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De motu cometarum in orbitis parabolicis, solem in foco habentibus

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XVIII.

De motu Cometarum in orbitis parabolicis, solem in foco habentibus.

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1. **Problema I.** Cometae in data orbita parabolica moti invenire locum heliocentricum ad datum tempus.

Solutio. (Fig. 214) Sit *MEAF* orbita cometae parabolica, in cujus foco F versetur sol, quae cum ponatur data, dabitur locus perihelii A in coelo ex sole visus, atque cometa ex sole cernetur in circulo maximo moveri, cujus planum pariter erit datum. Datum porro etiam sit tempus, quo cometa in perihelio versabitur, atque vel ante vel post hoc tempus quaeratur locus cometae M ex sole visus. Elapsum sit scilicet jam tempus T, postquam cometa in perihelio A extiterit, sitque hoc tempore M locus cometae, ita ut cometa in coelo appareat a loco perihelii A distare angulo ASM, qui angulus erit cometae anomalia vera. Sit iste angulus, qui quaeritur, $ASM = v$; atque ponatur distantia perihelii a sole $SA = a$, erit parameter parabolae $= 4a$. Ex loco cometae M in axem parabolae AS demittatur perpendiculum MP, et vocetur $AP = x$, $PM = y$, erit $yy = 4ax$, atque ob $PS = x - a$, erit radius $SM = x + a$. Hinc anguli $ASM = v$ sinus erit $= \frac{y}{a+x}$, et cosinus $= \frac{a-x}{a+x}$ posito sinu toto $= 1$. Cum igitur sit $\cos v = \frac{a-x}{a+x}$, erit

$$x = \frac{a(1 - \cos v)}{1 + \cos v}, \text{ et distantia cometae a sole } MS = a + x = \frac{2a}{1 + \cos v}.$$

Inventa ergo anomalia vera v innotescit distantia cometae a sole $MS = \frac{2a}{1 + \cos v}$. Quoniam vero tempus T, quo cometa a perihelio A ad locum M pertingit, est directe ut area ASM, et inverse ut radix quadrata ex latere recto seu parametro $4a$, aream ASM indagare oportet, quae est $=$ areae $APM - \triangle SPM$. At area APM ex natura parabolae est

$$= \tfrac{2}{3}xy, \text{ et } \triangle SPM = \tfrac{1}{2}y(x - a) = \tfrac{1}{2}xy - \tfrac{1}{2}ay;$$

quibus area ASM erit $= \frac{1}{6}\,xy + \frac{1}{2}\,ay$. Est vero $x = \frac{a\,(1-\cos\varphi)}{1+\cos\varphi}$ et $y = (a+x)\sin\varphi = \frac{2a\sin\varphi}{1+\cos\varphi}$. Hinc erit $\frac{1}{6}\,x + \frac{1}{2}\,a = \frac{2a + a\cos\varphi}{3\,(1+\cos\varphi)}$, ideoque area $ASM = \frac{2aa\,(2+\cos\varphi)\sin\varphi}{3\,(1+\cos\varphi)^2}$.

Ad hanc expressionem simpliciorem reddendam ponatur semissis anguli ASM tangens, seu $\operatorname{tang}\frac{1}{2}\varphi = t$, erit $\sin\frac{1}{2}\varphi = \frac{t}{\sqrt{(1+tt)}}$, $\cos\frac{1}{2}\varphi = \frac{1}{\sqrt{(1+tt)}}$, indeque $\sin\varphi = \frac{2t}{1+tt}$, $\cos\varphi = \frac{1-tt}{1+tt}$, atque porro $2+\cos\varphi = \frac{3+tt}{1+tt}$ et $1+\cos\varphi = \frac{2}{1+tt}$. Fiet itaque

$$\text{area } ASM = \tfrac{1}{3}\,aat\,(3+tt) = aa\left(t + \tfrac{1}{3}\,t^3\right).$$

Ponatur jam semiaxis major orbitae terrae, seu distantia media terrae a sole $= c$, atque planeta, qui circa solem circulum, cujus radius $= c$, describeret, periodum absoluturus esset uno anno sidereo, hoc est $365^d\,6^h\,8'\,31''$, quod tempus ponamus $= \theta$. Cum igitur hujus circuli area sit πcc, denotante $1:\pi$ rationem diametri ad peripheriam, et parameter diametro $2c$ sit aequalis, erit tempus unius revolutionis θ ut area πcc divisa per $\sqrt{2c}$, hoc est ut $\frac{\pi}{\sqrt{2}}\,c\sqrt{c}$. Simili vero modo est tempus T, quo cometa ex A in M pertingit, ut area $ASM = aa\,(t+\frac{1}{3}\,t^3)$ divisa per $\sqrt{4a}$, hoc est ut $(t+\frac{1}{3}\,t^3)\,\frac{a\sqrt{a}}{2}$; unde haec nascitur analogia $\theta : T = \frac{\pi c\sqrt{c}}{\sqrt{2}} : \frac{a\sqrt{a}}{2}(t+\frac{1}{3}\,t^3)$, ergo

$$t + \tfrac{1}{3}\,t^3 = \frac{\pi T c\sqrt{2c}}{\theta a\sqrt{a}} = 4{,}4428829381 \cdot \frac{T}{\theta} \cdot \frac{c\sqrt{c}}{a\sqrt{a}}.$$

Quia θ est annus sidereus, et T tempus datum, erit θ ad T ut 360^0 ad motum terrae medium tempori T convenientem. Si ergo ponatur motus terrae medius tempori T respondens $= m$, fiet $\frac{T}{\theta} = \frac{m}{360}$. Ex aequatione ergo cubica

$$t^3 + 3t = 13{,}3286488144 \cdot \frac{m}{360} \cdot \frac{c\sqrt{c}}{a\sqrt{a}}$$

inveniatur valor ipsius t, qui erit tangens semissis anguli ASM, hincque ad datum tempus vel ante vel post transitum cometae per perihelium A assignabitur locus cometae heliocentricus. Q. E. I.

2. **Coroll. 1.** Si tempus T sit spatium unius diei seu 24 horarum, erit $m = 59'\,8''$, unde calculo subducto fiet $t^3 + 3t = 0{,}036491289910 \cdot \frac{c\sqrt{c}}{a\sqrt{a}}$. Quare si T sit spatium n dierum, erit

$$t^3 + 3t = 0{,}036491289910 \cdot \frac{nc\sqrt{c}}{a\sqrt{a}} \quad \text{seu}$$

$$t + \tfrac{1}{3}t^3 = 0{,}012163763303 \cdot \frac{nc\sqrt{c}}{a\sqrt{a}}.$$

3. **Coroll. 2.** Pro quovis ergo dierum numero n reperitur numerus ipsi $t+\frac{1}{3}\,t^3$ aequalis, unde cum difficulter valor ipsius t, ex eoque valor anguli φ reperiatur, conveniet tabulam construi, quae pro singulis valoribus anguli φ exhibeat valores respondentes ipsius $t+\frac{1}{3}\,t^3$: hujus enim tabulae ope vicissim ex valore ipsius $t+\frac{1}{3}\,t^3$ dato angulus φ colligetur.

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4. **Coroll. 3.** Si ergo detur tempus, quo cometa in perihelio versatur, atque distantia perihelii a sole a, ad quodvis tempus, distantia cometae a perihelio ex sole visa determinari poterit ope tabellae. Scilicet propositum sit tempus n dierum vel ante vel post appulsum cometae ad perihelium, computetur valor $0{,}012163763303 \cdot \frac{nc\sqrt{c}}{a\sqrt{a}}$; hic valor quaeratur in tabula sub columna $t + \frac{1}{3}t^3$ ac respondens valor ipsius φ dabit angulum quaesitum.

5. **Exemplum.** Cometae, qui A. 1680 apparuit, Newtonus statuit latus rectum orbitae $4a = 236{,}8$, seu $a = 59{,}2$ existente $c = 10000$, atque istum cometam collegit in perihelio versatum esse A. 1680 decembr. die 8, $0^h\, 4'$ p. m. Hinc erit $0{,}012163763303 \cdot \frac{c\sqrt{c}}{a\sqrt{a}} = 26{,}70458$, atque n diebus vel ante vel post appulsum cometae ad perihelium erit $t + \frac{1}{3}t^3 = 26{,}70458\ n$, cui valori respondens angulus φ ostendet cometae distantiam a perihelio in orbita sua ex sole visam. Uno ergo die tam ante quam post perihelium hic cometa confecit angulum ASM plus quam 152^0; die autem vel praecedente vel antecedente tantum 6 circiter gradus absolvit. Postquam autem cometa perihelium transiisset, per spatium 90 dierum adhuc adparuit, toto ergo hoc tempore absolvit angulum ASM circiter 174^0.

6. **Coroll. 4.** Cognito angulo $ASM = \varphi$; innotescet cometae a sole distantia SM, quae est $= \frac{2a}{1 + \cos\varphi}$. Posito autem $t = \operatorname{tang} \frac{1}{2}\varphi$, erit $\cos\varphi = \frac{1 - tt}{1 + tt}$ et $1 + \cos\varphi = \frac{2}{1 + tt}$. Hinc erit distantia $SM = a(1 + tt) = a \sec^2 \frac{1}{2}\varphi$. Distantia ergo cometae a sole SM confecto angulo $ASM = 174^0$ ubi disparere coepit, ob $a = 59{,}2$ et $t = \operatorname{tang} 87^0$ erat 21613; non multum ergo diametrum orbis magni $2c$ excedebat.

7. **Coroll. 5.** Si ducatur tangens curvae in M, in eamque ex S perpendiculum demittatur, erit hoc perpendiculum $= a\sqrt{(1 + tt)}$, et celeritas cometae in puncto M erit ut $\frac{1}{a\sqrt{(1 + tt)}}$, seu ob a constans, ut $\cos \frac{1}{2}\varphi$.

8. **Scholion.** Si numerus, qui pro $t + \frac{1}{3}t^3$ resultat, non exacte reperiatur in tabula, tum per interpolationem consueto more investigabitur angulus φ in minutis primis et secundis, nisi forte numerus ille sit nimis magnus, atque numeri $t + \frac{1}{3}t^3$ angulis φ respondentes nimium a progressione arithmetica discrepent. Hoc igitur casu peculiari artificio opus erit, ex natura progressionis petito, ad angulum φ exactius determinandum. Quaeratur exempli gratia angulus ASM, quem cometa A. 1680 tempore 10 dierum confecerat: erit ergo $t + \frac{1}{3}t^3 = 267{,}0458$, unde apparet angulum contineri intra 167^0 et 168^0. Primum ergo more solito interpolatio instituatur:

167^0:	234,1492	267,0458
168^0:	296,6044	234,1492
$60'$:	62,4552	32,8966
	1,0409	1,0409
		32,8968
		1,3158
		296

angulus $v = 167^\circ\, 34'$ et $t = \text{tang}\, 83^\circ\, 47'$. Ponatur jam $t = \text{tang}\,(83^\circ\, 47' + m'')$ eritque

$$\begin{array}{rl} t = & 9{,}1802838 + 4145\, m \\ & 257{,}8974000 + 350000\, m \\ \hline & 267{,}0776838 + 354145\, m \end{array} \qquad \begin{array}{rl} lt & = 0{,}9628561 + 196\, m \\ lt^3 & = 2{,}8885683 + 588\, m \\ l3 & = 0{,}4771213 \\ \hline l\tfrac{1}{3}t^3 & = 2{,}4114470 + 588\, m \\ \text{num.} & = 257{,}8974 + 350\, m \end{array}$$

erit $267{,}0776 + 0{,}0354\, m = 267{,}0458$, hincque $354\, m = -318$, ergo m nequidem unum minutum secundum valet, ita ut vere sit $v = 167^\circ\, 34'$.

9. **Problema 2.** Ex datis tribus locis heliocentricis cometae ejus orbitam determinare.

Solutio. Fig. 215. Sit $LMNA$ orbita cometae, quae quaeritur; ac primo quidem planum, in quo existit, sponte innotescit ex duabus observationibus. Observetur primum cometa in directione SL; secundo in directione SM, et tertio in directione SN. Dantur ergo anguli LSM et LSN, itemque differentiae temporum inter has observationes. Sit tempus inter observationem primam et secundam $= m$ dierum, inter primam ac tertiam $= n$ dierum. Porro sit tangens semissis anguli $LSM = f$, tangens semissis anguli $LSN = g$. Ponatur tempus, quo cometa ex L in perihelium A perveniet $= z$ dierum; erit tempus inter cometae loca M et $A = z - m$, et inter loca N et $A = z - n$. Sit tangens semissis anguli $ASL = t$, erit

$$\text{tang}\,\tfrac{1}{2}\, ASM = \frac{t-f}{1+ft} \quad \text{et} \quad \text{tang}\,\tfrac{1}{2}\, ASN = \frac{t-g}{1+gt}.$$

Posito jam brevitatis gratia $N = 0{,}012163763303 \cdot \dfrac{c\sqrt{c}}{a\sqrt{a}}$, denotante a distantiam SA, et c distantiam mediam terrae a sole. His positis erit

$$t + \tfrac{1}{3}\, t^3 = Nz$$

$$\frac{t-f}{1+ft} + \frac{1}{3}\,\frac{(t-f)^3}{(1+ft)^3} = N(z-m)$$

$$\frac{t-g}{1+gt} + \frac{1}{3}\,\frac{(t-g)^3}{(1+gt)^3} = N(z-n)$$

Ex quibus tribus aequationibus tres incognitas N, z, et t determinari oportet. Per subtractionem secundae et tertiae a prima obtinentur hae duae aequationes

$$\frac{f(1+tt)}{1+ft} + \frac{ftt(1+tt) - fft(1-t^4) + \frac{1}{3}f^3(1+t^6)}{(1+ft)^3} = Nm$$

$$\frac{g(1+tt)}{1+gt} + \frac{gtt(1+tt) - ggt(1-t^4) + \frac{1}{3}g^3(1+t^6)}{(1+gt)^3} = Nn$$

$$\frac{f(1+tt)^2 + fft(1+tt)^2 + \frac{1}{3}f^3(1+tt)^3}{(1+ft)^3} = Nm$$

$$\frac{g(1+tt)^2 + ggt(1+tt)^2 + \frac{1}{3}g^3(1+tt)^3}{(1+gt)^3} = Nn$$

quarum una per alteram divisa dabit aequationem incognita N carentem, solamque t involventem

$$\frac{fn\,(1+\frac{1}{3}ff+ft+\frac{1}{3}fftt)}{(1+ft)^3}=\frac{gm\,(1+\frac{1}{3}gg+gt+\frac{1}{3}ggtt)}{(1+gt)^3}$$

$$\text{seu}\quad \frac{fn}{(1+ft)^2}+\frac{f^3n\,(1+tt)}{3\,(1+ft)^3}=\frac{gm}{(1+gt)^2}+\frac{g^3m\,(1+tt)}{3\,(1+gt)^3}$$

in qua aequatione incognita t ad quinque dimensiones ascendit. Quamobrem ut solutio facilior evadat, eligantur tres observationes a se invicem minimum distantes, ita ut f et g quasi infinite parva evadant. Tum vero prima observatio L non tantum a perihelio distet, ut t possit posteriorem terminum in utroque membro notabilis quantitatis efficere. Evanescent ergo in utroque membro termini posteriores, eritque

$$\frac{fn}{(1+ft)^2}=\frac{gm}{(1+gt)^2},\quad \text{unde fit}$$

$$(1+gt)\sqrt{fn}=(1+ft)\sqrt{gm},\quad \text{hincque}\quad t=\frac{\sqrt{gm}-\sqrt{fn}}{g\sqrt{fn}-f\sqrt{gm}}.$$

Hoc modo quidem tantum vero proxime valor ipsius t invenitur, quia minimus error in observationibus commissus ingentem aberrationem parit. Verum si hoc modo valor ipsius t prope verus fuerit inventus, tum aliae duae quaecunque observationes cum prima L conjungantur, atque tum aequatio, etsi quinque est dimensionum, tamen ob valorem ipsius t prope verum cognitum, verus valor non difficulter eruetur. Sit θ valor prope verus ipsius t, ponaturque $t=\theta+\psi$, ita ut ψ prae θ sit valde parvum, eritque

$$\frac{fn}{(1+f\theta)^2}+\frac{f^3n\,(1+\theta\theta)}{3\,(1+f\theta)^3}-\frac{2ffn\psi}{(1+f\theta)^3}-\frac{f^3n\psi\,(3f-2\theta+f\theta\theta)}{3\,(1+f\theta)^4}=$$

$$\frac{gm}{(1+g\theta)^2}+\frac{g^3m\,(1+\theta\theta)}{3\,(1+g\theta)^3}-\frac{2ggm\psi}{(1+g\theta)^3}-\frac{g^3m\psi\,(3g-2\theta+g\theta\theta)}{3\,(1+g\theta)^4},$$

ex qua aequatione ψ inventum dabit verum valorem tangentis $t=\theta+\psi$, et anguli ipsi respondentis duplum monstrabit angulum LSA, ideoque praebebit positionem axis AS parabolae quaesitae. Invento autem t erit

$$N=\frac{(1+tt)^2\,(f+fft+\frac{1}{3}f^3\,(1+tt))}{m\,(1+ft)^3}=0{,}0121637\cdot\frac{c\sqrt{c}}{a\sqrt{a}}$$

unde elicitur distantia $AS=a$, ac parabolae latus rectum $4a$. Denique colligetur tempus, quo cometa in perihelium perveniet, quod post observationem primam L eveniet z diebus, existente $z=\frac{3t+t^3}{3N}$. Cognitis ergo positione axis parabolae AS, ejus parametro $4a$, seu valore litterae N una cum tempore, quo cometa perihelium attingit, ad quodvis tempus locus cometae in orbita ejusque distantia a sole per problema praecedens definietur. Q. E. I.

10. **Coroll. 1.** Ex cognito tempore, quo cometa ad perihelium appellit, una cum valore numeri N ad datum tempus, distantia cometae a perihelio ex sole visa determinabitur; scilicet si haec distantia desideretur n diebus vel ante vel post appulsum ad perihelium, multiplicetur numerus N per n, ac productum quaeratur in tabula sub columna $t+\frac{1}{3}t^3$, cui respondebit angulus φ, distantiam cometae a perihelio e sole visam indicans.

11. **Coroll. 2.** Si valor numeri N inventus dividatur per 0,012163763303, quotus dabit unde reperietur distantia perihelii a sole $SA = a$, seu potius ejus ratio ad distantiam mediam terrae a sole. Hinc autem in quovis loco vera cometae a sole distantia colligetur. (6).

12. **Scholion.** Maxima difficultas posita est in inventione tangentis t ex aequatione

$$\frac{fn}{(1+ft)^2} + \frac{f^3 n(1+tt)}{3(1+ft)^3} = \frac{gm}{(1+gt)^2} + \frac{g^3 m(1+tt)}{3(1+gt)^3};$$

unde primum praecepimus ejusmodi observationes adhibere, in quibus f et g fiant vehementer parvae, ut t proxime tantum innotescat. At quoniam minimus error in observationibus nimium vicinis maxime deflectentem a veritate conclusionem producere potest, conveniet binas posteriores observationes neque nimis vicinas, neque nimis remotas a prima accipi, quo f et g neque sint vehementer parvae, neque ad unitatem appropinquent, quod eveniet, dummodo angulus LSN minor sit 60°. Tales observationes si eligantur, tum aequatio $\frac{fn}{(1+ft)^2} = \frac{gm}{(1+gt)^2}$ valorem ipsius t a vero aberrantem quidem, sed non multum, praebebit. Sit iste valor $t = \theta$, ita ut sit $\frac{fn}{(1+f\theta)^2} = \frac{gm}{(1+g\theta)^2}$, ac statuatur verus valor $t = \theta + \psi$, ubi ψ instar quantitatis vehementer parvae tractare licebit. Pervenietur autem, facta in solutione, ad hanc aequationem

$$\frac{f^3 n(1+\theta\theta)}{3(1+f\theta)^3} - \frac{ffn\psi(6+3ff+4f\theta+ff\theta\theta)}{3(1+f\theta)^4} = \frac{g^3 m(1+\theta\theta)}{3(1+g\theta)^3} - \frac{ggm\psi(6+3gg+4g\theta+gg\theta\theta)}{3(1+g\theta)^4}$$

quae, propter $fn : gm = (1+f\theta)^2 : (1+g\theta)^2$, abit in

$$\frac{ff(1+\theta\theta)}{1+f\theta} - \frac{f\psi(6+3ff+4f\theta+ff\theta\theta)}{(1+f\theta)^2} = \frac{gg(1+\theta\theta)}{1+g\theta} - \frac{g\psi(6+3gg+4g\theta+gg\theta\theta)}{(1+g\theta)^2}$$

ex qua aequatione si erutum fuerit ψ, habebitur satis prope $t = \theta + \psi$, qui tamen pari modo ulterius corrigi potest. Denique consultum erit tres observationes a se invicem maxime remotas adhibere, atque per aequationem quintae potestatis, qua t determinatur, exactissime valorem ipsius t determinare, id quod non difficulter praestabitur, cum valor ipsius t jam proxime sit notus. Sicque multiplicato observationum numero, orbita cometae continuo exactius cognoscetur.

13. **Problema 3.** Cognita cometae orbita, una cum temporis momento, quo in perihelio versatur, ad quodvis tempus cometae longitudinem ac latitudinem heliocentricam definire.

Solutio. Fig. 216. Quoniam cometa in plano per solem transeunte movetur, apparebit in circulo maximo incedere. Sit igitur sol in centro coeli siderei S, atque ☊MA☋ circulus maximus, in quo cometa ingredi cernitur, secundum ordinem litterarum ☊MA☋. Sit porro ☊ma☋ ecliptica secundum signorum ordinem distributa, et O polus eclipticae borealis, erit ☊ nodus ascendens orbitae cometae, et ☋ nodus descendens. Sit longitudo nodi ascendentis ☊ a prima stella arietis computata $= q$, atque inclinatio orbitae cometae ☊MA☋ ad eclipticam, seu angulus M☊$m = s$, qui si recto fuerit minor, motus cometae secundum signorum seriem fieri cernetur; contra, si angulus s recto sit major, motus cometae contra signorum seriem perficietur. Sit deinde A locus perihelii, per quem ex O ducatur circulus maximus OAa, erit a longitudo perihelii, cujus distantia a prima stella arietis sit $=p$, erit arcus $☊a = p - q$. Ex triangulo sphaerico $A☊a$ ad a rectangulo innotescit latitudo borealis perihelii Aa, erit nempe $\tang Aa = \tang s \,.\, \sin(p-q)$ et $\tang ☊A = \frac{\tang(p-q)}{\cos s}$. Sit a distantia cometae in perihelio a sole, et c distantia media terrae a sole, ac ponatur

$$N = 0{,}012163763303 \cdot \frac{c\sqrt{c}}{a\sqrt{a}}.$$

Jam quaeratur k diebus, antequam cometa ad perihelium appellit, locus cometae, qui sit in M, ad quem inveniendum sit angulus MSA seu arcus $MA = \varphi$ et $\tang \frac{1}{2}\varphi = t$, erit $t + \frac{1}{3}t^3 = Nk$, atque ope tabulae computatae ex numero Nk reperietur angulus φ. Sit arcus $☊A = A$, ita ut sit $\tang A = \frac{\tang(p-q)}{\cos s}$; erit arcus $☊M = A - \varphi$. Hinc in triangulo $☊mM$ ad m rectangulo erit $\sin Mm = \sin(A-\varphi)\sin s$ et $\tang ☊m = \tang(A-\varphi)\cos s$. Erit ergo Mm latitudo cometae heliocentrica, et si ad $☊m$ addatur q longitudo nodi, prodibit longitudo cometae a prima stella arietis computata. Q. E. I.

14. **Coroll. 1.** In triangulo sphaerico $M☊m$ latera Mm et $☊m$ et angulus $M☊m = s$ ita a se invicem pendent, ut sit $\tang Mm = \sin ☊m \tang s$. Haecque aequatio ex duabus inventis resultare debet, si arcus $A - \varphi$ eliminetur.

15. **Coroll. 2.** Quodsi autem ex duabus aequationibus inventis $\sin Mm = \sin(A-\varphi)\sin s$ et $\tang ☊m = \tang(A-\varphi)\cos s$ eliminetur angulus s, orietur haec aequatio

$$\cos(A-\varphi) = \cos Mm \,.\, \cos ☊m.$$

16. **Coroll. 3.** Quatuor ergo habentur aequationes, quae autem duabus tantum aequivalent, istae,

$$\sin Mm = \sin(A-\varphi)\sin s, \qquad \tang ☊m = \tang(A-\varphi)\cos s$$
$$\tang Mm = \sin ☊m \tang s, \qquad \cos(A-\varphi) = \cos Mm \cos ☊m$$

ex quibus binae, prouti commodum visum fuerit, ad usum adhiberi poterunt.

17. **Coroll. 4.** Data igitur, ex quibus longitudo ac latitudo heliocentrica cometae ad datum tempus assignatur, sunt primo ex orbita cometae desumta. Nempe distantia nodi ☊ a perihelio ex sole visa $= A$; deinde inclinatio orbitae cometae ad planum eclipticae, seu angulus $M☊m = s$. Porro, differentia inter tempus propositum et tempus, quo cometa perihelium attingit, quae in diebus expressa sit $= k$ dierum, ex qua ope tabulae ante datae sequitur angulus φ.

18. **Coroll. 5.** Quoniam cometae a sole distantia SM est $= a\sec^2\frac{1}{2}\varphi$, erit ejus distantia curtata, seu in ecliptica sumta $= a\sec^2\frac{1}{2}\varphi \,.\, \cos Mm$.

19. **Coroll. 6.** Si longitudo heliocentrica ponatur $= f$, et latitudo heliocentrica $= g$, erit $f = ☊m + q$, unde fiet

$$\text{I.}\ \sin g = \sin(A-\varphi)\sin s, \qquad \text{II.}\ \tang(f-q) = \tang(A-\varphi)\cos s$$
$$\text{III.}\ \tang g = \sin(f-q)\tang s, \qquad \text{IV.}\ \cos(A-\varphi) = \cos g\cos(f-q).$$

20. **Problema 4.** Fig. 217. Ex datis duobus cometae locis heliocentricis invenire inclinationem orbitae cometae ad eclipticam, atque positionem nodorum ☊ et ☋.

Solutio. Observetur primum cometa in L, sitque longitudo heliocentrica $= f$ et latitudo borealis $= g$; praeterea vero observetur cometa in M, sitque longitudo ejus $= f'$ et latitudo $Mm = g'$. Ponatur longitudo nodi ascendentis $☊ = q$, et inclinatio orbitae ad eclipticam, seu angulus $L☊l = s$. Erit igitur differentia longitudinum $lm = f' - f$. Ex aequatione ergo tertia (19) nascentur hae duae aequationes

$$\operatorname{tang} g = \sin (f - q) \operatorname{tang} s \quad \text{et} \quad \operatorname{tang} g' = \sin (f' - q) \operatorname{tang} s$$

quarum haec per illam divisa dabit

$$\frac{\operatorname{tang} g'}{\operatorname{tang} g} = \frac{\sin (f' - q)}{\sin (f - q)} = \frac{\sin (f - q + lm)}{\sin (f - q)}.$$

Cum autem sit $\sin (f - q + lm) = \sin (f - q) \cos lm + \cos (f - q) \sin lm$, fiet

$$\frac{\operatorname{tang} g'}{\operatorname{tang} g} = \cos lm + \frac{\sin lm}{\operatorname{tang} (f - q)}.$$

Hinc reperitur

$$\operatorname{tang} (f - q) = \frac{\operatorname{tang} g \sin lm}{\operatorname{tang} g' - \operatorname{tang} g \cos lm}.$$

Innotescit ergo differentia longitudinum $☊l = f - q$, unde ob longitudinem puncti l datam definietur longitudo nodi ascendentis ☊; qua cognita ob arcum $f - q$ datum erit $\operatorname{tang} s = \frac{\operatorname{tang} g}{\sin (f - q)}$, sicque inclinatio orbitae cometae ad planum eclipticae, seu angulus $L☊l = s$ invenitur. Q. E. I.

21. **Coroll. 1.** Quoniam est $\operatorname{tang} (f - q) = \frac{\operatorname{tang} f - \operatorname{tang} q}{1 + \operatorname{tang} f \operatorname{tang} q}$ et $lm = f' - f$, erit longitudo nodi quaesita

$$\operatorname{tang} q = \frac{\sin f \operatorname{tang} g' - \sin f' \operatorname{tang} g}{\cos f \operatorname{tang} g' - \cos f' \operatorname{tang} g}.$$

22. **Coroll. 2.** Si ponatur $\operatorname{tang} g = \alpha$, $\operatorname{tang} g' = \beta$, $\sin lm = \sin (f' - f) = \mu$ et $\cos (f' - f) = \nu$, reperietur

$$\operatorname{tang} s = \frac{\sqrt{(\alpha^2 + \beta^2 - 2\alpha\beta\nu)}}{\mu},$$

cujus fractionis numerator $\sqrt{(\alpha^2 + \beta^2 - 2\alpha\beta\nu)}$ est basis trianguli, cujus latera sunt α et β, angulum $= f' - f$ constituentia.

23. **Problema 5.** Si dato tempore, puta k dierum, vel ante vel post cometae appulsum ad perihelium, detur distantia cometae a perihelio e sole visa $= v$, invenire eandem distantiam tempore $k + x$ dierum vel ante vel post momentum, quo in perihelio versatur.

Solutio. Ponatur $\operatorname{tang} \frac{1}{2} v = t$, erit $t + \frac{1}{3} t^3 = Nk$, seu $\operatorname{tang} \frac{1}{2} v + \frac{1}{3} \operatorname{tang}^3 \frac{1}{2} v = Nk$. Sit jam angulus tempori $k + x$ dierum respondens $= v + \varphi$, erit, posito brevitatis gratia $\operatorname{tang} \frac{1}{2} v + \frac{1}{3} \operatorname{tang}^3 \frac{1}{2} v = V$, per calculum differentiarum finitarum

$$\operatorname{tang}\tfrac{1}{2}(\nu+\varphi)+\tfrac{1}{3}\operatorname{tang}^3\tfrac{1}{2}(\nu+\varphi)=V+\frac{\varphi\, dV}{d\nu}+\frac{\varphi^2\, ddV}{2d\nu^2}+\frac{\varphi^3\, d^3V}{6d\nu^3}+\text{etc.}=N(k+\varkappa)$$

Cum igitur sit $V=Nk$, erit

$$N\varkappa=\frac{\varphi\, dV}{d\nu}+\frac{\varphi\varphi\, ddV}{2d\nu^2}+\frac{\varphi^3\, d^3V}{6d\nu^3}+\text{etc.}$$

At est
$$\frac{dV}{d\nu}=\frac{1}{2\cos^2\frac{1}{2}\nu}+\frac{\operatorname{tang}^2\frac{1}{2}\nu}{2\cos^2\frac{1}{2}\nu}=\frac{1}{2\cos^4\frac{1}{2}\nu}$$

$$\frac{ddV}{d\nu^2}=\frac{\sin\frac{1}{2}\nu}{\cos^5\frac{1}{2}\nu}$$

$$\frac{d^3V}{d\nu^3}=\frac{1}{2\cos^4\frac{1}{2}\nu}+\frac{5\sin^2\frac{1}{2}\nu}{2\cos^6\frac{1}{2}\nu}=\frac{5}{2\cos^6\frac{1}{2}\nu}-\frac{2}{\cos^4\frac{1}{2}\nu}$$

etc.

Ex his ergo fiet
$$N\varkappa=\frac{\varphi}{2\cos^4\frac{1}{2}\nu}+\frac{\varphi^2\sin\frac{1}{2}\nu}{2\cos^5\frac{1}{2}\nu}+\frac{5\varphi^3}{12\cos^6\frac{1}{2}\nu}-\frac{\varphi^3}{3\cos^4\frac{1}{2}\nu}+\text{etc.}$$

Ponamus jam esse $\varphi=\alpha N\varkappa+\beta N^2\varkappa^2+\gamma N^3\varkappa^3+\text{etc.}$, atque facta substitutione habebimus

$$\begin{aligned}N\varkappa=&\frac{\alpha N\varkappa}{2\cos^4\frac{1}{2}\nu}+\frac{\beta N^2\varkappa^2}{2\cos^4\frac{1}{2}\nu}+\frac{\gamma N^3\varkappa^3}{2\cos^4\frac{1}{2}\nu}+\text{etc.}\\&+\frac{\alpha^2N^2\varkappa^2\sin\frac{1}{2}\nu}{2\cos^5\frac{1}{2}\nu}+\frac{\alpha\beta N^3\varkappa^3\sin\frac{1}{2}\nu}{\cos^5\frac{1}{2}\nu}+\text{etc.}\\&+\frac{5\alpha^3N^3\varkappa^3}{12\cos^6\frac{1}{2}\nu}\\&-\frac{\alpha^3N^3\varkappa^3}{3\cos^4\frac{1}{2}\nu}.\end{aligned}$$

His ad aequalitatem reductis erit

$$\alpha=2\cos^4\tfrac{1}{2}\nu$$

$$\beta=\frac{-\alpha^2\sin\frac{1}{2}\nu}{\cos\frac{1}{2}\nu}=-4\cos^7\tfrac{1}{2}\nu\sin\tfrac{1}{2}\nu$$

$$\gamma=\frac{-2\alpha\beta\sin\frac{1}{2}\nu}{\cos\frac{1}{2}\nu}-\frac{5\alpha^3}{6\cos^2\frac{1}{2}\nu}+\frac{2\alpha^3}{3},\quad\text{seu}\quad\gamma=\tfrac{4}{3}\cos^{10}\tfrac{1}{2}\nu\,(7-8\cos^2\tfrac{1}{2}\nu)$$

etc.

Ex his igitur reperitur

$$\varphi=2N\varkappa\cos^4\tfrac{1}{2}\nu-4N^2\varkappa^2\cos^7\tfrac{1}{2}\nu\sin\tfrac{1}{2}\nu+\tfrac{4}{3}N^3\varkappa^3\cos^{10}\tfrac{1}{2}\nu\,(7-8\cos^2\tfrac{1}{2}\nu)$$

quae expressio, nisi differentia temporis $\varkappa$ sit admodum magna, per approximationem satis prope praebet valorem ipsius φ. Primum enim si cometa a perihelio vehementer distet, angulus $\frac{1}{2}\nu$ non multum ab angulo recto differret, ideoque ejus cosinus fractio erit perquam exigua. Hinc terminus secundus multo minor erit primo, ac tertius secundo; ita ut plerumque primus terminus sufficere possit ad φ exprimendum, quo invento erit angulus quaesitus $=\nu+\varphi$. Q. E. I.

24. **Coroll.** Si igitur cometa tempore k dierum a perihelio movetur per angulum φ, tempore $k+\varkappa$ dierum movebitur proxime per angulum $\varphi + 2N\varkappa \cos^4 \frac{1}{2}\varphi$; vel si iste angulus propius desideretur, erit is $\varphi + 2N\varkappa \cos^4 \frac{1}{2}\varphi - 4N^2\varkappa^2 \cos^7 \frac{1}{2}\varphi \sin \frac{1}{2}\varphi$, atque tertius terminus

$$\tfrac{4}{3} N^3 \varkappa^3 \cos^{10} \tfrac{1}{2}\varphi \,(7 - 8\cos^2 \tfrac{1}{2}\varphi)$$

cum sequentibus semper tuto negligi poterit, dummodo utroque tempore cometa vel ante vel post perihelium versetur.

25. **Scholion 1.** Quo haec approximatio magis confirmetur, sumamus exemplum cometae A. 1680 visi, pro quo erat $N = 26{,}70458$, qui numerus, cum sit multo major unitate, terminos quod illius, valorem ipsius φ exhibentis, crescentes efficere videatur. Cum autem iste cometa uno die per angulum plus quam 152^0 a perihelio circa solem moveatur, quia cometam diu ante vel post perihelium observari ponimus, erit $\varphi > 152^0$ et $\cos\frac{1}{2}\varphi < \sin 14^0$ et proinde $\cos\frac{1}{2}\varphi < 0{,}2419219$, hoc est $< \frac{1}{4}$. Hinc erit $\cos^4 \frac{1}{2}\varphi < \frac{1}{256}$, quo valore terminus $2N\varkappa \cos^4 \frac{1}{2}\varphi$ valde redditur exiguus. Sumamus spatium k esse decem dierum, erit (8) $\varphi = 167^0\,34'$ et $\frac{1}{2}\varphi = 83^0\,47'$. Si jam hinc quaeramus angulum tempori $k+\varkappa$ dierum respondentem, sequens calculus instituatur:

$$\begin{array}{rl@{\qquad}rl}
l\cos\frac{1}{2}\varphi = & (-1){,}0345825 & lN = & 1{,}4265857 \\
l\sin\frac{1}{2}\varphi = & (-1){,}9974386 & l2 = & 0{,}3010300 \\
lN = & 1{,}4265857 & l\cos^4\frac{1}{2}\varphi = & (-4){,}1383300 \\
 & & & \overline{(-3){,}8659457}
\end{array}$$

$$\text{ergo } 2N\cos^4 \tfrac{1}{2}\varphi = 0{,}007344.$$

Pro secundo termino erit

$$\begin{array}{rl}
lN^2 = & 2{,}8531714 \\
l4 = & 0{,}6020600 \\
l\cos^7 \frac{1}{2}\varphi = & (-7){,}2420775 \\
l\sin\frac{1}{2}\varphi = & (-1){,}9974386 \\
 & \overline{(-4){,}6947475}
\end{array}$$

$$\text{ergo } 4N^2 \cos^7 \tfrac{1}{2}\varphi \sin \tfrac{1}{2}\varphi = 0{,}000495.$$

Pro termino tertio utrumque membrum seorsim computetur, nempe

$$\begin{array}{rl@{\qquad}rl}
lN^3 = & 4{,}2797571 & lN^3 = & 4{,}2797571 \\
l\frac{4}{3} = & 0{,}1249387 & l\frac{4}{3} = & 0{,}1249387 \\
l7 = & 0{,}8450980 & l8 = & 0{,}9030900 \\
l\cos^{10}\frac{1}{2}\varphi = & (-10){,}3458250 & l\cos^{12}\frac{1}{2}\varphi = & (-12){,}4149900 \\
 & \overline{(-5){,}5956188} & & \overline{(-7){,}7227758}
\end{array}$$

ergo $\frac{28}{3} N^3 \cos^{10} \frac{1}{2} v = 0{,}00003941$ ergo $\frac{32}{3} N^3 \cos^{12} \frac{1}{2} v = 0{,}0000005282$.

Cometa ergo tempore $10 + x$ dierum a perihelio movetur per angulum

$$167^\circ\, 34' + 0{,}007344\, x - 0{,}0004839\, x^2 + 0{,}00003888\, x^3$$

qui termini, nisi x decem dies superet, notabiliter decrescunt. Sit x spatium unius diei, erit

$$\begin{aligned} &+0{,}007344\\ &-0{,}000484\\ &+0{,}000039\\ \hline \varphi = &\;\;0{,}006899\\ &\;\;0{,}006690 \end{aligned} \qquad \begin{aligned} \sin 23' &= 0{,}006690\\ \sin 24' &= 0{,}006981\\ \hline &\quad\;\; 291 \end{aligned}$$

$$291 : 60'' = 209 : 43''$$

ergo tempori undecim dierum respondet angulus $167^\circ\, 57'\, 43''$.

26. **Scholion 2.** Si ponamus x negativum, tum omnes termini seriei valorem ipsius φ exhibentis iisdem signis erunt affecti, ideoque series eo magis convergit. Quodsi ergo tempori k dierum respondeat angulus v a perihelio sumtus, seu anomalia vera, tempori $k - x$ dierum respondebit anomalia vera $v - \varphi$, ita ut sit

$$\varphi = 2Nx \cos^4 \tfrac{1}{2} v + 4N^2 x^2 \cos^7 \tfrac{1}{2} v \sin \tfrac{1}{2} v + \tfrac{4}{3} N^3 x^3 \cos^{10} \tfrac{1}{2} v \,(7 - 8 \cos^2 \tfrac{1}{2} v)$$

quae primo, uti vidimus, vehementer convergit, si angulus $\frac{1}{2} v$ non multum deficiat ab angulo recto, hoc est si cometa adhuc longe a perihelio distet, etiamsi hoc casu N sit numerus satis magnus. Quodsi autem N sit numerus multo minor, quod evenit si perihelium cometae longius a sole sit remotum, tum haec series satis convergit, etiamsi cometa non tantopere a perihelio distet.

27. **Problema 6.** Ex datis tribus cometae longitudinibus ac latitudinibus heliocentricis orbitam ipsius determinare.

Solutio. Sit longitudo perihelii $= p$, distantia perihelii a sole $= a$, distantia terrae mediae a sole $= c$, ac ponatur $N = 0{,}012163763303 \cdot \frac{c \sqrt{c}}{a \sqrt{a}}$. Sit longitudo nodi ascendentis $= q$, inclinatio orbitae cometae ad eclipticam $= s$; capiatur angulus r, ut sit $\operatorname{tang} r = \frac{\operatorname{tang}(p - q)}{\cos s}$, erit r distantia perihelii a nodo. Sint tres observationes sumtae diu antequam cometa ad perihelium pertingit, seu statim atque apparere incipit. Sit pro observatione

		I.	II.	III.
longitudo cometae heliocentrica	$=$	f	f'	f''
latitudo cometae heliocentrica	$=$	g	g'	g''

tempus inter observationem I. et II. $= x$ dierum

inter I. et III. $= \lambda$ dierum.

Post primam observationem ponamus cometam ad perihelium pertingere spatio k dierum, et sit anomalia vera cometae tempore primae observationis $= \nu$, tempore secundae $= \nu - \varphi$, tempore tertiae $= \nu - \psi$, erit, uti vidimus,

$$\varphi = 2N\varkappa \cos^4 \tfrac{1}{2}\nu + 4N^2\varkappa^2 \cos^7 \tfrac{1}{2}\nu \sin \tfrac{1}{2}\nu + \text{etc.}$$

$$\psi = 2N\lambda \cos^4 \tfrac{1}{2}\nu + 4N^2\lambda^2 \cos^7 \tfrac{1}{2}\nu \sin \tfrac{1}{2}\nu + \text{etc.}$$

Jam erit per (19), posito r loco A

1) $\sin g = \sin(r-\nu)\sin s$
$\sin g' = \sin(r-\nu+\varphi)\sin s$
$\sin g'' = \sin(r-\nu+\psi)\sin s$

2) $\operatorname{tang}(f-q) = \operatorname{tang}(r-\nu)\cos s$
$\operatorname{tang}(f'-q) = \operatorname{tang}(r-\nu+\varphi)\cos s$
$\operatorname{tang}(f''-q) = \operatorname{tang}(r-\nu+\psi)\cos s$

itemque

3) $\operatorname{tang} g = \sin(f-q)\operatorname{tang} s$
$\operatorname{tang} g' = \sin(f'-q)\operatorname{tang} s$
$\operatorname{tang} g'' = \sin(f''-q)\operatorname{tang} s$

4) $\cos(r-\nu) = \cos g \cos(f-q)$
$\cos(r-\nu+\varphi) = \cos g' \cos(f'-q)$
$\cos(r-\nu+\psi) = \cos g'' \cos(f''-q)$

Ex aequationibus N° 3 sequitur

$$\frac{\sin(f'-q)}{\sin(f-q)} = \frac{\operatorname{tang} g'}{\operatorname{tang} g} = \frac{\sin f' \cos q - \cos f' \sin q}{\sin f \cos q - \cos f \sin q}$$

indeque

$$\operatorname{tang} q = \frac{\operatorname{tang} g' \sin f - \operatorname{tang} g \sin f'}{\operatorname{tang} g' \cos f - \operatorname{tang} g \cos f'}.$$

Idem valor pro longitudine nodi q prodire debet ex binis quibusvis aliis aequationibus ejusdem ordinis, siquidem observationes omni cura sunt institutae; erit ergo pariter

$$\operatorname{tang} q = \frac{\operatorname{tang} g'' \sin f' - \operatorname{tang} g' \sin f''}{\operatorname{tang} g'' \cos f' - \operatorname{tang} g' \cos f''}.$$

Inventa autem longitudine nodi ascendentis q, simul innotescit inclinatio orbitae cometae ad eclipticam s ex aequatione $\operatorname{tang} s = \frac{\operatorname{tang} g}{\sin(f-q)}$. Quia porro φ et ψ sunt anguli perquam exigui, erit

$$\sin(r-\nu+\varphi) = \sin(r-\nu) + \varphi\cos(r-\nu) \quad \text{et} \quad \sin(r-\nu+\psi) = \sin(r-\nu) + \psi\cos(r-\nu)$$

unde ex ordine aequationum primo habebitur

$$\frac{\sin g'}{\sin g} = 1 + \varphi \cot(r-\nu) \quad \text{et} \quad \frac{\sin g''}{\sin g} = 1 + \psi\cot(r-\nu)$$

$$\text{unde} \quad \frac{\sin g'' - \sin g}{\sin g' - \sin g} = \frac{\psi}{\varphi} = \frac{\lambda + 2N\lambda^2 \cos^3 \frac{1}{2}\nu \sin \frac{1}{2}\nu}{\varkappa + 2N\varkappa^2 \cos^3 \frac{1}{2}\nu \sin\frac{1}{2}\nu}$$

Praeterea vero cum sit $\sin(r-\nu) = \frac{\sin g}{\sin s}$, dabitur quoque $\cot(r-\nu)$, unde erit

$$\frac{\sin g' - \sin g}{\sin g \cot(r-\nu)} = \varphi = 2N\varkappa \cos^4 \tfrac{1}{2}\nu + 4N^2\varkappa^2 \cos^7 \tfrac{1}{2}\nu \sin\tfrac{1}{2}\nu$$

$$\text{et} \quad \frac{\sin g'' - \sin g}{\sin g \cot(r-\nu)} = \psi = 2N\lambda \cos^4 \tfrac{1}{2}\nu + 4N^2\lambda^2 \cos^7 \tfrac{1}{2}\nu \sin\tfrac{1}{2}\nu.$$

Ex his ergo aequationibus reperietur et valor numeri N, ex quo distantia perihelii a sole… tescit, et anomalia vera v pro prima observatione, ex qua tempus k, quo cometa perihelium att… innotescit. Deinde vero ex cognito v innotescit angulus r, hincque tandem longitudo perih… Q. E. I.

28. **Coroll. 1.** Quoniam invenimus

$$\varphi = \frac{\sin g' - \sin g}{\sin g \cot (r - v)} \quad \text{et} \quad \psi = \frac{\sin g'' - \sin g}{\sin g \cot (r - v)},$$

atque angulus $r - v$ datus est ex aequatione $\sin (r - v) = \frac{\sin g}{\sin s}$, dabuntur decrementa anomaliae verae v in observatione secunda et tertia, quae sunt φ et ψ.

29. **Coroll. 2.** Quoniam ergo dantur φ et ψ, erit ex (23)

$$N\varkappa = \frac{\varphi}{2 \cos^4 \frac{1}{2} v} + \frac{\varphi\varphi \sin \frac{1}{2} v}{2 \cos^5 \frac{1}{2} v} + \text{etc.}$$

$$N\lambda = \frac{\psi}{2 \cos^4 \frac{1}{2} v} + \frac{\psi\psi \sin \frac{1}{2} v}{2 \cos^5 \frac{1}{2} v} + \text{etc.}$$

hincque eliminando numerum N erit

$$\frac{\lambda}{\varkappa} = \frac{\psi \cos \frac{1}{2} v + \psi^2 \sin \frac{1}{2} v}{\varphi \cos \frac{1}{2} v + \varphi^2 \sin \frac{1}{2} v}, \quad \text{seu} \quad \lambda\varphi + \lambda\varphi^2 \operatorname{tang} \tfrac{1}{2} v = \varkappa\psi + \varkappa\psi^2 \operatorname{tang} \tfrac{1}{2} v,$$

ex qua expedite reperitur anomalia vera v, cum sit

$$\operatorname{tang} \tfrac{1}{2} v = \frac{\varkappa\psi - \lambda\varphi}{\lambda\varphi^2 - \varkappa\psi^2}.$$

30. **Coroll. 3.** Invento ergo hoc modo angulo v, ex eo statim angulus r, hincque longitudo perihelii p innotescit per aequationes $\sin (r - v) = \frac{\sin g}{\sin s}$ et $\operatorname{tang} (p - q) = \operatorname{tang} r \cos s$. Tum vero etiam numerus N definitur ex aequatione

$$N = \frac{\varphi}{2\varkappa \cos^4 \frac{1}{2} v} + \frac{\varphi^2 \sin \frac{1}{2} v}{2\varkappa \cos \frac{1}{2} v}$$

ex quo porro distantia perihelii a sole a determinatur.

31. **Scholion.** Si ob cosinum anguli $\frac{1}{2} v$ valde parvum, series, qua numerus N definitur, parum convergat, calculus sine approximatione, postquam φ et ψ sunt inventa, institui poterit hoc modo: Cum sit $Nk = \operatorname{tang} \frac{1}{2} v + \frac{1}{3} \operatorname{tang}^3 \frac{1}{2} v$, erit

$$N (k - \varkappa) = \operatorname{tang} \frac{v - \varphi}{2} + \tfrac{1}{3} \operatorname{tang}^3 \frac{v - \varphi}{2}$$

hincque $N\varkappa = \operatorname{tang} \frac{1}{2} v - \operatorname{tang} \frac{v - \varphi}{2} + \frac{1}{3} \operatorname{tang}^3 \frac{1}{2} v - \frac{1}{3} \operatorname{tang}^3 \frac{v - \varphi}{2}$. Sit $\operatorname{tang} \frac{1}{2} v = t$ et $\operatorname{tang} \frac{1}{2} \varphi = \mu$, itemque $\operatorname{tang} \frac{1}{2} \psi = \nu$, erit $\operatorname{tang} \frac{v - \varphi}{2} = \frac{t - \mu}{1 + \mu t}$ et $\operatorname{tang} \frac{v - \psi}{2} = \frac{t - \nu}{1 + \nu t}$. His substitutis erit

$$N\varkappa = \frac{\mu(1+tt)}{1+\mu t} + \frac{\mu tt(1+tt) - \mu^2 t(1-t^4) + \frac{1}{3}\mu^3(1+t^6)}{(1+\mu t)^3}$$

$$\text{et} \quad N\lambda = \frac{\nu(1+tt)}{1+\nu t} + \frac{\nu tt(1+tt) - \nu^2 t(1-t^4) + \frac{1}{3}\nu^3(1+t^6)}{(1+\nu t)^3}.$$

Ex his, eliminando N prodibit ista aequatio

$$\frac{\lambda(1+\nu t)^3}{\varkappa(1+\mu t)^3} = \frac{\nu(1+\nu t)^2 + \nu tt - \nu^2 t(1-tt) + \frac{1}{3}\nu^3(1-tt+t^4)}{\mu(1+\mu t)^2 + \mu tt - \mu^2 t(1-tt) + \frac{1}{3}\mu^3(1-tt+t^4)}$$

$$\text{sive} \quad \frac{\lambda(1+\nu t)^3}{\varkappa(1+\mu t)^3} = \frac{\nu + \nu^2 t + \frac{1}{3}\nu^3(1+tt)}{\mu + \mu^2 t + \frac{1}{3}\mu^3(1+tt)}.$$

Ex hac igitur aequatione etsi quinti gradus, si methodo praecedente jam prope valor ipsius t innotuerit, satis cito verus valor ipsius t colligi poterit. Quo invento tam numerus N quam tempus k dierum assignabitur.

§. 32. **Scholion 2.** Quanquam ante valores φ et ψ tantum per approximationem invenimus, observationes tres invicem proximas assumentes, tamen inveniri quoque possunt exacte, etiamsi observationes maxime a se invicem distent. Inventis enim q et s modo praescripto, qui nulla approximatione nitebatur, statim innotescit angulus $r - v$, cum sit

$$\text{vel} \quad \sin(r-v) = \frac{\sin g}{\sin s}, \quad \text{vel} \quad \operatorname{tang}(r-v) = \frac{\operatorname{tang}(f-q)}{\cos s}.$$

Hinc porro innotescit angulus φ ex aequatione $\sin(r - v + \varphi) = \frac{\sin g'}{\sin s}$, et angulus ψ ex aequatione $\sin(r - v + \psi) = \frac{\sin g''}{\sin s}$. Inventis ergo φ et ψ methodo in Scholio praecedente exhibita reperientur N et v. Ex datis ergo tribus quibuscunque locis cometae heliocentricis, longitudinibus scilicet ac latitudinibus, orbita cometae exactissime determinari poterit. Praestat tamen antequam iste modus adhibeatur, ex tribus observationibus invicem proximis et a perihelio longe remotis orbitam cometae vero proxime determinare; quo aequatio superior quinque dimensionum facilius resolvi possit. Cum enim hoc casu fiant μ et ν valde parva, erit proxime

$$\frac{\lambda(1+\nu t)^3}{\varkappa(1+\mu t)^3} = \frac{\nu}{\mu} \quad \text{et} \quad \frac{1+\nu t}{1+\mu t} = \frac{\sqrt[3]{\varkappa\nu}}{\sqrt[3]{\lambda\mu}}, \quad \text{unde} \quad t = \frac{\sqrt[3]{\lambda\mu} - \sqrt[3]{\varkappa\nu}}{\mu\sqrt[3]{\varkappa\nu} - \nu\sqrt[3]{\lambda\mu}};$$

ex quo valore prope vero facile valor exactior elicietur.

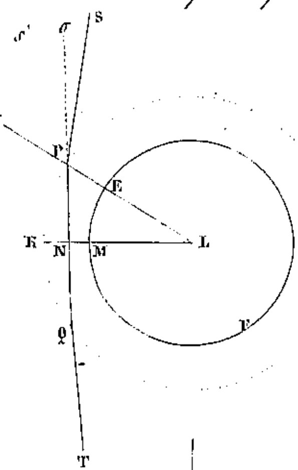

Fig. 211. p. 394.

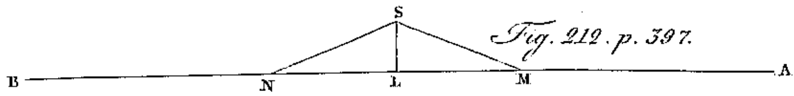

Fig. 212. p. 397.

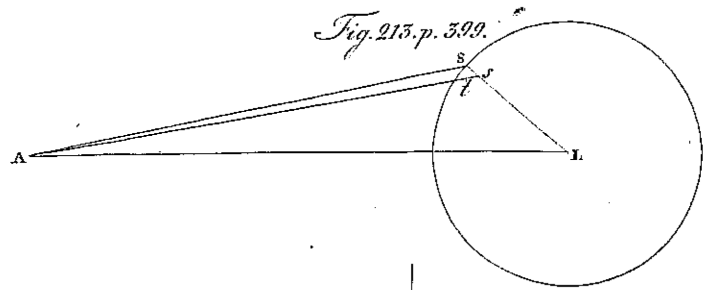

Fig. 213. p. 399.

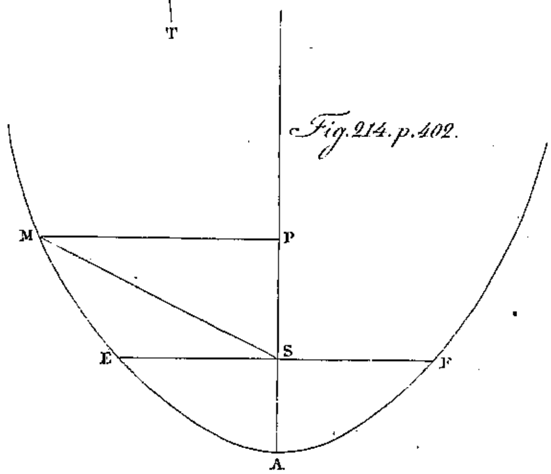

Fig. 214. p. 402.

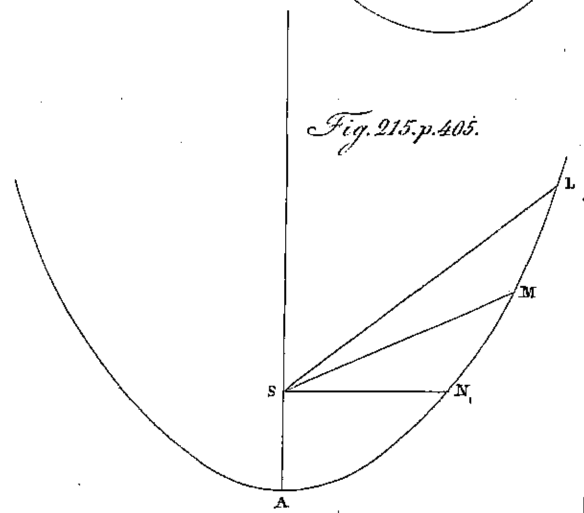

Fig. 215. p. 405.

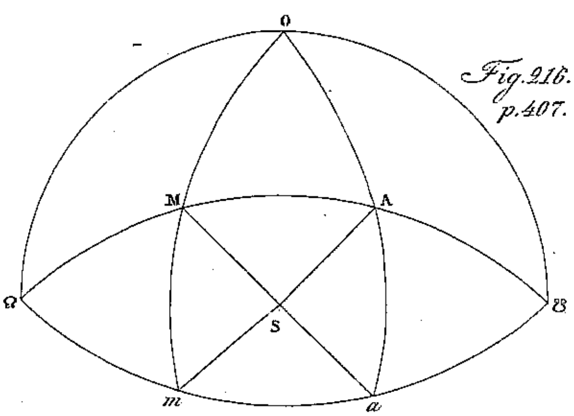

Fig. 216. p. 407.

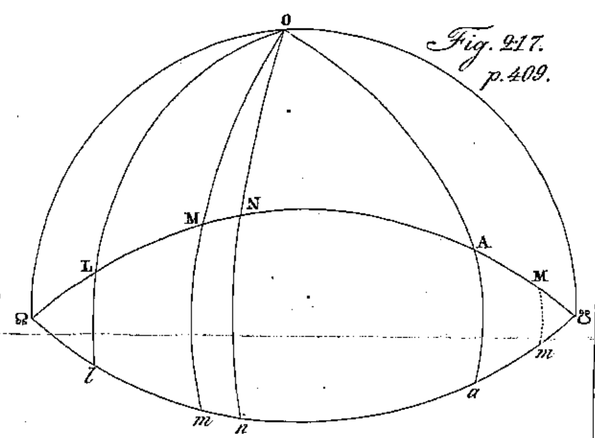

Fig. 217. p. 409.

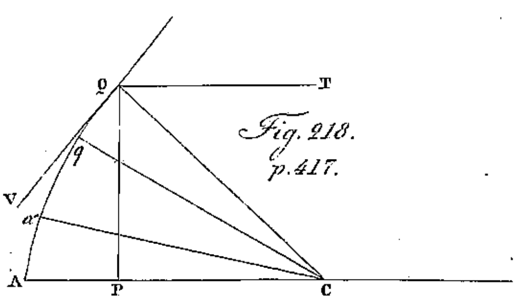

Fig. 218. p. 417.

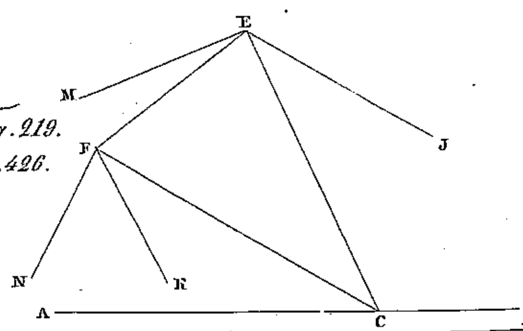

Fig. 219. p. 426.