly useful in studying phenomena or problems. These are the highest levels of abstraction and complexity.

1.10 KNOWLEDGE OF PRINCIPLES AND GENERALIZATIONS
Knowledge of particular abstractions which summarize observations of phenomena. These are the abstractions which are of value in explaining, describing, predicting, or in determining the causal relationships and relevant actions or actions to be taken.

1.11 KNOWLEDGE OF THEORIES AND STRUCTURES
Knowledge of the body of principles and generalizations together with the relationships which present a time, ranking and systematic view of a complex phenomenon, problem, or field. These are the most abstract formulations, and they can be used to show the interrelation and organization of a great expanse of knowledge.

INTELLECTUAL ABILITIES AND SKILLS

Abilities and skills refer to organized modes of operation and generalized techniques for dealing with materials and problems. The materials and problems may be of such a nature that little or no specialized and technical information is required. Such information as is required can be assumed to be part of the general fund of knowledge. Other problems may require specialized and technical information at a rather high level such that specific knowledge and skill in dealing with the problem and the materials are required. These abilities and skills objectives emphasize the mental processes of organizing and reorganizing material to achieve a particular purpose. The materials may be given or remembered.

2.0 COMPREHENSION
Comprehension is evidenced by the ease and accuracy with which the communication is paraphrased or rendered from one language or form of communication to another. Translation is based on the basis of familiarity and accuracy, that is, on the extent to which the material in the original communication is preserved, although the form of the communication has been altered.

2.1 TRANSLATION
Comprehension is evidenced by the ease and accuracy with which the communication is paraphrased or rendered from one language or form of communication to another. Translation is based on the basis of familiarity and accuracy, that is, on the extent to which the material in the original communication is preserved, although the form of the communication has been altered.

2.2 INTERPRETATION
Interpretation is the process of understanding. Interpretation involves the objective part-part-part rendering of a communication. Interpretation involves a rendering, rearrangement, or a reorganization of the material.

2.3 EXTRAPOLATION
The extension of trends or tendencies beyond the given data to determine implications, consequences, corollaries, effects, etc., which are in accordance with the conditions described in the original communication.

3.0 APPLICATION
The use of abstractions in particular and concrete situations. The abstractions may be in the form of theoretical or practical material. The abstractions may be used to make use of the material or idea both internally and externally without necessarily relating it to other material or seeing its fullest implications.

4.0 ANALYSIS
Analysis is the process of a breakdown into its constituent elements or parts such that the relative hierarchy of ideas is more clear and/or the relationships between the ideas expressed are more explicit. Such analyses are intended to clarify the nature of an abstraction, to indicate how the communication is organized, and the way in which it manages to convey its effects, as well as its basis and arrangement.

4.1 ANALYSIS OF ELEMENTS
Analysis of the elements included in a communication.

4.2 ANALYSIS OF RELATIONSHIPS
The connections and interactions between elements and parts of a communication.

4.3 ANALYSIS OF ORGANIZATIONAL PRINCIPLES
The organizational, systematic, arrangement, and structure which hold the communication together. This includes the "explicit" as well as the "implicit" structure. It includes the basis, necessarily arrangement, and the mechanisms which make the communication a unit.

5.0 SYNTHESIS
The putting together of elements and parts to form a whole. This involves the process of working with pieces, parts, elements, etc., arranging and combining them in such a way as to constitute a pattern or structure of the abstractions.

5.1 PRODUCTION OF A UNIQUE COMMUNICATION
The development of a communication in which the writer or speaker attempts to convey ideas, feelings, and/or experiences to others.

5.2 PRODUCTION OF A PLAN, OR PROPOSED SET OF OPERATIONS
The development of a plan of work or the proposal of a plan of operations. The plan should specify requirements of the task which may be given to the student or which he may develop for himself.

5.3 DERIVATION OF A SET OF ABSTRACT RELATIONS
The development of a set of abstract relations to clarify or express particular data or phenomena, or the translation of propositions and relations from a set of base propositions and symbolic notations.

6.0 EVALUATION
Judgments about the value of material and methods for given purposes. Quantitative and qualitative judgments about the extent to which material and methods satisfy criteria. These judgments are based on the belief that they may be determined by the student or those which are given to him.

6.1 JUDGMENTS IN TERMS OF INTERNAL EVIDENCE
Judgments of the accuracy of a communication from such evidence as logical accuracy, consistency, and other internal criteria.

6.2 JUDGMENT IN TERMS OF EXTERNAL CRITERIA
Judgments of material with reference to selected or remembered criteria.
## APPENDIX A.2

### DRUMHELLER'S MODIFICATION

Structure of a Modified Taxonomy of Educational Objectives

<table>
<thead>
<tr>
<th>MAJOR CATEGORIES</th>
<th>SUB-CATEGORIES</th>
<th>THE LEARNER WILL VERBALLY RECALL:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 KNOWLEDGE:</td>
<td>KNOWLEDGE OF</td>
<td>KNOWLEDGE OF</td>
</tr>
<tr>
<td>(Development of</td>
<td>SPECIFICS</td>
<td>WAYS &amp; MEANS OF</td>
</tr>
<tr>
<td>ability to recall</td>
<td>KNOWLEDGE</td>
<td>DEALING WITH</td>
</tr>
<tr>
<td>appropriate</td>
<td>OF SPECIFICS</td>
<td>SPECIFICS</td>
</tr>
<tr>
<td>information, with</td>
<td>KNOWLEDGE OF</td>
<td>KNOWLEDGE OF</td>
</tr>
<tr>
<td>or without</td>
<td>CLASSIFICATIONS</td>
<td>UNIVERSALS &amp;</td>
</tr>
<tr>
<td>comprehension)</td>
<td>&amp; SEQUENCES</td>
<td>ABSTRACTIONS</td>
</tr>
<tr>
<td></td>
<td>KNOWLEDGE OF</td>
<td>IN A FIELD</td>
</tr>
<tr>
<td></td>
<td>CRITERIA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>KNOWLEDGE OF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>METHODOLOGY</td>
<td></td>
</tr>
<tr>
<td></td>
<td>KNOWLEDGE OF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GENERALIZATIONS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>KNOWLEDGE OF</td>
<td></td>
</tr>
<tr>
<td></td>
<td>THEORIES AND</td>
<td></td>
</tr>
<tr>
<td></td>
<td>STRUCTURES</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2.0 COMPREHENSION:</th>
<th>The learner will give evidence of comprehension through:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Degree to which</td>
<td>2.1 Translation</td>
</tr>
<tr>
<td>one should be able</td>
<td>2.2 Interpretation</td>
</tr>
<tr>
<td>to manipulate</td>
<td>2.3 Extrapolation beyond data</td>
</tr>
<tr>
<td>meaningfully</td>
<td></td>
</tr>
<tr>
<td>Knowledge elements)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3.0 APPLICATION:</th>
<th>The learner will perform complex tasks in Application situations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Development of</td>
<td></td>
</tr>
<tr>
<td>complex functional</td>
<td></td>
</tr>
<tr>
<td>behaviors)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.0 ANALYSIS:</th>
<th>The learner will identify:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Development of</td>
<td>4.1 Elements</td>
</tr>
<tr>
<td>ability to identify</td>
<td>4.2 Relationships</td>
</tr>
<tr>
<td>relevant knowledge components in</td>
<td>4.3 Organizational principles relevant to specified application behavior</td>
</tr>
<tr>
<td>Application situations)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5.0 SYNTHESIS:</th>
<th>The learner will combine appropriate knowledge elements into the:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Development of</td>
<td>5.1 Production of a unique communication</td>
</tr>
<tr>
<td>ability to combine</td>
<td>5.2 Production of a plan or proposed set of operations</td>
</tr>
<tr>
<td>knowledge elements in fabricating appropriate Application behaviors)</td>
<td>5.3 Derivation of a set of abstract relations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6.0 EVALUATION:</th>
<th>The learner will appraise a communication or problem solution with:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Development of</td>
<td>6.1 Judgments in terms of internal evidence</td>
</tr>
<tr>
<td>ability to appraise</td>
<td>6.2 Judgments in terms of external criteria</td>
</tr>
<tr>
<td>the appropriateness of a proposed or applied Application behavior)</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX A.3

VERB LIST

SOME POSSIBLE VERBS FOR USE IN STATING COGNITIVE OUTCOMES

Bloom’s Taxonomy of Educational Objectives

<table>
<thead>
<tr>
<th>Cognitive</th>
<th>To promote abilities in thought and understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge</td>
<td>To recall and recognize.</td>
</tr>
<tr>
<td>Comprehension</td>
<td>To translate from one form to another.</td>
</tr>
<tr>
<td>Application</td>
<td>To apply or use information in a new situation.</td>
</tr>
<tr>
<td>Analysis</td>
<td>To examine a complex and break it down into its parts</td>
</tr>
<tr>
<td>Synthesis</td>
<td>To put together information in a unique or novel way to solve a problem.</td>
</tr>
<tr>
<td>Evaluation</td>
<td>To make a judgment about something in light of some criteria.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Knowledge</th>
<th>Comprehension</th>
<th>Application</th>
<th>Analysis</th>
<th>Synthesis</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>define</td>
<td>translate</td>
<td>interpret</td>
<td>distinguish</td>
<td>compose</td>
<td>judge</td>
</tr>
<tr>
<td>repeat</td>
<td>restate</td>
<td>apply</td>
<td>analyze</td>
<td>plan</td>
<td>appraise</td>
</tr>
<tr>
<td>record</td>
<td>discuss</td>
<td>employ</td>
<td>differentiate</td>
<td>propose</td>
<td>conclude</td>
</tr>
<tr>
<td>list</td>
<td>describe</td>
<td>use</td>
<td>appraise</td>
<td>design</td>
<td>rate</td>
</tr>
<tr>
<td>recall</td>
<td>explain</td>
<td>demonstrate</td>
<td>calculate</td>
<td>formulate</td>
<td>compare</td>
</tr>
<tr>
<td>name</td>
<td>express</td>
<td>dramatize</td>
<td>experiment</td>
<td>arrange</td>
<td>value</td>
</tr>
<tr>
<td>relate</td>
<td>identify</td>
<td>practice</td>
<td>test</td>
<td>assemble</td>
<td>revise</td>
</tr>
<tr>
<td>underline</td>
<td>locate</td>
<td>illustrate</td>
<td>compare</td>
<td>collect</td>
<td>score</td>
</tr>
<tr>
<td>recite</td>
<td>report</td>
<td>operate</td>
<td>contrast</td>
<td>construct</td>
<td>select</td>
</tr>
<tr>
<td>state</td>
<td>review</td>
<td>schedule</td>
<td>criticize</td>
<td>create</td>
<td>choose</td>
</tr>
<tr>
<td>give</td>
<td>tell</td>
<td>show</td>
<td>diagram</td>
<td>set up</td>
<td>assess</td>
</tr>
<tr>
<td>example</td>
<td></td>
<td>sketch</td>
<td>inspect</td>
<td>organize</td>
<td>estimate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>draw</td>
<td>debate</td>
<td>manage</td>
<td>measure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>determine</td>
<td>inventory</td>
<td>prepare</td>
<td>criticize</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>question</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>relate</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>solve</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>examine</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>categorize</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

WORDS WITH MEANING UNOBSERVABLE

Know  really understand
understand fully appreciate
appreciate have mastery of
discover acquire skills in
evaluate grasp significance of
comprehend develop understanding
realize have learned about
recognize be knowledgeable about
achieve mastery
APPENDIX B.

CONTENTS OF PHYSICS 53 PSI COURSE

MECHANICS

PART I: POINT-LIKE BODIES

Unit I: DIMENSIONS AND UNITS

Science and Physics
Fundamentals and Derived Physical Quantities
Dimensions and Systems of Units
Dimensional Consistency
Order of Magnitude
Conversion Factors

Unit II: BASIC KINEMATICS CONCEPTS

Position, Change of Position, and Distance
Traveled for a Body in Rectilinear Motion
Average Velocity, Velocity and Speed
Average Acceleration and Acceleration
Uniform Motion and Motion with Constant Acceleration

Unit III: MOTION IN ONE DIMENSION

Kinematics—Momentum—Energy

Instantaneous Velocity and Acceleration
Uniform Motion, Motion with Constant Acceleration and Motion with Acceleration Proportional to Time by using Calculus
Momentum and Energy Associated with Motion
Free Fall and Conservation of Mechanical Energy
Unit IV : VECTORS

Simple Properties of Vectors
Rectangular and Polar Components of a Vector
Position, Velocity and Acceleration Vectors
Relative Velocity and Acceleration
Products between Vectors

Unit V : MOTION IN TWO DIMENSIONS

Kinematics and Energy of Ballistic Problems
Circular Motion : Uniform and Accelerated
Relation between Linear and Angular Variables
Collision on a Plane Surface

Unit VI : DYNAMICS I : NEWTON'S LAWS

Force and Mass
Newton's Laws
Weight and Gravitational Force
Elastic Force
Contact Forces
Work done by a Constant Force and The
Work-Energy Theorem.

Unit VII : FORCES ON SYSTEMS

Motion of Systems under the Action of Forces
Pseudoforces
Equilibrium and its Conditions

Unit VIII : MOMENTUM AND ITS CONSERVATION

Conservation of Momentum Center-of-
Momentum-System.
Impulse and Change of Momentum
Systems with Variable Mass

Unit IX : ENERGY AND ITS CONSERVATION

More on the Work Done by a Force and The
Work-Energy Theorem
Conservative and Nonconservative Forces
Fields
Potential Energy and its Graphical
Representation
Conservation Law of Total Energy
Power
PART II : RIGID BODIES

Unit X : ROTATIONS ABOUT A FIXED AXIS

Torque and Rotation
Rotation about a Fixed Axis : the Moment of Inertia
Motion of Rigid Bodies : the Center of Mass
Momentum and Energy of Rotation
Conservation of Angular Momentum

Unit XI : MOMENT OF INERTIA AND THE CENTER OF MASS

Calculations of the Moment of Inertia of Simple Rigid Bodies
The Parallel Axis Theorem.
Finding the Center of Mass

Unit XII : THE COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF RIGID BODIES

Dynamics Equations of Combined Motion in Two Dimensions
Rolling Motion of Spheres and Cylinders

Unit XIII : OSCILLATIONS

Periodic Motion and the Free Harmonic Oscillator
S.H.O. and U.C.M.
Energy Considerations in Simple Harmonic Motion
# Appendix C

**Study Units for Mechanics of Point-Like Bodies**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit I</td>
<td>Dimensions and Units</td>
</tr>
<tr>
<td>Unit II</td>
<td>Basic Kinematics Concepts</td>
</tr>
<tr>
<td>Unit III</td>
<td>One Dimensional Motion: Kinematics-Momentum-Energy</td>
</tr>
<tr>
<td>Unit IV</td>
<td>Vectors</td>
</tr>
<tr>
<td>Unit V</td>
<td>Motion in Two Dimensions: Kinematics-Momentum-Energy</td>
</tr>
<tr>
<td>Unit VI</td>
<td>Dynamics I: Newton's Laws</td>
</tr>
<tr>
<td>Unit VII</td>
<td>Dynamics II: Forces on Systems</td>
</tr>
<tr>
<td>Unit VIII</td>
<td>Momentum and Its Conservation</td>
</tr>
<tr>
<td>Unit IX</td>
<td>Energy and Its Conservation</td>
</tr>
<tr>
<td>Remedial Unit I</td>
<td>Trigonometry</td>
</tr>
</tbody>
</table>
REMEDIAL UNIT I

TRIGONOMETRY

Introduction

The aim of this unit is to help you to refresh your basic Trigonometry concepts. These will be used in the Physics course of Mechanics. You probably already know this material, and here you are only going to review it and recall some concepts that you may have forgotten.

Objectives

To be able to:

1. Convert degrees to radians and radians to degrees.
2. Recall the definitions of trigonometric functions in relation to the unit circle and to a right triangle.
3. Compute values of the trigonometric relations for 30°-60°-90°, 45°-45°-90° and 37°-53°-90° triangle.
4. Use trigonometric relations to find sides and angles of triangles.
5. Calculate values of the trigonometric functions for angles > π/2.
6. Use simple trigonometric identities.
Remedial Unit I

Suggested Procedure

1. Go over the self-check test without using any aid except the table of trigonometric functions given at the end of this Unit.

2. Verify your answers against the answer sheet preceding the last page. If you answered 90% of the self-check test items you can skip the rest of this Unit and go over Unit 1.

3. If you had a lower percentage, then go in detail over the trigonometric review of pages 5 through 11 of this Unit.

4. Resolve the Pretest on Page 12 and check your answers.

5. If you have had difficulties with the Pretest, please work through this Remedial Unit once more, with the assistance of any general mathematics book including a chapter on trigonometry. For instance, *Mathematical Preparation for General Physics* by J. B. Marion and R. C. Davidson. (W. B. Saunders Company); or *Algebra and Trigonometry*, Third Edition, by P. K. Rees, F. C. Sparks, and C. Sparks Rees. (McGraw-Hill Book Company).
Remedial Unit I  SELF-CHECK TEST

1. Without the use of tables, convert degrees to radians and radians to degrees:
   (a) \(30^\circ = \) \(\text{ rad}\);  
   (b) \((3/4)\pi \text{ rad} = \) \(\text{ o}\);  
   (c) \(225^\circ = \) \(\text{ rad}\);  
   (d) \((5/3)\pi \text{ rad} = \) \(\text{ o}\).

2. (a) What is the maximum value for the sine of an angle, cosine of an angle, and tangent of an angle? Give at least one angle that has the maximum value for the named function.
   (b) Which angle in Problem 1 has the largest value for the sine, for the cosine, and for the tangent, respectively?

3. One acute angle of a right triangle is \(37^\circ\). The length of the side opposite the angle is 12.0 cm.
   (a) What are the ratios of the lengths of the sides of this triangle?
   (b) For this triangle, find the lengths of the other two sides of the triangle. (Show your work!)

4. One acute angle of a right triangle is \(40^\circ\). The length of the hypotenuse is 12.0 cm. Find the lengths of the other two sides.

5. In a right triangle the hypotenuse is \(2\sqrt{3}\) and one side is 3.
   (a) Find the missing side.
   (b) What are the angles?

6. A surveyor wishes to determine the distance between two points A and B, but he cannot make a direct measurement because a river intervenes. He sets off at a \(90^\circ\) angle to AB a line AC, which
Remedial Unit I

He measures to be 264 m. He measures an angle with his transit at point C to point B. Angle BCA is measured to be 62°. With this information, calculate AB.

7. Show that, for any angle θ,

\[
\sin^2 \theta + \cos^2 \theta = 1.
\]
Remedial Unit I

TRIGONOMETRY REVIEW

There are many natural phenomena in branches such as optics, heat, electronics, x-rays, acoustics, seismology and many other areas that are described in terms of periodic functions called trigonometric functions (or circular functions).

The Unit Circle

The unit circle is the circle of radius one unit (1) with center (0,0) whose equation is \( x^2 + y^2 = 1 \). With each point \((x,y)\) on the unit circle we can associate an arc of length \( s \), starting at the point \((1,0)\) and ending at the point \((x,y)\).

Positive arc lengths are measured in a counterclockwise direction from \((1,0)\), while negative arc lengths are measured in a clockwise direction.

The circumference of the unit circle is \( 2\pi \). For each value of \( s \), we have a unique value of \( x \) and a unique value of \( y \). The function \( \{(s,y)\} \) which associates with each arc length \( s \) the \( y \)-value of the corresponding point, is defined as:
sine function: \( \{(x,y) \mid y = \sin s\} \)

Identically,

\[
\text{cosine function: } \{(s,x) \mid x = \cos s\}
\]

Angles and the Unit Circle

Associated with each arc of length \( s \) on the unit circle is an angle \( \theta \). Such an angle, with the positive x-axis as initial side, the ray OP as terminal side and the origin as vertex, is said to be in standard position. The angle is positive when \( s \) is positive and negative when \( s \) is negative. There are two common ways to measure the size of the angles: degrees and radians. Many of the problems of planar and rotational motion and waves will depend upon your knowledge of radian measure of angles. The number/radians of an angle is the arc length cut off by the angle in standard position divided by the radius, or

\[
\text{angle in radians} = \frac{\text{arc length}}{\text{radius}}
\]

\[
\theta = \frac{s}{r}
\]

or \( \theta = s \) in unit circle.

So, one radian (1) is the measure of a positive angle that intercepts an arc of length 1 on a circle of radius 1.

On the other hand, one degree (1°) is the measure of a positive angle which is formed by \( 1/360 \) of one complete revolution.

Derive a formula to convert back and forth between angular measurements in degree and in radians. Use your formula to convert \( \frac{\pi}{4} \) radians to degree measure.
Right Triangle

Many of the applications of physics will require you to have a thorough knowledge of the basic properties of right triangles, i.e., triangles that have one angle equal to $90^\circ$.

The trigonometric functions are defined with respect to a right triangle as follows:

\[ \sin \theta = \frac{y}{r} \]
\[ \cos \theta = \frac{x}{r} \]
\[ \tan \theta = \frac{y}{x} \]
\[ \tan \theta = \frac{\sin \theta}{\cos \theta} \]

The values of the trigonometric sine, cosine, and tangent functions for a given $\theta$ can be determined from a table such as in the appendix to your text or the last page of this unit. You can also get the values by use of most slide rules ("S" and "T" scales) and many electronic calculators.

The $30^\circ$-$60^\circ$-$90^\circ$ and $45^\circ$-$45^\circ$-$90^\circ$ Triangles. It is also useful to remember the values of the functions for $\theta = 30^\circ$, $45^\circ$, and $60^\circ$ by means of the triangles below.
These triangles are right triangles, and you should check that the sides indeed satisfy the Pythagorean theorem. Note also that in any right triangle the longest side is the hypotenuse.

Using the basic definitions and the above triangles, one finds

\[
\begin{align*}
\sin 30^\circ &= 1/2 = 0.500, \\
\sin 45^\circ &= 1/\sqrt{2} = \sqrt{2}/2 = 1.414/2 = 0.707..., \\
\sin 60^\circ &= \sqrt{3}/2 = 1.732/2 = 0.866, \\
\cos 30^\circ &= \sqrt{3}/2 = 1.732/2 - 0.866, \\
\cos 45^\circ &= 1/\sqrt{2} = \sqrt{2}/2 = 0.707, \\
\cos 60^\circ &= 1/2 = 0.500, \\
\tan 30^\circ &= 1/\sqrt{3} = \sqrt{3}/3 = 1.732/3 - 0.577, \\
\tan 45^\circ &= 1/1 = 1.000, \\
\tan 60^\circ &= \sqrt{3} = 1.732.
\end{align*}
\]

The 3-4-5 Triangle. The 3-4-5 triangle (since the sides are in the ratio of 3 : 4 : 5) is known as the 37°-53°-90° triangle:

\[
\begin{align*}
\sin 37^\circ &= 0.6, & \sin 53^\circ &= 0.8, \\
\cos 37^\circ &= 0.8, & \cos 53^\circ &= 0.6, \\
\tan 37^\circ &= 0.75, & \tan 53^\circ &= 1.33.
\end{align*}
\]

You should memorize these three special triangles so that you can compute the values of sin, cosine, and tangent for the angles involved.

Angles > 90°

When calculating the products of vectors (See Vector Unit), one must determine the sine and cosine of angles greater than 90°, whereas most trig tables list values only for the angles less than or equal to 90°. Two alternative ways of remembering the necessary
relationships are as follows:

**METHOD I**: Recall the definitions of sine and cosine for general angles:

\[
\sin \theta = \frac{y}{r} \quad \text{and} \quad \cos \theta = \frac{x}{r},
\]

where \(x\) and \(y\) are the horizontal and vertical projections, respectively, of the radial distance \(r\), as shown in the figures below for angles in the various quadrants.

![Diagram](https://via.placeholder.com/150)

+---+
<table>
<thead>
<tr>
<th>r</th>
<th>y</th>
</tr>
</thead>
</table>
| x |---+---+

**(a)**

+---+
| r |
|---+---+
| y | x |

**(b)**

+---+
| r |
|---+---+
| y |

**(c)**

+---+
| r |
|---+---+
| y | x |

**(d)**

Figure 1
METHOD II: Recall the graphs of \( \sin \theta \) and \( \cos \theta \):

![Graph of \( \sin \theta \)](image)

![Graph of \( \cos \theta \)](image)

**Figure 2**

**Example**

Find \( \sin \theta \) and \( \cos \theta \) where \( \theta = 150^\circ = (180^\circ - 30^\circ) \).

**Solution I:** Comparison of Figures 1(a) and 1(b) together with the definitions of sine and cosine, shows that \( \sin 150^\circ = \sin 30^\circ \), \( \cos 150^\circ = -\cos 30^\circ \). Then \( \sin 30^\circ \) and \( \cos 30^\circ \) can be looked up in a table or on a slide rule (or, for this example, easily computed).

**Solution II:** Inspection of Figure 2(a) shows that \( \sin 150^\circ = \sin 30^\circ \); inspection of Figure 2(b) shows that \( \cos 150^\circ = -\cos 30^\circ \). The sine and cosine of \( 30^\circ \) are determined as in Solution I.
Useful Trigonometric Identities

In the solution of many problems in physics you may need to use a trigonometric identity. Listed below are some of the most useful ones:

\[ \sin^2 \theta + \cos^2 \theta = 1, \quad (1) \]
\[ \sin (A \pm B) = (\sin A)(\cos B) \pm (\sin B)(\cos A), \quad (2) \]
\[ \cos (A \pm B) = (\cos A)(\cos B) \mp (\sin A)(\sin B). \quad (3) \]

The tangent of the addition of angles follows directly from here.

You can develop the relationships for the sine and cosine of 2A by letting A equal B in Eqs. (2) and (3). Or the half-angle formulas can be derived by letting A = B = \( \theta \). Also, with the help of (1), (2) and (3) one can prove the inverse additional formulae.

\[ \sin A \pm \sin B = 2 \sin \left( \frac{A \pm B}{2} \right) \cos \left( \frac{A \pm B}{2} \right) \quad (4) \]
\[ \cos A + \cos B = 2 \cos \left( \frac{A + B}{2} \right) \cos \left( \frac{A - B}{2} \right) \quad (5) \]
\[ \cos A - \cos B = 2 \sin \left( \frac{A + B}{2} \right) \sin \left( \frac{A - B}{2} \right) \quad (6) \]

There are two formulae valid for any general triangle:

Law of sines:

\[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (7) \]

Law of cosines:

\[ c^2 = a^2 + b^2 - 2ab \cos C \quad (8) \]
Remedial Unit I

PRETEST

1. Convert the following angles to radian measures and give their sine, cosine, and tangent values:
   (a) 60°; (b) 53°; (c) 37°.

2. Find the unknowns of triangle A:
   \[ B = \frac{10}{7.0} \]
   \[ a = \frac{3.0}{10.0} \]
   \[ b = \frac{10}{7.0} \]

3. One acute angle of a right angle is 20°. The length of the hypotenuse is 6.0 in. Use trigonometry to calculate the lengths of the two sides.

4. In a 45°-45°-90° right triangle, what is the ratio of the hypotenuse to the sides?

5. State what is the value of \( a \) in the triangle without using a trigonometry table or the Pythagorean theorem.

6. A car was traveling exactly northeast. If it went a total distance of 42.4 km, how far north had it actually gone?
Remedial Unit I  SELF-CHECK TEST ANSWERS

1.  (a) $\pi/6$ rad;  (b) $135^\circ$;  (c) $5\pi/4$ rad;  (d) $300^\circ$.
2.  (a) $\sin \theta = 1$, $\theta = \pi/2$ or $90^\circ$; $\cos \theta = 1$, $\theta = 0$ or $0^\circ$;
    $\tan \theta \to \infty$, $\theta \to \pi/2$ or $90^\circ$.
3.  (a) The sides are in the ratio of $3 : 4 : 5$.  (b) 20 cm, 16 cm.
4.  7.7 cm, 9.2 cm.
5.  (a) $\sqrt{3}$,  (b) $30^\circ$, $60^\circ$, $90^\circ$; one side = (1/2) hypotenuse.
6.  497 m.
7.  $x/r = \cos \theta$, $y/r = \sin \theta$, $x^2 + y^2 = r^2$;
    or
    $x^2/r^2 + y^2/r^2 = 1$.
    Thus, $\cos^2 \theta + \sin^2 \theta = 1$. 
### NATURAL TRIGONOMETRIC FUNCTIONS

<table>
<thead>
<tr>
<th>Angle (Degrees)</th>
<th>Sine</th>
<th>Cosine</th>
<th>Tangent</th>
<th>Angle (Degrees)</th>
<th>Sine</th>
<th>Cosine</th>
<th>Tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>45°</td>
<td>0.707</td>
<td>0.707</td>
<td>1.000</td>
</tr>
<tr>
<td>1°</td>
<td>0.017</td>
<td>0.999</td>
<td>0.017</td>
<td>47°</td>
<td>0.731</td>
<td>0.682</td>
<td>1.062</td>
</tr>
<tr>
<td>2°</td>
<td>0.035</td>
<td>0.999</td>
<td>0.035</td>
<td>49°</td>
<td>0.743</td>
<td>0.669</td>
<td>1.141</td>
</tr>
<tr>
<td>3°</td>
<td>0.052</td>
<td>0.999</td>
<td>0.052</td>
<td>51°</td>
<td>0.755</td>
<td>0.656</td>
<td>1.176</td>
</tr>
<tr>
<td>4°</td>
<td>0.070</td>
<td>0.998</td>
<td>0.070</td>
<td>53°</td>
<td>0.766</td>
<td>0.643</td>
<td>1.202</td>
</tr>
<tr>
<td>5°</td>
<td>0.087</td>
<td>0.996</td>
<td>0.087</td>
<td>55°</td>
<td>0.777</td>
<td>0.630</td>
<td>1.228</td>
</tr>
<tr>
<td>6°</td>
<td>0.105</td>
<td>0.995</td>
<td>0.105</td>
<td>57°</td>
<td>0.788</td>
<td>0.616</td>
<td>1.253</td>
</tr>
<tr>
<td>7°</td>
<td>0.122</td>
<td>0.993</td>
<td>0.123</td>
<td>59°</td>
<td>0.799</td>
<td>0.603</td>
<td>1.278</td>
</tr>
<tr>
<td>8°</td>
<td>0.140</td>
<td>0.990</td>
<td>0.141</td>
<td>61°</td>
<td>0.810</td>
<td>0.590</td>
<td>1.302</td>
</tr>
<tr>
<td>9°</td>
<td>0.157</td>
<td>0.988</td>
<td>0.158</td>
<td>63°</td>
<td>0.821</td>
<td>0.577</td>
<td>1.326</td>
</tr>
<tr>
<td>10°</td>
<td>0.175</td>
<td>0.985</td>
<td>0.176</td>
<td>65°</td>
<td>0.832</td>
<td>0.564</td>
<td>1.350</td>
</tr>
<tr>
<td>11°</td>
<td>0.192</td>
<td>0.982</td>
<td>0.194</td>
<td>67°</td>
<td>0.842</td>
<td>0.551</td>
<td>1.374</td>
</tr>
<tr>
<td>12°</td>
<td>0.209</td>
<td>0.978</td>
<td>0.213</td>
<td>69°</td>
<td>0.852</td>
<td>0.539</td>
<td>1.397</td>
</tr>
<tr>
<td>13°</td>
<td>0.227</td>
<td>0.974</td>
<td>0.231</td>
<td>71°</td>
<td>0.862</td>
<td>0.526</td>
<td>1.421</td>
</tr>
<tr>
<td>14°</td>
<td>0.244</td>
<td>0.970</td>
<td>0.249</td>
<td>73°</td>
<td>0.872</td>
<td>0.514</td>
<td>1.444</td>
</tr>
<tr>
<td>15°</td>
<td>0.262</td>
<td>0.966</td>
<td>0.268</td>
<td>75°</td>
<td>0.882</td>
<td>0.501</td>
<td>1.467</td>
</tr>
<tr>
<td>16°</td>
<td>0.279</td>
<td>0.961</td>
<td>0.287</td>
<td>77°</td>
<td>0.892</td>
<td>0.489</td>
<td>1.490</td>
</tr>
<tr>
<td>17°</td>
<td>0.297</td>
<td>0.956</td>
<td>0.306</td>
<td>79°</td>
<td>0.902</td>
<td>0.476</td>
<td>1.513</td>
</tr>
<tr>
<td>18°</td>
<td>0.314</td>
<td>0.951</td>
<td>0.325</td>
<td>81°</td>
<td>0.912</td>
<td>0.464</td>
<td>1.536</td>
</tr>
<tr>
<td>19°</td>
<td>0.332</td>
<td>0.946</td>
<td>0.344</td>
<td>83°</td>
<td>0.922</td>
<td>0.452</td>
<td>1.559</td>
</tr>
<tr>
<td>20°</td>
<td>0.349</td>
<td>0.941</td>
<td>0.364</td>
<td>85°</td>
<td>0.932</td>
<td>0.440</td>
<td>1.582</td>
</tr>
<tr>
<td>21°</td>
<td>0.367</td>
<td>0.938</td>
<td>0.384</td>
<td>87°</td>
<td>0.942</td>
<td>0.429</td>
<td>1.604</td>
</tr>
<tr>
<td>22°</td>
<td>0.384</td>
<td>0.935</td>
<td>0.404</td>
<td>89°</td>
<td>0.952</td>
<td>0.417</td>
<td>1.627</td>
</tr>
<tr>
<td>23°</td>
<td>0.401</td>
<td>0.932</td>
<td>0.424</td>
<td>91°</td>
<td>0.962</td>
<td>0.406</td>
<td>1.650</td>
</tr>
<tr>
<td>24°</td>
<td>0.419</td>
<td>0.929</td>
<td>0.445</td>
<td>93°</td>
<td>0.972</td>
<td>0.394</td>
<td>1.672</td>
</tr>
<tr>
<td>25°</td>
<td>0.436</td>
<td>0.926</td>
<td>0.466</td>
<td>95°</td>
<td>0.982</td>
<td>0.382</td>
<td>1.695</td>
</tr>
<tr>
<td>26°</td>
<td>0.454</td>
<td>0.923</td>
<td>0.488</td>
<td>97°</td>
<td>0.992</td>
<td>0.370</td>
<td>1.717</td>
</tr>
<tr>
<td>27°</td>
<td>0.471</td>
<td>0.920</td>
<td>0.510</td>
<td>99°</td>
<td>1.002</td>
<td>0.359</td>
<td>1.739</td>
</tr>
<tr>
<td>28°</td>
<td>0.489</td>
<td>0.918</td>
<td>0.532</td>
<td>101°</td>
<td>1.012</td>
<td>0.348</td>
<td>1.760</td>
</tr>
<tr>
<td>29°</td>
<td>0.506</td>
<td>0.915</td>
<td>0.554</td>
<td>103°</td>
<td>1.022</td>
<td>0.336</td>
<td>1.781</td>
</tr>
<tr>
<td>30°</td>
<td>0.524</td>
<td>0.912</td>
<td>0.577</td>
<td>105°</td>
<td>1.032</td>
<td>0.325</td>
<td>1.802</td>
</tr>
<tr>
<td>31°</td>
<td>0.541</td>
<td>0.909</td>
<td>0.600</td>
<td>107°</td>
<td>1.042</td>
<td>0.314</td>
<td>1.823</td>
</tr>
<tr>
<td>32°</td>
<td>0.559</td>
<td>0.906</td>
<td>0.623</td>
<td>109°</td>
<td>1.052</td>
<td>0.302</td>
<td>1.844</td>
</tr>
<tr>
<td>33°</td>
<td>0.576</td>
<td>0.903</td>
<td>0.646</td>
<td>111°</td>
<td>1.062</td>
<td>0.291</td>
<td>1.865</td>
</tr>
<tr>
<td>34°</td>
<td>0.593</td>
<td>0.900</td>
<td>0.669</td>
<td>113°</td>
<td>1.072</td>
<td>0.279</td>
<td>1.885</td>
</tr>
<tr>
<td>35°</td>
<td>0.611</td>
<td>0.898</td>
<td>0.692</td>
<td>115°</td>
<td>1.082</td>
<td>0.268</td>
<td>1.905</td>
</tr>
<tr>
<td>36°</td>
<td>0.628</td>
<td>0.895</td>
<td>0.715</td>
<td>117°</td>
<td>1.092</td>
<td>0.256</td>
<td>1.925</td>
</tr>
<tr>
<td>37°</td>
<td>0.646</td>
<td>0.893</td>
<td>0.738</td>
<td>119°</td>
<td>1.102</td>
<td>0.245</td>
<td>1.945</td>
</tr>
<tr>
<td>38°</td>
<td>0.664</td>
<td>0.890</td>
<td>0.761</td>
<td>121°</td>
<td>1.112</td>
<td>0.233</td>
<td>1.965</td>
</tr>
<tr>
<td>39°</td>
<td>0.682</td>
<td>0.887</td>
<td>0.784</td>
<td>123°</td>
<td>1.122</td>
<td>0.222</td>
<td>1.985</td>
</tr>
<tr>
<td>40°</td>
<td>0.700</td>
<td>0.885</td>
<td>0.807</td>
<td>125°</td>
<td>1.132</td>
<td>0.210</td>
<td>2.005</td>
</tr>
<tr>
<td>41°</td>
<td>0.716</td>
<td>0.883</td>
<td>0.829</td>
<td>127°</td>
<td>1.142</td>
<td>0.200</td>
<td>2.024</td>
</tr>
<tr>
<td>42°</td>
<td>0.733</td>
<td>0.882</td>
<td>0.852</td>
<td>129°</td>
<td>1.152</td>
<td>0.191</td>
<td>2.043</td>
</tr>
<tr>
<td>43°</td>
<td>0.750</td>
<td>0.880</td>
<td>0.875</td>
<td>131°</td>
<td>1.162</td>
<td>0.181</td>
<td>2.062</td>
</tr>
<tr>
<td>44°</td>
<td>0.768</td>
<td>0.878</td>
<td>0.898</td>
<td>133°</td>
<td>1.172</td>
<td>0.171</td>
<td>2.081</td>
</tr>
<tr>
<td>45°</td>
<td>0.785</td>
<td>0.876</td>
<td>1.000</td>
<td>135°</td>
<td>1.182</td>
<td>0.158</td>
<td>2.098</td>
</tr>
</tbody>
</table>

Remedial Unit I: Trigonometry
UNIT I

UNITS - DIMENSIONS

Introduction

The first two Units of this course deal with some elementary matters that may partially review your preliminary physics course. As you can remember Physics is the science that studies natural phenomena and attempts to explain them in terms of a limited number of laws. The study of a simple physical observable such as the length of a pencil, to the study of a complex phenomenon such as nuclear fission, requires that man makes a series of measurements. That is, the experimental observation implies the actual measurement of certain physical observables.

You hear daily of measurement and its units—your height: 6 ft. 1 in.; your weight: 180 lbs; Stockton - San Francisco distance: 90 miles; maximum speed limit on highways: 55 mi/hr; average electric energy spent daily at home: 7.2 KWH; etc. The knowledge you can get about measurements, units, systems of units, dimensions of physical quantities, and dimensional analysis will enable you to feel more confident as you study later units and work on problems. Also, you will sometimes find it useful to calculate approximate solutions of a problem in terms of order of magnitude, before you make accurate operations. For all preceding reasons this unit is mainly concerned with Units and Dimensions of physical observables.
Learning Objectives

After you have achieved mastery of the content of this unit, you will be able to:

1. Describe the different systems of units used in mechanics by naming the fundamental quantities and their units.

2. Convert values of physical quantities from one system of units to another by using a table of Conversion Factors.

3. Determine whether or not a given equation is dimensionally consistent by using the fundamental dimensions or the units of all quantities involved in the equation.

4. Estimate the order of magnitude of physical quantities and express them by using the prefixes of powers of ten.

NOTE: If you think that you are already competent on these four objectives, you can skip directly to the Model Test (Pretest). You grade it yourself, and your score has to be 90% or better before you attempt taking the Test. The first two units have been purposely made somewhat less demanding than later units, and you may have already encountered this material. Try the Test of the Unit as soon as possible. The advantage of this method of instruction is that you do not have to spend any more time studying than necessary for you to attain mastery; on the other hand, taking a competency test before you are reasonably sure that you can pass is wasteful of both your time and the instructor's time.

1. You should read all Chapter 1 for general information. Study carefully Sections 3 and 4, Chapter 1, on Measurements, Units, and Dimensions of Physical Observables. In these two Sections are the necessary materials to accomplish all four objectives.

2. Objective 1 is partially satisfied with eqn (1-1) and context comments on page 7, page 9, and Appendix A on page 657. You should read the Supplementary Annotations following this Procedure in this Unit. It is worth your time to read: Ford*, Sections 2-1 through 2-8 (pp. 16-30); and Halliday-Resnick*, Section 1-5. Objective 2 is operationally shown in Examples 1 and 2 on pp. 9 and 10 of the text. See also Appendix B on p. 658 Conversion Factors. One of the best Conversion Factors Tables is in the Appendix H of David Halliday and Robert Resnick, Physics for Students of Science and Engineering, Combined Edition, (New York: John Wiley & Sons, Inc., 1965).

3. On p. 8 of ML* there is a discussion and an example on how to use dimensions to check out the correct relationship between the physical variables in an equation. This holds to accomplish objective 3. Also, you should read carefully H-R* Section 3-9 (pp. 35-36).
These are helpful for achieving objective 4.

5. After studying the text, read the Supplementary Annotations and study the Worked Problems attached to this Unit.
Make sure you work out your own answers and check them with those provided. The number in parenthesis heading each Worked Problem refers to the objective that is exemplified by the problem. If you have trouble with one or more of these problems, re-read the appropriate sections of the text and the Supplementary Annotations. Solve problems 1, 2, 3, 4, 9 and 12 on pp. 13-14 at the end of Chapter 1 of the textbook.

6. When you feel prepared, try the Pretest which has problems similar to the real Test. In the Pretest, the number of the problem or the number in parenthesis, when it appears, refers to the objective that is supposed to be evaluated by the problem. If you fail one or more of the problems in the Pretest go over it again in relation to objective material. You may wish to work an additional set of problems. Ask your proctor or instructor for them.

* The symbols used for bibliography in this unit and later units are as follows:


Unit I  
SUPPLEMENTARY ANNOTATIONS

The term Science distinguishes the areas of human knowledge which are based on experimental observation.

Physics is the science that correlates a set of experimental observations of the natural universe with the results of other observations, either made or yet to be made.

Measurement of a physical observation involves having a unit of measure and an instrument for comparison.

Physical quantities are divided into fundamental quantities and derived quantities. The fundamental quantities (often called undefinables) are the minimum numbers needed to give a consistent and unambiguous description of all the observables of physics.

The fundamental quantities are sometimes defined operationally by first, choosing a standard; and second, establishing procedures for obtaining units (multiples and submultiples of the standard) of the quantity. Dimensions are assigned to the fundamental quantities to use them in deriving other quantities because of the interrelation of the physical observables.

Dimensional Consistency for a physical equation means that each term in the equation has the same fundamental dimension.

Order of Magnitude of a physical observable is the power of ten more nearly to the value of the quantity.
# Unit I

## PREFIXES EQUIVALENT TO POWERS OF 10

<table>
<thead>
<tr>
<th>Power of 10</th>
<th>Symbol</th>
<th>Prefix</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12}$</td>
<td>T</td>
<td>tera</td>
</tr>
<tr>
<td>$10^9$</td>
<td>G</td>
<td>giga</td>
</tr>
<tr>
<td>$10^6$</td>
<td>M</td>
<td>mega</td>
</tr>
<tr>
<td>$10^3$</td>
<td>k</td>
<td>kilo</td>
</tr>
<tr>
<td>$10^2$</td>
<td>H</td>
<td>hekto</td>
</tr>
<tr>
<td>$10$</td>
<td>D</td>
<td>deka</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>d</td>
<td>deci</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>c</td>
<td>centi</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>m</td>
<td>milli</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>µ</td>
<td>micro</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>n</td>
<td>nano</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>p</td>
<td>pico</td>
</tr>
<tr>
<td>$10^{-15}$</td>
<td>f</td>
<td>femto</td>
</tr>
</tbody>
</table>

## SYSTEMS OF UNITS IN MECHANICS

<table>
<thead>
<tr>
<th>Fundamental Quantity</th>
<th>MKS (SI: Standard International)</th>
<th>CGS (Gaussian)</th>
<th>British (Engineering)</th>
<th>English System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Meter : m</td>
<td>Centimeter : cm</td>
<td>Yard : yd</td>
<td>Foot : ft</td>
</tr>
<tr>
<td>Length</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>Kilogram : kg</td>
<td>Gram : g</td>
<td>Slug : slug</td>
<td>Pound : lb</td>
</tr>
<tr>
<td>Time</td>
<td>Second : s</td>
<td>Second : s</td>
<td>Second : s</td>
<td>Second : s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## HYBRID UNITS

- **lb-m**: a mass that weighs one pound
- **oz (ounce)**: a mass that weighs one ounce
- **ton**: a mass that weighs one ton
- **g-f**: force exerted by the standard Earth's gravitational field over a mass of one gram.
- **kg-f**: force exerted by the standard Earth's gravitational field over a mass of one kilogram.
Unit I

SYMBOLS, DIMENSIONS, AND UNITS FOR PHYSICAL QUANTITIES IN MECHANICS

All units and dimensions are in the mks system. The primary units can be found by reading kilograms for M, meters for L, seconds for t, and coulombs for Q. The symbols are those used in the text.

<table>
<thead>
<tr>
<th>Physical Observable (Quantity)</th>
<th>Symbol</th>
<th>Dimensions</th>
<th>Derived MKS Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>a</td>
<td>LT⁻²</td>
<td>meter/second²</td>
</tr>
<tr>
<td>Angular acceleration</td>
<td>θ</td>
<td>T⁻²</td>
<td>radian/second²</td>
</tr>
<tr>
<td>Angular displacement, angle</td>
<td>θ</td>
<td></td>
<td>radian</td>
</tr>
<tr>
<td>Angular frequency and speed</td>
<td>ω</td>
<td>T⁻¹</td>
<td>radian/second</td>
</tr>
<tr>
<td>Angular momentum</td>
<td>L,J</td>
<td>ML²T⁻¹</td>
<td>kilogram-meter²/second</td>
</tr>
<tr>
<td>Area</td>
<td>A,S</td>
<td>L²</td>
<td>meter²</td>
</tr>
<tr>
<td>Displacement, position</td>
<td>r, d, s</td>
<td>L</td>
<td>meter</td>
</tr>
<tr>
<td>Energy, total</td>
<td>E</td>
<td>ML²T⁻²</td>
<td>joule = Newton-meter</td>
</tr>
<tr>
<td>kinetic</td>
<td>K,T</td>
<td>ML²T⁻²</td>
<td>joule = Newton-meter</td>
</tr>
<tr>
<td>potential</td>
<td>U</td>
<td>ML²T⁻²</td>
<td>joule = Newton-meter</td>
</tr>
<tr>
<td>Force</td>
<td>F</td>
<td>MLT⁻²</td>
<td>Newton = kilogram-meter/second</td>
</tr>
<tr>
<td>Frequency</td>
<td>f</td>
<td>T⁻¹</td>
<td>cycles/second = Hertz</td>
</tr>
<tr>
<td>Gravitational field strength</td>
<td>s</td>
<td>LT⁻²</td>
<td>Newton/kilogram</td>
</tr>
<tr>
<td>Gravitational potential</td>
<td>v</td>
<td>L²T⁻²</td>
<td>joules/kilogram</td>
</tr>
<tr>
<td>Length</td>
<td>l, s</td>
<td>L</td>
<td>meter</td>
</tr>
<tr>
<td>Mass</td>
<td>m</td>
<td>M</td>
<td>kilogram</td>
</tr>
<tr>
<td>Mass density</td>
<td>p</td>
<td>ML⁻³</td>
<td>kilogram/meter³</td>
</tr>
<tr>
<td>Momentum</td>
<td>p</td>
<td>MLT⁻¹</td>
<td>kilogram-meter/second</td>
</tr>
<tr>
<td>Period</td>
<td>T</td>
<td>T</td>
<td>second</td>
</tr>
<tr>
<td>Power</td>
<td>P</td>
<td>ML²T⁻³</td>
<td>watt = J/s</td>
</tr>
<tr>
<td>Pressure</td>
<td>p</td>
<td>ML⁻¹T⁻²</td>
<td>Newton/meter²</td>
</tr>
<tr>
<td>Rotational inertia</td>
<td>Π</td>
<td>ML²</td>
<td>kilogram-meter²</td>
</tr>
<tr>
<td>(Moment of inertia)</td>
<td></td>
<td></td>
<td>kg·m²</td>
</tr>
<tr>
<td>Solid angle</td>
<td>Ω</td>
<td>—</td>
<td>steradian</td>
</tr>
<tr>
<td>Time</td>
<td>t</td>
<td>T</td>
<td>second</td>
</tr>
<tr>
<td>Torque</td>
<td>τ</td>
<td>ML²T⁻²</td>
<td>Newton-meter</td>
</tr>
<tr>
<td>Velocity, speed</td>
<td>v, u</td>
<td>LT⁻¹</td>
<td>meters/second</td>
</tr>
<tr>
<td>Volume</td>
<td>V</td>
<td>L³</td>
<td>meter³</td>
</tr>
<tr>
<td>Wavelength</td>
<td>λ</td>
<td>L</td>
<td>meter</td>
</tr>
<tr>
<td>Work</td>
<td>W</td>
<td>ML²T⁻²</td>
<td>joule</td>
</tr>
<tr>
<td>Entropy</td>
<td>S</td>
<td>ML²T⁻²</td>
<td>joules/Kelvin degree</td>
</tr>
<tr>
<td>Internal energy</td>
<td>U</td>
<td>ML²T⁻²</td>
<td>joule</td>
</tr>
<tr>
<td>Heat</td>
<td>Q</td>
<td>ML²T⁻²</td>
<td>joule</td>
</tr>
<tr>
<td>Temperature</td>
<td>T</td>
<td>—</td>
<td>Kelvin degree</td>
</tr>
</tbody>
</table>
### Conversion Factors

**Length**
- 1 kilometer = 0.6214 mile
- 1 meter = 39.37 inches
- 1 inch = 2.54 centimeters

**Volume**
- 1 cubic meter = 35.31 cubic feet
- 1 cubic foot = 0.0283 cubic meters

**Mass**
- 1 kilogram = 2.2046 pounds
- 1 pound = 0.4536 kilograms

**Force**
- 1 newton = 0.2248 pound-force
- 1 pound-force = 4.448 newtons

**Energy**
- 1 joule = 0.2390 calorie
- 1 calorie = 4.184 joules

**Time**
- 1 second = 0.0167 minute
- 1 minute = 60 seconds

**Area**
- 1 square meter = 10.76 square feet
- 1 square foot = 0.0929 square meters

**Pressure**
- 1 atmosphere = 14.696 pounds per square inch (psi)
- 1 psi = 6894.76 Pascals

**Density**
- 1 kilogram per cubic meter = 0.6242 pounds per cubic foot
- 1 pound per cubic foot = 0.1602 kilograms per cubic meter

### Notes
- Some quantities in the table are not exact but are often used as such. When we speak of 1 meter, we usually mean 1.0000 meter, but we often use 1.0000 meter for convenience.
- The density of water at 4°C is 1.0000 gram per cubic centimeter, and at 25°C it is 0.9970 gram per cubic centimeter.

**Example Calculation**
- To convert 10 kilograms to pounds:
  
  $$10 \text{ kg} \times \frac{2.2046 \text{ lb}}{1 \text{ kg}} = 22.046 \text{ lb}$$

**Footnotes**
- The table above is intended for general use. For precise measurements, refer to the SI (International System of Units) standards.
### Unit I

<table>
<thead>
<tr>
<th>Unit</th>
<th>Definition</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 British thermal unit (Btu)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Btu</td>
<td>0.0001055</td>
</tr>
<tr>
<td>1 erg</td>
<td>foot-pound</td>
<td>ft-lb</td>
<td>0.001356</td>
</tr>
<tr>
<td>1 foot-pound</td>
<td>hour</td>
<td>hp-hr</td>
<td>1.336</td>
</tr>
<tr>
<td>1 horse-power-hour</td>
<td>joule</td>
<td>J</td>
<td>2545</td>
</tr>
<tr>
<td>1 joule</td>
<td>calorie</td>
<td>cal</td>
<td>1.055</td>
</tr>
<tr>
<td>1 calorie</td>
<td>kilowatt-hour</td>
<td>kwh</td>
<td>3413</td>
</tr>
<tr>
<td>1 electron volt (eV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>eV</td>
<td>2.39817667 X 10^6</td>
</tr>
<tr>
<td>1 million electron volts (MeV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MeV</td>
<td>2.6893740 X 10^-10</td>
</tr>
</tbody>
</table>

#### ENERGY, WORK, HEAT

Quantities in the shaded areas are not properly energy units but are included for convenience. They arise from the relativistic mass-energy equivalence formula \( E = mc^2 \) and represent the energy released if a kilogram or atomic mass unit (amu) is completely converted to energy.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Definition</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 erg</td>
<td>foot-pound</td>
<td>ft-lb</td>
<td>0.001356</td>
</tr>
<tr>
<td>1 foot-pound</td>
<td>hour</td>
<td>hp-hr</td>
<td>1.336</td>
</tr>
<tr>
<td>1 horse-power-hour</td>
<td>joule</td>
<td>J</td>
<td>2545</td>
</tr>
<tr>
<td>1 joule</td>
<td>calorie</td>
<td>cal</td>
<td>1.055</td>
</tr>
<tr>
<td>1 calorie</td>
<td>kilowatt-hour</td>
<td>kwh</td>
<td>3413</td>
</tr>
<tr>
<td>1 electron volt (eV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>eV</td>
<td>2.39817667 X 10^6</td>
</tr>
<tr>
<td>1 million electron volts (MeV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>MeV</td>
<td>2.6893740 X 10^-10</td>
</tr>
</tbody>
</table>

Reprinted with permission solicited
Unit I WORKED PROBLEMS

The number in parentheses refers to the number of the objective that will be exemplified by the problem.

1. (2) A car is running on a highway at a speed of 88 ft/s. Is it violating the actual maximum speed limit allowed? Express your answer in MKS Units.

Solution:

Looking up in the Conversion Factors Tables

1 ft = 1.894 x 10^{-4} mi

1 s = 1/60 min; 1 min = 1/60 hr

\[ u = 88 \text{ ft/s} = 88 \left( \frac{1.894 \times 10^{-4}}{1/60} \right) \text{ mi/min} = 1 \text{ mi/min} = 60 \text{ mi/hr} \]

If the maximum speed limit is 55 mi/hr, the driver has to get a ticket.

1 mi = 1.609 km

\[ = 60 \text{ mi/hr} = 60 \times (1.609) \text{ km/hr} = 96.54 \text{ km/hr} \]

\[ = 96.54 \left( \frac{1}{60} \right) \frac{\text{km}}{\text{min}} = 1.609 \text{ km/min} = 26.82 \text{ m/s} \]

2. (3) Determine whether or not the following equations are dimensionally consistent. Use the table of dimensions for physical quantities.

(a) \[ Y = u_o \sin \theta_0 t - \frac{1}{2} gt^2, \text{ where } Y = \text{position}, u_o = \text{speed}, t = \text{time}, g = \text{gravity acceleration} \]

(b) \[ E = \frac{1}{2} m u^2 + \frac{1}{2} k x^2, \text{ where } x = \text{position}, u = \text{speed}, m = \text{mass}, E = \text{energy}, k = \text{(kg/s}^2\text{)} \]
Solution:

(a) Using fundamental dimensions: \([L]; [M]; [T]\)

\[ Y = [L], \quad v_o = [L][T]^{-1}, \sin \theta_o = \text{a dimensional, } t = [T] \]

\[ g = [L][T]^{-2} \]

so that, dimensions of \(v_o \sin \theta_o t = [L][T]^{-1}[T] = [L] \)

dimensions of \(1/2 gt^2 = [L][T]^{-2}[T] = [L] \)

Hence, left-side, \(Y\), has dimensions of \([L]\) and each term on the righthand of the equation has the same dimension \([L]\), then the equation is dimensionally correct.

Notice that the factors \(\sin \theta_o\) and \(1/2\) are pure number, i.e., they do not have units.

(b) Using MKS Units:

\[ E : (J) = (N \cdot m) = (kg \cdot \frac{m}{s^2} \cdot m) = (kg \cdot \frac{m^2}{s^2}) \]

\[ \frac{1}{2} m \nu^2 : (kg) \left( \frac{m}{s} \right)^2 = \left( kg \cdot \frac{m^2}{s^2} \right) \]

\[ \frac{1}{2} kx^2 : \left( \frac{kg}{s^2} \right) (m)^2 = \left( kg \cdot \frac{m^2}{s^2} \right) \]

\[ E : \quad \frac{1}{2} m \nu^2 + \frac{1}{2} kx^2 \]

\[ \left( kg \cdot \frac{m^2}{s^2} \right) = \left( kg \cdot \frac{m^2}{s^2} \right) + \left( kg \cdot \frac{m^2}{s^2} \right) = \left( kg \cdot \frac{m^2}{s^2} \right) \]

Thus both sides of the equation are expressed in the same units, and the equation is said to be dimensionally consistent. Of course, you are not assured that the equation is correct; this method will not catch mistakes in signs or in numerical factors.
3. (4) Estimate the order of magnitude of the following quantities. Express them using prefixes:

(a) Atomic radius

(b) Mass of the earth

(c) Seconds in a day.

Solution

(a) Looking up in a table of physical constants, we find:

\[
\text{Bohr radius } a_0 = 0.529 \times 10^{-10} \text{ m} \sim 10^{-10} \text{ m} = 0.1 \text{ nm} = 1 \text{ Å}
\]

(b) Mass of the earth \( M_\oplus = 5.98 \times 10^{24} \text{ kg} \sim 10^{25} \text{ kg} = 10^{16} \text{ Tg} \)

(c) One day = 24 hr = 24 (60) min = 24 (60) (60) s = 86400 s =

\[
8.64 \times 10^4 \text{ s} \sim 10^5 \text{ s} = 0.1 \text{ Ms} = 10^2 \text{ ks}
\]
These Pretests usually have more detailed questions than you should expect the real test to have. Consider it as both a Review Test in which you evaluate yourself in how well you are doing so far; and a Model Test that is so representative of the actual examination that it could have been one of them. The numbers in parentheses are keyed to the objective's number that the problem is supposed to evaluate.

(1) Describe three systems of units by naming the fundamental quantities, their dimensions, and respective units.

(2) The speed of light in a vacuum is known to be about $3.00 \times 10^8 \text{ m/s}$. Find the time interval for the light to travel 1 foot of distance. Make all your calculations in CGS and British Systems. You can use a Conversion Factor Table.

(3) Determine whether or not the following expressions are dimensionally correct:

(a) $X = v_o t + 1/2at^2$ Use dimensions. $X = \text{position}, v_o = \text{speed}, a = \text{acceleration}, t = \text{time}$

(b) $I = (mv^2) + rp^2$ Use SI units. $I = \text{Rotational inertia}$

$m = \text{mass}$

$v = \text{speed}$

$r = \text{radius}$

$p = \text{linear momentum}$

You can use a Dimensions and Units Table.

(4) Estimate the order of magnitude of the following quantities. Express them using prefixes. You can use a Table of Physical Constants.

(a) Radius of the earth

(b) Mass of an electron

(c) Seconds in a year.
UNIT II

BASIC KINEMATICS CONCEPTS

Introduction

One of the areas of human knowledge based on accumulative learning is Physics. In order to understand and operate within the whole science of Physics or with one of its structures, you must know what and where each brick or cornerstone is in the building.

In this unit you will add a bit of information to your Physics knowledge. That which treats with the basic concepts of the geometric description of the motion of a body is called kinematics. When a train passes a station, the dispatcher says that the train and everybody in the train are in motion relative to the station. But the train's engineer might as well say that the station is in motion relative to the train, moving in the opposite direction, because the position of the station with respect to him is changing with the time. Furthermore, the engineer can say that his assistant, seated next to him, is at rest with respect to him because the assistant's position with respect to the engineer is not changing with the time. Therefore, when you talk about motion you are involving a physical body changing its position with time with respect to an observer, sometimes called frame of reference.

The physical body may be a person, an automobile, a planet, a
bird, or an electron. The actual motion of these bodies may be quite complicated, but when the dimensions of the body are much smaller than the path followed (trajectory), the physical body may be treated as an idealized point-like particle. This approach is applicable in many real situations. The simplest case of motion of a particle is when its trajectory is a straight line. Such motion is called rectilinear or one dimensional motion. The motion of that particle is described using the mathematics as a language for defining the concepts: position, change of position, distance traveled, time elapsed, average velocity, velocity, speed, average acceleration, acceleration, or for expressing relations between these concepts. Uniform motion and motion with constant acceleration are two particularly easy cases of rectilinear motion. For these two cases the functional relationships between variables are simple equations that may be represented by means of graphs.

In this unit only you may use formulas that may have been derived with the aid of calculus without deriving them yourself. You don't need any calculus approach at the moment. You will know and understand the definitions of the basic kinematic concepts and the formulas describing the uniform motion and the motion with constant acceleration. Also, you will show your skills in using those definitions and formulas to solve and analyze specific situations in the mentioned cases of rectilinear motion.
Unit II

Objectives

After you have mastered the content of this unit you will be able to:

1. Define and write mathematical expression of position, change of position, distance traveled, time interval, average velocity, instantaneous velocity, speed, average acceleration, instantaneous acceleration for rectilinear motion.

2. Use the formulas of uniform motion and motion with constant acceleration to find unknown kinematics quantities in problems.

3. Interpret graphically the resulting equations in particular uniform motions and motions with constant acceleration.

4. Analyze graphs of position versus time, velocity versus time, and acceleration versus time for uniform motion and motion with constant acceleration.

NOTE: This Unit is rather easy and less demanding than later units and you may have already encountered this material. If you think that you are already competent on these four objectives, you can skip directly to the Pretest. If your score is high (90% or more) try the Test as soon as possible.
Unit II

SUGGESTED PROCEDURE


1. Read and study, in detail, Chapter 2, Sections 1 through 4. At the first reading you can skip Section 2 and the discussion on pages 21 and 22.

2. Analyze Examples 2, 5, and 6 as stated in the textbook. Now, you must analyze carefully the Worked Problems 1, 2, 3, and 4 attached at the end of this Unit.

3. Solve Problems 1, 2, 3a, 3c, 4*, 5*, 8 and 9 on pages 38 and 39 of the textbook.

4. Prove to yourself that you are ready for the evaluation by taking the Pretest attached at the end of this Unit. If your auto-evaluation is less than good, go over the whole Unit again, and it is worth trying the Audiovisual Aids Section. Ask the instructor for a set of additional problems.

5. Try the test and Good Luck.
SUMMARY OF RELATIONS AND DEFINITIONS

Position: Coordinate(s) of a particle with respect to a particular frame of reference or system of coordinates

Motion: Motion of a body as point-like particle is, its change of position with time

Rectilinear Motion: The body's trajectory is a straight line

Change of Position:

Time Interval:

Velocity: Average: during a certain time interval is equal to the ratio of the change of position to the time interval

Instantaneous: at the exact time \( t \) is the limit of the average velocity in the time interval between \( t \) and \( t+\Delta t \) when the interval \( \Delta t \) becomes infinitesimally small

Acceleration: Average: during a certain interval is the ratio of the change of the instantaneous velocity to the time interval

Instantaneous: at the exact time \( t \) is the limit of the average acceleration in the time interval between \( t+\Delta t \) when the interval \( \Delta t \) becomes infinitesimally small

Uniform Motion:

Motion with constant acceleration:
Unit II WORKED PROBLEMS

1. (1)(2) A train of the Tokyo line moves with a constant velocity of 130 km/hr from Hiroshima toward Tokyo, 300 km away. How far is the train from Tokyo 70 minutes after it leaves Hiroshima?

Solution:

If \( \nu = \text{constant} \), then \( \frac{\Delta s}{\Delta t} = \text{constant} \), the constant velocity \( \nu = \nu_o \) and \( \Delta s = \Delta t \cdot \nu_o \). Here \( \nu_o = 130 \text{ km/hr} \); and \( \Delta t = 70 \text{ min} = \frac{70}{60} \text{ hr} \). Therefore \( \Delta s = \frac{70}{60} \text{ hr} \times 130 \text{ km/hr} = 152 \text{ km} \).

Note that \( \Delta s \) is the distance traveled, not the distance left to get to Tokyo, which is 300 km - 152 km = 148 km.

2. (1)(2) The Tokyo dispatcher then tells the engineer to decelerate with a constant deceleration of 0.1 km/min\(^2\)

a) What is the deceleration in units of km/hr/min and in units of km/hr\(^2\)?

b) What is the velocity of the train 1 minute later, and 1/2 hour later.

c) How far is the train from Hiroshima at the later time in (b)?

Solution:

(a) 1 hr = 60 min or 1 min = 1/60 hr

\[
0.1 \text{ km/min}^2 = 0.1 \left( \frac{\text{km}}{1/60 \text{ hr}} \right) \cdot \frac{1}{\text{min}} = (6 \text{ km/hr}) \left(1/\text{min}\right)
\]

and

\[
0.1 \text{ km/min}^2 = 0.1 \text{ km}/(1/60 \text{ hr})^2 = 360 \text{ km/hr}^2
\]
(b) For motion with uniform acceleration $= a$,

$v = v_0 + at$, where acceleration is a positive term.

Then, deceleration is a negative term, so that

$v = v_0 + (-6 \text{ km/hr/min})t$. The time here is the time measured from the moment at which

$u = v_0; \ t = 1 \text{ minute}$. Hence

$u = v_0 + (-6 \text{ km/hr/min}) (1 \text{ min}) = v_0 - 6 \text{ km/hr}$

$= 130 \text{ km/hr} - 6 \text{ km/hr} = 124 \text{ km/hr}$

(b) 2 $u = v_0 + at = 130 \text{ km/hr} + (-360 \text{ km/hr}^2) (\frac{1}{2} \text{ hr})$

$= 130 \text{ km/hr} - 180 \text{ km/hr} = -50 \text{ km/hr}$

The negative sign means the train is now going backwards (the engineer and dispatcher were later dismissed.)

(c) Before beginning this part, let us remember that when the train began to decelerate it was 152 km from Hiroshima, let us suppose we measure our distances from mile post 152 then

For motion with constant acceleration,

$s' = \frac{1}{2} at^2 + v_0 t$

We have $v_0 = 130 \text{ km/hr}$

$a = -360 \text{ km/hr}^2$

$t = \frac{1}{2} \text{ hr}$

so

$s' = \frac{1}{2} (-360 \text{ km/hr}^2) (\frac{1}{2} \text{ hr})^2 + 130 \text{ km/hr} (\frac{1}{2} \text{ hr})$

(distance moved due to deceleration) (distance train could have moved with no deceleration)
1.31

\[ \frac{1}{8} (-360) \text{ km} + \frac{1}{2} \times 130 \text{ km} \]

\[ = -45 \text{ km} + 65 \text{ km} \]

\[ = 20 \text{ km} \]

This is distance from the mile post.

The distance from Hiroshima is \( 152 \text{ km} + 20 \text{ km} = 172 \text{ km} \).

3. (3) Two trains are moving on the same track in the same direction.

The front one moves with a speed of 80 km/hr and the rear one with a speed of 100 km/hr. The engineer of the fast train sees the slow train in front of him when they are 1 km apart. What minimum constant braking deceleration must be applied so as to slow down and just reach the other train without collision? Interpret graphically your results:

Solution:

The fast train must change its speed from 100 km/hr to 80 km/hr in a time interval \( \Delta t = t \).

\[ a = \frac{\Delta u}{\Delta t} \quad \text{or} \quad u_f - u_o = at \]

where

\[ u_o = u_1 = 100 \text{ km/hr}; \quad u_f = u_2 = 80 \text{ km/hr} \quad \text{and} \]

\[ a = \text{deceleration} \]

\[ u_2 - u_1 = at \]

\[ t = \frac{u_2 - u_1}{a} \quad (1) \]

\[ d = 1 \text{ km} \]

\[ u_1 \rightarrow u_2 \rightarrow \text{meeting point} \]
However, we know that the train's position depends upon time. For the fast train, \( S_1 = u_1 t + \frac{1}{2}at^2 \) (constant deceleration)

For the slow train: \( S_2 = u_2 t \) (constant velocity)

And when they almost touch each other without collision

\[ d + S_2 = S_1 \]

Therefore,

\[ d + u_2 t = u_1 t + \frac{1}{2}at^2 \]  \hspace{1cm} (2)

Plugging (1) into (2)

\[ d = (u_1 - u_2) \left( \frac{u_2 - u_1}{a} \right) + \frac{1}{2}a \left( \frac{u_2 - u_1}{a} \right)^2 = - \frac{(u_1 - u_2)^2}{2a} \]

or

\[ a = - \frac{(u_1 - u_2)^2}{2d} \]

The negative sign shows explicitly that there is a deceleration.

So that, if \( |a| \geq \frac{(u_1 - u_2)^2}{2d} \), there will be no collision, but if \( |a| < \frac{(u_1 - u_2)^2}{2d} \), there will be a collision. Numerically

\[ a = - \frac{(100 - 80)^2}{2 \cdot 1} = - \frac{400}{2} = -200 \text{ km/hr}^2 \]

And the time elapsed since the engineer applied the brakes is represented by

\[ t = \frac{u_2 - u_1}{a} = \frac{80 - 100}{-200} = \frac{-20}{-200} \times 1/10 \text{ hr} = 6 \text{ min} \]
Geometrical Interpretation:

Train 1: \( T_1 \), deceleration constant = \(-a\)

Train 2: \( T_2 \), no acceleration = 0

So that, their graphs are straight horizontal lines as shown below

**Eqns of velocities are, \( T_2 \):** \( u_2 = 80 \text{ km/hr} \) = constant \( \Rightarrow \) horizontal line with slope zero

**Eqns of position are, \( T_2 \):** \( s_2 = u_2 t = 80t \), straight line with positive slope \( 80 \), intercept \( 0 \)

\( \begin{align*}
T_1: & \quad u_1 = u_0 - at = 100 - 200t, \text{ straight line with negative slope } (-200) \text{ and vertical intercept } (100) \\
T_1: & \quad s_1 = u_0 t - \frac{1}{2}at^2 = 100t - 100t^2, \text{ parabola with downward concavity.}
\end{align*} \)
4. (4) Free Vertical Motion under the Action of Gravity. This is the most important case of motion with constant acceleration. Assume that the gravitational acceleration downward has a value of 10 m/sec$^2$.

A boy on the top of a building 80 m high leans over the edge of the building and drops a ball. The corresponding graphs of that motion are shown below. Analyze them and obtain the motion equations.

**Solution:**

- **Horizontal Line:**
  
  ![Horizontal Line](image)
  
  Horizontal line = slope zero
  
  $a = 10 \text{ m/sec}^2$ for all time

- **Velocity Graph:innacle:**

  ![Velocity Graph](image)
  
  Straight line thru the origin
  
  slope $= \frac{40 - 10}{4 - 1} = 10 \text{ m/sec}^2$

  Equation of the line: $y = mx + b$
  
  $v = 10t \text{ (m/sec)}$

- **Distance Graph:**

  ![Distance Graph](image)
  
  Parabola with upward concavity and passing thru the origin
  
  $y = bx^2$

  $S = \frac{1}{2}at^2 = 5t^2$

  When the ball reaches the pavement, $S = 80 \text{ m}$

  \[ t^2 = \frac{80}{5}; \ t = 4 \text{ sec} \]

  The time required to reach the ground and the velocity at this moment is $v = 10(4) = 40 \text{ m/sec}$. 

Unit II
PRETEST

(1) Define and write the mathematical expressions of position, change of position, average velocity, instantaneous velocity, and average acceleration for rectilinear motion.

(2) An automobile started from rest and moved with constant acceleration. At a certain time it was traveling at 20 mi/hr, and 365 ft. farther on it was traveling at 30 mi/hr. Calculate:
   (a) The acceleration.
   (b) The time required to attain the final speed.

(3) The automobile of the previous problem continues with uniform motion after it reaches the final speed of 30 mi/hr. Sketch graph of $S$ vs $t$, $U$ vs $t$, and $a$ vs $t$ for the total motion from rest to the uniform motion.

(4) A particle's velocity is shown in the graph below. At $t=0$, its position is $S=0$.
   (a) Sketch the acceleration vs time graph corresponding to this $U$ vs $t$ graph.
   (b) Write equations describing the acceleration, velocity, and position of this particle motion.

![Graph of velocity vs time]

\[
\begin{align*}
\text{(m/sec)} U & \quad 2 \\
2 & \quad 8 \\
-2 & \quad 12 \\
\text{(t/sec)} & \end{align*}
\]

Answers:

Unit II

AUDIOVISUAL AIDS SECTION

Film-Loops

We urge and recommend that you select some of the following film-loops. They are very short, three or four minutes each, and will help you to understand some of the basic concepts in this unit. Also, they illustrate certain physical phenomena, and you can even take some measurements and get some results.

We suggest that you read carefully the film notes on the film-case before and after watching them. Also, try to answer the questions included in these film notes.

A. To supplement sections 1 and 2 of your textbook:

Distance, Time and Speed 80-3031/1

Using some pucks on an air table they introduce the relationship among distance, time and speed. You can calculate the speed of a puck and predict its position.

B. For sections 3 and 4:

1. One-Dimensional Acceleration 80-3064/1

Acquaints you with the relationship involved in uniformly accelerated motion. Use is made of the air table.

2. Constant Velocity and Uniform Acceleration 80-2728/1

Using the air track it shows to you the relationships involved when a body moves with constant velocity compared to when it moves with uniformly accelerated motion.
3. One-Dimensional Motion

Real time plots of displacement, velocity, and acceleration versus time are shown for a small cart being pulled back and forth along a track.
UNIT III

MOTION IN ONE DIMENSION

Kinematics-Momentum-Energy

Introduction

In the previous Unit the basic kinematics definitions were given and used in simple rectilinear motions. Now, we shall define and derive the same concepts but from a more general and useful standpoint: calculus. This mathematical tool, developed by a Physicist, will allow us to give a better interpretation to the kinematics relationships and to represent them graphically. Remember that a good graph is worth more than a hundred words. You will again analyze the cases of uniform motion and motion with constant acceleration but you will also investigate the case of rectilinear motion where the acceleration is proportional to the time.

Surely you have heard in the news the term "energy" used in connection with the "Energy Crisis". That is the same term you will find as one of the structures in the building of Physics. In both contexts, it refers to the property or ability of a system to produce change in its environment or in itself.

The physicist distinguishes between several types of energy. You will deal in this unit with kinetic energy (associated with the motion of a body), gravitational potential energy near the surface of the earth.
(associated with the relative body's position with respect to earth's surface) and mechanical energy (the sum of the former two).

Another term of daily use is "collision." You have heard about or have been involved in an automobile collision, or have had a head-on impact with another person in a crowded hall-way. In studying collisions you will find another of the structures of physics: momentum (quantity of motion). The study of collisions uses a fundamental physical tool, i.e. conservation laws. A conservation law means that something maintains the same value (is conserved) at all times. When two bodies in rectilinear motion collide there is conservation of rectilinear momentum and conservation of energy. Momentum and energy are by far the most important physical variables used for the quantitative description of physical phenomena. The total momentum and the total energy of an isolated physical system are always conserved.

Finally, in this Unit you shall study a more familiar rectilinear motion with constant acceleration: free fall. That is, we speak of "freely falling" objects near the earth's surface, and a study of their motion reveals that it is one-dimensional with constant acceleration and with mechanical energy conserved.

During this course we will continuously speak about these conservation laws and will find how they connect every specific part of the course.
Unit III

Objectives

After you have mastered the content of this unit, you will be able to:

1. Derive equations of average velocity, instantaneous velocity, average acceleration, instantaneous acceleration as functions of time for the rectilinear motions: uniform motion, motion with constant acceleration and/or motion with acceleration proportional to time, using calculus techniques.

2. Analyze graphs of position versus time, velocity versus time, and acceleration versus time for the rectilinear motion: uniform motion, motion with constant acceleration, motion with acceleration proportional to time, and/or combinations of these, from the standpoint of calculus.

3. Describe the concepts of linear momentum, kinetic energy, potential energy, and mechanical energy of one dimensional motion.

4. Examine one dimension collisions by using linear momentum conservation and kinetic energy conservation, and differentiate between elastic collision and inelastic collision.

5. Solve problems of bodies falling freely near the surface of the earth by using the concepts of distance traveled, position, velocity, acceleration, and/or conservation of mechanical energy.
Unit III  SUGGESTED PROCEDURE


As you can see from the Introduction, it is essential to know some basic techniques of calculus to master objectives 1 and 2 of this Unit and to derive other relationships in later Units. Hence, if your calculus knowledge is poor, go over the Remedial Unit II: Calculus, or study the Supplement I in the textbook, or study a self-study book such as Davidson and Marion, Mathematical Preparation for General Physics with Calculus, W. B. Saunders Co.

1. Study Chapter 2, Sections 1, 2, 3, and 4. Pay special attention to equations of definitions as (2-1), (2-2), (2-3), and (2-5'). Look at the calculus derivations for uniform motion and motion with constant acceleration in Section 4; study worked-problem 1 for motion with acceleration proportional to time. Analyze in detail examples 3 and 4, and solve the problems 3b, 6, 7* and 10*.

2. To achieve objective 2 study carefully the discussion at the end of Section 2, examples 1, 2 and the graphs given on page 22. Analyze the worked problems 2 and 3. Solve the problems 10, 11 and 12 of Halliday-Resnick.

3. The definitions necessary for objective 3 are in equations (2-13), (2-14), (2-20), and (2-21). Read the corresponding paragraphs.
4. Study Section 5 and examples 7 and 8. This section is the most important for the new concepts described there. Do not hesitate to ask questions of your proctor or instructor to be sure that you have understood these principles. Analyze worked problem 5 and solve problems 11, 13 and 14.

5. Objective 5 can be satisfied by studying Section 6, analyzing example 9 and worked-problem 4. Solve problem 16, 17 and 19.

6. Try the Pretest and have it graded by a proctor. Defend your solutions. If your score is above 90 percent, you are ready for the Test. If you failed your Pretest, go over this Unit again, especially over those failed objectives. Solve the optional problems 12, 15, 18 and 20. Also, you should try the Audiovisual Aids Section.
Unit III  
SUMMARY OF IMPORTANT RELATIONS AND CONCEPTS

Position

\[ S = S(t) \]

Instantaneous Velocity

\[ v(t) = \frac{ds}{dt} = \text{slope of the tangent line to the } S(t) \text{ versus } t \text{ curve} \]

Instantaneous Acceleration

\[ a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \text{slope of the tangent line to the } v(t) \text{ versus } t \text{ curve} \]

Linear Momentum (quantity of motion).

\[ p = mu \]

Energy

Kinetic

\[ T = \frac{1}{2}mu^2 \]

Gravitational Potential

(near the surface of the earth)

\[ U_g = mgh + U_0 \]

Mechanical

\[ E = T + U_g \]

Conservation

Momentum

\[ \Sigma p_i = \text{constant, or } p_{\text{before}} = p_{\text{after}} \]

Kinetic energy

\[ \Sigma T_i = \text{constant, or } T_{\text{before}} = T_{\text{after}} \]

Mechanical energy

\[ E = \text{constant, or } E_{\text{before}} = E_{\text{after}} \]

Elastic Collision: in which kinetic energy is conserved

Inelastic Collision: in which kinetic energy is not conserved. It may be either gained or lost during the collision.

Perfectly Inelastic Collision: in which the colliding objects stick together after the collision.
1. (1) A particle moves in a straight line and its position is given by

\[ x = -t^3 + 9t^2 - 23t + 15 \]

where \( x \) is in meters and \( t \) in seconds.

(a) Find \( v(t) \) and \( a(t) \).

(b) At what time does \( x = 0 \)? What is the position in \( t = 0 \)?

(c) At what time does \( v = 0 \)? When is the particle moving to the right and when is it moving to the left?

(d) Discuss the acceleration.

Solution:

(a) \( v(t) = x'(t) = \frac{dx}{dt} = -3t^2 + 18t - 23 \text{ (m/sec)} \)

\( a(t) = v'(t) = \frac{dv}{dt} = -6t + 18 \text{ (m/sec}^2) \)

(b) \( x = 0 \) means \(-t^3 + 9t^2 - 23t + 15 = 0\)

Using simple algebra (synthetic division) we find \((1-t)(t-3)(t-5) = 0\) which means that at \( t_1 = 1, t_2 = 3, \) and \( t_3 = 5 \text{ sec., the particle is at the origin of the one-dimensional coordinate. The position where the motion starts to be measured is } x_0 = 15 \text{ meters.} \)

(c) \( v = 0 \) means \(-3t^2 + 18t - 23 = 0\)

\( t_4 = 3 - \frac{2}{\sqrt{3}} \), and

\( t_5 = 3 + \frac{2}{\sqrt{3}} \)

i.e. \( t_4 \approx 1.85 \text{ sec, } t_5 \approx 4.15 \text{ sec or } v = - \left[ t - (3 - \frac{2}{\sqrt{3}}) \right] [ t - (3 + \frac{2}{\sqrt{3}}) ] \)

If \( t > (3 + \frac{2}{\sqrt{3}}) \) or \( t < (3 - \frac{2}{\sqrt{3}}) \) then \( v < 0 \) and the particle is moving to the left.
If \( (3 - \frac{2}{\sqrt{3}}) < t < (3 + \frac{2}{\sqrt{3}}) \) then \( v > 0 \) and the particle is moving to the right.

(d) \( a = -6t + 18 \),

and \( a = 0 \) when \( t = 3 \) sec.

If \( t < 3 \), then \( a \) is positive which means that between

\[ 0 < t < 3 \]

there is an increase in velocity.

If \( t > 3 \), then \( a \) is negative which means that there is a decreasing velocity.

2. (2) For the previous problem, graph position-vs-time, velocity-vs-time, and acceleration-vs-time curves by using the equations already obtained.

Solution:

\[ x = -t^3 + 9t^2 - 23t + 15, \text{ cubic eqn. is a cubic curve} \]

\[ x = (1-t) (t-3) (t-5) \]

\[ v = -3t^2 + 18t - 23 = - \left[t - (3 - \frac{2}{\sqrt{3}})\right] \left[t - (3 + \frac{2}{\sqrt{3}})\right], \text{ quadratic eqn is a parabola} \]

\[ a = -6t + 18, \text{ linear eqn is a line.} \]
3. (2) The graph of \( x \) versus \( t \) is for a particle in rectilinear motion. State for each interval whether the velocity is positive, negative, or zero and whether the acceleration is positive, negative or zero. The intervals are OA, AB, BC, CD.

Solution:
The slope of the tangent line to the curve position vs. time means physically the velocity.
A tangent line to OA is the same line OA which has a positive slope. The tangent line to curve AB at points like e or f has a positive slope. The tangent line to horizontal line BC is the same line BC which has slope zero. The tangent line to curve CD at points g and h has a negative slope.
The acceleration is interpreted as the change of velocity in time. As we always assume positive interval of time, then the sign of the acceleration is given by the change in velocity. If from 0 to A there isn't change in velocity, then acceleration is zero.

from A to B the velocity is positive but decreasing, then the acceleration is negative.

from B to C there is no change of velocity, then the acceleration is zero.

from C to D the velocity is negative but increasing (in h the slope of the tangent line is negative but less than g), then the acceleration is positive.

<table>
<thead>
<tr>
<th>Intervals</th>
<th>OA</th>
<th>AB</th>
<th>BC</th>
<th>CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( a )</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

4. (5) A tennis ball is dropped onto the floor from a height of 40 ft. It rebounds to a height of 30 ft. If the ball was in contact with the floor for 0.010 sec. what was its average acceleration during contact?

We have two ways of solving this problem:

I. Kinematic Solution: The ball is dropped from rest and falls freely with constant acceleration. Assuming negative the velocity directed downward

\[
\frac{v^2}{v_0^2} + 2as, \quad v_0 = 0, \text{rest}
\]

\[
v_1 = -\sqrt{2gs_1} = -\sqrt{2 \cdot 32 \cdot 4} = -16 \text{ ft/sec}.
\]
For the rebounding we can consider that the ball is thrown vertically upward from the ground with a speed of \( +V_2 \) and its highest point is \( S_2 = 3 \) ft.

\[
V_2 = +\sqrt{2gS_2} = +\sqrt{2 \cdot 32 \cdot 3} = 8\sqrt{3} \text{ ft/sec}
\]

Then, the average acceleration is defined as:

\[
a = \frac{\Delta u}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1} ; \quad a = \frac{8\sqrt{3} - (-16)}{10 - 2} = 2985 \text{ ft/sec}^2 \text{ upward}
\]

II. Energetic Solution: When the ball is located at rest at height \( S_1 \) it has a potential energy \( U_{10} = mgS_1 \) and a kinetic energy equal to zero, \( T_0 = 0 \). Now the ball is released and allowed to fall freely toward the ground. At the instant it hits the ground its potential energy \( U_{1f} = 0 \) and its kinetic energy \( T_{1f} = \frac{1}{2} mu_1^2 \).

So that we have: Total energy at \( S_1 \): \( U_{10} + T_{10} = mgS_1 + 0 = E_{10} \)

Total energy at ground: \( U_{1f} + T_{1f} = 0 + \frac{1}{2} mu_1^2 = E_{1f} \)

But the total energy is conserved so that

\[
E_{10} = E_{1f} ; \quad mgS_1 = \frac{1}{2} mu_1^2 \quad u_1 = -\sqrt{2gS_1}
\]

We choose the negative sign in the root square to point out movement downward. When the ball rebounds on the ground its kinetic energy is \( T_{20} = \frac{1}{2} mu_2^2 \) and its potential energy at this point is zero \( U_{20} = 0 \). At the highest point of its trajectory back upward the kinetic energy is \( T_{2f} = 0 \) and its potential
energy $U_{2f} = mgS_2$. So that all the kinetic energy is converted
in potential energy:

$$T_{20} = U_{2f} ; \quad 1_{2m}u_2^2 = mgS_2 ; \quad v_2 = \sqrt{2gS_2}$$

The average acceleration during contact

$$\bar{a} = \frac{v_2 - v_1}{\Delta t} = \frac{\sqrt{2gS_2} - (-\sqrt{2gS_1})}{\Delta t}$$

$$\bar{a} = 2985 \text{ ft/sec}^2 \text{ upward}$$

5. (4). In the previous problem it is assumed that the tennis ball has
a mass of 10 gm and that mass of the earth $M = 6 \times 10^{24}$ kg. Cal-
culate

(a) change of momentum of the tennis ball attributable to the
collision with earth;

(b) the recoil velocity of the earth;

(c) if the collision was inelastic.

Solution:

(a) $\Delta p_b = p_{bf} - p_{bi} = m u_2 - m u_1 = m (v_2 - v_1)$

$= 0.2045 \text{ (slug-ft/sec) upward}$

(b) The earth recoils during the collision with a velocity given by

$\Delta p_b = -\Delta p_e = -M (v_{2e} - v_{1e})$

where $u_{1e} = 0$ assuming the earth is at rest at initial

$u_{2e} = -\frac{m}{M} (v_2 - v_1)$
\( v_{2e} = -5 \times 10^{-25} \text{ ft/sec} \) downward. This is a negligible value of velocity.

(c) The kinetic energy of the ball changes during the collision: the kinetic energy at the instant the ball hits the floor is \( K_1 = \frac{1}{2}m \dot{u}_1^2 \), and the kinetic energy at the instant the ball leaves the floor is \( K_2 = \frac{1}{2}m \dot{u}_2^2 \)

\[
\Delta K = K_2 - K_1 = \frac{1}{2}m (\dot{u}_2^2 - \dot{u}_1^2)
\]

\[
= -0.22 \text{ lb-ft}
\]

This amount of energy is transformed into heat, sound vibration, rotation, and other forms of energy. So the collision was inelastic.
I. (1)(2) A body is moving along a straight line according to the equation $S = t^3 - t^2 - t + 1$, where $s$ is in meters and $t$ in seconds.

Find:

(a) the time(s) when the body passes the reference position zero.

(b) the instantaneous velocity at any given time. What is its value at $t = 0$?

(c) the general expression for the average acceleration for the time interval $t < t' < (t + \Delta t)$.

(d) the instantaneous acceleration at any time.

(e) the time interval(s) when the motion is accelerated and when the motion is retarded.

(f) Plot the $s$-$v$-$t$, $u$-$v$-$t$, and $a$-$v$-$t$ graphs.

II. (2) The following curve is the hodograph of a one-dimension motion ($u$-$v$-$t$). Analyze it and sketch the acceleration $v$-time, and position $v$-time graphs.

III. (3)(4) A railroad car of mass 1000 kg moves to the right along a rail track without friction at a speed of 10 m/sec and makes a head-on collision with another railroad car whose mass is unknown. The second railroad car was originally moving to the left at a speed of 6 m/sec. The railroad cars stick together after collision and move to the right at a speed of 4 m/sec.

(a) Make a sketch and describe in what way the conservation laws can be applied to find the unknown mass.

(b) What kind of collision is this? What fraction of the original kinetic energy is "lost" during the collision?
IV. (3)(5) A boy aims his toy gun vertically upward and shoots a small plastic bullet into the air. The bullet has a speed of 10 m/sec when it has reached one-half its maximum height. Assuming a uniform gravitational earth field with a constant acceleration \( g = -10 \text{ m/sec}^2 \). Calculate:

(a) the maximum height the bullet rises;

(b) the maximum speed of the bullet;

(c) the maximum values of kinetic energy, potential energy, and mechanical energy if the bullet's mass is 10 gm.

\[ h = \frac{v^2}{2g} \]

\[ v_{\text{max}} = \sqrt{2gh_{\text{max}}} \]

\[ K = \frac{1}{2}mv^2 \]

\[ U = mgh \]

\[ E_{\text{mech}} = K + U \]

\[ m = 0.01 \text{ kg} \]

\[ g = 10 \text{ m/sec}^2 \]

Answers:
Unit III  AUDIOVISUAL AIDS SECTION

Film Loops

A. 1. Conservation of Momentum: Elastic Collisions  80-2777/1

Provides several examples of elastic collisions, and shows the mathematical relations that exist between the momenta of the colliding masses before and after the collisions on the air track.

2. Conservation of Momentum: Inelastic Collisions  80-2751/1

Illustrates the relationships existing between masses and the velocities in an elastic collision using the air track.

B. 1. Velocity and Acceleration in Free Fall  80-2561/1

Depicts the relationships existing between displacement, velocity and acceleration for an object in free fall.

2. Acceleration Due to Gravity I  80-3452/1

Shows a bowling ball falling freely. Intervals are marked on the background so that you can time the ball as it falls through known distances. From these measurements you can calculate velocities and acceleration.

3. Acceleration Due to Gravity II  80-3460/1

This film is a variation on the previous experiment using a different approach, and you can determine for yourself the value of acceleration due to gravity.

4. Gravitational Potential Energy  80-3817

It shows the dependence of gravitational potential energy on mass and height.
Unit III

Autolecture

The Autolecture is a supplementary aid that is presented as an alternative way of study for those students who fail the Pretest or the regular Unit Test. Also, it could be interpreted as an extension of the regular procedure of study. The autolecture consists of: (a) an audio-cassette prerecorded with a short lecture, 30 minutes or less; and (b) a set of transparencies to be used on an overhead projector.

When you want to try the Autolecture go to the PSI room and ask the tutor in charge for Autolecture No. 1. You will receive an audio-cassette together with a set of transparencies, a cassette recorder, and an overhead projector that is conveniently placed in the room. If you do not know how to operate these instruments ask for instructions from the tutor. You will also need your textbook and some draft paper. Synchronize the audio cassette with the transparencies, and focus your attention on the lecture without taking notes, at least in the first run, except when you are asked to. Try to answer all the questions included in the autolecture. Discuss your answers and any doubts you have with the tutor. In a rerun of the autolecture you may take skeletal notes if you want to. When you go through, return all materials to the tutor in charge.

The Autolecture No. 1 contains a clear calculus derivation and geometrical interpretation of the basic kinematics concepts; a calculus derivation and interpretation of the rectilinear motion with constant acceleration; and the geometrical interpretation of rectilinear motion with acceleration proportional to time.
UNIT IV

VECTORS

Introduction

This unit is rather mathematical in the sense that you will learn how to operate with certain quantities called vectors. Velocity is an example of a vector quantity. When you specify the velocity at which you are moving when you drive home, you must not only specify how fast (the speedometer reading) but also indicate the direction which you are moving. That is, you must specify both magnitude and direction.

In addition to velocity, there are positions, displacements, accelerations, forces, moments, and other physical observables that are vector quantities. The description of physical phenomena in terms of vectors not only simplifies the calculations, but also brings out the essential features of the problem. Frequently a physical law is expressed as a relation between vectors and this has the advantage of not referring to a particular system of coordinates.

You will start by reviewing or learning the concept of vector and its simple vector algebra as addition, subtraction, multiplication by a number, components of a vector and magnitude of a vector. Then you will apply this vector algebra to define, from a more general point of view, the concepts of velocity and acceleration. Finally you will study two kinds of vector multiplication: the scalar and the vector products.
Unit IV

Objectives

To be able to:

1. Distinguish between vector and scalar quantities.
2. Add and subtract vectors.
3. Use vectors in rectangular form and in polar form.
4. Recognize and employ the concepts of velocity vector and acceleration vector.
5. Calculate the scalar (dot) product and the vector (cross) product between two vectors.
Unit IV

SUGGESTED PROCEDURE


1. This unit deals with the initial four sections of Chapter 3, and the Supplement III. The discussions in Section 1, Chapter 3 and Section 1, Supplement III help you to accomplish objective (1).

2. Study Section 2, Chapter 3 and Sections 2, 3 and 4 of Supplement III for objectives (2) and (3).

3. Analyze example 1a, c and the worked problems 1 and 2. Solve problems 1, 2 and 3 on page 66.

4. Study in detail Chapter 3, Sections 3 and 4 including the examples 2, 3, 4 and 5 and the worked problems 3 and 4. Attention must be paid to equation (3-6), definition of velocity vector, and equation (3-9), definition of acceleration vector. In example 3 and worked problem 3 there is implicitly the Chain Rule for adding relative velocities.

5. Solve problems 7, 8, 9 and 10*. In problem 7, the relative velocity and relative acceleration are \( \mathbf{v}_R = \mathbf{v}_B - \mathbf{v}_A \) and \( \mathbf{a}_R = \mathbf{a}_B - \mathbf{a}_A \). In problem 8, the angle of 10° is with respect to the vertical.

6. For objective (5) study Chapter 3, Section 2f, and Supplement III, Sections 5 and 6. Analyze the example 1 part b, and the worked problems 5, 6 and 7. Solve the problem 4.
7. Try the Pretest. If you need more practice go over the optional problems 6 and 11* and ask for a set of additional problems. Also, you must try to study the Audiovisual Section of this Unit.
Unit IV SUPPLEMENTARY ANNOTATIONS

The velocity of an object relative to a reference frame $3$ is the vector sum of the object's velocity relative to reference frame $1$, the velocity of reference frame $1$ relative to reference frame $2$, and the velocity of reference frame $2$ relative to reference frame $3$, $\vec{v}_{03} = \vec{v}_{01} + \vec{v}_{12} + \vec{v}_{23}$. (And so on if there are more reference frames (observers). The last equation illustrates the general rule for combining velocities: (1) Write each velocity with a double subscript in the proper order, meaning "velocity of (first subscript) relative to (second subscript)." (2) When adding relative velocities, the first letter of any subscript is to be the same as the last letter of the preceding subscript. (3) The first letter of the subscript of the first velocity in the sum, and the second letter of the subscript of the last velocity, are the subscripts, in that order, of the relative velocity represented by the vector sum. One more point should be noted. The velocity of body $A$ relative to reference $B$ (or observer $B$), $\vec{v}_{AB}$, is the negative of the velocity of $B$ relative to $A$, i.e.: $\vec{v}_{AB} = -\vec{v}_{BA}$. That is, interchanging two subscripts gives a vector equal in magnitude but opposite in direction.
Unit IV  SUMMARY OF RELATIONS AND DEFINITIONS

Vector \( \vec{r} = r \hat{u}_r \)

Magnitude = \( r = | \vec{r} | \)

Two-dimensional \( \vec{r} = x \hat{i} + y \hat{j} \)
\( r = \sqrt{x^2 + y^2} \) This is polar form or polar coordinates in two-dimensions.
\( \theta = \tan^{-1} \frac{y}{x} \)

Three-dimensional \( \vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \)
\( r = \sqrt{x^2 + y^2 + z^2} \) This is polar form in three-dimensions, but it is not polar coordinates.
\( \cos \alpha = \frac{x}{r}, \cos \beta = \frac{y}{r}, \cos \gamma = \frac{z}{r}, \tan \theta = \frac{y}{x} \)

Algebra of Vectors:

addition \( \vec{a} + \vec{b} = \vec{c} = \vec{b} + \vec{a} \)

additive inverse \( \vec{b} + (-\vec{b}) = 0 \)

subtraction \( \vec{d} = \vec{e} - \vec{b} = \vec{e} + (-\vec{b}) \)

multiplication by a number \( \vec{f} = \lambda \vec{b} \) in the same direction as \( \vec{b} \) if \( \lambda > 0 \)
in the opposite direction from \( \vec{b} \) if \( \lambda < 0 \)

dot product \( \vec{b} \cdot \vec{c} = |\vec{b}| |\vec{c}| \cos \theta_{(b,c)} = b_x c_x + b_y c_y + b_z c_z \)
cross product \( \vec{d} = \vec{b} \times \vec{c} \) with \( |\vec{d}| = |\vec{b}| |\vec{c}| \sin \theta_{(b,c)} \)

\[
\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\hat{i} & \hat{j} & \hat{k} \\
\hat{i} & \hat{j} & \hat{k} \\
\end{vmatrix}
\]

derivative \( \frac{db}{dt} = \frac{db_x}{dt} \hat{i} + \frac{db_y}{dt} \hat{j} + \frac{db_z}{dt} \hat{k} \)
Unit IV
Vector Kinematics

Position Vector
\[ \vec{r}(t) \]

Change in position = Displacement
\[ \Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{r}(t_2) - \vec{r}(t_1) \]

Velocity Vector
Average
\[ <\vec{V}> = \frac{\Delta \vec{r}}{\Delta t} \]

Instantaneous
\[ \vec{V}(t) = \frac{d\vec{r}}{dt} \text{ tangential to the trajectory} \]

Speed
\[ u = |\vec{V}(t)| \]

Acceleration Vector
Average
\[ <\vec{a}> = \frac{\Delta \vec{V}}{\Delta t} \]

Instantaneous
\[ \vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{r}}{dt^2} \text{, inward from the curvature of the trajectory} \]

Magnitude of acceleration
\[ a = |\vec{a}| \]

Relative Motion

relative position of A with respect to B
\[ \vec{r}_{AB} = \vec{r}_A - \vec{r}_B \]
\[ \vec{r}_{BA} = -\vec{r}_{BA} \]

relative velocity
\[ \vec{V}_{AB} = \vec{V}_A - \vec{V}_B \]

relative acceleration
\[ \vec{a}_{AB} = \vec{a}_A - \vec{a}_B \]

chain's rule for velocities
\[ \vec{V}_{AD} = \vec{V}_{AB} + \vec{V}_{BC} + \vec{V}_{CD} \]
1. (2) Using vector algebra, prove that the diagonals of a parallelogram bisect each other.

Solution:

Let $ABCD$ be the parallelogram with diagonals $\vec{c}$ and $\vec{d}$.

Since $\vec{c} = \vec{a} + \vec{b}$

$\vec{d} = \vec{b} - \vec{a}$ as shown in the figure.

If they bisect it must hold:

$\frac{\vec{c}}{2} + \frac{\vec{d}}{2} = \vec{b}$

and

$\frac{\vec{d}}{2} - \frac{\vec{c}}{2} = -\vec{a}$

but $\frac{\vec{a} + \vec{b}}{2} + \frac{\vec{b} - \vec{a}}{2} = \frac{\vec{b}}{2} + \frac{\vec{b}}{2} = \vec{b}$

and $\frac{\vec{b} - \vec{a}}{2} - \frac{\vec{a} + \vec{b}}{2} = -\frac{\vec{a}}{2} - \frac{\vec{a}}{2} = -\vec{a}$

2. (3) Given the points $A(4,-4,-2)$, $B(-2, 3, 1)$ and $C(4,3,0)$ in three dimension-space,

(a) Write the vector position of each point in rectangular and in polar form.

(b) Find the vector $\vec{R} = \vec{A} - \vec{B} + \vec{C}$
Solution:

(a) Rectangular form

Components of $\mathbf{A}$: $A_x = 4$, $A_y = -4$, $A_z = -2$

Vector $\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$

$\mathbf{A} = 4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$

Polar form:

magnitude: $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$A = \sqrt{4^2 + (-4)^2 + (-2)^2} = 6$

angle with $+X$ axis:

$\alpha_A = \text{Arc cos} \left( \frac{A_x}{A} \right) = \text{Arc cos} \left( \frac{4}{6} \right) = 48^\circ$

angle with $+Y$ axis:

$\beta_A = \text{Arc cos} \left( \frac{A_y}{A} \right) = 132^\circ$

angle with $+Z$ axis:

$\gamma_A = \text{Arc cos} \left( \frac{A_z}{A} \right) = 110^\circ$

Vector $\mathbf{B} = -2 \mathbf{i} + 3 \mathbf{j} + 1 \mathbf{k}$

Magnitude: $B = \sqrt{(-2)^2 + 3^2 + 1^2} = \sqrt{14}$

angle with $+X$ axis: $\alpha_B = \cos^{-1}(-2/\sqrt{14}) = 122.4^\circ$

angle with $+Y$ axis: $\beta_B = \cos^{-1}(3/\sqrt{14}) = 36.8^\circ$

angle with $+Z$ axis: $\gamma_B = \cos^{-1}(1/\sqrt{14}) = 74.5^\circ$

Vector $\mathbf{C} = 4 \mathbf{i} + 3 \mathbf{j} + 0 \mathbf{k}$ lies on $x$-$y$ plane
\[ C = \sqrt{4^2 + 3^2} = 5 \]

angle with +x axis: \[ \theta = \arctan \frac{\Delta y}{\Delta x} = \arctan \frac{3}{4} = 37^\circ \]

angle with +y axis: \[ \sigma = 90^\circ - \theta = 90^\circ - 37^\circ = 53^\circ \]

(b) \[ \mathbf{R} = \mathbf{A} - \mathbf{B} + \mathbf{C} \]

\[ = (4 \mathbf{i} - 4 \mathbf{j} - 2 \mathbf{k}) - (-2 \mathbf{i} + 3 \mathbf{j} + 1 \mathbf{k}) + (4 \mathbf{i} + 3 \mathbf{j}) \]

\[ \mathbf{R} = 10 \mathbf{i} - 4 \mathbf{j} - 3 \mathbf{k} \]

Magnitude \[ R = \sqrt{10^2 + (-4)^2 + (-3)^2} = \sqrt{125} \]

What are the angles with the axis?

3. (2)(3)(4) An airplane moves in a northwesterly direction at 125 mi/hr relative to the ground, due to the fact that there is a westerly wind of 50 mi/hr relative to the ground. How fast, and in what direction, would the plane have traveled if there were no wind?

Solution:

The first step is to do an illustrative vectorial graph using the fact that \( \mathbf{V}_{pg} = \) plane velocity with respect to ground

\( \mathbf{V}_{pw} = \) plane velocity with respect to wind

\( \mathbf{V}_{wg} = \) wind velocity with respect to ground

Then

\[ \mathbf{V}_{pg} = \mathbf{V}_{pw} + \mathbf{V}_{wg} \]

or

\[ \mathbf{V}_{pw} = \mathbf{V}_{pg} - \mathbf{V}_{wg} = \mathbf{V}_{pg} + (-\mathbf{V}_{wg}) \]
By using unit vectors:

\[ \vec{v}_{wg} = \vec{v}_{wg} \hat{j} \quad \text{and} \quad \vec{v}_{pg} = \vec{v}_{pg} \left( \frac{\sqrt{2}}{2} \hat{i} - \frac{\sqrt{2}}{2} \hat{j} \right) = \vec{v}_{pg} \frac{\sqrt{2}}{2} (\hat{i} - \hat{j}) \]

\[ \vec{v}_{pw} = \vec{v}_{pg} \frac{\sqrt{2}}{2} (\hat{i} - \hat{j}) - \vec{v}_{wg} \hat{j} = \vec{v}_{pg} \frac{\sqrt{2}}{2} \hat{i} - \vec{v}_{wg} \hat{j} \]

- (\vec{v}_{pg} \frac{\sqrt{2}}{2} + \vec{v}_{wg}) \hat{j} \]

Rectangular form:

\[ \vec{v}_{pw} = 125 \left( \frac{\sqrt{2}}{2} \right) \hat{i} - (125 \left( \frac{\sqrt{2}}{2} \right) + 50) \hat{j} = \]

\[ = 88 \hat{i} - 138 \hat{j} \text{ mi/hr} \]

Polar form:

magnitude \[ |\vec{v}_{pw}| = \vec{v}_{pw} = \sqrt{88^2 + 138^2} = 164 \text{ mi/hr} \]

and direction \[ \alpha = \arctan \left( \frac{88}{138} \right) = -33^\circ \] or \[ 33^\circ \] north of west

4. (3) (4) A particle moves along a curve whose parametric equations are \[ x = e^{-t}, \quad y = 2 \cos 3t, \quad z = 2 \sin 3t, \] where \( t \) is time in sec, and distances are in meters.

(a) Determine its velocity and acceleration at any time.

(b) Find the magnitudes of the velocity and acceleration at \( t = 0 \).

Solution:

(a) The position vector \( \vec{r} \) of the particle is

\[ \vec{r} = x \hat{i} + y \hat{j} - z \hat{k} \]

\[ \vec{r} = e^{-t} \hat{i} + 2 \cos 3t \hat{j} + 2 \sin 3t \hat{k} \text{ (m)} \]

Then the velocity is

\[ \vec{v} = \frac{d\vec{r}}{dt} = -e^{-t} \hat{i} - 6 \sin 3t \hat{j} + 6 \cos 3t \hat{k} \text{ (m/s)} \]
and the acceleration is
\[ \mathbf{a} = \frac{d\mathbf{V}}{dt} = e^{-t} \mathbf{i} - 18 \cos 3t \mathbf{j} - 18 \sin 3t \mathbf{k} \text{ (m/sec}^2\text{)} \]

(b) At \( t = 0 \),
\[ \frac{d\mathbf{r}}{dt} \bigg|_{t=0} = -\mathbf{i} + 6 \mathbf{k} \text{ and } \frac{d^2\mathbf{r}}{dt^2} \bigg|_{t=0} = \frac{d\mathbf{V}}{dt} \bigg|_{t=0} = \mathbf{i} - 18 \mathbf{j} \]

Then, magnitude of velocity at \( t=0 \) is
\[ V = \sqrt{(-1)^2 + (6)^2} = \sqrt{37} \text{ m/sec} \]

magnitude of acceleration at \( t=0 \) is
\[ a = \sqrt{1^2 + (-18)^2} = \sqrt{325} \text{ m/sec}^2 \]

Which are the corresponding directions?

5. (5) Using vector algebra, prove that the diagonals of a rhombus are perpendicular to each other.

Solution:

Diagonals are: \( \mathbf{c} = \mathbf{a} + \mathbf{b} \)

and \( \mathbf{d} = \mathbf{b} - \mathbf{a} \) as shown in the figure

\[ \mathbf{c} \cdot \mathbf{d} = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{b} - \mathbf{a}) = b^2 - a^2 \]

Since \( |\mathbf{b}| = |\mathbf{a}| \) it follows \( \mathbf{c} \cdot \mathbf{d} = 0 \)

But \( \mathbf{c} \cdot \mathbf{d} = 2cd \cos \theta = 0 \)

Since \( c \neq 0 \) and \( d \neq 0 \), it follows \( \cos \theta = 0 \), \( \theta = 90^\circ \)

at \( \mathbf{c} \perp \mathbf{d} \)
6. (5) Given the following position vectors \( \vec{D} = 2\hat{i} + 3\hat{j} + 1\hat{k}(m) \) and \( \vec{E} = 4\hat{i} - 5\hat{j} - 3\hat{k}, \) (m) calculate:

(a) \( \vec{D} \cdot \vec{E}; \)

(b) the angle between \( \vec{D} \) and \( \vec{E}. \)

(c) \( \vec{D} \times \vec{E} \)

(d) \( \vec{E} \times \vec{D} \) (show explicitly that \( \vec{D} \times \vec{E} = -\vec{E} \times \vec{D} \))

Solution:

(a) \( \vec{D} \cdot \vec{E} = D_xE_x + D_yE_y + D_zE_z = (2)(4) + (3)(-5) + (1)(-3) \ m^2 = (8 - 15 - 3)m^2 = -10m^2. \)

(b) We wish to find \( \theta; \) we could use either

\[ \vec{D} \cdot \vec{E} = |\vec{D}| \ |\vec{E}| \cos \theta, \text{ or } |\vec{D} \times \vec{E}| = |\vec{D}| \ |\vec{E}| \sin \theta. \]

Using the former, we have

\[ \cos \theta = \frac{\vec{D} \cdot \vec{E}}{|\vec{D}| \ |\vec{E}|}. \]

From above we have that the numerator \( \vec{D} \cdot \vec{E} = -10m^2. \) Now,

\[ |\vec{D}| = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{4 + 9 + 1} \ m = \sqrt{14} \ m = 3.75 \ m, \]

and

\[ |\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{16 + 25 + 9} \ m = \sqrt{50} \ m = 7.07 \ m. \]

(Note: We have preserved three significant figures because the calculation is not yet completed.) Therefore,

\[ \cos \theta = \frac{-10 \ m^2}{(3.75)(7.07) \ m^2} = -0.38. \]
The fact that the dot product (and therefore \( \cos \theta \)) is negative alerts us to the fact that the vectors are at an obtuse angle with each other and \( \theta > 90^\circ \), as shown. Looking up 0.38 in a trig table gives \( \phi = 67.8^\circ \). But the angle we want is \( \Theta = 180^\circ - \phi = 112.2^\circ \).

\[
\begin{align*}
(c) \quad \vec{D} \times \vec{E} &= (D_Ez - D_zE_y)\hat{i} + (D_zE_x - D_xE_z)\hat{j} + (D_xE_y - D_yE_x)\hat{k} \\
&= [(3)(-3) - (+1)(-5)] \hat{i} + [(+1)(4) - (2)(-3)]\hat{j} + \\
&\quad + [(2)(-5) - (3)(4)]\hat{k}, \\
\vec{D} \times \vec{E} &= -4\hat{i} + 10\hat{j} - 22\hat{k}.
\end{align*}
\]

\[
(d) \quad \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
E_x & E_y & E_z \\
D_x & D_y & D_z
\end{vmatrix} = \\
\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
4 & -5 & -3 \\
2 & 3 & 1
\end{vmatrix} = \\
[(-5)(1) - (3)(-3)]\hat{i} + [(2)(-3) - (4)(1)]\hat{j} + \\
\quad + [(4)(3) - (-5)(2)]\hat{k}
\]

\( \vec{E} \times \vec{D} = +4\hat{i} - 10\hat{j} + 22\hat{k} = -\vec{D} \times \vec{E} \) by comparison with (c).
Unit IV  
PRETEST

I. Answer the following questions:

(1) (a) Are there any differences between saying the average speed is the magnitude of the average velocity vector and saying the average speed is the total length of path traveled divided by the elapsed time?

(b) State which of the following quantities are vectors or scalars:
   1) The distance from Stockton to San Francisco.
   2) The reading of the car's meter in mi/hr.
   3) The location of a box in a storage area.

II. Two towns, A and B, are situated directly opposite each other on the banks of a river whose width is 8 miles and which flows at a speed of 4 mi/hr. A man located at A wishes to reach town C which is 6 miles upstream from and on the same side of the river as Town B. If his boat can travel at a maximum speed of 10 mi/hr and if he wishes to reach C in the shortest possible time, what course must he follow, and how long will the trip take?

III. Given \( \vec{A} \) with magnitude 15m pointing along the positive y axis, and \( \vec{B} = (4\hat{i} - 5\hat{k}) \) m, Calculate:

(a) \( \vec{A} \cdot \vec{B} \)

(b) The angle between \( \vec{A} \) and \( \vec{B} \)

(c) \( \vec{A} \times \vec{B} \)

(d) \( \vec{B} \times \vec{A} \) (show explicitly that \( \vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \).)

---

**Unit IV**  
PRETEST

I. Answer the following questions:

(1) (a) Are there any differences between saying the average speed is the magnitude of the average velocity vector and saying the average speed is the total length of path traveled divided by the elapsed time?

(b) State which of the following quantities are vectors or scalars:
   1) The distance from Stockton to San Francisco.
   2) The reading of the car's meter in mi/hr.
   3) The location of a box in a storage area.

II. Two towns, A and B, are situated directly opposite each other on the banks of a river whose width is 8 miles and which flows at a speed of 4 mi/hr. A man located at A wishes to reach town C which is 6 miles upstream from and on the same side of the river as Town B. If his boat can travel at a maximum speed of 10 mi/hr and if he wishes to reach C in the shortest possible time, what course must he follow, and how long will the trip take?

III. Given \( \vec{A} \) with magnitude 15m pointing along the positive y axis, and \( \vec{B} = (4\hat{i} - 5\hat{k}) \) m, Calculate:

(a) \( \vec{A} \cdot \vec{B} \)

(b) The angle between \( \vec{A} \) and \( \vec{B} \)

(c) \( \vec{A} \times \vec{B} \)

(d) \( \vec{B} \times \vec{A} \) (show explicitly that \( \vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \).)
Film-Loops

Reread the general instructions given in Unit II. If you have any questions contact your proctor. You should try to answer all the questions included in the film notes.

For Section 3:

1. The Velocity Vector 80-2512
   Shows the velocity vector, its properties, and its connection with the real motion of a moving particle.

2. Vector Addition: Velocity of a Boat 80-3478
   A boat crossing a river is viewed from a stationary point on a bridge and this way the addition of velocities is illustrated. You can see the idea and perform the related measurements and calculations.

For Section 4:

The Acceleration Vector 80-2538
Introduces the concept of the acceleration vector and shows the parallel between its relationship to the velocity vector and the relationship of the velocity vector to the displacement vector.
UNIT V

MOTION IN TWO DIMENSIONS

Introduction

The basic concepts related to the motion of objects in one dimension were explored in Units II and III. In this unit the discussion will be extended to the motion of physical bodies in two and three dimensions using the vector language developed in Unit IV. However, the emphasis will be placed on motion on a plane surface, since many interesting physical processes can be reduced and analyzed in two dimensions.

The concepts of velocity vector, acceleration vector, kinetic energy and potential energy will be applied to ballistic problems. Then, circular motion will be considered, including uniform and accelerated motion. Finally, collisions on a plane surface will be analyzed using the principles of conservation of momentum and energy. This unit can be hard because it deals mainly with applications where it is necessary to use the knowledge and analytic skills learned in the past units.
Unit V

Objectives

To be able to:

1. Apply the concepts of velocity vector, acceleration vector, kinetic energy and potential energy to ballistic problems (parabolic trajectory).
2. Solve problems of an object moving in uniform circular motion.
3. Relate circular variables to linear variables.
4. Solve problems of bodies moving in accelerated circular motion.
5. Analyze two-dimensional collisions (or explosions) by using the concepts of momentum and energy.
Unit V

SUGGESTED PROCEDURE


1. Go over Chapter 3, Section 5 of the textbook. It contains applications to ballistic problems including the use of concepts that you learned in Unit II -- namely potential and kinetic energy. On page 56 of the textbook there is a discussion of a very good example (§6) that you must understand. If you have difficulties ask the instructor or proctor. Also, you could read pages 45, 46, 47 and 48 of Fundamentals of Physics, Halliday and Resnick (New York: John Wiley & Sons, 1974).

2. Analyze worked problem 1 and solve problems 12, 13 and 15 in Chapter 3 of the textbook. Note that in problem 15, the shell velocity must be $v_0 = 300 \text{ m/sec}$.

3. Study Chapter 3, Section 7 dealing with physical bodies in a circular trajectory. It deserves special attention, but you should review the units used to measure angles (see Remedial Unit I: Trigonometry). It is recommended that you read carefully the contents of this Section 7, and also the Supplement IV, Section 1. Afterward try to go in detail over examples 8 and 9 in the textbook and analyze worked problem 2.

5. Study carefully Supplement IV, Section 2 to learn the derivation of velocity and acceleration in polar coordinates. Think about the physical meaning of each term involved in equations (S4-6) and (S4-7'). Also, study the relations between polar and lineal variables.

6. Analyze worked problem 3 and solve problems 21, 24*, and 28*.

7. Review the concepts of momentum and kinetic energy in Unit II and proceed to study Section 6 in the textbook entitled "Collisions on a Plane Surface". Read example 7 (page 59). It will help you. Note how every problem of this kind always starts with a vectorial equation (momentum equation). Study in detail worked problem 4.

8. Solve problems 16, 19 and 20 at the end of Chapter 3.

9. Try the Pretest. If you find that you need more practice, go again over the problems you found more difficult and ask for assistance from your proctor. Try the optional problems 14, 17, 18 and 26 of your textbook; and the problems 9, 11, 13, 37, and 39 of Fundamentals of Physics, Halliday & Resnick (New York: John Wiley & Sons, 1974). Also, it will be helpful to study the film-loops noted in the Audiovisual Aids Section of this Unit.
UNIT V  
SUMMARY OF RELATIONS AND DEFINITIONS  

Ballistic Problems
\[ a_y = \frac{d^2 y}{dt^2} = -g, \quad a_x = \frac{d^2 x}{dt^2} = 0 \]
\[ u_y = v_{y_0} - gt = u_0 \sin \theta_0 - gt, \quad u_x = u_{x_0} = u_0 \cos \theta_0 \]
\[ y = u_{y_0} t - \frac{1}{2} g t^2, \quad z = u_{x_0} t \]
\[ y = \left( \frac{v_{y_0}}{u_{x_0}} \right) x - \left( \frac{u_{x_0} \tan \theta_0}{u_{x_0}} \right) x^2 = \left( \frac{u_{x_0} \tan \theta_0}{2u_{x_0}} \right) x^2 \]
trajectory is a parabola
\[ z = y + \frac{v_{y_0}}{u_{x_0}} x = (1/2) u_{x_0}^2 \] if \( u_0 \) is reference zero

Circular Motion

<table>
<thead>
<tr>
<th>Angular Position</th>
<th>( \theta = \theta(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular Speed</td>
<td>( \omega = \frac{d\theta}{dt} )</td>
</tr>
<tr>
<td>Angular Acceleration</td>
<td>( \alpha = \frac{d\omega}{dt} = \frac{d^2 \theta}{dt^2} )</td>
</tr>
<tr>
<td>Period</td>
<td>( T = \frac{2\pi}{\omega} )</td>
</tr>
<tr>
<td>Frequency</td>
<td>( f = \frac{1}{T} = \frac{2\pi}{2\pi} )</td>
</tr>
<tr>
<td>Uniform Circular Motion</td>
<td>( \alpha = 0 )</td>
</tr>
<tr>
<td>( \omega = \text{constant} )</td>
<td>( \theta(t) = \theta_0 + \omega t )</td>
</tr>
<tr>
<td>With Constant Angular Acceleration</td>
<td>( \alpha = \text{constant} )</td>
</tr>
<tr>
<td>( \omega(t) = \omega_0 + \alpha t )</td>
<td>( \theta(t) = \theta_0 + \omega_0 t + (1/2) \alpha t^2 )</td>
</tr>
<tr>
<td>( \omega^2 \alpha^2 = 2(\theta_0 \theta_0) )</td>
<td></td>
</tr>
</tbody>
</table>

Relation between Linear and Angular Variables:
<table>
<thead>
<tr>
<th>arc length</th>
<th>( s = r\theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u = \omega r )</td>
<td>( \alpha = \omega^2 r = \frac{d^2 \theta}{dt^2} )</td>
</tr>
<tr>
<td>( c_x = \omega^2 r )</td>
<td>( c_y = \omega )</td>
</tr>
</tbody>
</table>

Arbitrary Motion on a Plane Surface:
\[ \vec{r} = \vec{r}_0 + \int \vec{v} \, dt \]
\[ \vec{v} = \frac{d\vec{r}}{dt} = \frac{d\theta}{dt} \hat{\theta} + r \frac{d\theta}{dt} \hat{\theta} \]
\[ \vec{a} = \frac{d^2 \vec{r}}{dt^2} = \frac{d^2 \theta}{dt^2} \hat{\theta} + \left( \frac{d\theta}{dt} \right)^2 \hat{\theta} + \left( \frac{d\theta}{dt} \right) \frac{d\theta}{dt} \vec{\omega} \]
for Circular Motion with constant angular acceleration,
\[ \frac{d\theta}{dt} = 0, \quad \frac{d^2 \theta}{dt^2} = \alpha \]
\[ \vec{v} = \omega \vec{\omega} \hat{r} \]
tangent to the circle
\[ \vec{a} = -\omega^2 \hat{r} + \vec{r} \vec{\omega} \]
radial pointing to the center of the circle
\[ \vec{a} = -\omega^2 \hat{r} + \vec{r} \vec{\omega} \]
tangent to the circle

Collision on a Plane Surface

| Always momentum conservation | \( \Delta T = 0 \), or \( L_T = \text{constant} \) |
| Elastic Collisions Only      | \( \Delta T = 0 \) or \( L_T = \text{constant} \) |
1. (1) A batter hits a pitched ball a height 4 ft. above ground so that its angle of projection is $45^\circ$, and its horizontal range is 350 ft. The ball is fair down the left line where a 24 ft. high fence is located 320 ft. from home plate. Will the ball clear the fence?

Solution:

The parabolic trajectory with maximum range ($\theta = 45^\circ$) starts 4 ft above the ground level. By placing the reference coordinates at the height 4 ft. it is easy to use the formulae (3-13) and (3-12)

From (3-13)

$$X_{\text{max}} = \frac{v_0^2}{g} \sin 2\theta$$

$$X_{\text{max}} = \frac{v_0^2}{g}, \text{ since } \theta = 45^\circ$$

$$v_0^2 = X_{\text{max}}(g)$$

From (3-12), the vertical position, above the x-axis when $X_1 = 320$ ft.

$$Y_1 = X_1 \tan \theta - \frac{X_1^2 g}{2v_0^2 \cos^2 \theta}$$

$$Y_1 = X_1 - \frac{X_1^2}{X_{\text{max}}^2}; \quad Y_1 = 320 \left(1 - \frac{320}{350}\right) = 27.5 \text{ ft.}$$
And the height of the ball above ground level

\[ h_1 = 4 + 27.5 = 31.5 \text{ ft.} \quad \Rightarrow \quad h_f = 24 \text{ ft.} \]

So that the ball clears the fence.

2. (2) A particle moves on the x-y plane doing a circular trajectory of radius \( r \) and with a constant angular speed \( \omega \). Show that:

(a) The velocity \( \vec{v} \) of the particle is perpendicular to \( \vec{r} \).

(b) The acceleration is directed toward the origin and has a magnitude proportional to the distance from the origin.

(c) \( \vec{r} \times \vec{v} \) is a constant vector.

Solution:

(a) \( w = \text{constant} = \frac{d\theta}{dt} \)

\[ \theta = \omega t \]

So that

\[ x = r \cos \theta = r \cos \omega t \]
\[ y = r \sin \theta = r \sin \omega t \]

\[ \vec{r} = x \hat{i} + y \hat{j} = r \cos \omega t \hat{i} + r \sin \omega t \hat{j} \]
\[ \vec{v} = \frac{d\vec{r}}{dt} = -w r \sin \omega t \hat{i} + wr \cos \omega t \hat{j}; \]

\[ |\vec{v}| = v = wr = \text{constant} \]

Then

\[ \vec{r} \cdot \vec{v} = [r \cos \omega t \hat{i} + r \sin \omega t \hat{j}] \cdot [-w r \sin \omega t \hat{i} + \]
\[ \quad + wr \cos \omega t \hat{j}] \]
\[ = (-wr^2 \cos \omega t \sin \omega t) + (wr^2 \sin \omega t \cos \omega t) \]
\[ = 0 \]

and \( \vec{r} \) and \( \vec{v} \) are perpendicular.
(b) \[ \ddot{\mathbf{r}} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} = -\omega^2 \mathbf{r} \cos \omega t \hat{i} - \omega^2 \mathbf{r} \sin \omega t \hat{j} \]

\[ \quad \quad \quad = -\omega^2 (r \cos \omega t \hat{i} + r \sin \omega t \hat{j}) \]

\[ \quad \quad \quad = -\omega^2 \mathbf{r} \quad \quad |\ddot{\mathbf{r}}| = a_n = \omega^2 r = \text{constant} \]

Then the acceleration is opposite to the direction of \( \mathbf{r} \), i.e. it is directed toward the origin. Its magnitude is proportional to \( |\mathbf{r}| \) which is the distance from the origin. The acceleration, directed toward the center of the circle is the centripetal acceleration.

(c) \[ \mathbf{r} \times \mathbf{v} = [r \cos \omega t \hat{i} + r \sin \omega t \hat{j}] \times [-w r \sin \omega t \hat{i} + w r \cos \omega t \hat{j}] \]

\[ \quad \quad = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r \cos \omega t & r \sin \omega t & 0 \\ -w r \sin \omega t & w r \cos \omega t & 0 \end{vmatrix} = wr^2 (\cos^2 \omega t + \sin^2 \omega t) \hat{k} \]

\[ \mathbf{r} \times \mathbf{v} = wr^2 \hat{k}, \quad \text{a constant vector} \]

What is the physical meaning of this vector?

3. (3) (4) A wheel starts from rest with a constant angular acceleration of 3.0 radians/sec\(^2\). The radius of the wheel is 0.50 meter. Calculate 2.0 sec later:

(a) The angular speed of the wheel.
(b) The angular displacement.
(c) The linear or tangential speed of a particle on the rim.
(d) The tangential acceleration of a particle on the rim.
(e) The centripetal acceleration of a particle on the rim.
(f) Are the preceding results the same for a particle halfway in from the rim?

Solution:

(a) By using Eq. (3-18):

\[ w = w_0 + \alpha t \]

Since \( w_0 = 0 \)

\[ w = 0 + (3.0)(2.0) = 6.0 \text{ rad/sec} \]

(b) Eq. (3-18'):

\[ \theta = \frac{1}{2} \alpha t^2 + w_0 t + \theta_0. \quad \theta = \frac{1}{2} \alpha t^2 \]

Since at \( t=0 \) we have

\[ w_0 = 0 \quad \theta_0 = 0 \]

Therefore, after 2.0 sec

\[ \theta(t = 2.0) = \frac{1}{2} (3.0)(2.0)^2 = 6.0 \text{ radians} = \frac{6.0}{2\pi} \text{ rev} = 0.96 \text{ rev} \]

(c) \( v = wr \)

\[ = (6.0)(0.50) = 3.0 \text{ m/sec} \quad \text{linear speed} \]

(d) \( a_T = \alpha r \)

\[ = (3.0)(0.50) = 1.5 \text{ m/sec}^2 \quad \text{(tangential acceleration)} \]

(e) \( a_R = \frac{v^2}{r} = w^2 r, \quad a_R = (6.0)^2(0.50) = 18 \text{ m/sec}^2 \quad \text{(centripetal acceleration)} \]

(f) Halfway in from the rim, \( r = 0.25 \text{ meter} \)

The angular variables are the same for this point as for a point on the rim. That is,

\[ \alpha = 3.0 \text{ rad/sec}^2, \quad w = 6.0 \text{ rad/sec} \]

But now the linear variables change,

\[ v = 1.5 \text{ m/sec}, \quad a_T = 0.75 \text{ m/sec}^2, \quad a_R = 9.0 \text{ m/sec}^2 \]
4. (5) A radioactive nucleus, initially at rest, decays by emitting an electron and a neutrino at right angles to one another. The momentum of the electron is $1.2 \times 10^{-22}$ kg-m/sec and that of the neutrino is $6.4 \times 10^{-23}$ kg-m/sec.

(a) Find the direction and magnitude of the momentum of the recoiling nucleus. The mass of the residual nucleus is $5.8 \times 10^{-26}$ kg.

(b) What is its kinetic energy of recoil?

Solution:

(a) We think of the system (residual nucleus + electron + neutrino) as initially bound and forming the radioactive nucleus. The system then fragments into three separate parts. The momentum of the system before fragmentation is zero. In the absence of external actions, the momentum after fragmentation is also zero. Hence, initial momentum = final momentum

$$\vec{P}_i = \vec{P}_f = 0$$

$$\vec{P}_N + \vec{P}_e + \vec{P}_\nu = 0$$

By using unit vectors

$$\vec{P}_\nu = P_\nu \hat{i}$$

$$\vec{P}_e = P_e \hat{j}$$

$$\vec{P}_N = P_{Nx} (-\hat{i}) + P_{Ny} (-\hat{j})$$
Therefore,

\[(P_v - P_{nx}) \hat{i} + (P_e - P_{ny}) \hat{j} = 0\]

\[\hat{P}_N = -P_v \hat{i} - P_e \hat{j}\]

or \[P_N = \sqrt{(P_v)^2 + (P_e)^2}\] and \[\theta = \arctan \frac{P_e}{P_v}\]

Using the numerical values:

\[P_N = 1.36 \times 10^{-22} \text{ kg} \cdot \text{m/sec}, \text{ and } \theta = 62^\circ\]

152° with respect to the electron or

-118° with respect to the neutrino

(b) Kinetic energy of the residual nucleus

\[K_N = \frac{1}{2} M_N V_N^2 = \frac{P_N^2}{2M_N}, \text{ } K_N = 1.6 \times 10^{-18} \text{ joules} = 1 \text{ eV}\]

Kinetic energy of the electron

\[K_e = \frac{1}{2} m_e V_e^2 = \frac{P_e^2}{2m_e}, \text{ } K_e = 0.79 \times 10^{-14} \text{ joules}\]

What is the kinetic energy of the neutrino?

From where does the energy of the fragments come?

Is this an elastic or inelastic collision?
PRETEST

Unit V

I. In a cathode-ray tube a beam of electrons is projected horizontally with a speed of $1.0 \times 10^9$ cm/sec into the region between a pair of horizontal plates 2.0 cm long. An electric field between the plates exerts a constant downward acceleration on the electrons of magnitude $1.0 \times 10^{17}$ cm/sec$^2$. Find:

(a) The vertical displacement of the beam in passing through the plates.
(b) The velocity of the beam as it emerges from the plates.
(c) The kinetic energy of an electron as it emerges from the plates.

II. An earth satellite moves in a circular orbit 400 miles above the earth's surface. The time for one revolution is found to be 98 min. Find the acceleration of gravity at the orbit.

III. A phonograph turntable is turning at 33 1/3 r.p.m. and has a radius of 0.150 m. After you turn it off, the bearing friction slows down the motion with a constant deceleration, and the turntable stops in 15 sec.

(a) What is the angular acceleration of the turntable?
(b) How many revolutions did it make before stopping?
(c) What are the radial and tangential accelerations of a point at the rim when $\omega = 3.00$ rad/sec?

IV. A shell of mass M is traveling along the X-direction with a velocity of 1000 m/sec; it explodes in mid-air into two fragments of equal mass. After the explosion one of the fragments has a velocity

$$\vec{v}_1 = 1200 \hat{i} + 900 \hat{j} \text{ (m/sec)}$$

(a) Find the vector velocity $\vec{v}_2$ of the other fragment and make a sketch of the situation.
(b) Calculate the kinetic energy before and after the explosion.
(c) What amount of energy is released in the explosion?

\[ I, II, III, IV \]
Unit V  AUDIOVISUAL AIDS SECTION

Film Loops:

For Section 5:

Trajectories  80-3064

The film acquaints you with the parabolic trajectory of a moving particle subject to a one-direction force. Aspects such as speed and acceleration in the two separate directions, collisions between moving objects, and range of a projectile are covered using the air table.

For Section 6:

Collisions in Two Dimensions  80-3155

The relationships involved in collisions on a plane surface are demonstrated. You can verify the conservation of momentum and determine an unknown mass. The air table is used.

For Section 7:

1. Velocity and Acceleration in Circular Motion  80-2546

This film demonstrates the relationships existing between displacement, velocity and acceleration in uniform circular motion.

2. Circular Motion  80-3080

Relationships between speed, velocity and acceleration for uniform circular motion are illustrated using the air table.
Auto-Lecture

Ask for the Autolecture No. 2 and study the new examples and try to answer the questions. If you have any doubts discuss them with your proctor and/or your instructor.

Autolecture No. 2 is longer than the first one because it covers the materials of Units IV and V which are dense in themselves. The autolecture contains a brief theoretical description of the vector-calculus approach for each major subject. Following each description there are one or two worked-out problems that show how to apply and use effectively the vector-calculus language in the solution and interpretation of physical interactions. Another main purpose of these solved-problems is to guide you in the proper line of approach to a problem and the "method of science."

The autolecture No. 2 starts with a description of the vector-calculus kinematics definitions. They are illustrated with a situation where they are used in summation and composition of vectors and the chain's rule for addition of vector velocities. There follows a vector presentation of motion equations of vertical motion with constant acceleration, horizontal constant motion, and an orthogonal superposition of those motions. There are three problems: vertical motion under the action of gravity, horizontal launching under the gravity action, and a projectile trajectory situation solved both kinematically and energetically. Finally, there is a vector treatment of circular motion equations and the relations between linear and angular kinematic variables. A problem showing the relationship between these two types of variables is solved.
Introduction

In the previous units, the motion of physical bodies was studied and analyzed. First, the study was limited to one-dimensional motion with the geometrical description of it together with the broader concepts of linear momentum and mechanical energy. Then the discussion was extended to two and even three dimensions and deepened using a more mathematical formulation with calculus and vectors. Yet, it was never asked what the cause is for the specific motion of a physical body. This question has puzzled man for centuries.

We are familiar with the fact that if we want a body to start to move, we have to push it or pull it, directly or indirectly, to change its state. Undoubtedly you have sometimes kicked a ball and have found that the particular ball's motion immediately after the kick (rectilinear translational, parabolic, rotational, spinning, "effect", etc.) depends on how you kicked it. That is, the interaction of the study object with other nearby or remote objects (its environment) determines motion's characteristics.

Galileo, early in the seventeenth century, was the first to state, by systematic application of the concepts of velocity and acceleration, that a body free of net external influences (forces) would continue to be at rest or continue to move with constant
velocity. This Principle of Galileo was later adopted as the First Newton Law. However, it was Sir Isaac Newton (1642-1727), born in England the year of Galileo's death, who made a connection between the interactions of a study-object with its environment and its motion. He built a beautiful architecture to cover an enormous range of experienced motions.

The aim of this unit is to familiarize ourselves with the sources of forces acting on a single body and the application technique of Newton's laws so that we can obtain parameters of motion of the study-object and in some cases even the equation of motion of the body. In the next unit we will treat systems having more than a physical body.
Unit VI

Objectives

To be able to:

1. Construct a free-body diagram of a body (as a particle representation) isolated from its environment showing the external forces acting on it and naming the source of each force.

2. Express correctly each pair of action-reaction forces between the study-body and its environment.

3. Find unknown quantities of motion (acceleration, velocity, displacement, position, period, frequency) of a study-body using the Newton's Laws when the external forces acting are constant, weight, gravitational, elastic, and/or contact forces.

4. Solve for acceleration, speed, period, orbital radius and/or mass of a study body moving in a circular orbit by using the universal gravitational force law together with the second law of motion.

5. Calculate the work done on a body by a constant force using either the definition of work done by a force or the work-kinetic energy theorem.

1. Study in detail Chapter 4, Sections 1 through 4. Note that Newton's First law is in Section 1; the statement of the Second Law together with its mathematical representations is in Section 2: general form in eqn. (4-2) and the particular case for constant mass in eqn. (4-4). The latter is used in all problems of this unit. Also, pay attention in Sections 2 and 3 to the definitions of inertial mass, gravitational mass and the principle of equivalence. In Section 4 the Third Law is discussed and its application illustrated using the universal gravitational force.

2. For objectives 1 and 2 beside the context, study Figures 4-1b, c, 4-3b, 4-5, 4-6, 4-8, 4-12, 5-6, 5-7, 5-9, and 5-10.

3. The forces that you will master are
   (a) Weight: the constant force of earth's gravitation near its surface, discussed in Chapter 4, Sections 3 (p.74), example 1, Section 5 (pp. 88-89).
   (b) Elastic force discussed in Chapter 4, Section 3 (pp. 76-77-78), example 2 where parameters of simple harmonic motion are found, and Chapter 5, Section 3.
   (c) Universal gravitational force discussed in Chapter 4, Sections 4 and 5, with special attention to the superposition principle, gravitational interaction on the surface of the earth, the simple pendulum and S.H.O.
accomplishes objective 4. Skip the Cavendish experiment.

(d) Contact forces: constraints (normals) and frictional forces are discussed thoroughly in Chapter 5, section 4, examples 2 and 3.

4. Analyze the worked-problems 1, 2, and 3 noting how the solving procedure outlined in the Summary of Relations and Definitions is followed. Solve problems in Chapter 4: 1, 2, 3, 4, 11, 18, 21, and 22, and in Chapter 5: 8, 12, and 21.

5. After reading Chapter 4, section 5, study worked problem 4 and solve the problems in Chapter 4: 4, 6, 9, 10, and 14. Chapter 5: 23.

6. The last objective is achieved studying Chapter 4, Section 6, paying special attention to context of eqn. (4-23) differential of work, eqn. (4-25) work done by a constant force, eqn. (4-24") work kinetic energy theorem, and example 4. Study worked problem 5 and solve problems 22 and 23 in Chapter 4.

7. Try the Pretest and self-grade it. If you feel you need more practice before attempting to take the Test, solve some of the additional problems referenced below.

Additional Problems:

Objective 1: B: 2-4, -7; 5-7, -9
H-R: 5-4, -5, -7, -11, -14
S-Z: 5-1

Objective 2: B: 5-1
H-R: 5-2
S-Z: 5-3, 2-1(a) - (g), 2-2.
Objective 3: B: 5-10, -11, -12, -13, -23.
H-R: 5-8, -9, -13, -16, -18, -21, -25, -35, -50
S-Z: 2-17, -22; 5-17, -21, -22, -24.

Objective 4: B: 10-12, -13, -14, -23, -24, -25.
H-R: 14-20, -22 to -27.
S-Z: 5-2, -4, -8 to -12; 6-43 to -47; 7-36(a), -50(e,f)

Objective 5: B: 8-1 to -8-14
H-R: 6-1 to -5, -12, -13, -14, -16 to -18
S-Z: 7-1 to -6; 7-16 to 19(a, b, d)
## Unit VI

### SUMMARY OF RELATIONS AND DEFINITIONS

#### Force:

arises from mutual interaction (by contact or at distance) between a body and another physical object.

#### Inertial Mass:

of a physical body is an intrinsic property of the body characterizing its reluctance to change its state of motion (inertia) under the action of a force (or forces).

#### Study-object:

(or standard body) is a single massive body chosen (isolated) to study the actions of other bodies upon it.

#### Environment:

the arrangement of other bodies surrounding (acting upon) the study-object.

#### Particle Model:

each body is considered as if it were concentrated at a point and had no extension in space.

### Newton's Laws:

#### First:

(Principle of Galileo or law of inertia): \[ \Sigma F = 0 \]

"Everybody continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it."

The inertial reference frame in which Newton's First law holds.

#### Second:

"The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

when \( m \) is constant

\[
\vec{F} = \frac{d\vec{r}}{dt} = \frac{d\vec{v}}{dt} = \frac{\vec{a}_{i.f.}}{m} = ma = m \frac{d\vec{v}}{dt} = m \frac{\vec{a}}{2} = m \frac{\vec{a}_{i.f.}}{2} = ma = m \frac{d\vec{v}}{dt} = m \frac{d^2\vec{r}}{dt^2} = m \frac{\vec{a}_{i.f.}}{2} = ma
\]
Unit VI

Third: "To every action there is always an equal reaction; or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts"

for isolated systems

\[ \sum_{i} \sum_{j \neq i} F_{ij} = 0 \]

\[ \frac{dF}{dt} = 0 \]

Types of Force

**Weight:** gravitational force near the earth's surface. Its equations of motion are of motion with constant acceleration

**Elastic:**

eqn. of motion:

\[ x(t) = A \cos(\omega t + \delta) \]

\[ \omega^2 = k/m, \ A \text{ and } \delta \text{ depend on initial conditions.} \]

period of oscillation:

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

simple pendulum:

\[ T = 2\pi \sqrt{\frac{l}{g}} \]

**Gravitational attraction:**

\[ F_{12} = \frac{GM_1m_2}{r_{12}^2} \]

relation between G and g

\[ g = \frac{GM}{R^2} \]

**Contact:**

**Friction:**

Static: \[ f_s \leq \mu_s N \]

Sliding: \[ f_k = \mu_k N \] always oppose the motion

Normal: \[ N = \text{Perpendicular force at the surfaces in contact.} \]
Unit VI

Principle of Superposition: When several forces act simultaneously on one body or system, each force acts independently of the existence of the others.

\[ \vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} \]

Work: differential done by a force
\[ dW = \vec{F} \cdot ds \]
done by a constant force on a body
\[ W = \vec{F} \cdot s \]
work-energy theorem
\[ W_{12} = \Delta T_{12} = T_2 - T_1 \]

Problem Solving Procedure

1. Identify the particular body (study-object) to whose motion the problem refers.
2. Identify all interactions (forces) between the study-body and its environment.
3. Choose a suitable inertial reference frame (position the origin and orient the coordinate axes so as to simplify the task of the next step as much as possible).
4. Draw a free-body diagram, i.e., draw the body as a particle, showing the reference frame and all the external forces acting on the body.
5. Resolve those forces not lying along a coordinate axis into their rectangular components.
6. Apply Newton's second law to each resultant component of force.

Generally, either the motion is known and the resultant force may be derived, or all the external forces are known in such a way that the acceleration is derived for constant mass and by integration to yield to the trajectory of the moving object.
Unit VI WORKED PROBLEMS

Note that one problem may accomplish several objectives at the same time. The number in parentheses is keyed to the objective's number. Springs, strings, ropes, and pulleys are considered, in general, to have negligible mass and to be frictionless.

1. Assume that block $m_2$ is moving to the right in the Figure, and all surfaces are rough.

(a) (1) Make a free-body diagram for block 2, specifying all the forces acting on it.

(b) (2) Express each pair action reaction forces.

Solution:

(a) We isolate block $m_2$ (our study-object)

- $T_3 =$ force that rope 3 exerts on $m_2$
- $T_2 =$ force that rope 2 exerts on $m_2$
- $W_2 =$ weight of $m_2$ or attraction of the earth on $m_2$
- $N_2 =$ normal force that the horizontal supporting table exerts on $m_2$
- $P =$ normal contact force on $m_2$ by $m_1$
- $f_{k1} =$ sliding frictional force on $m_2$ by the horizontal supporting table
- $f_{k2} =$ sliding frictional force on $m_2$ by the contact surface with $m_1$
(b) For each of the forces mentioned there is a reaction as follows:

Reaction to $\vec{T}_3$, $\vec{T}'_3 = \text{force that } m_2 \text{ exerts on rope } 3$

Reaction to $\vec{T}_2$, $\vec{T}'_2 = \text{force exerted by } m_2 \text{ on rope } 2$

Reaction to $\vec{W}_2$, $\vec{W}'_2 = \text{gravitational attraction by } m_2 \text{ on the earth}$

Reaction to $\vec{N}_2$, $\vec{N}'_2 = \text{force that } m_2 \text{ exerts on the horizontal supporting table}$

Reaction to $\vec{P}$, $\vec{P}' = \text{normal contact force on } m, \text{ by } m_2$

Reaction to $\vec{f}_{k1}$, $\vec{f}'_{k1} = \text{sliding force on the table by } m_2$

Reaction to $\vec{f}_{k2}$, $\vec{f}'_{k2} = \text{sliding force on } m_1 \text{ by the contact surface with } m_2$

Note that all reactions are forces done by $m_2$ on different bodies (its environment)

2. A flat plate of mass 5kg is acted upon by two forces

$\vec{F}_1 = 3 \hat{i} \text{ (N)}$ and $\vec{F}_2 = 2 \hat{i} + 2\hat{j} \text{ (N)}$. There is no friction

(a) (1) Draw a free body diagram

(b) (3) Calculate the acceleration of the plate

Solution:

(a)
(b) Resultant force acting on the plate

\[ \mathbf{F}_R = \sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = (3\mathbf{i}) + (2\mathbf{i} + 2\mathbf{j}) = 5\mathbf{i} + 2\mathbf{j} \text{ (N)} \]

Second Law:

\[ \mathbf{\hat{a}} = \frac{\mathbf{F}_R}{m} = \frac{1}{5} (5\mathbf{i} + 2\mathbf{j}) \text{ (m/s}^2 \text{)} \]

\[ \mathbf{\hat{a}} = \mathbf{i} + \frac{2}{5} \mathbf{j} \text{ (m/s}^2 \text{)} \text{ or } a_x = 1 \text{ m/s}^2 \text{ to right} \]

\[ a_y = \frac{2}{5} \text{ m/s}^2 \text{ upward} \]

3. (1)(3) A load of crates stands on the flat bed of a truck that is traveling along a highway at 60 mi/hr. The static coefficient of friction between the crates and the truck is 0.80. What is the shortest distance in which the truck can stop without letting his load slide?

Solution:

When the truck decelerates, the crates try to move ahead in the direction of motion (principle of inertia) so that the frictional force acts in the same direction of deceleration. Furthermore, in order for the crates not to slide, with maximum deceleration of the truck, they have to remain at rest with respect to the truck, i.e. they have to have the same deceleration as the truck.

Place a coordinate system fixed to earth and with its origin at the point where the truck begins to decelerate.

Free body diagram of the crate:

- \( \mathbf{N} \) = normal force
- \( \mathbf{W} \) = weight of crate = \( mg \)
- \( \mathbf{f} \) = frictional force
Newton's second law in components:
\[ \Sigma F_y = N - W = 0 \text{ no motion} \quad \therefore N = m \cdot g \]
\[ \Sigma F_x = -f = ma_x, \text{ deceleration} \]
but
\[ f = \mu_N N, \text{ maximum frictional force to allow maximum deceleration} \]
\[ a_x = -\mu_N g, \quad a_x = -25.6 \text{ (ft/s}^2) \]

and, initial speed \( u_o = 60 \text{ mi/hr} = 88 \text{ ft/s}; \) final speed \( u = 0 \)
\[ x = \frac{u^2 - u_o^2}{2a} = \frac{-u_o^2}{2a} \quad x = 151 \text{ ft} \]

4. (4) Jupiter has a moon with an approximately circular orbit of radius \( 4.2 \times 10^8 \text{ m} \) and a period of 42 hr.

(a) What is the magnitude of the gravitational field caused by Jupiter at the orbit of this moon?

(b) Find the mass of Jupiter.

Solution:
(a) Gravitation Law:
\[ F_G = \frac{GMm}{R^2} \quad (1) \]
Weight:
\[ F_g = mg \quad (2) \]
From: (1) and (2) \[ g = \frac{GM}{R^2} \quad (3) \]
Second Law \([F_R = Ma_{\text{centripetal}} = mw^2R] \quad (4)\]
From (1) and (4):
\[ \frac{R^3}{T^2} = \frac{GM}{4\pi^2} \quad (\text{Kepler's third law}) \quad (5) \]
Using (5) in (3) \[ g = \frac{4\pi^2R}{T^2}, \quad g = 0.73 \text{ m/s}^2 \]
5. (5) The constant forces \( \vec{F}_1 = \hat{i} + 2\hat{j} + 3\hat{k} \) (N) and \( \vec{F}_2 = 4\hat{i} - 5\hat{j} - 2\hat{k} \) (N) act simultaneously on a particle when a displacement from point

\[ A(20, 15, 0)(m) \] to the point \( B(0, 0, 7)(m) \) is done.

(a) What is the work done in the particle during that displacement?

(b) Suppose the same forces were acting but the motion went from \( B \) to \( A \). What is the work done on the particle in this situation?

(c) What is the final velocity in the latter situation if the particle of mass 4 kg was at rest at \( B \)?

Solution:

(a) Resultant force: \( \vec{F} = \vec{F}_1 + \vec{F}_2 = 5\hat{i} - 3\hat{j} + \hat{k} \)

Vectors position: \( \vec{r}_A = 20\hat{i} + 15\hat{j} + 0\hat{k} \) and \( \vec{r}_B = 0\hat{i} + 0\hat{j} + 7\hat{k} \)

From \( A \) to \( B \): \( \vec{r}_{AB} = \vec{r}_B - \vec{r}_A = -20\hat{i} - 15\hat{j} + 7\hat{k} \)

Work done by a constant force: \( W_{A\rightarrow B} = \vec{F} \cdot \vec{r}_{AB} \)

\[ W_{A\rightarrow B} = (5)(-20) + (-3)(-15) + (1)(7) = -48 \text{ J} \]

(b) Now, from \( B \) to \( A \): \( \vec{r}_{BA} = \vec{r}_A - \vec{r}_B = -\vec{r}_{AB} = 20\hat{i} + 15\hat{j} - 7\hat{k} \)

\[ W_{BA} = \vec{F} \cdot \vec{r}_{BA} = -\vec{F} \cdot \vec{r}_{AB} = 48 \text{ J} \]

(c) Work-energy theorem, \( W_{BA} = \Delta T_{BA} = T_A - T_B \) but \( T_B = 0 \), rest

\[ T_A = \frac{1}{2} m v_A^2 = W_{BA} \]

\[ v_A = 4.9 \text{ m/s} \]
Unit VI

PRETEST

I. A tractor climbs a rough hill of slope \( \theta \).
   (a) (1) Draw a free-body diagram of the tractor, identifying all external forces acting on it.
   (b) (2) Name each pair of action-reaction forces between the tractor and its environment.

II. A block of mass 2.0 kg is pressed against a vertical wall by a constant horizontal force of 50N. The coefficients of static friction and kinetic friction are respectively \( \mu_s = 0.6 \) and \( \mu_k = 0.4 \).
   (a) (1) Draw a free-body diagram for the block.
   (b) (3) Will the block start moving if it is at rest initially?
   (c) (2)(3) What is the force (magnitude and direction) on the block exerted by the wall?

III. (4) Assuming that the orbit of the earth about the sun is circular with orbital radius \( R = 1.5 \times 10^{11} \) (m), find the mass of the sun.

IV. (5) How much work is done by a 70 kg man walking up a 40 m high hill. If this work would be converted entirely into electrical energy, how long would the electrical energy keep a 100 watt bulb going? (\( \Omega \) Joule runs a 1 watt bulb for 1 second.)
Unit VI AUDIOVISUAL AIDS SECTION

Film-Loops

Reread the general instructions given in Units II and III.

We remind you to read the film notes and project the film on a sheet of white paper or on a board so you can take measurements and answer the questions attached to the film case.

For Chapter 4: Section 1:

Inertia I

It shows two simple experiments to demonstrate Newton's First Law of Motion—namely, every body in a state of rest or uniform motion in a right line continues in its state unless it is acted on by a resultant external force.

For Sections 2 and 3: 80-2736

Newton's First and Second Laws

By using the airtrack the motion of a body in an (almost) force free situation is illustrated. Also, the film shows the relationships existing when the body is subjected to a constant applied force.

For Section 4:

Newton's Third Law 80-2744

Shows how two gliders initially stationary on a horizontal air track exert forces on each other and how those are always equal and directed to opposite directions.
For Section 5:

Galileo's Experiment: On the Moon and On the Earth

A first demonstration was performed on August 2, 1971, at the Apollo 15 landing site near Hardley Rille and the Apennine Mountains on the moon. A second demonstration was performed by Astronaut Scott on November 25, 1974 at Edward, California.

Measurement of G -- The Cavendish Experiment 80-2124

The film loop demonstrates a standard technique—using the Cavendish balance—for determining the gravitational constant G. You can follow the instructions on the film case and using the Newton's Law of Universal Gravitation obtain the value of G.

Dynamics of a Pendulum 80-3130

Using a pendulum swinging on a tilted air table, the film makers show how its motion is affected by varying the parameter of acceleration of effective gravity, mass, and length. You can press the stop-motion button of the projector and record data to determine the period as a function of force, period as a function of mass, and period as a function of length.

Autolecture

This autolecture No. 3 will help you to gain an understanding of Newton's Law of Motion extrapolating real situations to ideal situations. It shows how to apply Newton's Laws to problems where certain conditions are idealized as having non fiction between surfaces and between bodies and air, springs and
ropes without mass, bodies as particles, and pulleys which are frictionless and massless. Although some of the situations chosen for analysis may seem simple and artificial, they are the prototypes of many interesting real situations. But more important is the method of analysis, which is applicable to all the modern and sophisticated situations of classical mechanics.

This autolecture contains a summary of Newton's Laws, and the technique of application is shown in four problems: (1) a spring scale serves to analyze action-reaction forces; (2) a system of cars in tandem serves to show application of the second law of motion; (3) a block moving on an inclined surface is a situation a little more complicated requiring application of all three laws of motion; and finally, (4) the mutual attraction between two spherical bodies requires the application of the universal gravitation law.
UNIT VII

DYNAMICS II: FORCES ON SYSTEMS

Introduction

This unit is an extension of the latter one in the sense that the same Newton's Laws are again consistently applied to physical situations to find unknown forces, masses, and variables, or equations of motion. The difference lies in that the situations are more complicated here. We will be concerned with systems of several bodies connected together, such as a plow to a horse, a train to a locomotive, a barge to a tugboat, or a body linked to a pulley. The motion of those systems (of one body or more bodies) may be uniform, with constant acceleration, and/or circular. Examples of dynamics of circular motion are twirling a weight at the end of a string, driving cars around curves, orbiting satellites, and riding merry-go-rounds or rotors.

However, a variation comes when instead of studying externally the motion of a car on a banked curve, you take the place of driver and analyze dynamically the situation. You will find that in an accelerated frame of reference you have to add a pseudo force to the real forces to be able to apply Newton's second law in the accelerated frame.

Finally, you observe in nature things that appear to us in no
motion or in uniform motion, such as a ladder propped against a wall or the structure of a bridge. These are equilibrium situations. You will study what the conditions for equilibrium are, how to use them to find unknown forces, application points of forces, or requirements of materials and dimensions that will assure equilibrium.
Unit VII

Objectives

To be able to:

1. Draw a free body diagram (as a particle representation) identifying the interaction forces for each body in a system of two or more interacting bodies and classify correctly each pair of action-reaction forces.

2. Solve for unknown parameters of motion, external and/or internal forces of a system of several bodies. The motion may be rectilinear uniform, rectilinear with constant acceleration, and/or circular.

3. Solve for parameters of motion problems of a body or system of bodies in an accelerated frame of reference by using the concept of pseudo-forces and Newton's Laws.

4. Analyze problems of equilibrium, either static or dynamic, of rigid bodies, making a free-body diagram, identifying all forces and torques, and applying the two equilibrium conditions to solve unknown parameters. The forces involved may be weight tension in ropes or wires, compressional forces on rods or hinges, constraints, and/or frictional forces.

1. Read Chapter 5 sections 1 through 4. Study in detail the Atwood's machine on pp. 105-108. This is a system of two bodies and their free-body diagrams shown in Figure 5-3. This example is analyzed dynamically and energetically, and it is found that the mechanical energy is conserved. Also, study with careful attention the discussion over the braking of an automobile on pp. 116-117. Work your own solutions for the worked problems 1 and 2, and solve problems, Chapter 5: 1, 2, 3, 5, 7 and 18. --All this holds for objectives 1 and 2.

2. For objective 3 study with analytic attention section 5 on Pseudo-forces. This is one of the most important because it introduces the concepts of inertial and accelerated frames of references and how there are two alternate interpretations to describe this effect of acceleration. Analyze examples 4, 5, and the accelerometer on pp. 121-124. Work your solution for worked-problem 3, and solve problems 4, 11, 13, 17, 19, 21 and 22.

3. Proceed to study section 6 and analyze examples 6 and 7 and worked problem 4. The two conditions for the equilibrium of a rigid body are on p. 126 before example 6. If you want to expand your study of this subject, go over Chapter 11, section 6. Solve problems 6, 15*, and 26 in
Chapter 5 and problem 17 in Chapter 11. This accomplishes objective 4.

4. As usual, we encourage you to solve the Pretest before attempting to take the competency test.
Unit VII SUMMARY OF RELATIONS AND DEFINITIONS

Newton's second Law (for \( m \) constant)
\[ \Sigma \vec{F} = \vec{F}_{\text{net}} = m \vec{a} = m \frac{d^2 \vec{r}}{dt^2} \]

Equations of motion:
\[ \vec{a}(t) = \frac{\vec{F}(t)}{m} \]
by integration
\[ \vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t) \, dt \]
\[ \vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t) \, dt \]

for \( \vec{F} \) = constant
\[ \vec{a} = \frac{\vec{F}}{m} = \text{constant} \]
\[ \vec{v} = \vec{v}_0 + \vec{a} t \]
\[ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \]

for elastic force \( \vec{F}_k = -k \vec{x} \)
\[ x = A \cos (\omega t + \delta), \quad \omega = \sqrt{k/m} \]
A, \( \delta \) depend on initial conditions

for circular motion
\[ \vec{F}_{\text{net}} = m \left[ r \omega^2 (-\hat{r}) + r \alpha \hat{\theta} \right] \]
circular uniform motion
\[ \vec{F}_{\text{net}} = m \vec{a}_\text{centripetal} = m r \omega^2 (-\hat{\theta}) \]

Noninertial reference frames
\[ \vec{a}_\text{Reference frame} \neq 0 \]

Pseudo-forces
\[ \vec{F}_p = -m \vec{a}_R \]

Newton's second law
\[ \Sigma \vec{F} + \vec{F}_p = \vec{ma} \quad \text{where} \ \vec{a} \ \text{is the body's acceleration in} \]
\[ \text{the noninertial frame} \]
in circular motion
\[ \vec{F}_p = m r \omega^2 (\hat{r}) \text{ centrifugal} \]
Unit VII

Equilibrium conditions:

1) the vector sum of all forces (including constraint forces) on the body must be zero
   \[ \sum \vec{F} = 0 \]

2) the sum of all torques acting on the body must be zero
   \[ \sum \tau = 0 \]

---

**static:** body at rest
**dynamic:** body with rectilinear uniform motion

torque exerted by a force
\[ \vec{\tau} = \vec{r} \times \vec{F} \]
\[ \tau = |\vec{r}| \cdot |\vec{F}| \cdot d_{\perp} \]

where \( d_{\perp} \) = "lever arm"

---

**Problem Solving Procedure**

1. Draw an imaginary boundary separating the system under consideration from its surroundings.
2. Draw a free body diagram showing vectors representing magnitude, direction, and point of application of all external forces acting on the system.
3. Choose a convenient coordinate frame, resolve all the external forces along these axes, and then apply the first equilibrium condition.
4. Choose a convenient axis of rotation, evaluate all the external torques around it, and apply the second equilibrium condition.

Then, the resulting simultaneous equations can be solved for the desired quantities.
1. Consider the system shown in the Figure. The kinetic coefficient of friction between block \( m_1 \) and the inclined plane is \( \mu_1 \) and between block \( m_2 \) and the horizontal table is \( \mu_2 \). The ropes and pulleys are massless, and the pulley is frictionless too. Take the following specific values: \( m_1 = 6 \text{ kg} \), \( m_2 = 5 \text{ kg} \), \( m_3 = 12 \text{ kg} \), \( \mu_1 = 0.2 \), \( \mu_2 = 0.1 \), \( \alpha = 60^\circ \) and \( g = 10 \text{ m/s}^2 \).

(a) (1) Draw a free body diagram of each body.

(b) (1) Identify all action-reaction pairs of all bodies in this system.

(c) (2) Find the acceleration of the system and the tension in the ropes.

Solution:

(a) Assuming motion to the right

For \( m_1 \):

\[ \vec{F}_{\text{net}} = m_1 g - T_1 - \mu_1 N_1 \]

For \( m_2 \):

\[ \vec{F}_{\text{net}} = m_2 g - T_2 - T_3 \]

For \( m_3 \):

\[ \vec{F}_{\text{net}} = m_3 g - T_3 \]
(b) \( \vec{f}_{1} \) = frictional force on \( m_1 \) by inclined
-\( \vec{f}_{1} \) = frictional force on \( m_1 \) by \( \vec{f}_{1} \)
\( \vec{N}_{1} \) = normal force on \( m_1 \) by inclined
-\( \vec{N}_{1} \) = normal force on \( m_1 \) by \( m_1 \)
\( \vec{T}_{1} \) = pulling tension on \( m_1 \) by rope 1
-\( \vec{T}_{1} \) = tension on rope 1 by \( m_1 \)
\( \vec{T}'_{1} \) = pulling tension on \( m_2 \) by rope 1
-\( \vec{T}'_{1} \) = tension on rope 1 by \( m_2 \)
\( \vec{f}_{2} \) = frictional force on \( m_2 \) by horizontal table
-\( \vec{f}_{2} \) = frictional force on \( \vec{f}_{2} \) by \( m_2 \)
\( \vec{N}_{2} \) = normal force on \( m_2 \) by table
-\( \vec{N}_{2} \) = normal force on \( \vec{N}_{2} \) by \( m_2 \)
\( \vec{T}'_{2} \) = pulling tension on \( m_2 \) by rope 2
-\( \vec{T}'_{2} \) = tension on rope 2 by \( m_2 \)
\( \vec{T}_{3} \) = pulling tension on \( m_3 \) by rope 2
-\( \vec{T}_{3} \) = tension on rope 2 by \( m_3 \)
\( \vec{W}_{1} \) = gravitational attraction on \( m_1 \) by earth
-\( \vec{W}_{1} \) = gravitational attraction on \( \vec{W}_{1} \) by \( m_1 \)
\( \vec{W}_{2} \) = gravitational attraction on \( m_2 \) by earth
-\( \vec{W}_{2} \) = gravitational attraction on \( \vec{W}_{2} \) by \( m_2 \)
\( \vec{W}_{3} \) = gravitational attraction on \( m_3 \) by earth
-\( \vec{W}_{3} \) = gravitational attraction on \( \vec{W}_{3} \) by \( m_3 \)
(c) As ropes and pulleys have no masses, then

\[ |T_1| = T_1 \quad \text{and} \quad |T_3| = T_3 \]

Each part of the system moves with the same acceleration \( a \).

Second Law for \( m_1 \):

\[ \Sigma F_y = N_1 - m_1 g \cos \theta = 0 \quad \text{[1]} \]

\[ \Sigma F_x = T_1 - m_1 g \sin \theta - f_1 = m_1 a \quad \text{[2]} \]

Second Law for \( m_2 \):

\[ \Sigma F_y = N_2 - m_2 g = 0 \quad \text{[3]} \]

\[ \Sigma F_x = T_3 - T_1 - f_2 = m_2 a \quad \text{[4]} \]

Second Law for \( m_3 \):

\[ \Sigma F_x = m_3 g - T_3 = m_3 a \quad \text{[5]} \]

Additionally, we know

\[ f_1 = \mu_1 N_1 \quad \text{[6]} \]

\[ f_2 = \mu_2 N_2 \quad \text{[7]} \]


\[ T_1 - m_1 g \sin \theta - \mu_1 m_1 g \cos \theta = m_1 a \quad \text{[8]} \]


\[ T_3 - T_1 - \mu_2 m_2 g = m_2 a \quad \text{[9]} \]

Adding [5], [8] and [9]

\[ m_3 g - m_1 g \sin \theta - \mu_1 m_1 g \cos \theta - \mu_2 m_2 g = \]

\[ = (m_1 + m_2 + m_3) a \quad \text{[10]} \]
This result could be obtained with less mathematical manipulation if we had realized that $T_1$ and $T_2$ are internal forces in the system of the three moving blocks, and the resultant external force along the motion of the system is exactly the left-side of eq [10], and the total mass in motion is $(m_1 + m_2 + m_3)$

$$a = \frac{g \left[ m_2 \mu_2 m_2 - m_1 (\sin \theta + \mu_1 \cos \theta) \right]}{(m_1 + m_2 + m_3)} \quad [11]$$


$$T_1 = m_1 g \left[ \frac{m_2 (\sin \theta + \mu_1 \cos \theta) - \mu_2}{(m_1 + m_2 + m_3)} + m_2 (\sin \theta + \mu_1 \cos \theta - 1) \right] \quad [12]$$


$$T_3 = m_3 g \left[ \frac{m_1 (1 + \sin \theta + \mu_1 \cos \theta) + m_2 (1 + \mu_2)}{(m_1 + m_2 + m_3)} \right] \quad [13]$$

Replacing numerical values yield to

$$a = 3.2 \text{ m/s}^2, \quad T_1 = 81.6 \text{ N}, \quad T_2 = 59.6 \text{ N}$$

2. A circular track of a highway is banked to allow traffic speed of 60 mi/hr without causing sideways friction on the tires. The radius of the curve is 650 ft. A car is moving at a speed of 80 mi/hr on the curve.

(a) (1) Draw a free-body diagram of the car negotiating the curve.

(b) (1) Identify the interactions responsible for the centripetal force.
(c) (2) What is the minimum coefficient of friction between tires and road that will allow the car to negotiate the turn without sliding off the road?

Solution:

(a) Free-body diagram

(b) The "centripetal" force is holding the car in the circular track so that it must be directed to the center of the circle (+\hat{i}). From the free-body diagram one sees that \vec{f} and \vec{N} have components in that direction; hence, the components are responsible for the net force pointing to the center.

Second Law for the car:

\[ \Sigma F_y = N \cos \Theta - W - f \sin \Theta = 0 \quad [1] \]
\[ \Sigma F_x = N \sin \Theta + f \cos \Theta = ma = mw^2R = m \frac{v^2}{R} \quad [2] \]

(c) If the car moves at 60 mi/hr, it does not depend upon friction to take the curve but only upon the normal force. Thus for \( f = 0 \)
N \cos \theta = mg
N \sin \theta = m \frac{u^2}{R}

Dividing member to member: \tan \theta = \frac{u^2}{gR}

\tan \theta = \frac{(88 \text{ ft/s})^2}{(32 \text{ ft/s}^2)(650 \text{ ft})} = 0.372, \quad \theta = 20.4^\circ

Now taking into account the friction for speeds greater than 60 mi/hr.

Using [1], [2] and \( f = \mu_s N \)

\begin{align*}
N \sin \theta + \mu_s N \cos \theta &= \mu u^2 / R, \\
N \cos \theta - \mu_s N \sin \theta &= mg
\end{align*}

\begin{align*}
\frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} &= \frac{u^2}{gR}
\end{align*}

Solving for \( \mu_s \):
\[
\mu_s = \frac{(u^2 / gR) \cos \theta - \sin \theta}{(u^2 / gR) \sin \theta + \cos \theta}
\]

\[
\mu_s = \frac{(0.662)(0.937) - (0.35)}{(0.662)(0.35) + (0.937)} = 0.23
\]

(3). Consider that the frictionless inclined plane shown in the figure is accelerating to the right at 3 m/s\(^2\). Will the small block \( m \) accelerate up or down the incline?

**Solution:**

If we fix a reference frame to the incline, then we have a noninertial coordinate system and the free-body diagram is
\( \vec{N} \) = normal force on \( m \) by inclined

\( \vec{W} \) = gravitational attraction on \( m \) by earth

\( \vec{F}_p = -ma_R \), pseudo-force because we have an accelerated frame.

So that, second law: \( \Sigma \vec{F} + \vec{F}_p = ma \)

in components:

\( \Sigma F_y = N - mg \cos \theta - ma_R \sin \theta = 0 \)

\( \Sigma F_x = mg \sin \theta - ma_R \cos \theta = ma_{\text{down}} \)

\( \therefore a = g \sin \theta - a_R \cos \theta \)

Using the numerical values \( a = 3.5 \text{ m/s}^2 \) down

Alternate Solution:

We solve the same problem without using pseudo-forces. We fix our coordinate system on earth and find the acceleration of block 1 with respect to earth.

Free-body diagram:

Second Law

\( \Sigma F_y = N - mg \cos \theta = 0 \)

\( \Sigma F_x = mgsin\theta = ma_{\text{le}} \)

Now the acceleration of the incline with respect to earth is \( a_R = 3 \text{ m/s}^2 \) horizontal, and its component parallel to the incline is
\[ \vec{a}_{ie} = \vec{a}_{Re} \cos \theta. \] Applying chain's rule for accelerations
\[ \vec{a}_{li} = \vec{a}_{le} + \vec{a}_{ei} = \vec{a}_{le} - \vec{a}_{ie} \]

\[ a_{li} = a_{le} - a_{ie} \]

\[ a_{li} = g \sin \theta - a_R \cos \theta \]

the same as above

(4). The system shown in the figure is at rest. The uniform beam weighs 40 lbs and the wire BC is massless. Find the force (magnitude and direction) exerted on the beam by the hinge A.

Solution

Free-body diagram is shown at the right side

Picking the conventional coordinates axis and resolving components of force. First equilibrium condition:
\[ \Sigma F = 0 \Rightarrow \Sigma F_x = 0 \text{ and } \Sigma F_y = 0 \]
\[ \Sigma F_x = R_x - T \cos \theta = 0 \]
\[ \Sigma F_y = R_y + T \sin \theta = m_1 g - m_2 g = 0 \]

Choosing an axis of rotation in A; Second equilibrium condition
\[ \Sigma \tau_A = 0 \]
\[ \Sigma \tau_A = m_2 g \left( \frac{\ell}{2} \cos \theta \right) + m_1 g (\ell \cos \theta) - T (\ell \sin \theta) = 0 \]
Solving [3] for \( T = (1/2 \ W_1 + 1/4 \ W_b) \ \frac{1}{\sin \Theta} \)

Substituting this value in [1] and [2]

\[ R_x = g \ (m_1 + 1/2 \ m_b) \ \frac{1}{2} \ cot \Theta = (1/2 \ W_1 + 1/4 \ W_b) \ cot \Theta \]

\[ R_y = g \ (1/2 \ m_1 + 3/4 \ m_b) = 1/2 \ W_1 + 3/4 \ W_b \]

Numerical values yield \( T = 62 \ lb, \ R_x = 48 \ lb, \ R_y = 60 \ lb \).

\[ \mathbf{R} = 48 \ \hat{i} + 60 \ \hat{j} \ (lb) \]

or

\[ R = 77 \ lb., \ \alpha = 51.4^\circ \]
Unit VII

PRETEST

I.(1)(2) Two blocks, \( m_1 = 2 \text{ kg} \) and \( m_2 = 1 \text{ kg} \) are in contact side by side on a horizontal rough table. The coefficient of sliding friction is 0.1.

(a) Draw a free-body diagram of each body if an external horizontal force of 6N is applied to \( m_1 \).
(b) Identify all action-reaction pairs of all bodies in this system.
(c) Find the force of contact between the blocks.
(d) Show that if the same magnitude of force 6N is applied to \( m_2 \) rather than \( m_1 \), the force of contact between the blocks is different than the previous value in (c).

II.(3) A pilot of a dive bomber who has been diving at a speed of 400 mi/hr pulls out of the dive by changing his course to a circle in a vertical plane.

(a) What is the minimum radius of the circle in order that the acceleration at the lowest point shall not exceed "7g"?
(b) How much does a 180 lb pilot apparently weigh at the lowest point of the pullout?

III.(4) For the situation shown, find the tension in the cable and the magnitude and direction of the force on the strut by the pivot A. Neglect the weight of the strut.
Unit VII AUDIOVISUAL AIDS SECTION

Film Loops

In general for the material covering this Unit and Chapter 5 of the textbook you can watch and study the following films:

Dynamics of Circular Motion 80-3114

The purpose of this film is to investigate the relationship among force, mass, and motion for circles of varying radii or, in other words, how Newton's Law applies to uniform circular motion. Three experiments are performed using a puck attached to a spring balanced on the air table. First, the period as a function of force; second, the period as a function of mass; and third, the period as a function of radius are investigated. Finally, you can combine the relationships derived in each part into a single equation which gives the force as a function of the parameters.

Central Forces: Iterated Blows 80-3627

The film shows the effect of various types of central forces on a moving point in space, using a computer-generated oscilloscope display. At equal time intervals the point is subjected to a central force blow which is either random, linear, or inverse square. The motion of the point can be followed projecting the film on a paper and marking the center and the blow positions. In this way you can verify the Law of Areas as a consequence of the Laws of Motion and of force always directed toward or away from a fixed point.
Kepler's Law

A computer program calculates the changing position of two planets moving under an inverse square force law around a sun and displays their positions on a cathode-ray tube at regular intervals. The force exerted by the planets on each other is ignored in the program. By projecting the film on paper and marking the successive positions of the planets, it can be verified that the motion of a body subject to an attractive inverse square force law satisfies Kepler's three laws. This film is a true "loop" in that the motion is continuous; there is no beginning and no end!

A Matter of Relative Motion

The concepts of relative motion and frame of reference are introduced using collisions as examples. The collision of two carts of equal mass is viewed as filmed in three distinct frames of reference: that of the moving cart, of the laboratory, and of the initially stationary carts. As photographed, the three events appear to be quite different, but in each the carts interact similarly, and laws of motion apply for each case.
UNIT VIII

MOMENTUM AND ITS CONSERVATION

Introduction

We have already dealt with linear momentum and its conservation for isolated systems. Hence, this Unit can be considered a review of that basic structure of physics; but, also we will expand its applications to more complicated situations. Unit VII is not a prerequisite for this one.

The description of physical systems in terms of energy and momentum "permits a much deeper insight into a physical problem." In previous units, momentum conservation was derived from Newton's second and third laws. However, physicists believe it to be a more fundamental principle than Newton's laws. The formalism of quantum mechanics is based on energy and momentum.

During our discussion of Newton's laws we showed that the total momentum of an isolated physical system remains always constant. However, the fact that momentum is conserved for isolated systems is not at all restricted. In many cases, for instance where collision problems are analyzed, the two interacting bodies can be treated as an isolated system. We already pointed out that momentum conservation is one of the most important and basic laws in our understanding of the physical world.
In this unit we will use the concept of momentum and its conservation to solve and analyze problems involving the interaction of physical bodies or systems where the forces are unknown or are too complex for a direct application of Newton's laws, or when the mass of one interacting body is changing in time as a moving rocket, a truck that lost its cargo enroute, or a conveyor belt.
Unit VIII

Objectives

To be able to:

1. Find the total linear momentum vector velocity and position of the center of momentum system (c.m.s.) of a system of interacting bodies (as particles) when given the masses, positions, and velocities of the bodies.

2. Describe the mutual interaction of a system of two or three bodies in the c.m.s. frame.

3. Solve for unknown parameters of a system of two or three interacting bodies (collisions, explosions, reactions, and decay processes) by recognizing the conditions under which the linear momentum is conserved and by applying momentum conservation.

4. Solve problems of systems of bodies where there are bodies whose mass changes substantially during the mutual interaction by using conservation of rectilinear momentum and/or its rate of change with time (rockets, conveyor belts, streams of water, and so on).
Unit VIII  
SUGGESTED PROCEDURE


This unit is mainly related to Chapter 6; however, Chapters 8 and 9 of H-R* provide an additional excellent discussion on the applications of momentum conservation. We recommend that you review carefully the concepts of linear momentum, kinetic energy, momentum conservation and Newton's Laws introduced in the previous units. For this you can use the Summary of Relations and Definitions of Units III, V and VI.

1. Study in detail Sections 1 and 2 of your textbook. They will give you a clear understanding of the motion of bodies in the laboratory system and the center-of-momentum-system (center-of-mass coordinate system). You will be able to analyze and understand the total vector linear momentum of the c.m.s. to apply momentum conservation to several simple problems. Analyze the examples shown in figures 6-1 through 6-6 (pp. 141 - 145), and read in detail example 1 on pp 145-147. Work your own solutions for worked problems I, II and III, and solve problems 1, 2, 3, 5, 11 and 18. All the preceeding holds for objectives 1 and 2.

2. For objective 3, proceed to study Section 3 on the concept of impulse including examples 2 and 3 on pp. 149-151 and worked problem IV. Solve problems 8, 10, and 13. You will realize that in collision problems, what
matters is not the force, but the integral of the force over the time interval in which the collision took place. That is the impulse that is equal to the change of momentum.

3. In previous units you used Newton's law most of the time in the form \( \vec{F} = ma \). However, the second law must be expressed as \( \vec{F} = \frac{dp}{dt} = \frac{d(mv)}{dt} \), because not always is the mass constant. So you must study Section 4 on systems with variable mass. We recommend that you read examples 4 and 5 on pages 153 and 154, but special attention must be given to the latter example and to the worked problem V. Solve problems 12, 14, 15, and 19. Proceed to study momentum conservation and rocket propulsion in Section 5. This is a special and spectacular application of momentum conservation with variable mass. Analyze figures 6-12 and 6-13 on pp. 155 and 156. Try to understand examples 6 and 7; they will be very helpful to you. Solve problems 22, 23, 24, and 25.

4. Be sure to see the Audiovisual Aids Section. Answer the Pre-test attached at the end of this unit. Remember to show your proctor or professor your worked problems and Pre-test. Now you are ready to take your test on Unit VIII. GOOD LUCK!
Unit VIII SUMMARY OF RELATIONS AND DEFINITIONS

Linear momentum of a particle \( \vec{p} = m\vec{v} \)

Conservation of linear momentum

for an isolated system of particles:

\[ \sum \vec{p}_i = \text{constant or} \]

"If the sum of the external forces acting on a system is zero"

\[ \frac{d\vec{p}}{dt} = 0 \]

Center-of-momentum system (c.m.s.)

is a system of coordinates in which the total momentum of an isolated system is zero

\[ \sum \vec{p}_i = 0 \]

c.m.s. = zero-momentum-system = rest frame = center of mass of the system if the velocities involved are small compare to the speed of light

velocity of c.m.s.

\[ \vec{U}_{cm} = \frac{\sum \vec{p}_i}{\sum m_i} \]

transformation from laboratory (unprimed variables) to c.m.s. (primed variables)

\[ \vec{v}_i' = \vec{v}_i - \vec{U}_{cm} \]

momentum of c.m.

\[ \vec{p}_{cm} = (\sum m_i) \vec{U}_{cm} \]

position vector of c.m. for a system of particles

\[ \vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \]

acceleration of c.m.

\[ \vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i} \]
Unit VIII

Impulse

\[ \mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}(t) \, dt = \Delta \mathbf{p} \]

is a measure of the cumulative effect of a force integrated over time, which produces a change in momentum.

<table>
<thead>
<tr>
<th>Formulation:</th>
<th>Differential Form</th>
<th>Integral Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>second law ( \mathbf{F} = \frac{d\mathbf{p}}{dt} )</td>
<td>( \mathbf{F} = \frac{d(m\mathbf{V})}{dt} = \mathbf{V} \frac{dm}{dt} + m \frac{d\mathbf{V}}{dt} )</td>
<td>Impulse eqn ( \mathbf{J} = \Delta \mathbf{p} )</td>
</tr>
<tr>
<td>third law ( \mathbf{F}<em>{12} = -\mathbf{F}</em>{21} )</td>
<td>Conservation of momentum ( \Delta \mathbf{p} = 0 )</td>
<td></td>
</tr>
</tbody>
</table>

Variable mass

Rocket propulsion

burn rate \( = \frac{dm}{dt} \)

exhaust velocity \( = \mathbf{u} \)

instantaneous mass and velocity of rocket \( M, \mathbf{V} \)

thrust \( = \mathbf{u} \left( \frac{dm}{dt} \right) = M \frac{d\mathbf{V}}{dt} \)

final velocity \( = \mathbf{v}_f = \mathbf{v}_o + \mathbf{u} \ln \left( \frac{M_o}{M_f} \right) \)
I. (3) A particle of mass \( m_A = m \) and velocity \( \vec{v}_A = v_0 \hat{i} \) collides elastically with a particle of mass \( m_B = 2m \) and of velocity \( \vec{v}_B = v_0 \hat{j} \). The collision is such that the relative velocity after the collision \( \vec{v}_A - \vec{v}_B \) is the negative of the relative velocity before the collision, \( \vec{v}_A - \vec{v}_B \), namely \( \vec{v}_A - \vec{v}_B = -(\vec{v}_A - \vec{v}_B) \). Find the velocities \( \vec{v}_A \) and \( \vec{v}_B \).

**Solution**

Let lower case letters, \( \vec{v} \) represent velocities and momenta before the collision and capital letters, \( \vec{V} \), the variables after collision.

Then, we have the condition:

\[
(\vec{v}_A - \vec{v}_B) = -(\vec{v}_A - \vec{v}_B)
\]

(c)

Conservation of momentum:

\[
\vec{p}_A + \vec{p}_B = \vec{P}_A + \vec{P}_B \tag{1}
\]

Kinetic energy conservation:

\[
t_A + t_B = T_A + T_B \tag{3}
\]

The equation (c) provides us with two scalar equations; equation (1) gives two additional equations because we are in two dimensions; and equation (3) gives one additional equation because energy is a scalar quantity.

In total we have 5 equations while we are seeking only 4 unknowns; the problem is overdetermined.

Using equations (c) and (1)

From (c)

\[
v_0 \hat{i} - v_0 \hat{j} = - \vec{v}_A + \vec{v}_B \tag{4}
\]

\[
\begin{cases}
-VA_x + VB_x = v_0 \\
-VA_y + VB_y = -v_0
\end{cases}
\]
From (1)

\[ m\hat{u}_0 \hat{i} + 2m\hat{u}_0 \hat{j} = m\hat{v}_A + 2m\hat{v}_B \]

\[
\begin{align*}
V_{Ax} + 2(V_{Ax} + \hat{u}_0) &= \hat{u}_0 \\
V_{Ay} + 2(V_{Ay} - \hat{u}_0) &= 2\hat{u}_0 \\
V_{Ax} + 2V_{Bx} &= \hat{u}_0 \\
V_{Ay} + 2V_{By} &= 2\hat{u}_0 \\
\end{align*}
\]

(5)

Using eqs (4) in eqs (5) we obtain

\[
\begin{align*}
V_{Ax} + 2(V_{Ax} + \hat{u}_0) &= \hat{u}_0 \\
V_{Ay} + 2(V_{Ay} - \hat{u}_0) &= 2\hat{u}_0 \\
\end{align*}
\]

Which yield the solutions

\[
\begin{align*}
V_{Ax} &= -\frac{\hat{u}_0}{3} ; & V_{Ay} &= \frac{4}{3} \hat{u}_0 \\
\end{align*}
\]

hence

\[
\begin{align*}
V_{Bx} &= 2/3 \hat{u}_0 ; & V_{By} &= \frac{\hat{u}_0}{3} \\
\end{align*}
\]

or

\[
\begin{align*}
\hat{V}_A &= \hat{u}_0 (-\frac{1}{3} \hat{i} + \frac{4}{3} \hat{j}) \\
\hat{V}_B &= \hat{u}_0 (\frac{2}{3} \hat{i} + \frac{1}{3} \hat{j}) \\
\end{align*}
\]

This solution is correct because eqs (c) and (1) are satisfied and also the kinetic energy is conserved as required for an elastic collision.

\[
\begin{align*}
t_A + t_B &= \frac{1}{2} m \hat{u}_0^2 + \frac{1}{2} (2m) \hat{u}_0^2 = \frac{3}{2} m\hat{u}_0^2 \\
T_A + T_B &= \frac{1}{2} m [(-\frac{\hat{u}_0}{3})^2 + \frac{4\hat{u}_0^2}{3} + \frac{1}{2} (2m) \left[ \frac{2\hat{u}_0^2}{3} + \frac{\hat{u}_0^2}{3} \right] \\
&= \frac{17}{18} m\hat{u}_0^2 + \frac{10}{18} m\hat{u}_0^2 = \frac{3}{2} m\hat{u}_0^2 \\
\end{align*}
\]

This should be, to some extent, a surprise, since in obtaining our solution we did not use the condition of kinetic energy conservation.
It can be shown (you must) that the condition (c) is such that kinetic energy conservation is automatically satisfied. Equation (c) in reality specifies only one condition, the scattering angle; in particular it requires in the c.m.s. frame that the particles scatter by 180° exactly.

II. (1) (2) Examine the preceding problem from the point of view of the c.m.s. system.

(a) Show that the magnitude of the relative velocity is the same in the laboratory and in the c.m.s. (it is invariant).

(b) Show that in the c.m.s. the magnitude of the relative velocity of two particles that collide elastically is the same before and after collision.

(c) From conclusions (a) and (b), show that the last problem as specified is correct, whereas stating \( \vec{v}_A - \vec{v}_B = -2(\vec{V}_A \vec{V}_B) \) is an unphysical condition.

Solution

Let us designate by primes the variables in the c.m.s. frame.

First, let us find the velocity of the center of momentum before collision:

\[
\vec{v}_{\text{cms}} = \vec{U}_{\text{cm}} = \frac{m_A \vec{v}_A + m_B \vec{v}_B}{m_A + m_B} ; \quad \vec{U}_{\text{cm}} = u_o \left( \frac{\hat{i} + 2\hat{j}}{3} \right)
\]

And the particles' velocities in the c.m.s. before collision:

\[
\vec{v}'_A = \vec{v}_B - \vec{U}_{\text{cm}} = u_o \left( \frac{2}{3} \hat{i} - \frac{2}{3} \hat{j} \right)
\]

\[
\vec{v}'_B = \vec{v}_B - \vec{U}_{\text{cm}} = u_o \left( -\frac{1}{3} \hat{i} + \frac{1}{3} \hat{j} \right)
\]

Note that \( p'_A + p'_B = 0 \) as it should be in the c.m.s. frame.
Also note that

\[ \vec{v}_A' - \vec{v}_B' = (\vec{v}_A - \vec{u}_{\text{cms}}) - (\vec{v}_B - \vec{u}_{\text{cms}}) = \vec{v}_A - \vec{v}_B \]

Namely, the difference in the velocity of two bodies is invariant under a Galilean transformation, such as performed here to transform to the c.m.s. frame. Therefore it follows that after collision

\[ \vec{v}_A' - \vec{v}_B' = \vec{v}_A - \vec{v}_B \]

Hence, the condition of Eq. (c), which is given in the laboratory system, is equally valid in that c.m.s. frame, namely, it must hold

\[ \vec{v}_A' - \vec{v}_B' = - (\vec{v}_A' - \vec{v}_B') \]  \hspace{1cm} (c)

This is very easily interpretable because it means simple reversal of the c.m.s. velocity (and momentum) vectors. And this automatically satisfies momentum and kinetic energy conservation.

Hence, the velocity vectors after collision are obtained by reversing the sign of eqns. of \( \vec{v}_A' \) and \( \vec{v}_B' \)

\[ \vec{v}_A' = -v_o \left( \frac{2}{3} \hat{i} - \frac{2}{3} \hat{j} \right) \]  \hspace{1cm} \[ \vec{v}_A \]

\[ \vec{v}_B' = -v_o \left( -\frac{1}{3} \hat{i} + \frac{1}{3} \hat{j} \right) \]  \hspace{1cm} \[ \vec{v}_B \]
Finally, we transform back to the laboratory system,

\[ \begin{align*}
\vec{v}_A &= \vec{v}_A' + \vec{u}_{\text{cms}} = v_o \left[ \left( -\frac{2}{3} + \frac{1}{3} \right) \hat{i} + \left( \frac{2}{3} + \frac{2}{3} \right) \hat{j} \right] = \\
&= v_o \left( -\frac{1}{3} \hat{i} + \frac{4}{3} \hat{j} \right) \\
\vec{v}_B &= \vec{v}_B' + \vec{u}_{\text{cms}} = v_o \left[ \left( \frac{1}{3} + \frac{1}{3} \right) \hat{i} + \left( -\frac{1}{3} + \frac{2}{3} \right) \hat{j} \right] = \\
&= v_o \left( \frac{2}{3} \hat{i} + \frac{1}{3} \hat{j} \right)
\end{align*} \]

in exact agreement with our previously obtained results.

III. (1)(3) An object of mass \( M_1 = 2\text{gm} \) with velocity \( \vec{v}_1 = 10 \hat{i} \text{ (cm/s)} \) collides with another object of mass \( M_2 = 5\text{gm} \) and velocity \( \vec{v}_2 = 3 \hat{i} + 5 \hat{j} \text{ (cm/s)} \). The objects become permanently attached to each other.

(a) What is the velocity of the center of mass?

(b) What is the final momentum of the combination in the laboratory system?

(c) What is the final momentum in the c.m.s. system?

(d) What fraction of the initial total kinetic energy is associated with the motion after collision?

Solution

(a) \[ \vec{u}_{\text{cms}} = \frac{M_1 \vec{v}_1 + M_2 \vec{v}_2}{M_1 + M_2} \quad \vec{u}_{\text{cms}} = 5 \hat{i} + \frac{25}{7} \hat{j} \text{ (cm/s)} \]

(b) Momentum is conserved in inelastic collision, so

\[ \vec{p}_F = \vec{p}_i = M_1 \vec{v}_1 + M_2 \vec{v}_2, \quad \vec{p}_F = 35 \hat{i} + 25 \frac{\text{gm - cm}}{\text{s}} \]

(c) By definition, the center-of-mass system is that system in which the center-of-mass is at rest and thus the total momentum,

\[ \vec{p}_{\text{cm}} = (M_1 + M_2) \vec{v}_{\text{cm}} = 0 \]
Note that the velocity of the center-of-mass is unchanged during the collision as no external forces act on the system. Thus,

\[ \left( \vec{p}_{\text{cm}} \right)_F = \left( \vec{p}_{\text{cm}} \right)_i = 0 \]

(d)

\[ (T_i)_{\text{Lab}} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \]

\[ (T_i)_{\text{Lab}} = \frac{1}{2} (2) (10)^2 + \frac{1}{2} (5) + (3^2 + 5^2) = 185 \text{ ergs} \]

Now, employing conservation of momentum in the lab system

\[ m_1 \vec{V}_1 + m_2 \vec{V}_2 = (m_1 + m_2) \vec{V}_F \]

where \( \vec{V}_F \) is the final velocity of the pair in the lab system. Note that \( \vec{V}_F \) is identical to \( \vec{V}_{\text{cm}} \) (in the lab system)

\[ (T_F)_{\text{Lab}} = \frac{1}{2} (m_1 + m_2) |\vec{V}_F|^2 \quad (T_F)_{\text{Lab}} = \frac{1}{2} (7) \left[ (5)^2 + (\frac{25}{7})^2 \right] = 132 \text{ ergs} \]

Thus

\[ \frac{(T_F)_{\text{Lab}}}{(T_i)_{\text{Lab}}} = \frac{132}{185} \approx 0.72 \]

IV. (3) Jack Nicklaus hits a golf ball giving it an initial speed of \( 5.0 \times 10^3 \) cm/sec directed 30° above the horizontal. The golf ball has a mass of 25 gm and the club and the ball are in contact for 10 milisec. Find:

(a) The impulse imparted to the ball.
(b) The impulse imparted to the club.
(c) The average force exerted on the ball by the club.
(d) The work done on the ball.
Solution

(a) We cannot calculate the impulse from the definition \( J = \int F \, dt \) because we do not know the force exerted on the ball as a function of time. However, we know that the change in momentum of an object acted on by an impulsive force is equal to the impulse. Hence

\[
\vec{J} = \Delta \vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)
\]

but \( \vec{v}_i \) of the golf ball is zero. Assuming positive \( \vec{v}_f = v_0 \hat{i} \)

\[
\vec{J} = m \vec{v}_f = \left(25 \times 10^{-3}\right)(5.0 \times 10) \text{ kg} - \text{m/s}
\]

along the direction of the initial velocity.

\[
\vec{J}_{\text{ball}} = 1.3 \hat{i} \text{ kg} - \text{m/s}
\]

(b) By using conservation of momentum between ball and club

\[
\Delta \vec{p}_{\text{ball}} = -\Delta \vec{p}_{\text{club}} = \vec{J}_{\text{ball}} = -\vec{J}_{\text{club}}
\]

\[
\vec{J}_{\text{club}} = -1.3 \hat{i} \text{ kg} - \text{m/s} . \text{ The minus sign shows that the direction of the impulse acting on the club is opposite that of the initial velocity of the ball.}
\]

(c) We cannot determine the exact force of the collision from the data given. Actually, any force whose impulse is \( 1.3 \text{ kg-m/s} \) will produce the same change in momentum. The average force during the collision time is:

\[
F_{av} = \frac{\Delta p}{\Delta t} ; \quad F_{av} = \frac{1.3 \text{ kg-m/s}}{10 \times 10^{-3} \text{s}} = 130 \text{ N}
\]
(d) The work-kinetic energy theorem says that the work done on the ball by the impulsive force is equal to the change of kinetic energy of the ball.

\[ W_F \text{ impulsive} = \Delta T_{\text{ball}} = T_f - T_i; \text{ but } T_i = 0 \]

\[ W_F = \frac{1}{2} (25 \times 10^{-3})(5 \times 10)^2 = 31.25 \text{ J} \]

V. (4) A heavy chain of mass \( \mu \) per unit length rests on the floor. At time \( t = 0 \) the end of the chain is pulled up along the vertical direction with a constant speed. Find the force that must be applied in order to maintain the above condition.

Solution

Here, we have a system with mass variable. The amount of chain that is being lifted is variable. Suppose a length \( x \) has already been picked off the floor. At the same time an incremental \( \Delta x \) of length of chain is being lifted from the floor and all is moving up with constant velocity. Hence, the external force \( F_a \) must balance the total weight \( x + \Delta x \) of the chain in order to keep the upward motion at constant velocity.

2nd Law: \( F_a - F_g = 0 \), motion at constant velocity

but \( F_g = (mg) \Delta x + (m'g) \Delta x = \mu gx + \frac{\Delta P}{\Delta t} \Delta x = \mu gx + \frac{\Delta m}{\Delta t} v \)
\[ F_a = F_g = \mu g x + \frac{\mu \Delta x}{\Delta t}, \text{ but } \frac{\Delta x}{\Delta t} = v \]

\[ F_a = \mu g x + \mu v^2 = \mu (g x + v^2) \]
Unit VIII

I. (1)(2) Two vehicles A and B are traveling west and south respectively toward the same intersection where they collide and lock together. Before the collision A (total weight, 9000 lb) is moving with a speed of 40 mph, and B (total weight, 12000 lb) has a speed of 60 mph.

(a) Find the magnitude and direction of the interlocked vehicles immediately after collision.
(b) Find the velocity and position at any time of the c.m.s. Describe the motion in the c.m.s. frame. Is it different from the laboratory system?

II. (3) When a bullet of mass 10 gm strikes a ballistic pendulum of mass 2 kg, the center of mass of the pendulum is observed to rise a vertical distance of 12 cm. The bullet remains embedded in the pendulum. Calculate the velocity of the bullet.

III. (3) A rifle of mass 8 kg fires a bullet of mass 20 gm. The rifle recoils initially with a velocity of 3 m/sec. The rifle is brought to rest at the gunner's shoulder in 50 millisec.

(a) What is the force experienced by the gunner?
(b) What is the muzzle velocity?

IV. (4) A rocket burns 50 gm of fuel per second, ejecting it as a gas with velocity of 500,000 cm/sec.

(a) What force does this gas exert on the rocket?
(b) Would the rocket operate in free space?
(c) If it would operate in free space, how would you steer it? Could you brake it?

\[ \text{Answers are provided at the end.} \]
Unit VIII  AUDIOVISUAL AIDS SECTION

Film-Loops

Reread the general instructions given in Unit II. We have several film-loops that introduce the laws of conservation of momentum and energy. We describe shortly the theme of each so you can choose what you want to watch and study. Don't forget to project the films on a white paper or board so you can take measurements of distance and time. Ask your proctor for a ruler stick, a stopwatch, and any other material you need to study quantitatively the films.

1. One-Dimensional Collisions: I  80-3650

Two different head-on collisions of a pair of steelballs are shown in this film. One ball at rest and the other moving are photographed in slow motion as they collide along a straight line. Detailed measurements can be taken, permitting you to make detailed analysis of the extent to which momentum and kinetic energy are conserved.

2. One-Dimensional Collisions: II  80-3668

It is an extension of the preceding film. Now, we have two situations: first, the two steelballs collide as they approach each other; and second, as one ball overtakes the other going in the same direction. You can make detailed measurements of the total momentum and kinetic energy of the balls before and after collision.

3. Inelastic One-Dimensional Collisions  80-3676

Here, the two steelballs are covered with clay so that they stick together after colliding. Hence, the collision is "inelastic".
In the first example one ball is at rest before collision. In the second example, both balls move toward each other. The total momentum and energy of the balls before and after the collision can be measured.

4. Two-Dimensional Collisions I 80-3684

Following the same setup as in the films before, two hard steelballs collide, but they do not move along the same straight line before or after collision; rather, they do two-dimensional collision. Slow-motion photography permits detailed analysis of the components of momentum, the vector momentum and the kinetic energy of the collision.

5. Two-Dimensional Collision II 80-3692

The film continues the study of two-dimensional collisions, showing two situations in slow motion: first, the steelball at rest is struck at a glancing angle by another ball of equal mass; the second experiment then shows the results when the two equal balls moving at the same initial speed collide obliquely.

6. Inelastic Two-Dimensional Collisions 80-3718

Slow-motion photography shows two clay balls of the same mass colliding in two-dimensions. The balls stick together after collision, moving as a single mass. Analysis of momentum conservation and energy before and after the collision can be made.

7. Scattering of a Cluster of Objects 80-3726

This film demonstrates six balls initially at rest, scattered by a moving, case-hardened steel ball. The slow-motion photography
allows you to make measurements of velocity and verify conservation of momentum.

8. Explosion of a Cluster of Objects 80-3734

Five balls are initially at rest, with a small cylinder containing gun powder in the center of the balls. The charge is exploded and each of the balls moves off in a different direction. The slow-motion sequences permit observation of the motion of the visible masses, prediction of the motion of the observed masses, and verification of the principle of conservation of momentum.


This film illustrates the ballistic pendulum and shows one way it may be used to determine the speed of the projectile. Extreme slow-motion photography (factor of 158) permits direct measurement of the pendulum following the impact by a rifle bullet; you will also be able to compute the kinetic energy of the bullet prior to impact.

10. Finding the Speed of a Rifle Bullet: II 80-3759

This film shows a second way that the ballistic pendulum can be used to determine the speed of a rifle bullet by measuring the change in elevation of the ballistic pendulum after impact by the bullet. The gain in potential energy and hence the loss in kinetic energy can be determined. Applying conservation of momentum then permits determination of the bullet's speed.
11. Recoil 80-3767
This film shows the recoil of a model cannon and given enough information to enable you to use the conservation laws to determine the recoil velocity of the gun.

12. Colliding Freight Cars 80-3775
This film presents a real life situation in which two freight-car-coupling-is-shown. The collisions, in some cases, were violent enough to break the couplings. Momenta and kinetic energies can be computed.

13. Collision with an Unknown Object 80-3973
In 1932 Chadwick discovered the neutron by analyzing collision experiments. This film demonstrates how the laws of conservation of momentum and energy may be used to determine the mass of an unknown object. The analysis is similar to Chadwick's although the film uses balls rather than elementary particles and nuclei.
UNIT IX

ENERGY AND ITS CONSERVATION

Introduction

We have reached our last Unit in the study of Mechanics of point-like bodies. We have already found and used the energy as a physical observable enabling us to analyze a process without a detailed knowledge of the forces involved in the interaction. In general, it is much easier to examine a process, taking into account the changes in energy that occur, than it is to examine the forces acting on the system. The fact that energy is a scalar observable (as contrasted to vector observables) is an added advantage. Therefore, we have several motives for studying more deeply the concept of energy; the present unit is rather extended to accomplish this.

So far you have learned that momentum and energy are the all-important observables for the description of the state of a physical body. Momentum was discussed extensively in the previous unit, and we already know that it represents the quantity of motion of a physical body and that the change in momentum of the body is the integrated effect over time of the resultant force acting on the body. We shall also find that there is another integral representation of Newton's second law, i.e. work-kinetic energy theorem. The technical definition of work is different from its every day usage and physiological meaning.
Rather, we can consider the work done by a force as a form of energy, and this is a first indication of the fact that the energy does not disappear and is not created. Every time we calculate the work done by a force acting on a body as its integrated effect over space, it will be possible to find a new form of energy which corresponds to that work. If the force is from the special class called conservative forces, it will be possible to define its work done as potential energy. We have: potential energy near the earth's surface, potential energy of universal gravitation, elastic potential energy, electric potential energy, etc.

In this unit we will be concerned mainly with mechanical energy, namely the kinetic energy of moving bodies and the potential energy associated with a body located in a region of space where a conservative force exits. In this way we can apply conservation of mechanical energy. However, we shall find systems where there are changes of mechanical energy, and these are produced by the action of so called nonconservative forces. But we will find other forms of energy (for example, chemical, thermal, nuclear, etc.); and if we take these into account, we say that the total energy is conserved. This is the law of conservation of energy. The total energy is always conserved in a system; energy may be transformed from one kind to another, but it cannot be created or destroyed.

Energy is often in the news lately. Sources of energy are of central importance in raising, or at least in sustaining, the standard of living of mankind above the subsistence level. Man's efforts in seeking sources of energy are based on the construction of machines to transform energy into forms that he can control. These are a few of the reasons why you must study and master the material in this Unit.
Unit IX

Objectives

To be able to:

1. Define and calculate the work done on a particle or a system of particles by a variable force using a line-integral.
2. Relate the work done by the resultant force acting on a body to its change in kinetic energy, and solve problems of finding particles' speed using the work-energy theorem.
3. Distinguish between conservative and nonconservative forces in problems.
4. Calculate the potential energy function given a conservative force, or conversely, given the potential energy function in one dimension, find the force.
5. Solve problems involving body motion acted upon by conservative forces by employing the law of conservation of mechanical energy.
6. Solve problems of moving motion which are acted upon by frictional forces using the law of conservation of total energy.
7. Find the power delivered or consumed by a body by means of its relationships to work, force, velocity and kinetic energy.
1. Review the concepts of kinetic energy and potential energy in Unit III, Chapter 2, Sections 5 and 6. Also, review the definition of work done by a force in Unit VI, Chapter 4, Section 6. Study Section 2 of Chapter 7 and pay special attention to Eq. (7-3) as a line integral. This is the key to accomplishing objective 1. Analyze carefully the examples 1, 2, and Worked Problems I and II attached at the end of this Unit. We recall that you used Eq. (7-2) (it is the same Eq. (4-25)) to evaluate the work done by a constant force, and now it appears as a special case of the general Eq (7-3), where the force can be variable in position or time. It is important to remember that in applying those Eqs. the force may be the resultant force acting on the physical body, or it may be a particular force of interaction of the body-study with another object. Also, note in examples 1 and 2 that constraint forces do no work. Solve problems 2, 3, 4, and 6. It is helpful to have at hand the Summary of Important Relations and Definitions given at the end of this Unit.

2. Objective 2 is stated in Eq. (7-5) and illustrated by examples 3 and 4. You shall note that using the work-energy theorem you will be able to solve dynamics problems, namely,
to obtain parameters of motion without an explicit solution of the equation of Newton's 2nd Law. In other cases, using the work-energy theorem along with Newton's Law will help you to solve diverse problems where it is necessary to know forces that do work. Here you must again remember to distinguish between the resultant force on a moving body (this accelerates the body and affects its kinetic energy) and the individual forces of interaction with other separate objects that may do positive or negative work on the moving body. Free body diagrams will help you to keep this in mind. Study Worked Problems II and IV and work out the problems 1, 5, 10, and 18.

3. Proceed to study Section 3 of Chapter 7. This is self-explanatory in helping you attain objective 3. But this objective is satisfied with the following alternative statements:

(a) A force is conservative if the work done by it on a body that moves between two positions depends only on these positions and not on the path followed. A force is nonconservative if the work done by that force on a body that moves between two positions depends on the path taken between those positions. (b) A force is conservative if the work done by the force on a body that moves through any round trip is zero. A force is nonconservative if the work done by the force on a body that moves through any round trip is not zero. Study Example 5, and the solution of Worked Problem V. Solve Problems 7 and 12.
4. Study in detail Sections 4, 5a, b, c, (skipping the examples) and Section 7 dealing with Potential (or Positional) Energy. The Eqs (7-8) and (7-20) are the keys to objective 4. You should note how powerful and useful it is to visualize these arguments by thinking of a graph of potential energy as a function of position. Study the solutions of Example 11 and Worked Problems VI and VII. Work out solutions for problems 11, 12, 15, 27 and 29.

5. Now, go in detail over Examples 6, 7, 8 and Section 7 that illustrate applications of the law of Energy Conservation. Eq. (7-9) is a statement of conservation of mechanical energy and is applicable to problems that do not involve friction. Eq. (7-16) is a particular application of the conservation of total energy. We encourage you to study Section 7-7 and 7-8 on pages 123 to 126 of Fundamental of Physics, Halliday, D. and Resnick, R. (Wiley, 1974) and pay attention to Eq. (7-14) and the last Eq. on page 125 as statements of conservation of total energy. Study the solutions of Worked Problems VIII and IX and solve problems 1, 14, 15, 16, and 18.

6. Read Section 6 and direct special study to Eq. (7-17) that is the key to objective 7. Examples 9 and 10 illustrate applications with constant forces. But if a car or other vehicle has an engine that delivers constant power and the velocity changes, then the resultant force acting on it must vary; if the resultant force is constant and the
velocity varies, then the power must vary. Keep all the possibilities in mind and look closely at the Worked Problem X. Solve problems 20, 21, 23, 24, and 26.

7. See the Audiovisual Aids Section in this Unit. Now you should try the Pretest before attempting the Test. Discuss the Pretest and Problems with your professor or proctor. If you need more practice, you should work some optional problems that you can get from your professor.
Unit IX  
SUMMARY OF RELATIONS AND DEFINITIONS

Infinetesimal work done by a force
\[ dW = \mathbf{F} \cdot d\mathbf{s} \]

Work done by a force is a measure of the accumulative effect of a force integrated over space, which produces a change in kinetic energy.

\[ W_{1,2} = \int_{s_1}^{s_2} \mathbf{F} \cdot d\mathbf{s} \]

Work-energy theorem

\[ \int_{A}^{B} \mathbf{F} \cdot d\mathbf{s} = \Delta T \]

Conservative forces

Potential Energy
with respect to point B
\[ U(x) = U(B) - \int_{B}^{x} \mathbf{F} \cdot dx \]

Mechanical energy conservation
\[ E = U(x) + T(x) \]

Constant force of gravity
\[ U_g(x,y) = mgy \]

Coil spring force
\[ U_k(x) = \frac{1}{2}kx^2 \]

Force of gravitational attraction
\[ U_G(r) = -\frac{Gm_1m_2}{r} \]

Energy Conservation
\[ W_{\text{nc}} = \Delta T + \Sigma \Delta U \]

Power
\[ p = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} \]

Force from potential energy
\[ F(x) = -\frac{du}{dx} \quad \text{(one dimension only)} \]
Unit IX  
WORKED PROBLEMS

I. (1) A particle moves along the x-axis from $x_1$ to $x_2$ according to the force expression given below. Find the work done on the particle and the work done by the particle in each case. The constant $x_0$ equals 3m.

(a) $F(x) = \left(\frac{x}{x_0}\right)^2 - 16 \text{ (N)}$; $x_1 = -x_0$; $x_2 = 2x_0$

(b) $F(x) = \left(\frac{x}{x_0}\right)^3 - 3 \left(\frac{x}{x_0}\right)^2 + 6 \left(\frac{x}{x_0}\right) + 3 \text{ (N)}$;

$x_1 = 0$; $x_2 = 2x_0$

Solution

$W = \int_{x_0}^{x_2} F \cdot dx$ is the work done by the force. $-W$ is work done by the particle.

(a) $W = \int_{-x_0}^{2x_0} \left[\left(\frac{x}{x_0}\right)^2 - 16\right] dx = \left[\frac{1}{3} \left(\frac{x}{x_0}\right)^3 - 16 \left(\frac{x}{x_0}\right)\right]_{-x_0}^{2x_0} = -45x_0 = -135 \text{ J}$

Since $W$ is negative and $(-W)$ is positive, work is done on the particle.

(b) $W = \int_{0}^{2x_0} \left[\left(\frac{x}{x_0}\right)^3 - 3 \left(\frac{x}{x_0}\right)^2 + 6 \left(\frac{x}{x_0}\right) + 3\right] dx$,

Let $u = \frac{x}{x_0}$; $du = x_0 dx$

$= \int_{0}^{2} \left[u^3 - 3 u^2 + 6 u + 3\right] x_0 du = \left[\frac{1}{4}(2)^2 - (2)^3 + 3(2)^2 + 3(2)\right] x_0 = 14 x_0 = 42 \text{ J}$

Since $W$ is positive, work is done on the particle.
II (1) A large box of 25 kg is suspended from the end of a rope 5 m long. The box is then pulled aside (1/2)m from the vertical and held there.

(a) What is the force needed to keep the box in this position?
(b) Is work being done in holding it there?
(c) Was work done in moving it aside? If so, how much?
(d) Does the tension in the rope perform any work on the box?

Solution

(a) Free body diagram shows three forces.

Equilibrium: \( \Sigma F_x = F - T \sin \theta = 0 \)
\( \Sigma F_y = T \cos \theta - mg = 0 \)

\( F = mg \tan \theta, \quad F = (25 \times 10) \frac{0.5}{5} = 25 \text{N} \)

(b) No. At this point there is no displacement of the application point of \( F \) so that there is not work done.

(c) Yes.

\( W_F = \int F \cdot ds, \text{ but } F \text{ is variable, } F = \frac{mg}{L} x \hat{\imath}. \text{Since } \sin \theta = \tan \theta = \frac{x}{L} \)

\( W_F = \int F \cdot ds = \frac{mg}{L} \int_0^x x \, dx = \frac{mg}{2L} \frac{x^2}{2} \); \( W_F = \frac{25}{4} = 6.25 \text{ Joules} \)

(d) No. It is a constraint.

III (1)(2) Consider a simple pendulum: that is, a small body of mass \( m \) suspended from a weightless string of length \( L \). This pendulum swings over a small arc so that its maximum angular displacement is \( \theta_m \) as shown in the Figure. Find the work done by each force and analyze the energy.

Solution:

A free-body force diagram for the mass in an arbitrary position is shown.
There are only two forces $T = \text{tension in the cord}$ and $F_g = mg$ the weight of the bob
The work done in a displacement $ds$ by the force $T$ is

$$dW_T = T \cdot ds$$

$T$ is directed along the radius and $= T \hat{j}$
$ds$ is directed along the tangent to the arc $= ds \hat{i}$

$$dW_T = T ds \hat{j} \cdot \hat{i} = 0, \quad \text{since} \hat{j} \parallel \hat{i}$$

or by definition of scalar product

$$dW_T = T ds \cos \alpha = 0, \quad \text{since} \alpha = \pi/2$$

Now, the work done by the force $F_g$ in a displacement $ds$

$$dW_{F_g} = \vec{F}_g \cdot ds = F_g ds \cos \beta = F_g ds (-\sin \Theta)$$

or using vectors

$$\vec{F}_g = (F_g \sin \Theta)(-\hat{i}) + (F_g \cos \Theta)(-\hat{j}); \quad ds = ds \hat{i}$$

$$\therefore \vec{F}_g \cdot ds = - (F_g \sin \Theta)ds$$

The $y$-component of $F_g$ does no work on the pendulum.

To determine the work done in a displacement from $\Theta = 0$ to $\Theta = \Theta_m$ we integrate along the arc.
We use the relation
\[ ds = Ld\theta \]
\[ W = \int_{\theta_0}^{\theta_m} \vec{F}_g \cdot ds = -mgL_0^m \sin \theta_0 \, d\theta = -mgL(1-\cos \theta_m) \]
but the geometry of the Figure says
\[ h = L(1-\cos \theta_m) \]
So that we obtain the simple result
\[ W_{Fg} = -mgh \]
It is negative since \( \vec{F}_g \) has opposite direct to \( ds \). In another approach, we calculate the resultant force acting on the bob
\[ \vec{F}_R = \vec{F}_g + T = \vec{F}_g \sin \theta (-\hat{i}) + \vec{F}_g \cos \theta (-\hat{j}) + T \hat{j} \]
2nd Law:
\[ \Sigma F_x = -F_x = m\alpha_x = F_{\text{res}} \]
\[ \Sigma F_y = T - F_y = 0 \text{ no motion along this axis} \]
\[ \vec{F}_R = -F_g \sin \theta \hat{i} \]
Work done by the resultant force
\[ W_{F_R} = \int \vec{F}_R \cdot ds = -\int F_g \sin \theta \, ds = -mgh \]
Now the work-energy theorem says
\[ W_{\text{Resultant}} = \Delta T = T_{\text{final}} - T_{\text{initial}} \]
The change in kinetic energy will be decreased by \( mgh \)
\[ \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = -mgh \]
For \( \theta_m \) the speed \( v_f = 0 \) because it is the returning point and the pendulum is momentarily at rest.
\[ -\frac{1}{2} m v_o^2 = -mgh \quad v_o = \sqrt{2gh} \quad \text{or} \quad h = \frac{v_o^2}{2g} \]
Also we remember that potential energy was defined

\[ U = mgh + U_0 \]

where \( U_0 \) is the reference potential or

\[ \Delta U = mgh \]

\[ W_{FR} = \Delta T = -\Delta U \quad \Delta T + \Delta U = 0 \]

There is conservation of mechanical energy!

IV (2) A 2000-kg car travels at 20.0 m/s on a level road. The brakes are applied long enough to do \( 1.20 \times 10^5 \) J of work.

(a) What is the final speed of the car?

(b) What further distance is required to stop the car completely if the brakes are applied again and the constant decelerating force on the car is \( 4.0 \times 10^3 \) N? (Use two methods and compare the results.)

(c) What ultimately happens to the car's initial kinetic energy?

Solution

(a) The kinetic energy of the car (mass \( M = 2000 \) kg) is converted to thermal energy of the brake drums and linings by friction. The work done \( W = 1.20 \times 10^5 \) J reduces the car's kinetic energy from the initial \( T_i \) to the final \( T_f = T_i - W \). The speed drops from the initial \( v_i = 20.0 \) m/s to the final \( v_f = ? \), where

\[ T_i = \frac{1}{2}Mv_i^2, \quad T_f = \frac{1}{2}Mv_f^2 \]

\[ v_f = \sqrt{\frac{2T_f}{M}} = \sqrt{\frac{2(T_i - W)}{M}} = \sqrt{v_i^2 - 2\frac{W}{M}} \]

\[ = \sqrt{400 - \frac{2(1.2 \times 10^5)}{2 \times 10^3}} = \sqrt{400 - 120} = \sqrt{280} = 16.7 \text{ m/s} \]
(b) With applied force \( F = 4.0 \times 10^3 \) N, the car moves a
distance \( d \), so work in the amount \( W' = Fd = T_f \) is
done to stop the car.

\[
d = \frac{T_f}{F} = \frac{1}{2} \frac{Mv_f^2}{F} = \frac{1}{2} \times \frac{2 \times 10^3 \times 280}{4.0 \times 10^3} = 70 \text{ m.}
\]

Find deceleration \( a \), time to stop \( t \), and distance moved \( d \) at the average speed \( v_{av} = (1/2) v_f \). Use Newton's second law:

\[
a = \frac{F}{M} = -\frac{4000}{2000} = -2 \text{ m/s}^2; \quad t = \frac{v_f}{a} = \frac{16.7}{-2.0} = 8.35 \text{ s};
\]

\[
d = v_{av} \cdot t = \frac{1}{2} (16.7) (8.35) = 70 \text{ m.}
\]

(c) Thermal energy in brakes - see part (a).

V (3) A certain force is given by the expression \( F = \hat{u}_0/r \), where

\( \hat{u}_0 \) is a unit vector along the tangential direction. The
lines of constant force are, as shown in the figure, circles centered on the origin.

(a) Calculate the work done along the closed path (1) shown
in the figure, and show that it is zero.

(b) Calculate the work done along the path (2) which encloses
the origin, and show that it is different from zero. Is
this force conservative?
Solution

(a) \[ W = \int \mathbf{F} \cdot d\mathbf{s} \]

for circular segments of path \( ds = r_o d\theta \)

\[
W = \int_{\theta_1}^{\theta_2} \frac{1}{r} ds = \int_{\theta_1}^{\theta_2} \frac{1}{r_o} r_o d\theta = \theta_2 - \theta_1
\]

In the inner circular path, the work has the same absolute value but opposite sign, thus they cancel.

Along the radial paths \( \mathbf{F} \cdot d\mathbf{s} = 0 \). Thus the total work is zero.

(b) For a path encircling the origin

\[
W = \int_0^{2\pi} \frac{1}{r} d\theta = \int_0^{2\pi} \frac{1}{r} rd\theta = 2\pi
\]

where the sign of \( W \) depends on the sense in which the trajectory is followed. The force is not conservative.

VI (4) (A) A body moving along the x axis is subject to a force given by \( F(x) = -kx + cx^2 \). Find the potential energy function for this force. Let \( U(x) = 0 \) at \( x = 0 \).

Solution

The potential energy is defined as

\[
U(x) = -\int_{x_0}^{x} F(x) dx + U(x_0)
\]

In this case \( x_0 = 0 \) and \( U(x_0) = 0 \).

\[
U(x) = \int_0^x kx dx - \int_0^x ck^2 dx; \quad U(x) = \frac{1}{2} (kx^2) - 1/3(cx^3)
\]

(B) What force corresponds to a potential \( U = 20x^2 - 35z^3 \)

Solution

\[
F_x = -\frac{\partial U}{\partial x} = -40x
\]

\[
F_y = -\frac{\partial U}{\partial y} = 0 \quad \therefore \quad \mathbf{F} = -40 \mathbf{x} \hat{i} + 105z^2 \mathbf{k}
\]

\[
F_z = -\frac{\partial U}{\partial z} = +105z^2
\]
VII (4) (Halliday-Resnick-Combined Edition, Page 164). The potential energy function for the force between two atoms in a diatomic molecule can be expressed approximately as follows:

\[ U(x) = \frac{a}{x^2} - \frac{b}{x^6} \]

where \(a\) and \(b\) are positive constants and \(x\) is the distance between atoms.

(a) At what values of \(x\) is \(U(x)\) equal to zero? At what value of \(x\) is \(U(x)\) a minimum? In Figure (a) we show \(U(x)\) versus \(x\). The values of \(x\) at which \(U(x)\) equals zero are found from

\[ \frac{a}{x^2} - \frac{b}{x^6} = 0 \]

Hence,

\[ x^6 = \frac{a}{b} \quad x_1 = \sqrt[6]{\frac{a}{b}} \]

\(U(x)\) also becomes zero as \(x \to \infty\) (see figure or put \(x = \infty\) into equation for \(U(x)\)), so that \(x = \infty\) is also a solution.
Figure: (a) The potential energy and
(b) The force between two atoms in a diatomic molecule
as a function of the distance x between atoms.

The value of x at which $U(x)$ is a minimum is found from

$$\frac{d}{dx} U(x) = 0$$

That is,

$$\frac{-12a}{x^{13}} + \frac{6b}{x^{7}} = 0$$

or

$$x^6 = \frac{2a}{b}, \quad x_2 = \sqrt[6]{\frac{2a}{b}}$$

(b) Determine the force between the atoms.

$$F(x) = -\frac{d}{dx} U(x)$$

$$F = -\frac{d}{dx} \left[ \frac{a}{x^{12}} - \frac{b}{x^6} \right] = \frac{12a}{x^{13}} - \frac{6b}{x^7}$$

We plot the force as a function of the separation between the atoms in Figure (b). When the force is positive (from $x = 0$ to $x_2 = \sqrt[6]{\frac{2a}{b}}$), the atoms are repelled from one another (force directed toward increasing x). When the force is negative (from $x_2 = \sqrt[6]{\frac{2a}{b}}$ to $x = \infty$), then atoms are attracted to one another (force directed toward decreasing x). At $x_2 = \sqrt[6]{\frac{2a}{b}}$ the force is zero; this is the equilibrium point and is a point of stable equilibrium.

(c) Assume that one of the atoms remains at rest and that the other moves along x. Describe the possible motions.
From the analysis of this section it is clear that the atom oscillates about the equilibrium separation at \( x = \frac{\sqrt{2a/b}}{b}, \) much as a particle might slide up and down the frictionless hills of the potential valley.

(d) The energy needed to break up the molecule into separate atoms is called the dissociation energy. What is the dissociation energy of the molecule?

If one atom has enough kinetic energy to get over the potential hill, it will no longer be bound to the other atom. Hence, the dissociation energy \( D \) equals the change in potential energy from the minimum value at \( x = \frac{\sqrt{2a/b}}{b} \) to the value at \( x = \infty \).

This is simply

\[
U(x = \infty) - U(x = -\frac{\sqrt{2a/b}}{b}) = 0 - \left( \frac{a}{4a^2/b^2} - \frac{b}{2a/b} \right) = \frac{b^2}{4a}
\]

If the kinetic energy at the equilibrium position is equal to greater than this value, the molecule will dissociate.

VIII (5) A toy car of mass \( m \) slides along the frictionless loop-the-loop track shown in the figure.

(a) At what height above the bottom of the loop should the car be released so that it does not exert force against the track at the top of the loop (point C)?

(b) What is the car's speed at point A? At point B?

(c) What is the resultant force acting on the car at B?
Solution

(a) At point C the only force is the car's weight according to the condition; there is no force against the track.

2nd Law \[ F = ma \]
\[ mg = m \frac{v_c^2}{R} \]
\[ v_c^2 = gR, \text{ critical speed.} \]

Conservation of mechanical energy between points (1) and (2)

\[ E_1 = E_C; \quad mgh = mg(2R) + \frac{1}{2} m v_c^2 \]
\[ h = 2R + \frac{1}{2} R = \frac{5}{2} R \]

(b) Conservation of mechanical energy between points (1) and (A)

\[ E_1 = E_A; \quad mgh = \frac{1}{2} m v_A^2 \quad v_A^2 = 2gh = 2g \frac{5}{2} R; \]

\[ v_A = \sqrt{5gR} \]

And between points (1) and (B)

\[ E_1 = E_B; \quad mgh = mg R + \frac{1}{2} m v_B^2 \quad v_B = \sqrt{5gR} \]

(c) \[ F_{\text{centripetal}} = N = \frac{m v_B^2}{R} = 3mg \]

\[ F_R = W^2 + N^2 = mg \sqrt{10} \]

\[ \alpha = \arctan 3 = 71.6^\circ \text{ with respect to the vertical} \]

IX (6) The cable of a 4000 lb elevator in the Fig. below snaps when the elevator is at rest at the first floor so that the bottom whose is a distance \( d = 12 \text{ ft} \) above a cushioning spring constant is \( k = 10000 \text{ lb/ft} \). A safety device clamps the guide rails so that a constant friction force of 1000 lb opposes the motion of the elevator.
(a) Find the speed of the elevator just before it hits the spring.
(b) Find the distance $s$ that the spring is compressed.
(c) Find the distance that the elevator will "bounce" back up the shaft.
(d) Using the conservation of energy principle, find the total distance that the elevator will move before coming to rest.

**Solution**

(a) From conservation of energy between the first floor and the top of the spring

\[ W_f = \Delta T + \Delta U_g = -f \cdot d = \frac{1}{2} m v^2 + mg(-d) \]

\[ v^2 = \frac{2(-f \cdot d + mgd)}{m} \]

\[ v^2 = 2\left(-10^3 \cdot 12 + 4 \times 10^3 \cdot 12\right) = 4 \times 10^3 / 32 \]

\[ v^2 = 3^2 \cdot s^2 \]

\[ V_2 = 24 \text{ ft/sec} \]

(b) From conservation of energy between the position at the top of the spring and the maximum compression $s$. (positive downward)

\[ W_f = \Delta T + \Delta U_g + U_k \]

\[ -fs = -\frac{1}{2} m v^2 + mg(-s) + \frac{1}{2} k s^2 \]

\[ \frac{1}{2} k s^2 + (f-mg)s - \frac{1}{2} m v^2 = 0 \]

Numerical substitution yields

\[ s = 3 \times 10^{-1}(1\pm 9), \quad s_1 = 3 \text{ ft} \]
(c) From conservation of energy between the latter position (called 3) and the back up position (called 4)

\[ W_f = \Delta T_{34} + \Delta U_{g34} + \Delta U_{k34} \]

\[ -f y = 0 + mgy - \frac{1}{2} k S^2 \]

Numerical substitution

\[ y = \frac{-\frac{1}{2} kS^2}{f + mg} \]

The negative sign means distance upward.

(d) Conservation of energy between the initial situation (elevator at rest at the first floor) and the final situation (elevator at rest compressing the spring).

\[ W_f = \Delta E = \Delta T + \Delta U_g + \Delta U_k \]

But \( \Delta T = 0 \)

And in the final situation

\[ k s_2 = mg \quad s_2 = mg/k \]

\[ -f \cdot d_T = -mg (d + s_2) + \frac{1}{2} k s_2^2 \]

\[ d_T = \frac{(mg)^2}{2f} + \frac{mg}{f} \quad d_T = 49 \text{ ft} \]

The following questions deal with a sports car of mass 1500 kg (including the driver and a companion). Its maximum power is \( 7.2 \times 10^4 \) W:

(a) How long would it take the car to accelerate from 15.0 m/s to 30.0 m/s on a straight, level road, if the engine delivered maximum power continuously?

(b) What is the speed \( v(t) \) of the car as function of time after the accelerator is "floored" (i.e., maximum power \( P \), initial speed \( v_o = 15 \text{ m/s} \))?
(c) What is the accelerating force \( F(t) \) exerted on the car in (b)?

Solution

(a) Call speeds \( v_0 = 15.0 \, \text{m/s} \) and \( v_1 = 30.0 \, \text{m/s} \). Gain in kinetic energy \( T_1 - T_0 \) comes from power delivered up to time \( t \). Maximum power is \( P = 7.2 \times 10^4 \, \text{W} \).

\[
T_1 - T_0 = Pt; \quad t = \frac{(T_1 - T_0)/P}{1/2 M(v_1^2 - v_0^2)/P};
\]

\[
t = \frac{1}{2} \left( \frac{1500}{(30^2 - 15^2)/7.2 \times 10^4} \right) = 7.0 \, \text{s}.
\]

(b) \( v(t) \) can be found from \( T(t) \) (kinetic energy):

\[
\frac{1}{2} M v^2 = T(t) = T_0 + Pt; \quad v(t) = \sqrt{2(T_0 + Pt)/M} = \sqrt{v_0^2 + 2Pt/M}
\]

\[
v(t) = \sqrt{225 + 2(7.2 \times 10^4) t/(1.5 \times 10^3)} = 15 \sqrt{1 + 0.43t} \, \text{m/s}
\]

(c) \( P = Fv \) or \( F = \frac{P}{v} = \frac{7.2 \times 10^4}{15\sqrt{1 + 0.43t}} = \frac{4.8 \times 10^3}{\sqrt{1 + 0.43t}} \, \text{N}
\]

\( F \) directed along the road
Unit IX

PRETEST

I. A certain peculiar spring is found not to conform to Hooke's Law. The force it exerts when stretched a distance $x$ (in meters) is found to have magnitude $(52.8x + 38.4x^2)$ N in the direction opposing the stretch.

(a) (1) Compute the total work required to stretch the spring from $x = 0.50$ to $x = 1.00$ meter.

(b) (2) With one end of the spring fixed, a particle of mass 2.17 kg is attached to the other end of the spring when it is extended by an amount $x = 1.00$ meter. If the particle is then released from rest, compute its speed at the instant the spring has returned to the configuration in which the extension is $x = 0.50$ meter.

(c) (3) Is the force exerted by the spring conservative or non-conservative? Explain.

II. A particle of mass 16.0 kg constrained to move along the $z$ axis is subject to a conservative force field given by $F(z) = -Az^3 + Bz$, as in Figure 1. ($F$ is in Newton's; the numerical values of $A$ and $B$ are $A = 8.0, B = 1.00$.)

(a) What are the dimensions of $A$ and $B$?

(b) (4) Find the potential energy as a function of $z$ and sketch it. $U(0) = 0$.

(c) (5) With what speed will the particle arrive at $z = 0$ if it starts from rest at $z_0 = 4.0$ m?

(d) (5) Do the same for the particle starting at $z_0 = 0.100$ m.
III. (6) A 16.0 kg block traveling at 6.0 m/s in a horizontal direction collides with a horizontal weightless spring of force constant 5.0 N/m. The block compresses the spring a distance \( s \). When the spring is back to the uncompressed position, the block is traveling with a speed of 2.00 m/s. If the coefficient of friction between the block and surface is 0.40, determine the energy expended by nonconservative forces. (See Figure 2)

IV. (7) Show that when friction is present in an otherwise conservative mechanical system, the rate at which mechanical energy is dissipated equals the frictional force times the speed at that instant, or \( \frac{d}{dt} (K + U) = -fv \).
Film Loops

For sections 2 and 3 of the textbook.

A Method of Measuring Energy. Nails Driven into Wood 80-3791

If a moving object strikes a nail, the object will usually lose all of its moving energy. The energy the object had, just before collision, becomes work done on the nail, driving it into the block of wood. This fact is shown in the film using a falling object and a nail. By finding relations between the number of blows and the penetration depth of the nail into the wood, you may be led to discover a useful proportionality to the energy of the falling mass.

For section 4:

Gravitational Potential Energy 90-3817

Hammers of different masses are dropped from different heights on nails stuck into a long block of wood. Two types of measurements are possible in this film. In the first scenes it is possible to determine how gravitational potential energy depends upon weight. Objects of different weight fall from the same distance. The latter scenes provide information for studying the relationship between gravitational potential energy and position. Bodies of equal masses are raised to different heights and allowed to fall.

For section 5:

Conservation of Energy 80-2769

A series of three experiments illustrates some form of being converted into kinetic energy of an air-track glider. The three experiments are: mechanical work converted to kinetic energy,
gravitational potential energy converted to kinetic energy, and elastic potential energy converted to kinetic energy.

Conservation of Energy. Pole Vault 80-3833

The film demonstrates the various types of energy present in a pole vault and shows that the total energy is a constant. This is accomplished by using a stop-frame technique for velocity determination. The distortion of the pole determines its elastic potential energy.

Conservation of Energy. Aircraft Takeoff 80-3841

The film demonstrates the conservation of mechanical energy during an aircraft maneuver. The maneuver chosen is an altitude gain from level flight just above a runway to level flight a few hundred feet up. On the assumption that the air resistance is just matched by the constant throttle output of the engine, the total mechanical energy is compared at three points in the flight.
1. Without any aid, name the fundamental quantities, dimensions and units of the MKS, CGS, and English systems.

2. A light cube receptacle containing 2 pints of particular fluid weights 4 lb. What is the density of the fluid? What is the side of the cube? Express your answers in CGS system. You can use a Table of Conversion Factors.

3. Determine whether or not the following equations are dimensionally correct. Use a Table of Dimensions and Units for Physical Quantities.
   (a) \( E = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \delta) + \frac{1}{2} kA^2 \cos^2(\omega t + \delta) \). Use dimensions.
   \( \omega = \) angular frequency, \( A = \) Amplitude (m), \( m = \) mass; \( k = \) (N/m); \( E = \) energy.
   (b) \( L = rp + Ir \) Use MKS units. \( L = \) angular momentum; \( I = \) Rotational inertia; \( p = \) linear momentum; \( r = \) radial position.

4. Estimate the order of magnitude of the following quantities. Express them using prefixes whenever possible. Use Table of Physical Constants.
   (a) Mean distance sun-earth in MKS system.
   (b) Age of the Universe \( 3 \times 10^9 \) years in seconds
   (c) Mass of a proton in CGS system.
1. Name the fundamental quantities, dimensions and units of: MKS, CGS, and British systems.

2. The earth has an equatorial radius $R_e = 6.37 \times 10^8$ cm and gives a completed revolution about its axis in 24 hours. Determine the tangential speed of a point on the equator with respect to the center of the earth. Express your answer in English units. Use a Table of Conversion Factors.

3. Find if the following questions are dimensionally correct. Use a Table of Dimensions and Units for Physical Quantities.
   
   (a) $\frac{E}{t} = 5 \frac{Ep}{ms} - 7 \text{ ap.}$
   
   Use fundamental dimensions. $E =$ energy; $p =$ linear momentum; $a =$ acceleration; $t =$ time; $m =$ mass; $s =$ position.

   (b) $T = \frac{1}{2} m u^2 + \frac{1}{2} I \omega^2$
   
   Use MKS Units. $T =$ kinetic energy; $m =$ mass; $u =$ speed; $I =$ rotational inertia; $\omega =$ angular speed

4. Using a Table of Physical Constants to find the order of magnitude of the following quantities. Express them with prefixes whenever is possible.
   
   (a) The ratio of Earth's mass to the Moon's mass.
   
   (b) The length of a light year.
   
   (c) Period of a radio frequency signal of 101 MHz.
1. Give the three fundamental quantities, dimensions and units for the systems: MKS, CGS, English and British.

2. In experiments with radioactive materials, lead bricks are used for shielding purpose. A typical brick is in the shape 2 in x 4 in x 8 in and its density 11.3 g/cm³. What is the mass of such a brick? Express your answer in MKS and British units.

3. Determine if there is dimensional consistency in the equations:

   Use dimension for:
   (a) \( W = mF \cdot u^3 \)  
       \( W = \) work; \( a = \) acceleration; \( m = \) mass; 
       \( F = \) force; \( u = \) speed; \( p = \) linear momentum

   Use MKS units for:
   (b) \( F = ma + \frac{W + U}{s} \)  
       \( U = \) potential energy, \( s = \) displacement

   Use a Table of Dimensions and Units of Physical Quantities.

4. Using a Table of Physical Constants to find the order of magnitude of the following quantities. Express them with prefixes whenever possible.

   (a) Diameter of a nucleus if it is \( 10^{-4} \) of the diameter of an atom.

   (b) The number of atoms of hydrogen that there is in the mass of the sun assuming that the latter is formed mainly by hydrogen.

   (c) Your age in seconds.
## UNIT I

### GRADING KEY FOR TEST 1

#### Solutions

<table>
<thead>
<tr>
<th></th>
<th>Dimension</th>
<th>Units</th>
<th>Verify that:</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>L</td>
<td>m cm ft</td>
<td>(objective 1)</td>
</tr>
<tr>
<td>mass</td>
<td>M</td>
<td>kg g --</td>
<td>student really knows this and it wasn't a copy of</td>
</tr>
<tr>
<td>time</td>
<td>T</td>
<td>s s s</td>
<td>the Table.</td>
</tr>
<tr>
<td>force</td>
<td>F</td>
<td>-- lb</td>
<td></td>
</tr>
</tbody>
</table>

2. 8 pints = 231 in³; \( \rho = \frac{M}{V} = 0.0693 \text{ lb/m³} \)  
   1 lb·m = 453.6 g·ft = 16.39 cm³ (objective 2)  
   there are the correct conversion factors.

3. (a) \( E = [ML^2T^{-2}], \quad m = [M], \quad k = [MLT^{-2}], \quad A = [L] \)  
   \( 1/2, \sin^2(\omega t + \delta), \sin^2(\omega t + \delta) \) are nondimensionals  
   \( 1/2 \omega^2 A^2 \sin^2 (\omega t + \delta) = [ML^2T^{-2}] \)  
   \( 1/2 k A^2 \cos^2 (\omega t + \delta) = [ML^2T^{-2}] \)  
   \( [ML^2T^{-2}] = [ML^2T^{-2}] + [ML^2T^{-2}] \)  
   it is dimensionally consistent

(b) \( L: (\text{kg·m}^2/\text{s}), \quad r: (\text{m}), \quad p: (\text{kg·m}/\text{s}), \quad I: (\text{kg} \cdot \text{m}^2), \quad r_p: (\text{kg·m}^2/\text{s}), \quad I: (\text{kg}/\text{m}) \)  
   \( (\text{kg·m}^2/\text{s}) = (\text{kg·m}^2/\text{s}) + (\text{kg}/\text{m}) \)  
   it is not consistent

Check each term has the correct MKS units, and the units for both sides of the equation are worked out.
4. (a) Mean distance sun-earth:

\[ R_{\oplus} = 1.49 \times 10^{11} \text{ m} \]
\[ \sim 10^{11} \text{ m} = 0.1 \text{ Tm} \]

(b) Age of the Universe:

\[ 3 \times 10^9 \text{ yr.} = (3 \times 10^9) (3.2 \times 10^7) = \]
\[ = 9.6 \times 10^{16} \text{ s} \]
\[ \sim 10^{17} \text{ s} = 10^5 \text{ Ts} \]

(c) Mass of proton:

\[ m_p = 1.673 \times 10^{-27} \text{ kg} = 1.673 \times 10^{-24} \text{ g} \]
\[ \sim 10^{-24} \text{ g} = 10^{-9} \text{ fg} \]
UNIT I

GRADING KEY FOR TEST 2

Solutions

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Dimension</th>
<th>System of Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>[L]</td>
<td>MKS cm CGS m yd</td>
</tr>
<tr>
<td>Mass</td>
<td>[M]</td>
<td>kg g slug</td>
</tr>
<tr>
<td>Time</td>
<td>[T]</td>
<td>s s s s</td>
</tr>
</tbody>
</table>

Verify that:

Objective 1

That student knows the system and that this answer was not a copy of the Table.

Objective 2

Are the correct conversions factors used?

Objective 3

Use of dimension in each term and that both sides of the equation are worked out

Objective 4

This started with the numerical values. Use of prefixes.

2. \( R_\odot = 6.37 \times 10^8 \text{ cm}, T_\odot = 24 \text{ hr} = 8.64 \times 10^9 \text{ s} \)

\[ v = \frac{2\pi R_\odot}{T_\odot} = 464 \text{ m/s} = 1522 \text{ ft/s} \]

3. We drop the parentheses for simplicity writing wherever is possible:

(a) \[ \frac{[ML^2T^{-2}]}{[T]} = 5 \frac{[ML^2T^{-2}][MLT^{-1}]}{[M][L]} = 7 [LT^{-2}][MLT^{-1}] \]

It is not dimensional consistent

(b) \[ (J) = \frac{1}{2} (\text{kg})(m/s)^2 + \frac{1}{2} (\text{kg}m^2)(s^{-1})^2 \]

\[ (\text{kg}m^2/s^2) = (\text{kg}m^2/s^2) + (\text{kg}m^2/s^2) \]

it is o.k.

4. (a) \( M_\odot = 5.93 \times 10^{24} \text{ kg}, M_q = 7.35 \times 10^{22} \text{ kg} \)

\[ \frac{M_\odot}{M_q} = 0.81 \times 10^2 \sim 10^2 \]

(b) \( s = ut = (3 \times 10^8 \text{ m/s})(3.2 \times 10^7 \text{ s}) = 9.6 \times 10^{15} \text{ m} \sim 10^{16} \text{ m} = 10^4 \text{ Tm} \)

(c) \[ T = \frac{1}{f} = \frac{1}{101 \times 10^6} \text{ (s)} = 0.999099 \times 10^{-8} \sim 10^{-8} \text{ s} = 10 \text{ ns} \]
UNIT I

GRADING KEY FOR UNIT TEST 3

Solutions

1. Quantity Dimension MKS CGS British English
   length L m cm yd ft
   mass M kg g slug --
   time T s s s s
   force F -- -- lb

2. \(1 \text{ in}^3 = 16.39 \text{ cm}^3\) so \(V = 64 \text{ in}^3 = 1049 \text{ cm}^3\)
   \(M = \rho V = 11.85 \text{ kg}\)
   \(1 \text{ lbm} = 453.59 \text{ g}\) \(M = 26.12 \text{ lbm}\)

3. (a) \([\text{ML}^2\text{T}^{-2}] [\text{LT}^{-2}] [\text{MLT}^{-1}] = \)
   \([\text{M}] [\text{MLT}^{-2}] [\text{LT}^{-1}]^3 \)
   \([M^2L^4T^{-5}] = [M^2L^4T^{-5}]\) \(\text{It's o.k.}\)

   (b) \((N) = (\text{kg})(\text{m/s}^2) + \frac{(J \text{+ J})}{(m)}\)

   \((N) = (N) + \frac{(N \times m)}{m} = (N)\)
   or \((\text{kg-m/s}^2) = (\text{kg-m/s}^2) + (N)\) \(\text{It's o.k.}\)

4. (a) \(da = 2 \cdot (0.529 \times 10^{-10}) \text{m}\)
   \(dn = 10^{-4} (1.058 \times 10^{-10}) \text{m}\)

   \(dn = 1.058 \times 10^{-14} \text{m} + 10^{-14} \text{m} = 10 \text{ fm}\)

Verify that:

Objective 1
Student can defend his answers, and that it is not a guess or copy.

Objective 2
There are the correct conversion factors.

Objective 3
Use of dimensions in each term and that both sides of the equation are worked out
Use of MKS units.

Objective 4
Start with the values. Use prefixes.
(b) \[ m_H = 1.008 \text{ amu} = 1.67 \times 10^{-27} \text{ kg} \]

\[ M_0 = 1.99 \times 10^{30} \text{ kg} \]

\[ \frac{M_0}{m_H} = 1.19 \times 10^{57} \sim 10^{57} \]

(c) Average age 20 yrs. = 20 \((3.2 \times 10^7 \text{s})\) = \(6.4 \times 10^8 \text{s}\) \(\sim 10^9 \) = 1 Gs
1. A towing truck is pulling a car up a hill having a slope of 15°. The pulling chain makes an angle of 30° with respect to the ground as shown in the figure. The car has a mass of 1500 kg, and the coefficient of friction between its tires and the ground is 0.200.

   (a) Draw a free body diagram of the car.

   (b) What is the reaction force to each of the forces acting on the car?

   (c) What force must the towing truck exert on the car if it is moving up at constant speed?

2. Because of an accident on a space flight, a 70 kg man is left in deep space, 1.0 x 10^4 m from the spherical asteroid Juno of mass 1.5 x 10^{14} kg.

   (a) How fast must he move, propelled by his rocket pack, to achieve a circular orbit around the asteroid at this distance, rather than crashing to its surface? (You must start from the fundamental gravitational and centripetal force (or acceleration) equations.)

   (b) It takes 8 h and 35 min for the rescue ship to arrive. Where should they look for him relative to the place of the accident?

3. If the car of problem 1 makes a displacement of 4 m along the incline, find:

   (a) The work done on the car by the resultant force.

   (b) The work done by each force (pulling chain, weight, friction and normal) on the car.
1. Given the situation pictured in the figure.

   (a) Specify all the forces acting on the body B and draw a free body diagram for B.

   (b) List all pairs action-reaction forces between B and its environment.

2. A hockey puck having a mass of 0.110 kg slides on the ice for 15.2 m before it stops.

   (a) Draw a free-body diagram of the puck.

   (b) If its initial speed was 6.1 m/s, what is the force of friction between the puck and ice? Use Laws of Motion.

   (c) What is the coefficient of kinetic friction.

   (d) Solve part (b) using energetic considerations.

3. Assuming that earth's orbit around the sun is circular find the orbital velocity of the earth and its radial acceleration toward the sun (You must start from the fundamental gravitational and centripetal force equations).
1. A book rests on a horizontal table, and a paperweight is lying on top of the book.
   (a) Draw a free body diagram of the book and identify all forces acting on it.
   (b) Identify the reaction force to each force acting on the book.

2. A roof of a house is inclined 37°, and a chunk of ice begins to slide down from a position 5 m from the edge. The coefficient of kinetic friction is \( \mu_k = 0.1 \) and the edge of the roof is 12 m above the ground.
   (a) Draw a free-body diagram of the chunk of ice sliding on the roof.
   (b) How far from the house will the ice land?

3. Determine the mass of the earth from the period \( T \) and the radius \( r \) of the moon's orbit about the earth.
   \[ T = 27.3 \text{ days}; \quad r = 3.85 \times 10^5 \text{ km}; \quad G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

4. A football player tries to stop an adversary player by placing his right shoulder against the adversary's chest at an angle 37° upward. What average force does the first player do to stop, in this way, a 200 lb adversary in a distance of 3 ft if the adversary player is moving with a speed of 8 ft/s.
Solutions

(a) Verify:
Objective 1
All the forces are present and well depicted.

(b) Action Reaction
\[ \vec{F} \] Car exerts a pull on the chain \((-\vec{F})\)
\[ \vec{N} \] Car exerts a normal for on the ground \((-\vec{N})\)
\[ \vec{W} \] Car attracts the earth \((-\vec{W})\)
\[ \vec{f} \] Car exerts frictional force on the ground \((-\vec{F})\)

(c) Newton's second law to the car:
\[ \sum F_y = N + F \sin 30^\circ - W \cos 15^\circ = 0 \]
\[ \sum F_x = F \cos 30^\circ - W \sin 15^\circ - f = 0 \]
\[ f = \mu N \]
Solving simultaneous equations for \( F \)
\[ F = mg \frac{\sin 15^\circ + \mu \cos 15^\circ}{\cos 30^\circ + \mu \sin 30^\circ} \]
\[ F = 6900 \text{ N}, \ \theta = 30^\circ \text{ up the ground.} \]
\[ \vec{F} = 6000 \hat{i} + 3450 \hat{j} \text{ N} \]
2. (a) \[ F_{\text{grav}} = F_{\text{centrip}} \text{ or } \frac{GMm}{r^2} = \frac{mv^2}{r} \]

\[ v = \left(\frac{GM}{r}\right)^{1/2}, \quad v = 1.0 \text{ m/s} \]

(b) \[ T = \frac{2\pi r}{v}, \quad T = 6.3 \times 10^4 \text{s} \]

\[ t = 8 \text{h} + 45 \text{ min} = 3.15 \times 10^4 \text{s} \]

They should, therefore, look for him almost exactly on the other side of Juno.

3. (a) Resultant Force:

\[ \vec{F}_R = \vec{F} + \vec{N} + \vec{W} + \vec{f} = 0 \]

since the car moves at constant speed.

Total work \( W = \vec{F}_R \cdot \vec{s} = 0 \)

(b) Work done by the chain:

\[ W_F = \vec{F} \cdot \vec{s} = F \cos \Theta \, d \]

\[ = (6900) \left(\frac{3}{2} \right) 4 = 23900 \text{ J} \]

\[ W_{(mg)} = (mg \cdot \vec{s}) = -mg (\sin 15^\circ) \, d \]

\[ = -15500 \text{ J} \]

\[ W_f = \vec{f} \cdot \vec{s} = -fd \]

\[ = -8.400 \text{ J} \]

\[ W_N = \vec{N} \cdot \vec{s} = 0 \]
Solutions:

1. (a)

Verify:

Objective 1

All the forces are present including friction force

(a) Action:

(b)

- \( \vec{T}_1 \) = force done by rope 1 on B
- \( \vec{T}_2 \) = force done by rope 2 on B
- \( \vec{N} \) = force done by horizontal surface on B
- \( \vec{f} \) = frictional force done by horizontal surface on B
- \( \vec{w} \) = weight or attraction of the earth on B

Reaction:

- \( -\vec{T}_1 \) = pulling tension on rope 1 done by B
- \( -\vec{T}_2 \) = pulling tension on rope 2 done by B
- \( -\vec{N} \) = normal force on the table done by B
- \( -\vec{f} \) = frictional force on the table done by B
- \( -\vec{w} \) = gravitational attraction on the earth done by B

Objective 2

Each force should be given and described
2. (a)

![Diagram with vectors and motion](attachment:image)

(b) Assuming constant deceleration

\[
a = -\frac{\nu_0^2}{2s},
\]

Second law:

\[
\Sigma F_y = N - W = 0
\]
\[
\Sigma F_x = -f = ma = + m \frac{\nu_0^2}{2s}
\]

\[|\vec{f}| = 0.130 \text{ N}\]

(c) \[\mu = \frac{f}{N} = \frac{f}{W}, \quad \mu = 0.120\]

(d) Work done by friction:

\[W = \vec{f} \cdot \vec{S} = -fs\]

Change in kinetic energy:

\[\Delta T = T - T_o = -T_o = -\frac{1}{2} m\nu_o^2\]

Work-energy theorem: \[-fs = -\frac{1}{2} m\nu_o^2\]

\[f = \frac{m\nu_o^2}{2s}, \text{ same as (b)}\]

Objective 1

Check the negative sign. Deceleration is decreasing speed.

Negative friction according to axes.

Objective 3

Negative work because \(f\) is opposite to displacement.

Negative because there is decreasing in speed.

Objective 5
3. \( F_{\text{grav.}} = F_{\text{centr.}} \)

\[
\frac{GMm}{R^2} = ma_c = m \frac{u^2}{R}
\]

\[
a_c = \frac{GM}{R^2}, \quad u = \left(\frac{GM}{R}\right)^{1/2}
\]

\[
u = 2.97 \times 10^4 \text{ m/s} = 106900 \text{ km/hr} = 66428 \text{ mi/hr}
\]

\[
a = 5.9 \times 10^{-3} \text{ m/s}^2 = 0.59 \text{ cm/s}^2 = 0.0194 \text{ ft/s}^2
\]

Objective 4

student really started by equating gravitational force to centripetal force. If some other (correct) equation was used, make him derive it!
UNIT VI

GRADING KEY FOR TEST 3

Solutions

1. (a) 

\[ \begin{align*} 
N & \uparrow \\
P & \downarrow \\
W & \downarrow 
\end{align*} \]

(b) Action: Reaction:

\[ \vec{W} \]

Book attracts the earth with a force \((-\vec{W})\)

\[ \vec{N} \]

Book exerts a downward force on the table \((-\vec{N})\)

\[ \vec{P} \]

Book exerts upward force on the paper-weight \((-\vec{P})\)

Verify:

Objective 1

That all forces are present and well depicted

Objective 2

Each force should be given and said on what by what.

2. (a) 

\[ \begin{align*} 
\vec{F}_k & \hat{i} \\
\vec{N} & \hat{j} \\
\vec{W} & \hat{k} \\
37^\circ 
\end{align*} \]

(b) Applying Newton's second law to the chunk

\[ \begin{align*} 
\Sigma F_y & = N - W \cos \Theta = 0 \\
\Sigma F_x & = W \sin \Theta - f_x = ma_x \\
f_x & = \mu_k N 
\end{align*} \]

Objective 1

That all forces are present. Frictional force opposes to motion

Objective 3

No motion in \(\hat{j}\) direction. Acceleration along the roof.
Solving simultaneously all three equs.:

\[ a_x = g (\sin \Theta - \mu_k \cos \Theta) \]

Velocity at the edge of the roof

\[ v_o^2 = 2a_x l, \quad v_o = \sqrt{2 g l (\sin \Theta - \mu_k \cos \Theta)} \]

In components

\[ v_{ox} = v_o \cos \Theta, \quad v_{oy} = v_o \sin \Theta \]

Parabolic motion yields

\[ y = \frac{v_{oy}}{v_{ox}} s - \frac{1}{2} \frac{g}{v_{ox}^2} s^2 \]

\[ s = \frac{v_{ox} v_{oy}}{g} - \frac{v_{oy}^2}{g} \sqrt{\frac{2}{g} v_{oy} - 2gy} \]

Using \( g = 10m/s^2 \) and \( \gamma = -12 \),
numerical solutions give:

\[ a_x = 5.2 \text{ m/s}^2; \quad v_o = 7.2 \text{ m/s}; \]
\[ S = 7.6 \text{ m} \]

3. \[ F = \frac{GM_{em}m_{em}}{R_{em}^2} = m_a m = m \omega^2 R_{em} \]

\[ M_e = \frac{4\pi^2}{G} \cdot \frac{R_{em}^2}{T_m^2} \quad \text{(Kepler's law)} \]

\[ M_e = 6.07 \times 10^{24} \text{ kg} \]

4. Work done by \( F \):

\[ W = \overrightarrow{F} \cdot \overrightarrow{s} = -F \cos \Theta x \]

Objective 4

\( F_G \) acts as a centripetal force.
Correct relation between centripetal acceleration and period.

Objective 5

Starts with vector eq.
\( F \) opposes to motion.
Negative sign. Final speed = 0.
Work-energy theorem:

\[ W_F = \Delta T = - T_o \]

\[ F = \frac{(mg) v_o^2}{xg \cos \theta} \quad , \quad F = 167 \text{ N} \]

Don't use weight for mass in the numerical values. What happened with the energy? Is there a violation to the conservation principle?
APPENDIX E

AUTOLECTURES

Autolecture No 1

Autolecture No 2

Autolecture No 3
AUTOLECTURE No 1

BASIC KINEMATICS DEFINITIONS

Problem 1: Mathematical and Geometrical Interpretation.

Problem 2: Rectilinear Motion with Acceleration Proportional to Time.

Problem 3: Rectilinear Motion with Constant Acceleration.

Transparencies: 6.
AUTOLECTURE No. 1

This lecture is prepared to help you understand and complement Units II and III of our course in Physics 53 P.S.I. We recommend that you have at hand: first, the package of transparencies that you will be using on the overhead projector; second, the collection of the slides that goes with Chapter 2 of your textbook; third, you must have your textbook ready (and of course some paper to take notes).

This autolecture #1 deals mainly with the study of motion in one dimension. During our regular lecture we defined average velocity of a physical body in a certain interval of time delta t as the ratio of the distance traveled over delta t. Also, we defined during our lecture instantaneous velocity. We mentioned that the instantaneous velocity in the time interval between t and t + delta t is the limit of average velocity when the interval delta t becomes very, very small. That is, \( v = \frac{ds}{dt} \). You can check on page 17 of your book for the mathematical expression labeled (2-2).

Now, if you want, read over the concepts of average velocity and instantaneous velocity in the textbook. Once you are ready, proceed with the next concept of acceleration. In our lecture we define also the average acceleration as the ratio of the change of the instantaneous velocity, that we label as \( \Delta v = v_2 - v_1 \) divided by the time interval \( \Delta t = t_2 - t_1 \) over which the change occur. You can look
now on page 20 expression (2-3). (gap)

In exact analogy with the definition of instantaneous velocity, mathematically we can write that the acceleration is equal to the derivative of the velocity with respect to time. This is labeled as expression (2-4). We note, however, that the acceleration also could be expressed in terms of the position s of the object. For that we use the symbol called second derivative and say: the acceleration is the second derivative of the position with respect to time.

We recommend now, that you read over the concept of acceleration since we will use an example in which you might obtain a better understanding. (gap)

Place transparency #1 on your overhead projector.

We are considering a bug that moves on a straight line back and forth and we measure its position and time. Of course you will notice that in Figure 1, we are using an arbitrary reference for the distance traveled. We can now make a graph of the position versus time. And we obtain the curve that you see on the transparency.

Let us now select two points: P with coordinates \((s_1, t_1)\) and Q with coordinates \((s_2, t_2)\). Evidently the change of position between P and Q is given by \(\Delta s_{1,2} = s_2 - s_1\). It takes place in the time \(\Delta t_{1,2} = t_2 - t_1\). Then, the average velocity is given by relation (2-1). Now let us make a geometrical interpretation of the line \(P,Q\) as you see it in the transparency. By using geometry the slope of the line \(P,Q\) is given by \(m_{PQ}\).

Evidently this is \(\Delta s_{1,2}\) over \(\Delta t_{1,2}\), which is the average velocity. Now let's consider that point Q approaches point P such that \(s_2\) approaches \(s_1\). In other words delta s is smaller.
Delta t approaches zero in such a way that delta s approaches zero. Then, the limit is our definition of instantaneous velocity given by the relation labeled (2-2). What is the geometrical interpretation? Place on your overhead projector, transparency $\#2$. (gap)

Now let's talk about the geometrical interpretation. The line $P,Q$ approaches the tangent line $P,T$ as $Q$ approaches $P$ or as delta t approaches zero. Then, the limit of the line $P,Q$ as delta t approaches zero, is equal to the tangent line $P,T$. What is the conclusion? The slope of the tangent line to the position curve is the instantaneous velocity or mathematically the derivative of the position with respect to time. Using this last fact we plot the instantaneous velocity versus time. Think how you could do it. It will result in graph 3. By using an analogous procedure you can obtain the average acceleration which is the slope of the line $R,S$. (gap)

Place transparency $\#3$ on the overhead projector.

We use the same approach as we did before with the instantaneous velocity. That is, the instantaneous acceleration at time $t$ is the limit of the average acceleration when the interval delta t becomes infinitesimally small. In order words, it is the derivative of the velocity with respect to time. Geometrically it means that the limit of the line $R,S$, when delta t approaches zero is equal to the tangent line labeled $S,W$. The conclusion is evident, the slope of the tangent to the instantaneous velocity versus time curve gives us the instantaneous acceleration. (gap)

We are now going to use another example because in this way you will understand better the concepts of acceleration and velocity. You can stop the machine at any time and use your slide projector with
the slide that corresponds to Figure 2-5 of your textbook or if you prefer open your textbook on page 22 Figure 2-5. You will have three graphs. One graph is position versus time. Second, velocity versus time, and the third one is acceleration versus time of a body moving in one dimension. Note that when the tangent to the position curve in the first figure, position versus time, is parallel to the axis that represent time, the velocity is zero. Now you look on the second graph which represent velocity versus time. You will note that zero velocity corresponds to the point when the curve crosses or touches the time axis. Move now to the third graph, acceleration versus time. You will find that when the velocity is zero the acceleration may be different from zero. Draw tangent lines to the position curve at points before and after the point where the velocity is zero. Select the points. Think and discuss. (gap)

Mathematically speaking inflection point is the point where the curve changes its concavity. Find an inflection point, in the position-versus-t graph. To this inflection point corresponds a relative maximum or minimal value of the velocity and the acceleration is zero. Draw tangent lines at some points before and after the inflection points on the position curve, s-versus-t. Think again and discuss with your partner, instructor or proctor. (gap)

Place transparency #4 - Motion with constant acceleration, on the overhead projector. Read example 3 on your transparency.

The general solution can be obtained in two different ways. One, we can call it the mathematical solution. The other, is the so-called geometrical approach. For the mathematical solution we will
follow the direct approach given during our regular lecture. Read carefully the mathematical solution in your transparencies #4 and #5. (gap)

Let us now consider the geometrical solution. If the car is moving with constant acceleration the instantaneous acceleration is equal to the average acceleration. Note on your transparency and compare the equation of the instantaneous velocity with the equation of a straight line. Look at Figure 4 and think. (gap)

Place transparency #6 on your overhead projector.

One can obtain a special simple equation for the average velocity in this special case of uniform acceleration. We take the total interval and divide it into several very small intervals. We make these time intervals small enough so that the velocity is not changed greatly during each one of the small intervals. Follow the mathematical calculation given on transparency #6. (gap)

As you can see each $\Delta s$ represents geometrically the area of each rectangle. And the addition of all these displacements is the total displacement moved.

If the $\Delta t$'s are made very small the sum of the areas of the rectangles will differ negligibly from the total area under the line from the initial time to the final time. That is, the shaded area.

And finally, we've arrived to the same equations as in the first method. It is worth to note that in this new approach we have come out with an important result, i.e. the area under the velocity curve represents physically the change of position, and mathematically the operation integration.

Using an analogous procedure you can show that the area under the acceleration curve represents physically the change of velocity.
UNIT I

EXAMPLE 1

A bug moves on a straight line "back and forth" and its position is timed.

\[ \text{Fig. 1 - One dimensional trajectory} \]

\[ \text{Fig. 2 Position vs. time} \]

The points \( P(s_1, t_1) ; Q(s_2, t_2) \)

Change of position \( \Delta s_{12} = s_2 - s_1 \) takes place in an interval \( \Delta t_{12} = t_2 - t_1 \)

Average velocity:
\[ \bar{v}_{12} = \frac{\Delta s_{12}}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1} \tag{2-1} \]

Slope of the line \( PQ \):
\[ m_{PQ} = \frac{\Delta s_{12}}{\Delta t_{12}} = \overline{v_{12}} \]

When \( \Delta t \to 0 \), \( t_2 \to t_1 \) and point \( Q \) approaches point \( P \)

such that \( s_2 \to s_1 \) and \( \Delta s \to 0 \)

The instantaneous velocity is:
\[ v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \tag{2-2} \]
UNIT 1

TRANSPARENCY 2

Geometrical Interpretation:

The line PQ approaches the tangent line PT.

That is, as Q→P or Δt→0, then:

\[ \lim_{\Delta t \to 0} \text{(line PQ)} = \text{tangent line PT} \]

Hence: Slope of the tangent to the curve

\[ m_{\text{tang}} = \frac{ds}{dt} \]

Using this fact we can plot instantaneous velocity versus time

![Figure 3 - Velocity vs. Time](image)

Using an analogous procedure:

Change of velocity \( \Delta v = v_2 - v_1 \) between points \( R(v_1, t_1) \) and \( S(v_2, t_2) \)

Time interval \( \Delta t = t_2 - t_1 \)

Average acceleration \( \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \) \hspace{1cm} (2-3)

Slope of the line \( RS \):

\[ m_{RS} = \frac{\Delta v}{\Delta t} = \bar{a} \]
And similarly the instantaneous acceleration at the time \( t \) is the limit of the average acceleration when the interval \( \Delta t \) becomes infinitesimally small,

\[
a = \lim_{\Delta t \to 0} \left( \frac{\Delta v}{\Delta t} \right) = \frac{dv}{dt}
\]

which geometrically means:

\[
\lim_{\Delta t \to 0} (\text{line } RS) = \text{tangent line } SW
\]

acceleration = the slope of the tangent to the instantaneous velocity curve.

= the derivative of the velocity with respect to time.

Example 2

Project the slide corresponding to Fig. 2-5 or open your textbook to page 22.

Graphs: \( s\text{-vs. } t \), \( v\text{-vs. } t \), \( a\text{-vs. } t \), of a body in one dimension motion.

Note the points of velocity zero in each graph.

Draw tangent lines at points before and after the points where velocity is zero. Discuss the corresponding points in each of the other graphs.

Draw tangent lines at points before and after the inflection point in the position curve. Analyze the corresponding points in the velocity and acceleration curves.
UNIT I

Motion with Constant Acceleration

Example 3

According to the performance data for a certain car it is capable of accelerating from 2 m/sec to 12 m/sec in 10 sec. Its velocity as a function of time is shown in the Fig 4.

A) Find the general expression for acceleration, velocity and position as functions of time. Assume constant acceleration.

B) Calculate its acceleration and the distance it goes in 10 sec.

\[ v(t) = 2 + 1 \times t \]

\[ x(t) = 2t + 0.5 \times t^2 \]

\[ a(t) = 1 \]

Fig 4.

A) Mathematical Solution:

By definition: \( a = \frac{dv}{dt} \) where \( a = \text{constant} \)

Or in differential form: \( dv = a \, dt \)

Conditions: initial time: \( t_0 = 0 \), initial velocity: \( v_0 \)
final time: \( t_f = t \), final velocity: \( v_f = v \)

Integrating both sides

\[ \int_{v_0}^{v} dv = \int_{t_0}^{t} a \, dt = a \int_{t_0}^{t} dt \]

Since \( a \) is constant

\[ \therefore \int_{v_0}^{v} = a \int_{0}^{t}, v - v_0 = \Delta v = a (t - 0) \therefore v = v_0 + a t \]
UNIT I

Now, by definition: \( v = \frac{ds}{dt} \) or \( ds = v \, dt \)

By taking: \( s_o = \text{initial position for } t_o = 0 \)
\( s = \text{final position for } t_f = t \)
\[
\int_{s_o}^{s} ds = \int_{t_o}^{t} v \, dt = \int_{t_o}^{t} (u_o + at) \, dt = u_o \int_{t_o}^{t} dt + a \int_{t_o}^{t} t \, dt
\]

Since \( v_o \) and \( a \) are constants.
\[
\Delta s = s - s_o = v_o (t - 0) + a (t^2 - t_o^2) = v_o t + \frac{1}{2} a t^2
\]
\[
s = s_o + v_o t + \frac{1}{2} a t^2
\]

2) **Mathematic Geometrical Solution**

Since the acceleration is constant, the average acceleration = the instantaneous acceleration

\[
a = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_o}{t - t_o} \implies v = v_o + a \cdot t
\]

**Equation of a straight line**: \( y = mx + b \) where

\( b = \text{vertical intercept} \)
\( m = \text{slope of the line} \)

![Diagram](image-url)
Divide the interval $\Delta t = t_f - t_i$ into $N$ very small intervals.

The distance moved by the object during the successive time intervals will be:

$$
\Delta s_1 = v_1 \Delta t_1 \\
\Delta s_2 = v_2 \Delta t_2 \\
\vdots \\
\Delta s_N = v_N \Delta t_N
$$

Since the sum of all these displacements is approximately the total distance moved, we have

$$
\Delta S \approx \Delta s_1 + \Delta s_2 + \ldots + \Delta s_N = v_1 \Delta t_1 + v_2 \Delta t_2 + \ldots + v_N \Delta t_N
$$

But as $\Delta t \to 0$, $N \to \infty$ and

$$
\Delta s = \lim_{\Delta t \to 0} \left( \sum \Delta s_i \right) = \int v \, dt = \text{Area}
$$

And this comes out as the shaded area in Fig. 6.

This area is found from the geometry of the Fig. to be

$$
\text{Area} = v_0 t + \frac{1}{2} (v - v_0) t = \Delta s
$$

And:

$$
\overline{v} = \frac{1}{2} \left( v_0 + v \right)
$$

$$
\Delta s = s - s_0 = \frac{1}{2} \left[ v_0 + (v_0 + at) \right] t
$$

$$\therefore \quad s = s_0 + v_0 t + \frac{1}{2} at^2.$$
VECTOR KINEMATICS

Problem 1: Chain's Rule for Addition Velocity Vectors.

EQUATIONS OF: FREE-FALL, HORIZONTAL CONSTANT MOTION, AND PARABOLIC TRAJECTORY

Problem 2: Vertical Launching under Gravity Action.
Problem 3: Horizontal Launching under Gravity Action.
Problem 4: Parabolic Launching under Gravity Action.
Problem 5: Energetic Considerations for Parabolic Launching.

CIRCULAR MOTION

Relations between Linear and Angular Kinematics Variables.

Problem 6: Disk Rotating by Action of a Falling-Weight.

Transparencies: 11.
This autolecture deals mainly with motion on a plane surface. We know from our common experience that the average velocity has both direction and magnitude. We must know not only how fast a body is moving but also in what direction. Physical quantities, such as the average velocity, that have both direction and magnitude are frequently vectors. At this point we must clarify that not all physical quantities that have direction and magnitude are vectors. We recommend that you go to supplement III on page 577 of your textbook which contains a more complete discussion of vector algebra.

Place transparency #1 on your overhead projector. (gap)

If we accept that the average velocity is a vector, we have to redefine it in terms of the so called vector-position. In fact, look at Figure 3-1 of your textbook, page 41. The average velocity will be defined as the change in the vector position per unit time. You may notice in your book that they use boldface letters for vector quantities. In your transparency we will place a small arrow on each of the physical quantities that are vectors. You may recognize that in addition to velocity, momentum and acceleration are vectors. We have other type of physical observables and we may consider as a scalar like kinetic or potential energy and mass of an object. By a scalar we mean that these observables have no direction associated with them and one number is enough to specify them completely. Vector observables, we insist, must be thought of as a step in a space...
with a specific direction and magnitude.

Look now at Figure 3-5 in your textbook that describes the trajectory of a physical body moving on a plane surface. We define or redefine the instantaneous velocity vector as the limit of the average velocity vector when \( \Delta t \) approaches zero. That is, \( \mathbf{v} \) is equal to the derivative of the vector position with respect to time. Note in Figure 3-5 that the velocity vector is always tangential to the trajectory of the motion. Many times we hear the term speed. We will use the term speed for the magnitude of the velocity vector.

Read over page 46 of your textbook. (gap)

We are going to apply now the concepts that we obtained of average velocity and velocity vector to different types of problems. But first we recommend that you go into detail through pages 42, 43, and 44 of your textbook that deal with simple properties of vectors. You can stop the machine at any time. (gap)

Place transparency #2 on your overhead projector and read carefully problem 1. (gap)

From your analysis of the problem we see that we have to consider three vectors. The first, is the velocity of the plane with respect to earth. The second vector is the velocity of the plane with respect to the wind. And the third vector will represent the velocity of the wind with respect to earth. See your figure and think on it carefully. (gap)

Using your knowledge of addition of vectors we conclude that the velocity of the plane with respect to earth is equal to the vector addition of the velocity of plane with respect to the wind plus the velocity of the wind with respect to earth. A simple calculation will
tell you that you can find the velocity of the plane with respect to the wind by subtraction of two vectors. See the equations on your transparency. (gap)

Notice in your calculations that we have used the vector notation. That is, in the case of the velocity of the plane with respect to earth the vector velocity has only one component the x-component represented then by its numerical value times the unit vector in the x-direction. The vector that represents the velocity of the wind with respect to earth has two components; x and y components. See it in detail on your transparency. (gap)

Now to obtain the velocity of the plane with respect to the wind you must refer to equation (1) and subtract two vectors: The velocity of the plane with respect to earth, and the velocity of the wind with respect to earth. Notice that the velocity of the plane with respect to wind comes out as equation (4). Also, you can obtain the magnitude of the velocity vector of the plane with respect to wind and its direction by using simple trigonometry relations. (gap)

Let us go over the concept of the acceleration vector. The acceleration vector measures the rate at which the velocity vector changes with time in both direction and magnitude. It is important to note that acceleration exists whenever the velocity vector changes. The velocity may be changing only in magnitude in which case the acceleration vector lies along the velocity vector. But the velocity may be changing only in direction — (Constant speed) in which case the acceleration vector is directed perpendicular to the velocity vector and toward the center of curvature of the instantaneous trajectory. Finally, the velocity vector may be changing both in magnitude and
direction as in the case represented in Figure 3-8 of your textbook.

Open your textbook on page 49 and review the concept of acceleration vector. (gap)

It is suggested that you go into detail over Figure 3-8. At part (a) we represent the trajectory of the motion with the velocity vectors tangent to the trajectory at each point. At part (b) the velocity vectors are plotted from a common origin and they define a curve which is called the hodograph. Notice that the acceleration vectors are tangent to the hodograph at each point. At part (c) the acceleration vectors are indicated for several positions along the original trajectory. Note that the acceleration vector always points inward.

Now place transparency #3 on your overhead projector. (gap)

We are now going to use our general concepts of vector velocity and vector acceleration and apply them to motion in one dimension and two dimensions. First, let us consider the motion of an object under the action of gravity near the surface of the earth. That is, an object under free fall. We assume that vectors are positive when they are directed upward and negative when the vectors are directed downward. Then the acceleration, the velocity, and the position of the object are given by the vector equations shown on your transparency. (gap)

Second suppose that our physical body is just moving in the x-direction with constant uniform motion in a straight line. Evidently the vectors equations of motion are those given in part 2 of your transparency. (gap)

Third, assume now that your physical body has uniform motion
in the x-direction and in addition is submitted to the action of gravity and has accelerated motion in the y-direction. Now, the resultant motion will be the superposition of the two orthogonal motions. See in detail the equations of motion. (gap)

Place transparency #4 on your overhead projector. (gap)

It represents a case of a free fall vertical motion. You may remember that in Units II and III you did not use vector notation. In this case we are going to show you how the vector notation will help you to understand better the motion of an object even in one dimension. Read your problem carefully. (gap)

In order to use vector notation we place a coordinate system at the top of the building so the vectors pointing vertically upward will be positive and they will be negative for vectors that are pointing downward. Go into detail over this problem. Take your time and try to understand it. If you have difficulties ask your professor or proctor for assistance. (gap)

Place transparency #5 on your overhead projector. (gap)

Let us consider that the ball in the previous problem is thrown horizontally from the top of the building with a velocity of 10 m/sec directed to the right as we indicated in the figure. We want to know now where will the ball hit the ground. Again, we will place our system of coordinates at the top of the building. It is also indicated in the figure. Read the solution of the problem step by step. You can stop your machine at any time and ask questions from your professor or proctor. (gap)

Now place transparency #6 on your overhead projector.

Here, we are combining the two problems that we solved before.
Suppose that the ball had been thrown upward but in such a way it has an initial horizontal component of the velocity of 10 m/sec. and at the same time an initial component of the velocity in a vertical direction of 20 m/sec. Read the questions. (gap)

To solve the problems we will place our system of coordinates at the top of the building in the direction indicated in the figure. You can take enough time to read the solution and to stop the machine at any time that is convenient for you. Do not hesitate to ask questions from your professor or proctor and think if you really understood the problem.

Place transparency #9 on your overhead projector. (gap)

We are going to analyze the previous problem by using energetic considerations. We can think that the ball has a mass of 10 grams. The building has the same height and of course, we place our system of reference in the same position that it was before.

You will then go step by step through transparency #9 that will show you how to solve the same problem by using the principle of conservation of mechanical energy. Remember that you can stop your cassette player at any time and of course ask questions from your professor or proctor. (gap)

You can see that by using the principle of conservation of mechanical energy the solution to your problem is easier, besides you don't have to worry about vector consideration. You're working with scalar quantities and you are obtaining the same results. Think carefully, why can you do it?

We are now going to work on circular motion, but before we do this we will make some general considerations.
Please place transparency #10 on your overhead projector. (gap)

We are considering a particle or physical object moving along a curved path. Evidently at any time $t$ its position is given by the vector $\mathbf{r}$ and the velocity by the derivative of $\mathbf{r}$ with respect to $t$. Remember that the velocity vector is always tangent to the path. The vector $\mathbf{r}$ will be given by the magnitude of vector $\mathbf{r}$ times a unit vector along the radial direction. We are considering two unit vectors: $\mathbf{u}_0$, the unit vector tangent to the path, and $\mathbf{u}_r$, the unit vector along the radius vector. The acceleration will then be given by an equation with two terms. In other words, when the particle moves and the velocity changes then the acceleration is the result of changing the magnitude of the velocity and/or changing the direction of the velocity.

It can be shown that the rate of $\mathbf{u}_0$ is a vector pointing in the opposite direction of $\mathbf{u}_r$. Please read supplement IV on page 593 of your textbook. (gap)

We arrive to a very interesting conclusion. It is that the acceleration has two components. One, that we call tangential acceleration represented by the change in magnitude of the velocity per unit time; and another, it is the normal or centripetal acceleration, that represents the change in direction of the velocity per unit time. (gap)

We will now be able to find relations between angular variables and linear variables. Read carefully the derivations on your transparency. (gap)

Place transparency #12 on your overhead projector.

It has a very interesting problem. We urge you to read it in detail and go through the solution. Please ask your professor
or proctor if you have any questions. (gap)

I am sure that this autolecture will help you. Again we have to mention that this is not a substitution for the regular lecture. It will supplement it. GOOD LUCK.
Unit II

**Vectors Quantities: Both Direction and Magnitude**

**Vector Position: \( \mathbf{r} \).** It measures the position of a body at every instant of time from a fixed (arbitrary) point \( O \) (origin of measurements).

![Diagram showing vector position and trajectory](image)

Average Velocity Vector:
\[
\mathbf{v}_{\text{ave}} = \frac{\mathbf{r}(t_2) - \mathbf{r}(t_1)}{t_2 - t_1} = \frac{\Delta \mathbf{r}}{\Delta t} \quad (3-1)
\]

Instantaneous Velocity Vector:
\[
\mathbf{v} = \lim_{\Delta t \to 0} \mathbf{v}_{\text{ave}} = \frac{d \mathbf{r}}{dt} \quad (3-6)
\]

It is always tangential to the trajectory of the motion and measures the rate at which the position vector changes with time (in both direction and magnitude).

**Speed = Magnitude of the Velocity Vector**: \( v = |\mathbf{v}| \)

Acceleration Vector:
\[
\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2 \mathbf{r}}{dt^2} \quad (3-8)
\]

It measures the rate at which the velocity vector changes with time (in both direction and magnitude).

In Cartesians Components:
\[
\mathbf{r} = x \mathbf{\hat{e}} + y \mathbf{\hat{j}} + z \mathbf{\hat{k}}
\]
\[
\mathbf{v} = v_x \mathbf{\hat{e}} + v_y \mathbf{\hat{j}} + v_z \mathbf{\hat{k}}
\]
\[
\mathbf{a} = a_x \mathbf{\hat{e}} + a_y \mathbf{\hat{j}} + a_z \mathbf{\hat{k}}
\]
UNIT II

PROBLEM 1

The pilot of an airplane wishes to reach a town 200 miles east of his present position. A wind blows 30 miles/hr from the northwest. Calculate his velocity with respect to the moving air mass if his schedule requires him to arrive at his destination in 40 min.

**Solution**

\[ \vec{v}_{pe} = \text{plane's velocity with respect to earth} \]

\[ \vec{v}_{pw} = \text{plane's velocity with respect to wind} \]

\[ \vec{v}_{we} = \text{wind's velocity with respect to earth} \]

\[ \vec{v}_{pe} = \vec{v}_{pw} + \vec{v}_{we} \]

\[ \vec{v}_{pw} = \vec{v}_{pe} - \vec{v}_{we} \]

Time of flight \( T = 40 \text{ min} = \frac{2}{3} \text{ hr} \)

\[ v_{pe} = |\vec{v}_{pe}| = \frac{200}{\sqrt{3}} = 300 \text{ mi/hr to East} \]

\[ \vec{v}_{pe} = 300 \hat{e} \]

\[ v_{we} = v_{we} \cos 45^\circ \hat{e} - v_{we} \sin 45^\circ \hat{j} = 30 \frac{v_{we}}{\sqrt{2}} (\hat{e} - \hat{j}) \]

\[ \vec{v}_{pw} = 279 \hat{e} + 21 \hat{j} \text{ (mi/hr)} \]

or:

\[ |\vec{v}_{pw}| = \sqrt{(279)^2 + (21)^2} = 280 \text{ mi/hr} \]

\[ \alpha = \arctan \frac{21}{279} \approx 4.3^\circ \text{ North of East} \]
Unit II

1) Vertical Motion under the action of gravity

\[ \ddot{a}_y = g \ (\downarrow) \quad \text{but} \quad \ddot{a}_x = \frac{d \ddot{v}_x}{dt} \]

by integration:

\[ \dot{v}_y = \ddot{v}_y - gt \ (\downarrow) \quad \text{but} \quad \dot{v}_x = \frac{d \ddot{v}_x}{dt} \]

\[ \ddot{y} = \ddot{y}_0 + \dot{v}_y t - \frac{gt^2}{2} \ (\downarrow) \]

2) Horizontal Constant Motion

\[ \ddot{u}_x = u_{x0} \ (\downarrow) \]

\[ \ddot{x} = x_0 + u_{x0} t \ (\downarrow) \]

3) By Superposition of the before two orthogonal motions:

\[ \ddot{a} = a_x \ (\downarrow) + a_y \ (\downarrow) = -g \ (\downarrow) \]

\[ \ddot{r} = u_x \ (\downarrow) + u_y \ (\downarrow) \]

\[ \ddot{v} = u_{x0} \ (\downarrow) + (u_{y0} - gt) \ (\downarrow) \]

\[ \ddot{r} = \ddot{x} + \ddot{y} \]

\[ \ddot{r} = (x_0 + u_{x0} t) \ (\downarrow) + (y_0 + u_{y0} t - \frac{1}{2} gt^2) \ (\downarrow) \]
Problem 2
A ball is thrown upward with a speed of 20 m/sec from the top of a 20 m high building.

a) Where will the ball be at 1 sec after it is thrown?
b) Is it moving up or down at that time?
c) At t=3 sec what are its position and velocity?

Solution
Fix conventional coordinates at the top

\[ \vec{d}_y = -g \hat{j} \]
\[ \vec{v}_y = (v_{y0} - gt) \hat{j} \]
\[ \vec{y} = (y_0 + v_{y0} - \frac{1}{2}gt^2) \hat{j} \]

At \( t = 0 \):
\[ y_0 = 0 \], \( v_{y0} = +20 \text{ m/sec} \), \( g = 9.8 \text{ m/sec}^2 \)
\[ \therefore \vec{y} = (20t - \frac{1}{2}gt^2) \hat{j} \text{ (m)} \]

At \( t = 1 \text{ sec} \), \( \vec{y}(t=1) = +15 \hat{j} \text{ (m)} \)
Since \( y(t=1) \) is positive, the ball is above the throwing point.

b) \( \vec{v}_y = (20 - gt) \hat{j} \text{ (m/sec)} \)
At \( t = 1 \text{ sec} \), \( \vec{v}_y(t=1) = +10.2 \hat{j} \text{ (m/sec)} \)
The plus sign says the ball is still moving upward.

c) At \( t = 3 \text{ sec} \).
\[ \vec{y}(t=3) = -15.9 \hat{j} \text{ (m)} \]
\[ \vec{v}_y(t=3) = -5.3 \hat{j} \text{ (m/sec)} \]
The negative \( \vec{y} \) says the ball is below the throwing point.
The negative \( \vec{v}_y \) says the ball is now moving downward.
Problem 3
Let us consider that the ball in the previous problem is thrown horizontally from the top of the building with a speed of 10 m/sec to the right. Where does the ball strike the ground?

Solution
Consider the horizontal and the vertical motion separately

Vertical Motion:
\[ \vec{v}_0 = 0 \quad ; \quad \vec{y}_{\text{final}} = -20 \, \uparrow \]
\[ \vec{y} = (v_{y0} \cdot t - \frac{1}{2} g \cdot t^2) \, \uparrow \Rightarrow -20 \, \uparrow = -\frac{1}{2} \cdot 9.8 \cdot t^2 \, \uparrow \]
Hence, time spent to reach ground
\[ t_f = \left( \frac{20 \cdot 2}{9.8} \right)^{\frac{1}{2}} = 2.02 \, \text{sec} \]

Horizontal Motion:
\[ \vec{x} = \vec{v}_{x0} \cdot t = \vec{v}_{x0} \cdot t \, \hat{\ell} \]
\[ \vec{x}_{\text{max}} = (10 \cdot 2.02) \hat{\ell} = 20.2 \, \ell \quad (\text{m}) \]
Unit II

Problem 4

The same ball had been thrown in such a way that has a horizontal component of velocity of +10 m/sec and a vertical component of velocity of +20 m/sec.

a) What is the time when the ball is 15 m above the top of the building?
b) At what time does the ball reach the highest point of its trajectory?
c) How high does the ball go?
d) At what time will it return to its original elevation?
e) What is the horizontal range of the ball up to this point?
f) What is its velocity?
g) What is the ball's velocity when it strikes the ground?
h) How long is the ball in the air?
i) Where does the ball land?

Solution

\[ \mathbf{v}_0 = 10 \hat{i} + 20 \hat{j} \]

or \[ \text{speed} = v_0 = \sqrt{10^2 + 20^2} = 22.4 \text{ m/sec} \quad \text{and} \quad \theta_0 = \arctan \frac{20}{10} = 63.4^\circ \]

Fix coordinates axis at the top
Separate the motion into two orthogonal motions
Use scalars equation with proper use of sign to mean directions

A) \[ \mathbf{v} = \mathbf{v}_0 + \mathbf{v}_0t - \frac{1}{2} \mathbf{g} t^2 \]
becomes \[ y = v_{0y} t - \frac{1}{2} g t^2 \] since \( y_0 = 0 \)
When \( y = 15 \) then \[ 15 = 20t - \frac{1}{2} g t^2 \]
Quadratic eq. with solutions: \( t_1 = 1 \text{ sec.}, \quad t_2 = 3 \text{ sec.} \]
At \( t = 1 \) sec., \( y_1 = +15 \text{ m} \); \( \vec{v}_y(t=1) \) is positive. The ball is still going up.

At \( t = 3 \) sec.; \( y_2 = +15 \text{ m} \); \( \vec{v}_y(t=3) \) is negative. The ball is falling down.

b) At the highest elevation, \( \vec{v}_y = 0 \); \( \vec{v}_{y0} - gt \)

\[ t_3 = \frac{\vec{v}_{y0}}{g}, \quad t_3 = 2.04 \text{ sec.} \]

c) Maximum height is at \( t_3 = 2.04 \text{ sec.} \)

\[ y_{\text{max}} = \frac{\vec{v}_{y0}^2}{2(-g)} \quad \text{or} \quad y_{\text{max}} = \frac{\vec{v}_{y0}^2 - \vec{v}_{y0}^2}{2(-g)} \quad y_{\text{max}} = 20.4 \text{ m}. \]

d) On the original elevation \( \vec{y}_0 = 0 \); \( \vec{y}_{0} = \vec{v}_{y0}t - \frac{1}{2}gt^2 \)

That gives: \( t_0 = 0 \) initial condition

\[ t_4 = \frac{2\vec{v}_{y0}}{g}, \quad t_4 = 4.08 \text{ sec.} \]

Note \( t_4 = 2t_3 \)

Time required for the ball to reach its maximum height from starting level is the same as is required to reach the original level from its maximum height.

e) The range is obtained by evaluating \( X(t_4) \):

\[ X_R = \vec{v}_{x0}t_4 \quad \text{or} \quad X_R = \frac{\vec{v}_{x0} \vec{v}_{y0}}{g} \quad ; \quad X_R = 40.8 \text{ m} \]

f) \( \vec{v}_{y4} = \vec{v}_{y0} - gt_4 \quad ; \quad \vec{v}_{y4} = -20 \text{ m/sec} \)

The negative sign means that vector \( \vec{v}_y \) is pointed down.

Horizontal component of velocity: \( \vec{v}_x = \vec{v}_{x0} = 10 \text{ m/sec.} \)

So that the velocity at the range: \( \vec{v}_4 = 10 \hat{i} -20 \hat{j} \text{ (m/sec)} \)

or \( \vec{v}_4 = 22.4 \text{ m/sec.} \quad \text{and} \quad \theta_4 = -63.4^\circ \) (fourth quadrant)

The same speed as the initial velocity but different direction!
Unit II

G) WHEN THE BALL STRIKES THE GROUND \( y_s = -20 \) m.

So that, \( v_{ys}^2 = v_{yo}^2 + 2(-g)(-y_s) \)

\[ v_{ys} = -28 \text{ m/sec. pointing down}. \]

AND \( v_{xs} = 10 \text{ m/sec} \)

or \( v_x = 30 \text{ m/sec}; \quad \theta_s = -70.4^\circ \)

H) To FIND THE TIME OF FLIGHT WE CAN USE \( y = v_{yo}t - \frac{1}{2}gt^2 \)

Which gives a quadratic equation for \( t \). To AVOID USE OF THAT, WE USE:

\[ v_{ys} = v_{yo} - gt_s \quad \therefore \quad t_s = \frac{v_{ys} - v_{yo}}{-g} \quad ; \quad t_s = 4.9 \text{ sec} \]

1) Plug \( t_s \) in \( x \):

\[ x_s = v_{xo}t_s \quad ; \quad x_s = 49 \text{ m} \]

PROBLEM 5

ANALYSE THE PREVIOUS PROBLEM USING ENERGETIC CONSIDERATIONS IF THE MASS OF THE BALL IS KNOWN.

SOLUTION

At the level of the top of the building WE CHOOSE OUR REFERENCE LEVEL TO MEASURE POTENTIAL ENERGIES.

INITIAL CONDITIONS: \( U_o = 0 \); \( K_o = \frac{1}{2}mv_o^2 \); \( E_o = \frac{1}{2}mv_o^2 \)

CONDITIONS AT MAXIMUM HEIGHT: \( U_3 = mgY_{max} \); \( K_3 = \frac{1}{2}mv_3^2 \); \( E_3 = mgY_{max} + \frac{1}{2}mv_o^2 \)

We know that \( v_3^2 > v_o^2 \)

Hence \( \Delta K_{o3} = K_3 - K_o = \frac{1}{2}m(v_o^2 - v_3^2) \) is negative, Which MEANS there is a DECREASE IN KINETIC ENERGY:

\[ \Delta K_{o3} = -\frac{1}{2}m(v_o^2) \]
UNIT II

TRANSPARENCY 9

\[ \Delta U_{o3} = U_3 - U_0 = mg y_{max} \]

There is an increase

\[ = \frac{1}{2} m v_y^2 \quad \text{since} \quad y_{max} = \frac{v_y^2}{2g} \]

And finally we get

\[ \Delta U_{o3} = -\Delta K_{o3} \quad \text{or} \quad \Delta U_{o3} + \Delta K_{o3} = 0 \]

There is conservation of mechanical energy \( E_3 = E_0 \)

Now, when the ball comes back to the original level, \( U_4 = 0 \); \( K_4 = \frac{1}{2} m v_4^2 \)

\[ \Delta U_{34} = U_4 - U_3 = -mg y_{max} = -mg \left( \frac{v_y^2}{2g} \right) \]

\[ \Delta K_{34} = K_4 - K_3 = \frac{1}{2} m (v_{f4}^2 - v_{f3}^2) \]

Applying conservation of energy:

\[ \Delta U_{34} + \Delta K_{34} = 0 \]

\[ \therefore \quad v_4 = \sqrt{v_{f0}^2 + v_{y0}^2} = v_0 \]

The speed at \( t_4 \) (magnitude of velocity \( v_4 \)) is the same as on the initial point.

When the ball lands:

\[ K_5 = \frac{1}{2} m v_5^2 \quad ; \quad U_5 = mg (-y_5) \quad ; \quad E_5 = \frac{1}{2} m v_5^2 - mg y_5 \]

\( U_5 \) is negative since the ball is below the zero reference of potential energies.

From the conservation of mechanical energy:

\[ E_5 = E_0 \]

\[ \frac{1}{2} m v_5^2 - mg (-y_5) = \frac{1}{2} m v_0^2 \]

or

\[ v_5 = \sqrt{v_0^2 + 2gy_5} \]
Unit II

Circular Motion

Particle moving along a curved path

Position: \( \vec{r} = r \hat{u}_r \)
\( \hat{u}_r = \text{unit vector along the radius vector} \)

Velocity: \( \vec{v} = \frac{d\vec{r}}{dt} = \nu \hat{u}_\theta \)
\( \hat{u}_\theta = \text{unit vector tangent to the trajectory,} \)
\( \nu = \text{speed} \)

Acceleration: \( \vec{a} = \frac{d\vec{v}}{dt} = \hat{u}_\theta \frac{dv}{dt} + \nu^2 \frac{d\hat{u}_\theta}{dt} \)

It can be shown that rate of \( \hat{u}_\theta \) with respect of time is a vector pointing in the opposite direction of \( \hat{u}_r \). See Supplement IV on page 593.

i.e. \( \frac{d\hat{u}_\theta}{dt} = -\frac{\nu}{R} \hat{u}_r \)

Hence \( \vec{a} = \hat{u}_\theta \frac{dv}{dt} + \frac{\nu^2}{R} (-\hat{u}_r) = \vec{a}_\theta + \vec{a}_r \)

or \( \vec{a}_\theta = \frac{dv}{dt} (\hat{u}_\theta), \quad \vec{a}_r = \frac{\nu^2}{R} (-\hat{u}_r) \)

\( \vec{a}_\theta = \text{tangential acceleration: change in magnitude of velocity} \)
\( \vec{a}_r = \text{normal or centripetal acceleration: change in direction of velocity} \)

Relation between linear and angular kinematics variables.

\( S: \text{length of arc of the path of radius } R \)
\( \theta = \frac{S}{R}, \quad \frac{d\theta}{dt} = \frac{1}{R} \frac{dS}{dt} = \omega; \quad \frac{d\omega}{dt} = \frac{1}{R} \frac{dv}{dt} \)
\( \omega = \frac{\nu}{R}, \quad \alpha_r = \omega^2 R, \quad \alpha = \frac{\alpha_r}{R} \)

Try to show using vector notation and vector product that:
\( d\vec{s} = d\theta \times \vec{r} \)
\( \vec{v} = \vec{\omega} \times \vec{r} \)
\( \vec{a}_\theta = \vec{\alpha} \times \vec{r} \)
\( \vec{a}_r = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) \)
UNIT II

PROBLEM 6

A disk D can rotate about its horizontal axis. A light cord is wrapped around the outer circumference of the disk and a body A, attached to the card, falls under the action of gravity. At initial time, the velocity of A is 0.04 m/sec downward and 2 sec later A has fallen 0.2 m.

a) Find the tangential and normal acceleration at any instant of any point on the rim of the disk.

b) Find the equations of motion with angular variables.

SOLUTION

A) Fixing the origin of coordinates at the position when \( t = 0 \), \( \dot{y}_0 = 0.04 \) (m/sec)

The equation of uniform accelerated motion for the body A is

\[
y = \dot{y}_0 t + \frac{1}{2} a_A t^2
\]

Setting \( y = 0.2 \) m when \( t = 2 \) sec, then we have \( a_A = 0.06 \) m/sec²

so that, \( y_A = 0.04t + 0.03t^2 \) m.

The velocity of A:

\[
\dot{y}_A = \frac{dy_A}{dt} = 0.04 + 0.06t \text{ m/sec}
\]

Acceleration of A:

\[
a_A = \frac{d^2 y_A}{dt^2} = 0.06 \text{ m/sec}^2
\]

The last two eons, also gives the velocity and the tangential acceleration of any point B on the rim because \( a_0 = a_A = 0.06 \) m/sec²

The normal acceleration

\[
a_r = \frac{\dot{r}^2}{R}, \quad a_r = 0.016 + 0.048t + 0.036t^2 \text{ m/sec}^2
\]

b) Angular variables

\[
\alpha = \frac{a_A}{R}, \quad \alpha = 0.6 \text{ (rad/sec}^2)\]

Integrating:

\[
\omega_0 = 0.4 \text{ (rad/sec)}, \quad \omega = 0.4 + 0.6t \text{ (rad/sec)}
\]

\[
\theta = 0.4t + 0.3t^2 \text{ (rad)}
\]
NEWTON'S LAWS

Problem 1: Application of Second and Third Laws to a Static System.

Problem 2: Application of Second Law to a Moving System of Bodies Interconnected.

Problem 3: Inclined Surface.

Problem 4: Gravitational Force and Superposition Principle.

Transparencies: 7.
AUTOLECTURE No. 3

In Units I thru V you studied motion in one and two-dimensions. You learned the general expressions using vector notation for velocity, acceleration and so forth. Finally you went through circular motion, but we did not analyze what the cause is for the motion of a physical body. This autolecture deals mainly with the concept of force and Newton's Laws.

For centuries man was puzzled with the question related to the cause of motion. We know that Galileo realized that a physical body, free of external influences, would continue to be at rest, or continue to move with constant velocity. However, it was Sir Isaac Newton who gave a comprehensive interpretation of the reason for motion. We can consider him the architect of classical mechanics. His three laws of motion were first presented in his *Principia Mathematica Philosophiae Naturalis* (in 1686).

Let us now try to understand Newton's first law. Place transparency #1 on your overhead projector. (gap)

Suppose we consider a block on a rigid horizontal surface as indicated on your transparency. If we let the block slide along this plane we see that it gradually slows down and stops. Now suppose you repeat the experiment using a smoother block, a smoother surface, and providing a lubricant. You will notice that the velocity decreases more slowly than before, and that the block travels farther before it finally stops.
Now think of your experiment on the air track using a glider. It is not possible for the glider to slide on the track if we are not providing it with an air cushion. Evidently we can extrapolate our results and say that if all friction could be eliminated the physical body would continue moving indefinitely in a straight line with constant speed. Newton stated his first law with these words "Every physical object persists in its state of rest, or of uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it."

Newton's first law is often called the "law of inertia".

But now we have a problem. We have used the word "force" and we haven't explained the causes of force. We normally think in terms of force when we give a pull or push to a physical object. When a force acts on a body, the body will change its state of motion. For example, a freely falling body accelerates. The earth moves around the sun in an elliptical orbit and so on. We also observe that different forces acting on the same object may cause accelerations. Moreover, the same force acting on different bodies produce different accelerations. That is, we can think of as a relation between applied forces and acceleration. But this relation must include also the reluctance of the body of change its state of motion. Newton stated his second law and we use his own words in this way "The change of motion is proportional to the motive force impressed; and is made in the direction of the right line of which that force is impressed."

See transparency #1 for the mathematical expression for Newton's second law. (gap)

Now in terms of the second law the expression change of the
motion means rate of change of the momentum, as you can see on transparency #1. If the quantity of matter was assumed to be constant then Newton's second law could be expressed as $\vec{F} = m\vec{a}$.

As you can see from the mathematical expression $F = ma$ (1) we have included what we call quantity of matter which in fact is the mass of an object. What can we say then about mass? Well, we could say that mass is the property of a physical body that characterizes its response in terms of acceleration to a given force. That is, we define the ratio of the force to the resulting acceleration as the inertial mass of the physical body. We explicitly use the term "inertial mass" to emphasize that this property of the physical body characterized its reluctance to change its motion (inertia) under the action of a given force.

Now there is a question that we ask you to think for the real answer. That is, how is inertial mass related to the weight of an object? Well, read carefully page 73 of your textbook and think.

In mechanics we will be concerned mainly with forces of the following types: First, the gravitational interaction between macroscopic objects; second, the restoring forces exerted by springs when they are moved from their equilibrium position; third, frictional forces; and fourth, forces of constraint which arise when the moving body is not completely free but attached in some way to other bodies and/or to a physical system.

In some cases we will mention or use some electromagnetic forces when it is relevant. But we will normally work with the four types that we have mentioned above.
In the future you will see that there are also other types of forces. The so-called nuclear forces, that are of very short range, and weak forces.

Let us consider now Newton's third law. Many times we will refer to it as the law of action and reaction.

The effects of action and reaction are part of our every day experience. We find by experiment that when one body exerts a force on a second body the second body always exerts a force on the first.

Furthermore, we find that these forces are equal in magnitude but opposite in direction. Newton’s third law of motion was stated in this way. "To every action there is always opposed an equal reaction, or the mutual actions of two bodies upon each other are always equal and directed to contrary parts."

Perhaps an example will illustrate it. This example is taken from the book FUNDAMENTALS OF PHYSICS by David Halliday and Robert Resnick on page 66.

Visualize a person kicking open a door. The force exerted by the person on the door accelerates the door (it flies open); at the same time, the door exerts an equal but opposite force on the person, which decelerates the person (his foot loses forward velocity). The person will be painfully aware of the "reaction" force to his "action."

Now, let us consider another special example, in which we encourage you to think about it.

A horse is urged to pull a wagon. The horse refuses to try, citing Newton's third law as his defense: "'The pull of the horse on the wagon is equal but opposite to the pull of the wagon on the horse.' And the horse says: If I can never exert a greater force on
the wagon than it exerts on me, how can I ever start the wagon moving?"
Well, how would you reply to the horse? Do you think he really under-
stood the principle of action-reaction? Do you understand it? If
you have difficulties discuss them with your proctor or instructor.
(gap)

Place transparency #2 on the overhead projector. But be sure
that you cover the solution of the problem with a sheet of paper.

Now, read carefully the problem. In the figure, there are

a spring scale and two weights of 5 Newtons attached in the way in-
dicated in the figure. Think that there is no motion of any type, the
pulleys are frictionless and massless, and the mass of the spring is
very very small as compared with the weights attached at the ends.
Try the answer the question, what does the scale read in this situ-
tion? (gap) Read the solution. (gap)

Do you agree with the solution? If not, discuss it with
your proctor or professor. You can stop the machine at any time to
think and read over and over the solution.

Now place transparency #3 on the overhead projector.

In the figure, there are three railroad cars connected together
by ropes. They are on a horizontal frictionless rail track; and they
are being pulled to the right with a force of 60 Newtons. Cover the
solution, and find for yourself the tensions T_1 and T_2 as indicated
in the figure. (gap)

After you have tried to solve it look at the solution and
again, see if you are in agreement with the solution that we presented
to you.

Place transparency #4 on the overhead projector. (gap)
In this case, there is a physical body (it would be a crate) of mass 1000 kilograms moving up hill along a street inclined 30°. We are assuming the street is frictionless. We ask you to determine the force necessary to move the body (a) with uniform motion, (b) with acceleration of 0.5 m/s². And finally, you have to find the force exerted on the body by the street in each case.

Before you look the solution we urge you, first, think in all interactions between the body and its environment; second, draw a diagram indicating all the forces acting on the body; third, resolve the forces on a convenient coordinate system; and fourth, apply Newton's second law. (gap)

Do you agree with our solutions? If you didn't understand the way we did, or if your calculations are different from ours, please consult your proctor or instructor.

Now place transparency #6 on the overhead projector. (gap)

This is an example in which we will make use of the universal law of gravitation. But also, you will be dealing with the Principle of Superposition as mentioned in your textbook on page 83.

There are two masses, according to the problem, of 800 kilograms and 600 kilograms separated by a distance of 0.25 meter. We want to know the net gravitational force due to those masses acting on one kilogram at a point 0.2 meters from the 800 kg mass and 0.15 meters from the 600 kg mass. It is evident that this point is not on the straight line joined the two masses of 800 kg and 600 kg.

Again, we urge you to try to solve the problem before looking directly to the solution. (gap)
We hope that this autolecture helps you to understand Unit VI. Don't forget to try to watch the film-loops as indicated in the Audiovisual Aid Section of Unit VI. GOOD LUCK. Buena Suerte.
Newton's First Law

If the quantity of matter was assumed to be constant then:

\[ F = \frac{d\vec{p}}{dt} \]  

(1)

Newton's Second Law

\[ F = \frac{d}{dt} (m\vec{v}) = m \frac{d\vec{v}}{dt} \]

Newton's Third Law:

\[ F_{12} = -F_{21} \]  

(3)
**Problem 1**

Two weights of 5 Nt are attached to an ideal massless spring scale as shown in the Fig. Assume that the pulley has no mass and is frictionless and that it merely serves to change the direction of the tension in the massless string. The whole system has no motion. What does the scale read?

**Solution:**

Since no body is accelerating, all the forces on any body will add vertically to zero.

To the body A:

\[ \overrightarrow{T_1} = \text{tension in the string, pulling vertically up on the mass A} \]

\[ \overrightarrow{W_A} = \text{the pull of the earth acting vertically down on the body A, called its weight} \]

\[ \overrightarrow{F_i} = \text{sum of all the forces acting on body A} \]

Newton's 2nd Law: \[ \overrightarrow{F_i} = \overrightarrow{T_1} + \overrightarrow{W_A} = m_A a_A \]

The body A is at rest so \( a_A = 0 \), so \( \overrightarrow{T_1} = -\overrightarrow{W_A} \)

Or \( T_1 = W_A \) since the forces act along the same line.

To the body B: In identical way, \( \overrightarrow{T_2} = \overrightarrow{W_B} \)

To the hook:

\( \overrightarrow{T_1'} = \text{pull of the body A on the hook of the spring and is the reaction force of the action force} \)

These are not an action-reaction pair because they act on the same body.
2nd Law: \( \vec{F}_k = \) Elastic force of the spring on the hook. This is the force that the scale reads to the spring balance. 

\[ \sum T_i + \vec{F}_k = 0 \] 

\( \therefore \vec{F}_k = T_1 = w_A \)

3rd Law: \( \vec{F}_k = -T_2 \)  

2nd Law: \( \vec{F}_k + T_2' = 0 \)  

\( \therefore T_2' = T_2 \)  

SO THAT: \( T_2 = w_A = w_B \)

Therefore, the pull of the holding point on the spring has to be equal to the weight measured if it has to be at rest. Classify all the forces in this problem according to action and reaction pairs.
Problem 2  Suppose three railroad cars are being pulled in tandem by an
engine as shown in the fig. The cars move on a horizontal frictionless
track and are pulled to the right with a force of $F_3 = 60 \text{ N}$.
If $m_1 = 10 \text{ kg}$, $m_2 = 20 \text{ kg}$, $m_3 = 30 \text{ kg}$ find the tensions $T_1$, $T_2$ and the
net force on each car.

Solution:

\[ F_3 = \text{resultant external force acting on the system of the three cars} \]
\[ M = \text{total mass of the system} \]
\[ a_5 = \text{acceleration of the system} \]

2nd law: \[ F_3 = M \ddot{a} \]
\[ F_3 = ma_5 \]
\[ a_5 = F_3 = 60 \]
\[ M = (10 + 20 + 30) \]

Each part of the system moves with that acceleration. Hence we
calculate what force is necessary to move each part of the system.

\[ T_2 = (m_1 + m_2) a_5 \]
\[ T_2 = 30 \text{ Newton} \]
\[ T_1 = m_1 a_5 \]
\[ T_1 = 10 \text{ Newton} \]

Then the net force on each car:

Car 1: \[ F_1 = T_1 = 10 \text{ Newton} \]
Car 2: It is acted for $T_2$ to the right, but 3rd law says that it
also feels a force to the left and equal to $T_1$ acted by car 1.

Hence:

\[ F_2 = T_2 - T_1 = 20 \text{ Newton} \]

\[ \text{check:} \ a_2 = \frac{F_2}{m_2} = 1 \text{ m/sec}^2 \]
\[ a_2 = a_5 \]

And Car 3:

\[ F_3 = T_3 - T_2 = 30 \text{ Newton} \]

\[ \text{check:} \ a_3 = \frac{F_3}{m_3} = 1 \text{ m/sec}^2 = a_5 \]
A physical body of mass \( m = 1000 \text{ kg} \) moves uphill along a street inclined 30\(^\circ\). Assuming that the street is frictionless, determine the force necessary to move the body (a) with uniform motion (b) with an acceleration of 0.5 m/sec\(^2\). Find the force exerted on the body by the street in each case.

**SOLUTION**

You will solve this problem applying Newton's Law.

The mass of the physical body is \( m \); and the forces acting on the body are indicated in the figure. They are, the weight of the physical body \( w = mg \) acting downward; the force \( F \) that we are applying to the physical body pointing uphill and the force \( N \) exerted by the street on the body. Our system of coordinates is indicated in the figure.

Since the object is moving along the x-axis we write Newton's 2nd Law.

\[ F = ma \]  

But:

\[ F \text{ resultant} = F - mg \sin \alpha \]

So:

\[ F - mg \sin \alpha = ma \]  

is the equation of motion.

(a) If the physical body moves with uniform motion \( a = 0 \)

Hence

\[ F = mg \sin \alpha \]

\[ F = 1000 \times 9.8 \times \sin 30^\circ \]

\[ F = 4900 \text{ Newtons} \]
(b) If the physical body moves with an acceleration $a = 0.5 \text{ m/sec}^2$ and there is no motion along the y-axis we use equation (1) so:

\[
F = ma + mg \sin \alpha
\]

\[
F = m (a + g \sin \alpha)
\]

\[
F = 1000 (0.5 + 9.8 \sin 30°)
\]

\[
F = 5400 \text{ Newtons}
\]

To find the force exerted on the body by the street in each case, we can think in terms of Newton’s 3rd Law or practically a combination of all three Newton’s Laws, so

\[
N = mg \cos \alpha
\]

Then

\[
N = 1000 \times 9.8 \cos 30°
\]

\[
N = 4900 \sqrt{3} \text{ Newtons}
\]
EXAMPLE

An 800 kg mass and a 600 kg mass are separated by 0.25 meter. What is the net gravitational force due to these masses acting on a 1 kg mass at a point 0.2 meters from the 800 kg mass and 0.15 meters from the 600 kg mass?

SOLUTION

According to the problem, the masses are situated as indicated in the figure:

\[ \text{Since the three bodies are present, the force acting on each body is the vector sum of the forces exerted by the two other bodies (Superposition Principle)} \]

\[ \overrightarrow{F}_3 = \overrightarrow{F}_{31} + \overrightarrow{F}_{32} \]

That is, the net force on mass \( m_3 \) is equal to the vector addition of the forces exerted by \( m_1 \) and \( m_2 \) on \( m_3 \).

Force exerted by \( m_1 \) on \( m_3 \) (magnitude):

\[ F_{31} = \frac{G m_1 m_3}{r_{31}^2} = \frac{6.67 \times 10^{-11} \times 800 \times 1}{(0.2)^2} = 1.33 \times 10^{-6} \text{ Newtons} \]

Force exerted by \( m_2 \) on \( m_3 \) (magnitude):

\[ F_{32} = \frac{G m_2 m_3}{r_{32}^2} = \frac{6.67 \times 10^{-11} \times 600 \times 1}{(0.15)^2} = 1.78 \times 10^{-6} \text{ Newtons} \]
It is very easy to prove that $F_{31}$ and $F_{32}$ are at right angle, because

$$
\begin{align*}
\mathbf{r}_{12}^2 &= \mathbf{r}_{31}^2 + \mathbf{r}_{32}^2 \\
(0.25)^2 &= (0.2)^2 + (0.15)^2 \\
0.625 &= 0.625
\end{align*}
$$

So, the total force on $M_3$ is:

$$
F_3 = \sqrt{F_{31}^2 + F_{32}^2}
$$

$$
F_3 = 2.22 \times 10^{-6} \text{ N} \quad \text{(magnitude)}
$$

How can you obtain the direction?
APPENDIX F

FILM-LOOPS USED IN PHYSICS 53 PSI

BFA-EALING-SCIENCE FILM LOOPS

MECHANICS ON AN AIR TABLE

Distance, Time and Speed
One-Dimensional Acceleration
Trajectories
Circular Motion
Simple Harmonic Motion
Dynamics of Circular Motion
Dynamics of Pendulums
The Center of Mass
Collisions in Two Dimensions

MECHANICS ON AN AIR TRACK

Constant Velocity and Uniform Acceleration
Newton's First and Second Laws
Newton's Third Law
Conservation of Momentum: Inelastic Collisions
Conservation of Energy
Conservation of Momentum: Elastic Collision
Simple Harmonic Motion: The Stringless Pendulum
Center-of-Mass Pendulum
How an Air Track Works

VECTOR KINETICS

The Velocity Vector (B/W)
Velocity in Circular and Simple Harmonic Motion (B/W)
The Acceleration Vector (B/W)
Velocity and Acceleration in Simple Harmonic Motion (B/W)
Velocity and Acceleration in Circular Motion (B/W)
Velocity and Acceleration in Free Fall (B/W)

INDIVIDUAL FILM LOOPS

Conservation of Linear and Angular Momentum
Coupled Oscillators: Equal Masses
Coupled Oscillators: Unequal Masses
Coupled Oscillators: Part I. Energy Transfer
Coupled Oscillators: Part II. Other Oscillators
Coupled Oscillators: Part III. Normal Modes
Measurement of "G": The Cavendish Experiment
One Dimensional Motion
The Wilberforce Pendulum
PROYECT PHYSICS

UNIT 1: CONCEPTS OF MOTION

Acceleration Due to Gravity I
A Matter of Relative Motion
Acceleration Due to Gravity II
Vector Addition: Velocity of a Boat

UNIT 2: MOTION IN THE HEAVENS

Central Forces: Iterated Blows
Kepler's Laws

UNIT 3: THE TRIUMPH OF MECHANICS

One-Dimensional Collisions: I
One-Dimensional Collisions: II
Inelastic One-Dimensional Collisions
Two-Dimensional Collisions: I
Two-Dimensional Collisions: II
Inelastic Two-Dimensional Collisions
Scattering of a Cluster of Objects
Explosion of a Cluster of Objects
Finding the Speed of a Rifle Bullet: I
Recoil
Colliding Freight Cars
Dynamics of a Billiard Ball
A Method of Measuring Energy: Nails Driven into Wood
Gravitational Potential Energy
Kinetic Energy
Conservation of Energy: Pole Vault
Conservation of Energy: Aircraft Take-off
Reversibility of Time

UNIT 6: THE NUCLEUS

Collisions with an Unknow Object

DEMONSTRATIONS IN PHYSICS

Inertia I
Inertia II