



1830

Solutio problematis ad analysin infinitorum indeterminatorum referendi

Leonhard Euler

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Record Created:

2018-09-25

Recommended Citation

Euler, Leonhard, "Solutio problematis ad analysin infinitorum indeterminatorum referendi" (1830). *Euler Archive - All Works*. 779.
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SOLUTIO PROBLEMATIS
AD ANALYSIN INFINITORUM INDETERMINATORUM
REFERENDI.

Conventui exhibita die 20. Aug. 1781.

Problema, cujus heic solutionem tradere animus est, ita enunciatur: Propositis quotcunque functionibus p, q, r, s, t , etc., ejusdem variabilis x , invenire functionem x , ita comparatam, ut omnes, quos ecce formulae differentiales:

$$p\partial x, q\partial x, r\partial x, s\partial x, t\partial x, \text{ etc.}$$

evadant integrabiles, cujus solutio, breviter exposita ita se habet.

Postquam functiones datae pro lubitu certo ordine fuerint dispositae, veluti hoc modo: p, q, r, s , etc. ex iis deriventur sequentes functiones primi gradus:

$$q' = \frac{\partial q}{\partial p}, \quad r' = \frac{\partial r}{\partial p}, \quad s' = \frac{\partial s}{\partial p}, \quad t' = \frac{\partial t}{\partial p}, \quad \text{etc.}$$

Ex his simili modo formentur sequentes secundi gradus:

$$r'' = \frac{\partial r'}{\partial q}, \quad s'' = \frac{\partial s'}{\partial q}, \quad t'' = \frac{\partial t'}{\partial q}, \quad \text{etc.}$$

quarum numerus jam unitate minor est quam numerus praecedentium. Hinc porro deducantur functiones gradus tertii, quae erunt $s''' = \frac{\partial s''}{\partial r'}$, $t''' = \frac{\partial t''}{\partial r'}$, etc. quarum numerus iterum unitate minor est praecedentium numero, et ita porro, ita ut si functionum propositarum numerus fuerit 5, ultima sit $t'''' = \frac{\partial t'''}{\partial s''}$.

Jam simili ratione ex functione quaesita x formemus alias per similes gradus, quae sint:

$$x = \frac{\partial x'}{\partial p}, \quad x' = \frac{\partial x''}{\partial q}, \quad x'' = \frac{\partial x'''}{\partial r''}, \quad x''' = \frac{\partial x''''}{\partial r'''}, \quad \text{etc.}$$

unde vicissim habebimus sequentes determinaciones:

$$\partial x' = x \partial p, \quad \partial x'' = x' \partial q, \quad \partial x''' = x'' \partial r'', \quad \text{etc.}$$

His formulis cum praecedentibus conjunctis sequentes determinaciones seu reductiones formularum primi gradus:

$$q' \partial x' = x \partial q; \quad r' \partial x' = x \partial r; \quad s' \partial x' = x \partial s; \quad t' \partial x' = x \partial t; \quad \text{etc.}$$

Eodem modo formulae secundi gradus ad primum reducentur, cum sit etiam

$$r'' \partial x'' = x' \partial r', \quad s'' \partial x'' = x' \partial s', \quad t'' \partial x'' = x' \partial t', \quad \text{etc.};$$

porro formulae tertii gradus ad secundum, ob

$$s''' \partial x''' = x'' \partial s'', \quad t''' \partial x''' = x'' \partial t'', \quad \text{etc.}$$

et ita porro.

Cum jam in ordine litterarum x, x', x'', x''', x'''' , etc. perventum fuerit ad ultimam, quae casu quinque functionum erit x^v , pro ea accipiatur ad libitum functio quaecunque ipsius v , quae sit V , ita ut habeamus $x^v = V$, atque hinc praecedentes omnes sponte determinabuntur, cum sit:

$$x^{iv} = \frac{\partial V}{\partial r^{iv}}, \quad x''' = \frac{\partial x^{iv}}{\partial s'''}, \quad x'' = \frac{\partial x'''}{\partial r''}, \quad x' = \frac{\partial x''}{\partial q}, \quad x = \frac{\partial x'}{\partial p}.$$

His jam valoribus inventis integralia omnium formularum quae requiruntur, ita se habebunt:

$$\int p \partial x = px - x'$$

$$\int q \partial x = qx - q'x' + x''$$

$$\int r \partial x = rx - r'x' + r''x'' - x'''$$

$$\int s \partial x = sx - s'x' + s''x'' - s'''x''' + x^{iv}$$

$$\int t \partial x = tx - t'x' + t''x'' - t'''x''' + t^{iv}x^{iv} - x^v$$

etc.

etc.

quarum formularum veritas per differentiationem sponte elucet.

Ex his jam tota Problematis solutio est manifesta, sive numerus functionum propositarum fuerit major sive minor. Quovis enim casu valor ipsius x semper per differentialia ejusdem gradus, quot fuerint functiones propositae exprimatur, ita ut, si duae tantum proponantur quantitas x ad differentialia secundi gradus ascendet, si ternae ad differentialia tertii ordinis et ita porro.

Hic denique observandum occurrit, prouti functiones propositae alio atque alio modo disponantur, ad integralia maxime diversa perventum iri, quae tamen omnia inter se convenire necesse est siquidem quilibet ordo ad solutionem generalem perducit. Revera autem quaelibet horum integralium forma ad quamlibet aliam reduci potest, si loco functionis V assumamus TV . Semper enim littera T ita determinari potest ut quaelibet forma integralium ad quamlibet aliam reducat.
