1790

Innumerae aequationum formae ex omnibus ordinibus, quarum resolutio exhiberi potest

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INNVMERAE AEQVATIONVM FORMAE, EX OMNIBVS ORDINIBVS, QVARVM RESOLVTIO EXHIBERI POTEST.

Auctore
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Conuent. exhib. d. 6 Maii 1776.

§. 1.

Quoniam regulae generales pro resolutione aequationum non vltra quartum gradum extenduntur, plurimum intererit, eiusmodi aequationum formas notasse, quas resoluere liceat. Hic autem de eiusmodi aequationibus loquor, quae neque radices habeant rationales, neque per factores in aequationes ordinum inferiorum resolui queant; quandoquidem facillimum foret innumerabiles huiusmodi aequationes resolubiles proferre. Hanc ob rem eiusmodi aequationum species attentione dignae sunt censendae, quarum resolutio necessario extractionem radicum eiusdem ordinis, cuius est ipsa aequatio, postulat.

§. 2. Huiusmodi aequationes iam olim a Moivraeo pro singulis ordinibus in medium sunt prolatae, quibus Scientia analytica non parum amplificata merito est putanda; deinde vero etiam ipse plures tales aequationes in lucem protraxi: nuper autem se mihi obtulit methodus innumerabiles alias huius indolis aequationes eliciendi, quas spero Geometris haud ingratas esse futuras.

§. 3. Iſtas igitur aequationum formas, prouti ad eas ſum deductus, hic ordine proponam.

I. Si $x^2 = ab$, erit $x = \sqrt{ab}$.

II. Si $x^3 = 3abx + ab(a+b)$, erit $x = \sqrt[3]{aab} + \sqrt[3]{abb}$

III. Si $x^4 = 6abxx + 4ab(a+b)x + ab(aa+ab+bb)$, erit

$$x = \sqrt[4]{a^3b} + \sqrt[4]{aabb} + \sqrt[4]{ab^3}.$$

IV. Si $x^5 = 10abx^3 + 10ab(a+b)xx + 5ab(aa+ab+bb)x + ab(a^3+aab+abb+b^3)$, erit

$$x = \sqrt[5]{a^4b} + \sqrt[5]{a^3bb} + \sqrt[5]{aab^3} + \sqrt[5]{ab^4}.$$

V. Si $x^6 = 15abx^4 + 20ab(a+b)x^3 + 15ab(aa+ab+bb)xx + 6ab(a^3+aab+abb+b^3)x + ab(a^4+a^3b+aabb+ab^3+b^4)$, erit

$$x = \sqrt[6]{a^5b} + \sqrt[6]{a^4b^2} + \sqrt[6]{a^3b^3} + \sqrt[6]{aab^4} + \sqrt[6]{ab^5}.$$

VI. Si $x^7 = 21abx^5 + 35ab(a+b)x^4 + 35ab(aa+ab+bb)x^3 + 21ab(a^3+aab+abb+b^3)xx + 7ab \times$
$\times (a^4+a^3b+aabb+ab^3)x + ab(a^5+a^4b+a^3bb+aab^3+ab^4+b^5)$, erit

$$x = \sqrt[7]{a^6b} + \sqrt[7]{a^5bb} + \sqrt[7]{a^4b^3} + \sqrt[7]{a^3b^4} + \sqrt[7]{aab^5} + \sqrt[7]{ab^6}.$$

§. 4. Hinc iam facile colligitur in genere pro ordine quocunque

$$x^n = \frac{n(n-1)}{1.\ 2}abx^{n-2} + \frac{n(n-1)(n-2)}{1.\ 2.\ 3}ab(a+b)x^{n-3} + \frac{n(n-1)(n-2)(n-3)}{1.\ 2.\ 3.\ 4}ab(aa+ab+bb)x^{n-4} + \frac{n(n-1)(n-2)(n-3)(n-4)}{1.\ 2.\ 3.\ 4.\ 5}ab^3(aab+abb+b^3)x^{n-5} + \text{etc.}$$

fore

$$x = \sqrt[n]{a^{n-1}b} + \sqrt[n]{a^{n-2}bb} + \sqrt[n]{a^{n-3}b^3} + \sqrt[n]{a^{n-4}b^4} + \text{etc.}$$

Vel ſi loco illorum coëfficientium ſcribamus breuitatis gratia n^{II}, n^{III}, n^{IV}, n^{V}, n^{VI}, etc. iſta aquatio generalis ſuccinctius ita exprimi

exprimi poterit:

$$x^n = n^{II}\,ab\left(\frac{a-b}{a-b}\right)x^{n-2} + n^{III}\,ab\left(\frac{a^2-b^2}{a-b}\right)x^{n-3} + n^{IV}\,ab\left(\frac{a^3-b^3}{a-b}\right)x^{n-4}$$
$$+ n^{V}\,ab\left(\frac{a^4-b^4}{a-b}\right)x^{n-5} + n^{VI}\,ab\left(\frac{a^5-b^5}{a-b}\right)x^{n-6} + \text{etc.}$$

tum vero etiam ipſa radix ita concinnius exprimi poterit, vt ſit

$$x = \frac{a\sqrt[n]{b} - b\sqrt[n]{a}}{\sqrt[n]{a} - \sqrt[n]{b}},$$

quae ergo eſt aequatio generalis ad omnes ordines patens.

§. 5. Has aequationes in aliam formam transfundere licet, qua artificium, quod eo manuduxit, magis occultatur. Ponamus ſcilicet litterarum a et b productum $ab = p$, earumque ſummam $a + b = s$, hasque duas litteras p et s loco illarum a et b in calculum introducamus; tum autem erit $a = \frac{s + \sqrt{(ss - 4p)}}{2}$ et $b = \frac{s - \sqrt{(ss - 4p)}}{2}$. His iam nouis valoribus introductis aequationes ſuperiores ſpeciales ſequentes formas induent:

I. Si $x^2 = p$ erit, $x = \sqrt{p}$.

II. Si $x^3 = 3px + ps$ erit $x = \sqrt[3]{aab} + \sqrt[3]{abb} = \sqrt[3]{ap} + \sqrt[3]{bp}$.

III. Si $x^4 = 6pxx + 4psx + p(ss - p)$, erit
$$x = \sqrt[4]{aap} + \sqrt[4]{abp} + \sqrt[4]{bbp}.$$

IV. Si $x^5 = 10px^3 + 10psxx + 5p(ss-p)x + p(s^3 - 2sp)$, erit
$$x = \sqrt[5]{a^3p} + \sqrt[5]{ap^2} + \sqrt[5]{bp^2} + \sqrt[5]{b^3p}.$$

V. Si $x^6 = 15px^4 + 20psx^3 + 15p(ss-p)xx + 6p(s^3 - 2ps) + p(s^4 - 3ps^2 + pp)$, erit
$$x = \sqrt[6]{a^4p} + \sqrt[6]{aapp} + \sqrt[6]{p^3} + \sqrt[6]{bbpp} + \sqrt[6]{b^4p}.$$

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VI. Si $x^7 = 21px^5 + 35psx^4 + 35p(ss-p)x^3 + 21p(s^3-2ps)xx$
$+ 7p(s^4-3pss+pp)x + p(s^5-4ps^3+3pps)$, erit
$$x = \sqrt[7]{a^5p} + \sqrt[7]{a^3pp} + \sqrt[7]{ap^3} + \sqrt[7]{bp} + \sqrt[7]{b^3pp} + \sqrt[7]{b^5p}.$$
etc.

§. 6. Quo nunc hanc formam generalem reddamus, obſeruandum eſt nouos coëfficientes litteris p et s contentos ſeriem conſtituere recurrentem, cuius ſcala relationis eſt $s-p$. Si enim ponamus:
$$Q = \frac{a^\lambda - b^\lambda}{a-b},\quad Q' = \frac{a^{\lambda+1} - b^{\lambda+1}}{a-b} \text{ et } Q'' = \frac{a^{\lambda+2} - b^{\lambda+2}}{a-b},$$
manifeſto erit $Q'' = sQ' - pQ$, namque ob $s = a+b$ erit
$$sQ' = \frac{a^{\lambda+2} + b\,a^{\lambda+1} - a\,b^{\lambda+1} - b^{\lambda+2}}{a-b},$$
at ob $p = ab$ erit
$$pQ = \frac{a^{\lambda+1}b - a\,b^{\lambda+1}}{a-b},$$
qua forma ab illa ablata remanebit
$$sQ' - pQ = \frac{a^{\lambda+2} - b^{\lambda+2}}{a-b}.$$

Hac igitur lege obſeruata habebimus ſequentes transformationes:

$$\frac{a-b}{a-b} = 1;$$
$$\frac{a^2-b^2}{a-b} = s;$$
$$\frac{a^3-b^3}{a-b} = ss-p;$$
$$\frac{a^4-b^4}{a-b} = s^3-qsp;$$
$$\frac{a^5-b^5}{a-b} = s^4-3pss+pp,$$
$$\frac{a^6-b^6}{a-b} = s^5-4ps^3+3pps,$$
$$\frac{a^7-b^7}{a-b} = s^6-5ps^4+6ppss-p^3,$$
$$\frac{a^8-b^8}{a-b} = s^7-6ps^5+pps^3-4p^5s,$$
etc.

§. 7. Ordo, quo iſtae formulae progrediuntur, iam ſatis eſt perſpicuus. Primo enim poteſtates ipſius s continuo binario de-

decrescunt, contra vero ipsius p potestates vnitate crescunt, signis alternantibus; coëfficientes autem numerici cuiusque termini conueniunt cum iis, quos iidem termini in euolutione binomii essent habiturae, vel, quod eodem redit, ii omnes permutationes litterarum p et s indicant, ita vt coëfficiens termini $p^\alpha\ s^\beta$ fit $= \frac{1.2.3.\ldots\ldots(\alpha+\beta)}{1.2.3.\ldots\ldots\alpha \times 1.2.3.\ldots\ldots\beta}$. Hinc ergo deducimus transformationem sequentem generalem:

$$\frac{a^{\lambda+1}-b^{\lambda+1}}{a-b} = s^\lambda - \frac{(\lambda-1)}{1} p\, s^{\lambda-2} + \frac{(\lambda-2)\,\lambda-3)}{1.\ 2.} p\, p\, s^{\lambda-4}$$

$$- \frac{(\lambda-3)(\lambda-4)(\lambda-5)}{1.\ \ 2.\ \ 3} p^3\, s^{\lambda-6} + \frac{(\lambda-4)(\lambda-5)(\lambda-6)(\lambda-7)}{1.\ \ 2.\ \ 3.\ \ 4} p^4\, s^{\lambda-8}$$

$$- \frac{(\lambda-5)(\lambda-6)(\lambda-7)(\lambda-8)(\lambda-9)}{1.\ \ 2.\ \ 3.\ \ 4.\ \ 5} p^5\, s^{\lambda-10} + \text{etc.}$$

§. 8. Quodsi ergo hos valores in aequatione generali supra §. 4. data substituamus, aequatio generalis, cuius resolutionem hac methodo exhibere licet, talem habebit formam:

$$x^n = \frac{n(n-1)}{1.\ 2} p\, x^{n-2} + \frac{n(n-1)(n-2)}{1.\ 2.\ \ 3} p\, s\, x^{n-3} + \frac{n(n-1)(n-2)(n-3)}{1.\ \ 2.\ \ 3.\ \ 4} p\,(ss-p)\,\lambda^{n-4}$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{1.\ \ 2.\ \ 3.\ \ 4.\ \ 5} p\,(s^3 - 2\,p\,s)\,x^{n-5}$$

$$+ \frac{n.\ldots.(n-5)}{1.\ 2\ \ldots\ 6} p\,(s^4 - 3\,p\,s\,s + p\,p)\,x^{n-6} + \frac{n.\ldots.(n-6)}{1.\ 2\ \ldots\ 7} p\ \times$$

$$\times\ (s^5 - 4\,p\,s^3 + 3\,p\,p\,s)\,x^{n-7} + \text{etc.}$$

Huius scilicet aequationis resolutio, quicunque numeri pro p et s accipiantur, semper erit in potestate, eius quippe radix, postquam ex numeris p et s isti fuerint deriuati:

$$a = \frac{s+\sqrt{(ss-4p)}}{2} \text{ et } b = \frac{s-\sqrt{(ss-4p)}}{2},$$

ita exprimetur vt fit $x = \frac{a\sqrt[n]{b} - b\sqrt[n]{a}}{\sqrt[n]{a} - \sqrt[n]{b}}$.

§. 9. Haec quidem formula vnicam radicem aequationis propositae nobis largitur, verum tamen facile hinc omnes plane radices eiusdem aequationis deducuntur, quarum quidem numerus

 est

eſt $=n$. Primo enim ponendo $b=ak$, radix illa reuocabitur ad vnicum ſignum radicale, cum hinc fiat $x=\frac{a\sqrt[n]{k}-b}{1-\sqrt[n]{k}}$. Nunc vero iſta radix, nempe $\sqrt[n]{k}$, valores diuerſos numero n admittit, quemadmodum etiam radix poteſtatis n ex vnitate, ſcilicet $\sqrt[n]{1}$, totidem diuerſos valores recipit, quorum vnus ſemper ipſi vnitati aequatur. Vnde ſi quilibet horum valorum deſignetur littera ϱ, ita vt ſit $\varrho^n=1$, iſta littera ϱ inuoluet n diuerſos valores, quorum quemlibet cum formula $\sqrt[n]{k}$ coniungere licet, ſcribendo ſcilicet eius loco $\varrho\sqrt[n]{k}$, quamobrem omnes plane radices aequationis propoſitae in hac formula continebuntur: $x=\frac{\varrho a\sqrt[n]{k}-b}{1-\varrho\sqrt[n]{k}}$, ſiue $x=\frac{\varrho a\sqrt[n]{b}-b\sqrt[n]{a}}{\sqrt[n]{a}-\varrho\sqrt[n]{b}}$; tum vero ſi haec formula per diuiſionem euoluatur, prodibit iſta expreſſio:

$$x=\varrho\sqrt[n]{a^{n-1}b}+\varrho^2\sqrt[n]{a^{n-2}bb}+\varrho^3\sqrt[n]{a^{n-3}b^3}+\text{etc.}$$

cuius expreſſionis numerus terminorum eſt $n-1$, vltimo exiſtente $\varrho^{n-1}\sqrt[n]{ab^{n-1}}$.

§. 10. Operae pretium erit hanc rem exemplo illuſtraſſe. Sumamus igitur $n=5$, $s=1$ et $p=-1$, vt proponatur iſta aequatio quinti gradus:

$$x^5=-10x^3-10xx-10x-3, \text{ ſiue}$$

$$x^5+10x^3+10xx+10x+3=0.$$

Ad huius ergo aequationis radices inueſtigandas, capiantur hi valores: $a=\frac{1+\sqrt{5}}{2}$; et $b=\frac{1-\sqrt{5}}{2}$, quibus inuentis erit quaelibet radix $x=\frac{\varrho a\sqrt[5]{b}-b\sqrt[5]{a}}{\sqrt[5]{a}-\varrho\sqrt[5]{b}}$, vel introducendo litteram $k=\frac{b}{a}=-\frac{3+\sqrt{5}}{2}$,

erit

erit $x = \frac{\varrho a \sqrt[5]{k} - b}{1 - \varrho \sqrt[5]{k}}$. Sin autem hanc formam euoluere velimus, ob $ab = p = -1$ reperiemus:

$$x = -\varrho \sqrt[5]{a^3} + \varrho^2 \sqrt[5]{a} + \varrho^3 \sqrt[5]{b} - \varrho^4 \sqrt[5]{b^3},$$

quae expreſſio penitus in numeris euoluta praebet

$$x = -\varrho \sqrt[5]{(2+\sqrt{5})} + \varrho^2 \sqrt[5]{\left(\frac{1+\sqrt{5}}{2}\right)} + \varrho^3 \sqrt[5]{\left(\frac{1-\sqrt{5}}{2}\right)} - \varrho^4 (2-\sqrt{5})$$

Demonſtratio.
formularum ſupra datarum.

§. 11. Analyſis, quae ad iſtas aequationes perduxit, maxime eſt obuia, ita vt vix quicquam in receſſu habere videatur: tota enim petita eſt ex hac aequatione ſimpliciſſima: $\frac{(a+x)^n}{(b+x)^n} = \frac{a}{b}$. Cum enim hinc fiat $\frac{a+x}{b+x} = \sqrt[n]{\frac{a}{b}}$, inde colligitur incognita:

$$x = \frac{a - b\sqrt[n]{\frac{a}{b}}}{\sqrt[n]{\frac{a}{b}} - 1} = \frac{a\sqrt[n]{b} - b\sqrt[n]{a}}{\sqrt[n]{a} - \sqrt[n]{b}}$$

quae eſt ea ipſa radix quam pro aequationibus ſuperioribus aſſignauimus.

§. 12. Quodſi vero aequationem illam aſſumtam euoluamus, quoniam inde fieri debet $a(x+b)^n = b(x+a)^n$, ſiue $a(x+b)^n - b(x+a)^n = 0$, hinc deriuabitur ſequens aequatio:

$$\left.\begin{array}{l} a x^n + \frac{n}{1} a b x^{n-1} + \frac{n(n-1)}{1.\ 2} a b b x^{n-2} \\ \qquad + \frac{n(n-1)(n-2)}{1.\ 2.\ 3} a b^3 x^{n-3} + \text{etc.} \\ -b x^n - \frac{n}{1} a b x^{n-1} - \frac{n(n-1)}{1.\ 2} a a b x^{n-2} \\ \qquad - \frac{n(n-1)(n-2)}{1.\ 2.\ 3} a^3 b x^{n-3} - \text{etc.} \end{array}\right\} = 0,$$

vbi

vbi membra fecunda fe mutuo tollunt. Iam quia primum membrum afficitur per $a - b$, reliqua membra in alteram partem transferantur, ac per $a - b$ diuidantur, ficque emerget fequens aequatio;

$$x^n = \frac{n(n-1)}{1.\ 2}\, a\, b \left(\frac{a-b}{a-b}\right) x^{n-2} + \frac{n(n-1)(n-2)}{1.\ 2.\ 3}\, a\, b \left(\frac{aa-bb}{a-b}\right) x^{n-3}$$
$$+ \frac{n(n-1)(n-2)(n-3)}{1.\ 2.\ 3.\ 4}\, a\, b \left(\frac{a^3-b^3}{a-b}\right) x^{n-4} + \text{etc.}$$

quae eft ipfa aequatio generalis fupra tractata, cuius ergo radix eft $x = \frac{a\sqrt[n]{b} - b\sqrt[n]{a}}{\sqrt[n]{a} - \sqrt[n]{b}}$.

§. 13. Hinc forte quispiam expectare poffet, fimili modo huiusmodi aequationes generaliores obtineri poffe, fi loco illius formulae fimpliciffimae haec formula latius patens: $\frac{(f+x)^n}{(g+x)^n} = \frac{a}{b}$, fundamenti loco conftituatur, fiquidem hic quatuor quantitates arbitrariae a, b, f et g, in computum introducuntur, cum ante binae tantum a et b ineffent; verum tamen quomodocunque litterae f et g a litteris a et b diuerfae accipiantur, tamen cafus femper ad priorem fimpliciorem reduci poteft. Ad hoc oftendendum ponamus $x = \alpha + \beta z$, et aequatio noftra fiet $\frac{(\alpha + f + \beta z)^n}{(\alpha + g + \beta z)^n} = \frac{a}{b}$, fiue

$\frac{\left(\frac{\alpha+f}{\beta} + z\right)^n}{\left(\frac{\alpha+g}{\beta} + z\right)^n} = \frac{a}{b}$; atque nunc manifeftum eft, quantitates α et β femper ita capi poffe, vt fiat $\frac{\alpha+f}{\beta} = a$ et $\frac{\alpha+g}{\beta} = b$, quandoquidem hinc deducitur $\alpha = \frac{bf-ag}{a-\beta}$, ideoque $\beta = \frac{f-g}{a-b}$. Sicque formula illa, quae multo generalior videbatur, femper ad fimpliciffimam illam fupra tractatam reuocari poteft, neque idcirco quicquam noui inde eft expectandum.

Anno-

Annotatio.

in aequationes ſupra euolutas.

§. 14. Si formas, quas pro radicibus harum aequationum ſupra aſſignauimus, accuratius perpendamus, haec omnia egregie conuenire deprehenduntur, cum coniectura illa, quam olim in medium proferre ſum auſus, dum pro reſolutione aequationis cuiuscunque gradus, in qua ſecundus terminus deſit, veluti

$$x^n = p\,x^{n-2} + q\,x^{n-3} + r\,x^{n-4} + \text{etc.}$$

affirmaui, ſemper dari aequationem reſoluentem vno gradu inferiorem, huius formae:

$$y^{n-1} - A\,y^{n-2} + B\,y^{n-3} - C\,y^{n-4} + D\,y^{n-5} - \text{etc.} = 0,$$

cuius radices, numero $n - 1$, ſi fuerint α, β, γ, δ, ε, etc. futurum ſit

$$x = \sqrt[n]{\alpha} + \sqrt[n]{\beta} + \sqrt[n]{\gamma} + \sqrt[n]{\delta} + \text{etc.}$$

§. 15. Cum igitur pro forma generali, quam ſupra tractauimus, radix inuenta ſit

$$x = \sqrt[n]{a^{n-1}\,b} + \sqrt[n]{a^{n-2}\,b\,b} + \sqrt[n]{a^{n-3}\,b^3} - - - + \sqrt[n]{a\,b^{n-1}},$$

hinc ſequitur aequationis reſoluentis ordinis $n - 1$ radices fore $a^{n-1}\,b$; $a^{n-2}\,b\,b$; $a^{n-3}\,b^3$; $a^{n-4}\,b^4$; - - - - $a\,b^{n-1}$, quae ergo erunt valores ipſius y. Quare cum coëfficiens A ſit ſumma omnium harum radicum, erit $A = \frac{a\,b\,(a^{n-1} - b^{n-1})}{a - b}$, poſtremum autem huius aequationis membrum abſolutum erit productum ex omnibus his radicibus, quod ergo erit $= a^{\frac{nn-n}{2}} \times b^{\frac{nn-n}{2}}$. Pro reliquis terminis percurramus aequationes particulares ſupra expoſitas.

I. Pro aequatione tertii gradus:

$$x^3 = 3\,a\,b\,x + a\,b\,(a + b)$$

vbi erat radix

$$x = \sqrt[3]{a\,a\,b} + \sqrt[3]{a\,b\,b}.$$

Hic ſi aequatio reſoluens ſtatuatur

$$y\,y - A\,y + B = 0,$$

eius radices erunt $a\,a\,b$ et $a\,b\,b$, ideoque $A = a\,b\,(a + b)$ et $B = a^3\,b^3$.

II. Pro aequatione quarti gradus:

$$x^4 = 6\,a\,b\,x\,x + 4\,a\,b\,(a + b)\,x + a\,b\,(a\,a + a\,b + b\,b)$$

Hic eſt radix

$$x = \sqrt[4]{a^3\,b} + \sqrt[4]{a\,a\,b\,b} + \sqrt[4]{a\,b^3},$$

vnde ſi aequatio reſoluens ſtatuatur

$$y^3 - A\,y\,y + B\,y - C = 0,$$

eius radices erunt $a^3\,b$; $a\,a\,b\,b$; $a\,b^3$; quocirca habebimus

$$A = a\,b\,(a\,a + a\,b + b\,b),$$
$$B = a^3\,b^3\,(a\,a + a\,b + b\,b) \text{ et}$$
$$C = a^6\,b^6.$$

III. Pro aequatione quinti gradus:

$$x^5 = 10\,a\,b\,x^3 + 10\,a\,b\,(a + b)\,x\,x + 5\,a\,b\,(a\,a + a\,b + b\,b)\,x + a\,b\,(a^3 + a\,a\,b + a\,b\,b + b^3),$$

Hic igitur erit

$$x = \sqrt[5]{a^4\,b} + \sqrt[5]{a^3\,b\,b} + \sqrt[5]{a\,a\,b^3} + \sqrt[5]{a\,b^4},$$

vnde ſi aequatio reſoluens ſtatuatur:

$$y^4 - A\,y^3 + B\,y\,y - C\,y + D = 0,$$

eius radices erunt, $a^4\,b$; $a^3\,b\,b$; $a\,a\,b^3$; $a\,b^4$; vnde colligitur fore

fore $A = ab(a^3 + aab + abb + b^3)$,

$B = a^3 b^3 (a^4 + a^3 b + 2 aabb + ab^3 + b^4)$,

$C = a^6 b^6 (a^3 + aab + abb + b^3)$,

$D = a^{10} b^{10}$.

IV. Pro aequatione ſexti gradus:

$$x^6 = 15\, ab x^4 + 20\, ab (a+b) x^3 + 15\, ab (aa + ab + bb) xx$$
$$+ 6ab (a^3 + aab + abb + b^3) + ab (a^4 + a^3 b + aabb + ab^3 + b^4),$$

Hic igitur habebitur

$$x = \sqrt[6]{a^5 b} + \sqrt[6]{a^4 bb} + \sqrt[6]{a^3 b^3} + \sqrt[6]{aab^4} + \sqrt[6]{ab^5}.$$

vnde ſi aequatio reſoluens ſtatuatur

$$y^5 - Ay^4 + By^3 - Cyy + Dy - E = 0,$$

eius radices erunt $a^5 b$; $a^4 bb$; $a^3 b^3$; aab^4; ab^5, vnde colligitur fore

$A = ab(a^4 + a^3 b + aabb + ab^3 + b^4)$,

$B = a^3 b^3 (a^6 + a^5 b + 2 a^4 bb + 2 a^3 b^3 + 2 aab^4 + ab^5 + b^6)$,

$C = a^6 b^6 (a^6 + a^5 b + 2 a^4 bb + 2 a^3 b^3 + 2 aab^4 + ab^5 + b^6)$,

$D = a^{10} b^{10} (a^4 + a^3 b + aabb + ab^3 + b^4)$,

$E = a^{15} b^{15}$.

vbi formulae mediae B et C ita concinnius exprimi poſſunt:

$B = a^3 b^3 (aa + bb)(a^4 + a^3 b + aabb + ab^3 + b^4)$ et

$C = a^6 b^6 (aa + bb)(a^4 + a^3 b + aabb + ab^3 + b^4)$,

quae determinationes fortaſſe aliquam lucem accendere poſſunt ad reſolutionem aequationum generalem feliciori ſucceſſu tractandam.