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Innumerae aequationum formae ex omnibus ordinibus, quarum resolutio exhiberi potest

Leonhard Euler

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INNVMERAE
AEQVATIONVM FORMAE,
EX OMNIBVS ORDINIBVS,
QVARVM RESOLVTIO EXHIBERI POTEST.

Auctore
L. EVLERO.

Conuent. exhib. d. 6 Maii 1776.

§. 1.

Quoniam regulae generales pro resolutione aequationum non ultra quartum gradum extenduntur, plurimum intererit, eiusmodi aequationum formas notasse, quas resolvere liceat. Hic autem de eiusmodi aequationibus loquor, quae neque radices habeant rationales, neque per factores in aequationes ordinum inferiorum resolui queant; quandoquidem facillimum foret innumerabiles huiusmodi aequationes resolubiles proferre. Hanc ob rem eiusmodi aequationum species attentione dignae sunt censendae, quarum resolutio necessario extractionem radicum eiusdem ordinis, cuius est ipsa aequatio, postulat.

§. 2. Huiusmodi aequationes iam olim a Moivraeo pro singulis ordinibus in medium sunt prolatae, quibus Scientia analytica non parum amplificata merito est putanda; deinde vero etiam ipse plures tales aequationes in lucem protraxi: nuper autem se mihi obtulit methodus innumerabiles alias huius indolis aequationes eliciendi, quas spero Geometris haud ingratas esse futuras.

Nova Acta Acad. Imp. Sc. T. VI.

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§. 3.

§. 3. Iftas igitur aequationum formas , prouti ad eas sum deductus , hic ordine proponam.

I. Si $x^2 = ab$, erit $x = \sqrt{ab}$.

II. Si $x^3 = 3 abx + ab(a+b)$, erit $x = \sqrt[3]{aab} + \sqrt[3]{abb}$

III. Si $x^4 = 6 abxx + 4 ab(a+b)x + ab(aa+ab+bb)$, erit
 $x = \sqrt[4]{a^3b} + \sqrt[4]{aabb} + \sqrt[4]{ab^3}$.

IV. Si $x^5 = 10 abx^3 + 10 ab(a+b)xx + 5 ab(aa+ab+bb)x + ab(a^3+aab+abb+b^3)$, erit
 $x = \sqrt[5]{a^4b} + \sqrt[5]{a^3bb} + \sqrt[5]{aabb^3} + \sqrt[5]{ab^4}$.

V. Si $x^6 = 15 abx^4 + 20 ab(a+b)x^3 + 15 ab(aa+ab+bb)xx + 6 ab(a^3+aab+abb+b^3)x + ab(a^4+a^3b+aabb+ab^3+b^4)$, erit

$$x = \sqrt[6]{a^5b} + \sqrt[6]{a^4b^2} + \sqrt[6]{a^3b^3} + \sqrt[6]{aabb^4} + \sqrt[6]{ab^5}$$

VI. Si $x^7 = 21 abx^5 + 35 ab(a+b)x^4 + 35 ab(aa+ab+bb)xx^3 + 21 ab(a^3+aab+abb+b^3)xx + 7 ab \times$
 $\times (a^4+a^3b+aabb+ab^3)x + ab(a^5+a^4b+a^3bb+aabb^3+ab^4+b^5)$, erit

$$x = \sqrt[7]{a^6b} + \sqrt[7]{a^5bb} + \sqrt[7]{a^4b^3} + \sqrt[7]{a^3b^4} + \sqrt[7]{aabb^5} + \sqrt[7]{ab^6}$$

§. 4. Hinc iam facile colligitur in genere pro ordine quocunque

$$x^n = \frac{n(n-1)}{1 \cdot 2} ab x^{n-2} + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} ab(a+b) x^{n-3} \\ + \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} ab(aa+ab+bb) x^{n-4} \\ + \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} ab^3(aab+abb+b^3) x^{n-5} + \text{etc.}$$

fore

$$x = \sqrt[n]{a^{n-1}b} + \sqrt[n]{a^{n-2}bb} + \sqrt[n]{a^{n-3}b^3} + \sqrt[n]{a^{n-4}b^4} + \text{etc.}$$

Vel si loco illorum coefficientium scribamus breuitatis gratia n^{II} , n^{III} , n^{IV} , n^{V} , n^{VI} , etc. ista aquatio generalis succinctius ita exprimi

exprimi poterit:

$$x^n = n^{II} ab \left(\frac{a-b}{a-b}\right) x^{n-2} + n^{III} ab \left(\frac{a^2-b^2}{a-b}\right) x^{n-3} + n^{IV} ab \left(\frac{a^3-b^3}{a-b}\right) x^{n-4} \\ + n^V ab \left(\frac{a^4-b^4}{a-b}\right) x^{n-5} + n^{VI} ab \left(\frac{a^5-b^5}{a-b}\right) x^{n-6} + \text{etc.}$$

tum vero etiam ipsa radix ita concinnius exprimi poterit, vt sit

$$x = \frac{a \sqrt[n]{b} - b \sqrt[n]{a}}{\sqrt[n]{a} - \sqrt[n]{b}},$$

quae ergo est aequatio generalis ad omnes ordines patens.

§. 5. Has aequationes in aliam formam transfundere licet, qua artificium, quod eo manuduxit, magis occultatur. Ponamus scilicet litterarum a et b productum $ab=p$, earumque summam $a+b=s$, hasque duas litteras p et s loco illarum a et b in calculum introducamus; tum autem erit $a = \frac{s + \sqrt{(ss-4p)}}{2}$ et $b = \frac{s - \sqrt{(ss-4p)}}{2}$. His iam nouis valoribus introductis aequationes superiores speciales sequentes formas induent:

I. Si $x^2 = p$ erit, $x = \sqrt{p}$.

II. Si $x^3 = 3px + ps$ erit $x = \sqrt[3]{aab} + \sqrt[3]{abb} = \sqrt[3]{ap} + \sqrt[3]{bp}$.

III. Si $x^4 = 6p x x + 4ps x + p(ss-p)$, erit

$$x = \sqrt[4]{aap} + \sqrt[4]{abp} + \sqrt[4]{bbp}.$$

IV. Si $x^5 = 10p x^3 + 10ps x x + 5p(ss-p)x + p(s^3 - 2sp)$, erit

$$x = \sqrt[5]{a^3 p} + \sqrt[5]{a p^2} + \sqrt[5]{b p^2} + \sqrt[5]{b^3 p}.$$

V. Si $x^6 = 15p x^4 + 20ps x^3 + 15p(ss-p) x x + 6p(s^3 - 2ps) + p(s^4 - 3ps^2 + pp)$, erit

$$x = \sqrt[6]{a^4 p} + \sqrt[6]{aapp} + \sqrt[6]{p^3} + \sqrt[6]{bbpp} + \sqrt[6]{b^4 p}.$$

VI. Si $x^7 = 21px^5 + 35psx^4 + 35p(ss-p)x^3 + 21p(s^3-2ps)x^2 + 7p(s^4-3pss+pp)x + p(s^5-4ps^3+3ppps)$, erit
 $x = \sqrt[7]{a^5p} + \sqrt[7]{a^3pp} + \sqrt[7]{ap^3} + \sqrt[7]{bp} + \sqrt[7]{b^3pp} + \sqrt[7]{b^5p}$.
 etc

§. 6. Quo nunc hanc formam generalem reddamus, obseruandum est nouos coëfficientes litteris p et s contentos seriem constituere recurrentem, cuius scala relationis est $s-p$. Si enim ponamus:

$$Q = \frac{a^\lambda - b^\lambda}{a-b}, \quad Q' = \frac{a^{\lambda+1} - b^{\lambda+1}}{a-b} \quad \text{et} \quad Q'' = \frac{a^{\lambda+2} - b^{\lambda+2}}{a-b},$$

manifesto erit $Q'' = sQ' - pQ$, namque ob $s = a + b$ erit

$$sQ' = \frac{a^{\lambda+2} + ba^{\lambda+1} - ab^{\lambda+1} - b^{\lambda+2}}{a-b},$$

at ob $p = ab$ erit

$$pQ = \frac{a^{\lambda+1}b - ab^{\lambda+1}}{a-b},$$

qua forma ab illa ablata remanebit

$$sQ' - pQ = \frac{a^{\lambda+2} - b^{\lambda+2}}{a-b}.$$

Hac igitur lege obseruata habebimus sequentes transformationes:

$\frac{a-b}{a-b} = 1;$	}	$\frac{a^5 - b^5}{a-b} = s^4 - 3ps^3 + pp^3,$
$\frac{a^2 - b^2}{a-b} = s;$		$\frac{a^6 - b^6}{a-b} = s^5 - 4ps^3 + 3ppps,$
$\frac{a^3 - b^3}{a-b} = ss - p;$		$\frac{a^7 - b^7}{a-b} = s^6 - 5ps^4 + 6ppps - p^3,$
$\frac{a^4 - b^4}{a-b} = s^3 - qsp;$		$\frac{a^8 - b^8}{a-b} = s^7 - 6ps^5 + pp^3s - 4p^5s,$
etc.		

§. 7. Ordo, quo istae formulae progrediuntur, iam satis est perspicuus. Primo enim potestates ipsius s continuo binario de-

decreſcunt, contra vero ipſius p poteſtates unitate creſcunt, ſignis alternantibus; coëfficiens autem numerici cuiusque termini conueniunt cum iis, quos iidem termini in euolutione binomiali eſſent habiturae, vel, quod eodem reſultat, ii omnes permutationes litterarum p et s indicant, ita vt coëfficiens termini $p^\alpha s^\beta$ ſit $\frac{1 \cdot 2 \cdot 3 \cdot \dots \cdot (\alpha + \beta)}{1 \cdot 2 \cdot 3 \cdot \dots \cdot \alpha \times 1 \cdot 2 \cdot 3 \cdot \dots \cdot \beta}$. Hinc ergo deducimus transformationem ſequentem generalem:

$$\begin{aligned} \frac{a^{\lambda+1} - b^{\lambda+1}}{a - b} &= s^\lambda - \frac{(\lambda-1)}{1} p s^{\lambda-2} + \frac{(\lambda-2)(\lambda-3)}{1 \cdot 2} p p s^{\lambda-4} \\ &- \frac{(\lambda-3)(\lambda-4)(\lambda-5)}{1 \cdot 2 \cdot 3} p^3 s^{\lambda-6} + \frac{(\lambda-4)(\lambda-5)(\lambda-6)(\lambda-7)}{1 \cdot 2 \cdot 3 \cdot 4} p^4 s^{\lambda-8} \\ &- \frac{(\lambda-5)(\lambda-6)(\lambda-7)(\lambda-8)(\lambda-9)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} p^5 s^{\lambda-10} + \text{etc.} \end{aligned}$$

§. 8. Quodſi ergo hos valores in aequatione generali ſupra §. 4. data ſubſtituamus, aequatio generalis, cuius reſolutionem hac methodo exhibere licet, talem habebit formam:

$$\begin{aligned} x^n &= \frac{n(n-1)}{1 \cdot 2} p x^{n-2} + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} p s x^{n-3} + \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} p (s s - p) x^{n-4} \\ &+ \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} p (s^3 - 2 p s) x^{n-5} \\ &+ \frac{n \cdot \dots \cdot (n-5)}{1 \cdot 2 \cdot \dots \cdot 6} p (s^4 - 3 p s s + p p) x^{n-6} + \frac{n \cdot \dots \cdot (n-6)}{1 \cdot 2 \cdot \dots \cdot 7} p \times \\ &\times (s^5 - 4 p s^3 + 3 p p s) x^{n-7} + \text{etc.} \end{aligned}$$

Huius ſcilicet aequationis reſolutio, quicumque numeri pro p et s accipiantur, ſemper erit in poteſtate, eius quippe radix, poſtquam ex numeris p et s iſti fuerint deriuati:

$$a = \frac{s + \sqrt{(s s - 4 p)}}{2} \quad \text{et} \quad b = \frac{s - \sqrt{(s s - 4 p)}}{2},$$

ita exprimetur vt ſit $x = \frac{a^{\frac{n}{2}} b - b^{\frac{n}{2}} a}{\sqrt{a} - \sqrt{b}}$.

§. 9. Haec quidem formula vnicam radicem aequationis propoſitae nobis largitur, verum tamen facile hinc omnes plane radices eiusdem aequationis deducuntur, quarum quidem numerus

est $=n$. Primo enim ponendo $b=ak$, radix illa reuocabitur ad vnicum signum radicale, cum hinc fiat $x = \frac{a\sqrt[n]{k} - b}{1 - \sqrt[n]{k}}$. Nunc vero ista radix,

nempe $\sqrt[n]{k}$, valores diuerfos numero n admittit, quemadmodum etiam radix potestatis n ex vnitata, scilicet $\sqrt[n]{1}$, totidem diuerfos valores recipit, quorum vnus semper ipsi vnitati aequatur. Vnde si quilibet horum valorum designetur littera ϱ , ita vt fit $\varrho^n = 1$, ista littera ϱ inuoluet n diuerfos valores, quorum quemlibet cum formula $\sqrt[n]{k}$ coniungere licet, scribendo scilicet eius loco $\varrho\sqrt[n]{k}$, quamobrem omnes plane radices aequationis propositae in hac formula continebuntur: $x = \frac{\varrho a\sqrt[n]{k} - b}{1 - \varrho\sqrt[n]{k}}$,

siue $x = \frac{\varrho a\sqrt[n]{b} - b\sqrt[n]{a}}{\sqrt[n]{a} - \varrho\sqrt[n]{b}}$; tum vero si haec formula per diuisionem euoluatur, prodibit ista expressio:

$$x = \varrho\sqrt[n]{a^{n-1}b} + \varrho^2\sqrt[n]{a^{n-2}bb} + \varrho^3\sqrt[n]{a^{n-3}b^3} + \text{etc.}$$

cuius expressionis numerus terminorum est $n-1$, vltimo existente $\varrho^{n-1}\sqrt[n]{ab^{n-1}}$.

§. 10. Operae pretium erit hanc rem exemplo illustrasse. Sumamus igitur $n=5$, $s=1$ et $p=-1$, vt proponatur ista aequatio quinti gradus:

$$x^5 = -10x^3 - 10xx - 10x - 3, \text{ siue}$$

$$x^5 + 10x^3 + 10xx + 10x + 3 = 0.$$

Ad huius ergo aequationis radices inuestigandas, capiantur hi valores: $a = \frac{1+\sqrt{5}}{2}$; et $b = \frac{1-\sqrt{5}}{2}$, quibus inuentis erit quaelibet radix

$$x = \frac{\varrho a\sqrt[5]{b} - b\sqrt[5]{a}}{\sqrt[5]{a} - \varrho\sqrt[5]{b}}, \text{ vel introducendo litteram } k = \frac{b}{a} = -\frac{3+\sqrt{5}}{2},$$

erit

erit $x = \frac{\rho a \sqrt[5]{k-b}}{1-\rho \sqrt[5]{k}}$. Sin autem hanc formam euoluere velimus, ob $a b = p = -1$ reperiemus:

$$x = -\rho \sqrt[5]{a^3} + \rho^2 \sqrt[5]{a} + \rho^3 \sqrt[5]{b} - \rho^4 \sqrt[5]{b^3},$$

quae expressio penitus in numeris euoluta praebet

$$x = -\rho \sqrt[5]{(2+\sqrt{5})} + \rho^2 \sqrt[5]{\left(\frac{1+\sqrt{5}}{2}\right)} + \rho^3 \sqrt[5]{\left(\frac{1-\sqrt{5}}{2}\right)} - \rho^4 (2-\sqrt{5})$$

Demonstratio. formularum supra datarum.

§. 11. Analysis, quae ad istas aequationes perduxit, maxime est obuia, ita vt vix quicquam in recessu habere videatur: tota enim petita est ex hac aequatione simplicissima: $\frac{(a+x)^n}{(b+x)^n} = \frac{a}{b}$. Cum enim hinc fiat $\frac{a+x}{b+x} = \sqrt[n]{\frac{a}{b}}$, inde colligitur incognita:

$$x = \frac{a - b \sqrt[n]{\frac{a}{b}}}{\sqrt[n]{\frac{a}{b}} - 1} = \frac{a \sqrt[n]{b} - b \sqrt[n]{a}}{\sqrt[n]{a} - \sqrt[n]{b}}$$

quae est ea ipsa radix quam pro aequationibus superioribus assignauimus.

§. 12. Quodsi vero aequationem illam assumtam euoluamus, quoniam inde fieri debet $a(x+b)^n = b(x+a)^n$, siue $a(x+b)^n - b(x+a)^n = 0$, hinc deriuabitur sequens aequatio:

$$\left. \begin{aligned} & a x^n + \frac{n}{1} a b x^{n-1} + \frac{n(n-1)}{1 \cdot 2} a b b x^{n-2} \\ & + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} a b^3 x^{n-3} + \text{etc.} \\ - & b x^n - \frac{n}{1} a b x^{n-1} - \frac{n(n-1)}{1 \cdot 2} a a b x^{n-2} \\ & - \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} a^3 b x^{n-3} - \text{etc.} \end{aligned} \right\} = 0,$$

vbi

vbi membra secunda se mutuo tollunt. Iam quia primum membrum afficitur per $a - b$, reliqua membra in alteram partem transferantur, ac per $a - b$ diuidantur, sicque emerget sequens aequatio :

$$x^n = \frac{n(n-1)}{1 \cdot 2} a b \left(\frac{a-b}{a-b}\right) x^{n-2} + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} a b \left(\frac{a-b}{a-b}\right)^2 x^{n-3} \\ + \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} a b \left(\frac{a-b}{a-b}\right)^3 x^{n-4} + \text{etc.}$$

quae est ipsa aequatio generalis supra tractata, cuius ergo radix est $x = \frac{a\sqrt[n]{b} - b\sqrt[n]{a}}{\sqrt[n]{a} - \sqrt[n]{b}}$.

§. 13. Hinc forte quispiam expectare posset, simili modo huiusmodi aequationes generaliores obtineri posse, si loco illius formulae simplicissimae haec formula latius patens: $\frac{(f+x)^n}{(g+x)^n} = \frac{a}{b}$, fundamenti loco constituatur, siquidem hic quatuor quantitates arbitrariae a, b, f et g , in computum introducuntur, cum ante binæ tantum a et b inessent; verum tamen quomocunque litterae f et g a litteris a et b diuersae accipiantur, tamen casus semper ad priorem simpliciozem reduci potest. Ad hoc ostendendum ponamus $x = a + \beta z$, et aequatio nostra fiet $\frac{(a+f+\beta z)^n}{(a+g+\beta z)^n} = \frac{a}{b}$, siue

$$\frac{\left(\frac{a+f}{\beta} + z\right)^n}{\left(\frac{a+g}{\beta} + z\right)^n} = \frac{a}{b}; \text{ atque nunc manifestum est, quantitates } a$$

et β semper ita capi posse, vt fiat $\frac{a+f}{\beta} = a$ et $\frac{a+g}{\beta} = b$, quandoquidem hinc deducitur $a = \frac{bf-ag}{a-\beta}$, ideoque $\beta = \frac{f-g}{a-b}$. Sicque formula illa, quae multo generalior videbatur, semper ad simplicissimam illam supra tractatam reuocari potest, neque idcirco quicquam noui inde est expectandum.

Annotatio.

in aequationes supra euolutas.

§. 14. Si formas, quas pro radicibus harum aequationum supra assignauimus, accuratius perpendamus, haec omnia egregie conuenire deprehenduntur, cum coniectura illa, quam olim in medium proferre sum ausus, dum pro resolutione aequationis cuiuscunque gradus, in qua secundus terminus desit, veluti

$$x^n = p x^{n-2} + q x^{n-3} + r x^{n-4} + \text{etc.}$$

affirmaui, semper dari aequationem resoluentem vno gradu inferiori, huius formae:

$$y^{n-1} - A y^{n-2} + B y^{n-3} - C y^{n-4} + D y^{n-5} - \text{etc.} = 0,$$

cuius radices, numero $n - 1$, si fuerint $\alpha, \beta, \gamma, \delta, \epsilon, \text{etc.}$ futurum fit

$$x = \sqrt[n]{\alpha} + \sqrt[n]{\beta} + \sqrt[n]{\gamma} + \sqrt[n]{\delta} + \text{etc.}$$

§. 15. Cum igitur pro forma generali, quam supra tractauimus, radix inuenta fit

$$x = \sqrt[n]{a^{n-1} b} + \sqrt[n]{a^{n-2} b b} + \sqrt[n]{a^{n-3} b^3} - - - + \sqrt[n]{a b^{n-1}},$$

hinc sequitur aequationis resoluentis ordinis $n - 1$ radices fore $a^{n-1} b; a^{n-2} b b; a^{n-3} b^3; a^{n-4} b^4; - - - - a b^{n-1}$, quae ergo erunt valores ipsius y . Quare cum coefficientis A sit summa omnium harum radicum, erit $A = \frac{a b (a^{n-1} - b^{n-1})}{a - b}$,

postremum autem huius aequationis membrum absolutum erit productum ex omnibus his radicibus, quod ergo erit

$= a^{\frac{n n - n}{2}} \times b^{\frac{n n - n}{2}}$. Pro reliquis terminis percurramus aequationes particulares supra expositas.

I. Pro aequatione tertii gradus:

$$x^3 = 3abx + ab(a+b)$$

vbi erat radix

$$x = \sqrt[5]{a^2ab} + \sqrt[5]{ab^2b}$$

Hic si aequatio resoluens statuatur

$$yy - Ay + B = 0,$$

eius radices erunt aab et abb , ideoque $A = ab(a+b)$
et $B = a^3b^3$.

II. Pro aequatione quarti gradus:

$$x^4 = 6abxx + 4ab(a+b)x + ab(aa+ab+bb)$$

Hic est radix

$$x = \sqrt[4]{a^3b} + \sqrt[4]{aabb} + \sqrt[4]{ab^3},$$

vnde si aequatio resoluens statuatur

$$y^3 - Ayy + By - C = 0,$$

eius radices erunt a^3b ; $aabb$; ab^3 ; quocirca habebimus

$$A = ab(aa+ab+bb),$$

$$B = a^3b^3(aa+ab+bb) \text{ et}$$

$$C = a^6b^6.$$

III. Pro aequatione quinti gradus:

$$x^5 = 10abx^3 + 10ab(a+b)xx + 5ab(aa+ab+bb)x + ab(a^3+aab+abb+b^3),$$

Hic igitur erit

$$x = \sqrt[5]{a^4b} + \sqrt[5]{a^3bb} + \sqrt[5]{aabb^3} + \sqrt[5]{ab^4},$$

vnde si aequatio resoluens statuatur:

$$y^4 - Ay^3 + Byy - Cy + D = 0,$$

eius radices erunt, a^4b ; a^3bb ; aab^3 ; ab^4 ; vnde colligitur
fore

fore $A = ab(a^3 + aab + abb + b^3),$
 $B = a^3b^3(a^4 + a^3b + 2aabb + ab^3 + b^4),$
 $C = a^6b^6(a^3 + aab + abb + b^3),$
 $D = a^{10}b^{10}.$

IV. Pro aequatione sexti gradus:

$$x^6 = 15abx^4 + 20ab(a+b)x^3 + 15ab(aa+ab+bb)xx + 6ab(a^3+aab+abb+b^3) + ab(a^4+a^3b+aabb+ab^3+b^4),$$

Hic igitur habebitur

$$x = \sqrt[6]{a^5b} + \sqrt[6]{a^4bb} + \sqrt[6]{a^3b^3} + \sqrt[6]{aabb^4} + \sqrt[6]{ab^5}.$$

vnde si aequatio resolvens statuatur

$$y^5 - Ay^4 + By^3 - Cy^2 + Dy - E = 0,$$

eius radices erunt $a^5b; a^4bb; a^3b^3; aabb^4; ab^5$, vnde colligitur fore

$$A = ab(a^4 + a^3b + aabb + ab^3 + b^4),$$

$$B = a^3b^3(a^6 + a^5b + 2a^4bb + 2a^3b^3 + 2aabb^4 + ab^5 + b^6),$$

$$C = a^6b^6(a^6 + a^5b + 2a^4bb + 2a^3b^3 + 2aabb^4 + ab^5 + b^6),$$

$$D = a^{10}b^{10}(a^4 + a^3b + aabb + ab^3 + b^4),$$

$$E = a^{15}b^{15}.$$

vbi formulae mediae B et C ita concinnius exprimi possunt:

$$B = a^3b^3(aa+bb)(a^4+a^3b+aabb+ab^3+b^4) \text{ et}$$

$$C = a^6b^6(aa+bb)(a^4+a^3b+aabb+ab^3+b^4),$$

quae determinationes fortasse aliquam lucem accendere possunt ad resolutionem aequationum generalem feliciori successu tractandam.