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De motu libero plurium corporum filis colligatorum super plano horizontali

Leonhard Euler

DE MOTV LIBERO
PLVRIVM CORPORVM FILIS COLLIGATORVM SVPER PLANO HORIZONTALI.

Auctore
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Problema 1.

§. 1.

Si duo corpora A *et* B, *filo* $AB = a$ *colligata, super plano horizontali vtcunque proiiciantur, eorum motum determinare.* Tab. IV. Fig. 1.

Solutio.

Elapso tempore t habeant corpora situm in figura repraesentatum, et pro vtroque ponantur coordinatae

$$OP = x,\ PA = y \text{ et } OQ = x' \text{ et } QB = y';$$

porro ponatur angulus $BAp = p$, eritque

$$Ap = a\cos. p \text{ et } Bp = a\sin. p,$$

vnde fit

$$x' = x + a\cos. p \text{ et } y' = y + a\sin. p.$$

 Iam

Iam sit tensio fili $AB = P$, qua corpus A secundum directiones suarum coordinatarum protrahitur viribus $P \cos. p$ et $P \sin. p$; corpus vero B iisdem viribus retrahitur; vnde principia motus sequentes praebent aequationes:

$$\text{I. } \frac{A\, ddx}{2g\, dt^2} = P \cos. p;$$

$$\text{II. } \frac{A\, ddy}{2g\, dt^2} = P \sin. p;$$

$$\text{III. } \frac{B\, ddx'}{2g\, dt^2} = -P \cos. p;$$

$$\text{IV. } \frac{B\, ddy'}{2g\, dt^2} = -P \sin. p.$$

Hinc iam statim prima ac tertia additae dant

$$A\, ddx + B\, ddx' = 0$$

et secunda et quarta dat

$$A\, ddy + B\, ddy' = 0;$$

ex quibus integratis colligitur

$$Ax + Bx' = \alpha t + \beta;$$

$$Ay + By' = \gamma t + \delta.$$

Hinc cognoscimus, ambo corpora ita moueri, vt eorum commune centrum grauitatis in linea recta vniformiter progrediatur. Quod si nunc toto spatio aequalem motum in directionem contrariam mente imprimamus, centrum grauitatis in quiete manebit, quod ergo ponamus esse in ipso puncto O, ac pro hoc casu motum corporum inuestigemus, quo inuento, centro grauitatis iterum motus vniformis rectilineus, quem dempsimus, imprimatur, et prodibit verus motus amborum corporum, hocque modo obtinebimus vt fiat:

$$Ax + Bx' = 0 \text{ et } Ay + By' = 0.$$

Cum igitur sit

$$x' = x + a \cos. p \text{ et } y' = y + a \sin. p;$$

hinc

hinc fiet

$$(A+B)x+Ba\,\text{cof.}\,p=0 \text{ et } (A+B)y+Ba\,\text{fin.}\,p$$

vnde porro colligitur

$$(A+B)^2(xx+yy)=BBaa, \text{ ideoque}$$

$$\sqrt{(xx+yy)}=\frac{Ba}{A+B},$$

vbi $\sqrt{(xx+yy)}$ denotat diſtantiam corporis A a centro O, quae ergo manet conſtans, perinde ac diſtantia alterius corporis B ab O. Sit igitur A O B ſitus amborum corporum poſt tempus $=t$, eritque A O B linea recta $=a$; ac diſtantiae Tab. IV. Fig. 2.

$$AO=\frac{Ba}{A+B} \text{ et } BO=\frac{Aa}{A+B}.$$

Supereſt ergo tantum vt angulus A O P vel B O Q, quem filum A B cum axe conſtituit, definiatur; hic vero angulus cum ſit p, ex aequationibus prima et ſecunda definiri poteſt, vnde fit:

$$A\,ddx\,\text{fin.}\,p - A\,ddy\,\text{cof.}\,p=0, \text{ ſiue}$$

$$ddx\,\text{fin.}\,p - ddy\,\text{cof.}\,p=0.$$

Quare, cum ex ſupra inuentis ſit

$$x=-\frac{Ba\,\text{cof.}\,p}{A+B} \text{ et } y=-\frac{Ba\,\text{fin.}\,p}{A+B},$$

his valoribus ſubſtitutis fiet

$$-\text{fin.}\,p.\,dd.\,\text{cof.}\,p+\text{cof.}\,p.\,dd.\,\text{fin.}\,p=0.$$

Eſt vero:

$$dd.\,\text{cof.}\,p=-ddp\,\text{fin.}\,p-dp^2\,\text{cof.}\,p \text{ et}$$

$$dd.\,\text{fin.}\,p=ddp\,\text{cof.}\,p-dp^2\,\text{fin.}\,p,$$

vnde fit $ddp=0$, ſicque adipiſcimur $p=\alpha t+\beta$; vnde diſcimus, celeritatem angularem fili A B, quae eſt $\frac{dp}{dt}=\alpha$, eſſe conſtantem, quocirca ſolutio noſtri problematis ita ſe habet: Quomodocunque noſtra corpora filo A B colligata

 pro-

proiiciantur, eorum motus ita erit comparatus, vt eorum commune centrum grauitatis g vniformiter in directum progrediatur, interea vero ambo corpora circa hoc ipsum punctum g vniformiter gyrentur, prouti scilicet motus primo impressus postulat.

Problema 2.

Tab. IV. Fig. 3. §. 2. *Si tria corpora* A, B, C, *filis* AB $=$ a *et* BC $=$ b *connexa, vtcunque super plano horizontali proiiciantur, eorum motum inuestigare.*

Solutio.

Ponantur pro singulis corporibus coordinatae

$$OP = x,\ PA = y;\ OQ = x',\ QB = y';$$
$$OR = x'',\ RC = y'';$$

tum vero inclinatio filorum, scilicet anguli $BAp = p$ et $CBq = q$, hincque statim fit

$$x' = x + a\,\text{cof.}\,p;\qquad x'' = x + a\,\text{cof.}\,p + b\,\text{cof.}\,q$$
$$y' = y + a\,\text{fin.}\,p\qquad y'' = y + a\,\text{fin.}\,p + b\,\text{fin.}\,q.$$

Porro denotet P tensionem fili AB et Q tensionem fili BC, ex quibus oriuntur sequentes aequationes:

$$\text{I.}\ \frac{A\,ddx}{2g\,dt^2} = P\,\text{cof.}\,p;$$
$$\text{II.}\ \frac{A\,ddy}{2g\,dt^2} = P\,\text{fin.}\,p;$$
$$\text{III.}\ \frac{B\,ddx'}{2g\,dt^2} = -P\,\text{cof.}\,p + Q\,\text{cof.}\,q;$$
$$\text{IV.}\ \frac{B\,ddy'}{2g\,dt^2} = -P\,\text{fin.}\,p + Q\,\text{fin.}\,q;$$
$$\text{V.}\ \frac{C\,ddx''}{2g\,dt^2} = -Q\,\text{cof.}\,q;$$
$$\text{VI.}\ \frac{C\,ddy''}{2g\,dt^2} = -Q\,\text{fin.}\,q;$$

qua-

quarum prima, tertia et quinta additae manifesto praebent

$$A\,dd\,x + B\,dd\,x' + C\,dd\,x'' = 0;$$

similique modo II. IV et VI additae praebent

$$A\,dd\,y + B\,dd\,y' + C\,dd\,y'' = 0;$$

quibus vt ante motus aequabilis rectilineus centri grauitatis communis indicabitur, qui motus cum iam vt cognitus spectari possit, centrum grauitatis quasi in O fixum iam concipiamus, hincque habebimus has aequationes:

$$A\,x + B\,x' + C\,x'' = 0 \text{ et } A\,y + B\,y' + C\,y'' = 0;$$

hincque porro colligimus sequentes aequationes:

$$(A+B+C)\,x + (B+C)\,a \cos. p + C\,b \cos. q = 0 \text{ et}$$

$$(A+B+C)\,y + (B+C)\,a \sin. p + C\,b \sin. q = 0.$$

vnde ipsae coordinatae sequenti modo exprimentur:

$$x = \frac{-(B+C)\,a \cos. p - C\,b \cos. q}{A+B+C}; \quad y = \frac{-(B+C)\,a \sin. p - C\,b \sin. q}{A+B+C}$$

$$x' = \frac{A\,a \cos. p - C\,b \cos. q}{A+B+C}; \quad y' = \frac{A\,a \sin. p - C\,b \sin. q}{A+B+C}$$

$$x'' = \frac{A\,a \cos. p + (A+B)\,b \cos. q}{A+B+C}; \quad y'' = \frac{A\,a \sin. p + (A+B)\,b \sin. q}{A+B+C}$$

Nunc igitur superest vt bini anguli p et q definiantur. Hunc in finem ex aequationum I. et II. eliminemus tensionem P, vnde fit $dd\,x \sin. p - dd\,y \cos. p = 0$; similique modo ex V et VI, eliminando tensionem Q, habebimus

$$dd\,x'' \sin. q - dd\,y'' \cos. q = 0;$$

quarum prior, restitutis valoribus, abit in sequentem:

$$\left.\begin{array}{l} +(B+C)\,a\,(\cos. p.\, dd. \sin. p - \sin. p.\, dd. \cos. p) \\ +C\,b\,(\cos. p.\, dd. \sin. q - \sin. p.\, dd. \cos. q) \end{array}\right\} = 0$$

posterior vero eodem modo tractata praebet:

+ (A-

$$\left.\begin{array}{l}+(A+B)b(\sin. q.\, dd.\cos. q-\cos. q.\, dd.\sin. q)\\ +Aa(\sin. q.\, dd.\cos. p-\cos. q.\, dd.\sin. p)\end{array}\right\}=0.$$

Ad has aequationes resoluendas notemus esse:

$$\cos. p.\, dd.\sin. p-\sin. p.\, dd.\cos. p=ddp$$

$$\cos. p.\, dd.\sin. q-\sin. p.\, dd.\cos. q=ddq\cos.(q-p)$$
$$-dq^2\sin.(q-p)$$

$$\sin. q.\, dd.\cos. q-\cos. q.\, dd.\sin. q=-ddq$$

$$\sin. q\, dd.\cos. p-\cos. q.\, dd.\sin. p=-ddp\cos.(q-p)$$
$$-dp^2\sin.(q-p).$$

Hi ergo valores in superioribus aequationibus substituti praebent istas:

$$(B+C)a\,ddp+Cb(ddq\cos.(q-p)-dq^2\sin.(q-p))=0$$

et

$$-(A+B)b\,ddq-Aa(ddp\cos.(q-p)+dp^2\sin(q-p))=0.$$

Ponamus

$$\frac{(B+C)a}{Cb}=m \text{ et } \frac{(A+B)b}{Aa}=n,$$

vt habeamus has duas aequationes:

$$1^\circ.\; m\,ddp+ddq\cos.(q-p)-dq^2\sin.(q-p)=0$$

$$2^\circ.\; n\,ddq+ddp\cos.(q-p)+dp^2\sin.(q-p)=0$$

ex quibus ambos angulos incognitos p et q elicere oportet, id quod sequenti modo succedet.

Integrentur hae duae aequationes, quod fieri licet more solito, ac reperietur:

$$m\,dp+dq\cos.(q-p)-\int dp\,dq\sin.(q-p)=\text{const.}$$

$$n\,dq+dp\cos.(q-p)+\int dp\,dq\sin.(q-p)=\text{const.}$$

vnde patet, summam harum formularum a formulis integralibus fore liberam, ita vt hinc adipiscamur hanc ae-

qua-

quationem integratam:

$$3^{o}.\ m\,dp + n\,dq + (dp + dq)\,\mathrm{cos}.\,(q-p) = \tfrac{1}{2}\alpha\,dt.$$

Deinde vero ista combinatio: $1^a.\ dp + 2^a.\ dq$ fit integrabilis et praebet hanc aequationem:

$$\tfrac{1}{2} m\,dp^2 + \tfrac{1}{2} n\,dq^2 + dp\,dq\,\mathrm{cos}.\,(q-p) = \tfrac{1}{8}\beta\,dt^2,$$

ita vt nunc loco binarum aequationum differentialium secundi gradus habeamus sequentes duas aequationes tantum primi gradus:

$$\text{I. } 2m\,dp + 2n\,dq + 2(dp + dq)\,\mathrm{cos}.\,(q-p) = \alpha\,dt;$$

$$\text{II. } 4m\,dp^2 + 4n\,dq^2 + 4\,dp\,dq\,\mathrm{cos}.\,(q-p) = \beta\,dt^2;$$

quarum tamen vlterior resolutio non parum dexteritatis postulat. Sequenti autem modo negotium expediri poterit.

Faciamus scilicet sequentes substitutiones: Primo fiat $q - p = \Phi$, vt fit $dq - dp = d\Phi$; ac ponamus porro

$$dp = (\tfrac{u-1}{2})\,d\Phi \text{ et } dq = (\tfrac{u+1}{2})\,d\Phi;$$

denique vero etiam ponatur $dt = \theta\,d\Phi$, vt omnia elementa ad idem differentiale $d\Phi$ reducamus, hocque modo nostrae aequationes induent formas sequentes:

$$\text{I. } (m+n)\,u + n - m + 2u\,\mathrm{cos}.\,\Phi = \alpha\,\theta$$

$$\text{II. } (m+n)\,uu + 2(n-m)\,u + m + n + 2(uu-1)\,\mathrm{cos}.\,\Phi = \beta\,\theta\,\theta$$

ex quarum priore colligitur $u = \frac{\alpha\theta + m - n}{m + n + 2\,\mathrm{cos}.\,\Phi}$.

Iam cum aequatio altera sit

$$uu(m + n + 2\,\mathrm{cos}.\,\Phi) + 2u(n-m) + m + n - 2\,\mathrm{cos}.\,\Phi = \beta\,\theta\,\theta,$$

in hac loco u valor modo inuentus substituatur et prodibit

$$\beta\,\theta^2 = \frac{\alpha\alpha\,\theta\theta - 2(n-m)\,\alpha\,\theta + (n-m)^2}{m+n+2\,\mathrm{cos}.\,\Phi} + \frac{2(n-m)\,\alpha\,\theta - 2(n-m)^2}{m+n+2\,\mathrm{cos}.\,\Phi} + m + n - 2\,\mathrm{cos}.\,\Phi\,,$$

quae porro reducitur ad hanc formam:

$$\alpha\alpha\,\theta\theta + 4\,m\,n - 4\,\mathrm{cos}.\,\Phi^2 = \beta\,\theta\theta\,(m+n+2\,\mathrm{cos}.\,\Phi),$$

ex qua aequatione commode elicitur

$$\theta\,\theta = \frac{4\,(m\,n - \mathrm{cos}.\,\Phi^2)}{\beta\,(m+n+2\,\mathrm{cos}.\,\Phi) - \alpha\,\alpha},$$ ita vt sit

$$\theta = \frac{2\,\sqrt{(m\,n - \mathrm{cos}.\,\Phi^2)}}{\sqrt{(\beta\,(m+n+2\,\mathrm{cos}.\,\Phi) - \alpha\,\alpha)}}.$$

Quia ergo posuimus $d\,t = \theta\,d\,\Phi$, erit

$$d\,t = \frac{2\,d\,\Phi\,\sqrt{(m\,n - \mathrm{cos}.\,\Phi^2)}}{\sqrt{(\beta\,(m+n+2\,\mathrm{cos}.\,\Phi) - \alpha\,\alpha)}}.$$

Sicque iam habemus relationem inter tempus t et angulum Φ, ita vt inde ad quoduis tempus angulus Φ definiri possit.

Quodsi iam loco θ hunc valorem substituamus, nanciscemur pro u istam formulam:

$$u = \frac{(n-m)}{m+n+2\,\mathrm{cos}.\,\Phi} + \frac{2\,\alpha\,\sqrt{(m\,n - \mathrm{cos}.\,\Phi^2)}}{m+n+2\,\mathrm{cos}.\,\Phi\,\sqrt{(\beta\,(m+n+2\,\mathrm{cos}.\,\Phi) - \alpha\,\alpha)}},$$

ita vt hic u per solum angulum Φ definiatur. Hinc ergo quaeratur integrale $\int u\,d\,\Phi$, quo inuento innotescent ambo anguli p et q: erit enim

$$p = \tfrac{1}{2}\int u\,d\,\Phi - \tfrac{1}{2}\Phi \quad \text{et} \quad q = \tfrac{1}{2}\int u\,d\,\Phi + \tfrac{1}{2}\Phi,$$

vbi integrale $\int u\,d\,\Phi$ nouam quantitatem constantem includit; quemadmodum etiam $t \int \theta\,d\,\Phi$ constantem arbitrariam complectitur, ita vt cum literis α et β omnino quatuor constantes arbitrariae in nostra solutione contineantur, prorsus vt integratio completa postulat. Statim enim deducti sumus ad sex aequationes differentiales, quarum duae autem inseruiebant vtrique tensioni P et Q definiendis, ita vt tantum quatuor ipsam solutionem contineant; at at vero duae aequationes integrales initio ſtatim inuentae

$$A x + B x' + C x'' = \mathfrak{A} t + \mathfrak{B} \text{ et}$$
$$A y + B y' + C y'' = \mathfrak{C} t + \mathfrak{D}$$

iam continebant quatuor conſtantes arbitrarias, etiamſi eas nihilo aequales aſſumſimus, vt commune centrum grauitatis ad quietem redigeremus; vnde patet, per quatuor illas conſtantes nunc introductas ſolutionem completam reddi.

Quod autem ad iſtas conſtantes attinet, manifeſtum eſt conſtantem β neque euaneſcentem neque negatiuam accipi poſſe, quia aliquoquin formula pro tempore fieret imaginaria; quin etiam ſemper eſſe debet

$$\beta > \frac{\alpha\alpha}{m + n + 2 \operatorname{cof.} \Phi};$$

ac ſi angulus Φ vsque ad 180° augeri poſſit, tum eſſe oportet $\beta > \frac{\alpha\alpha}{m + n - 1}$. Circa quantitates autem m et n notaſſe iuuabit eſſe $m n = \frac{(A + B)(B + C)}{A C}$, quae quantitas ſemper vnitate maior eſt, niſi fuerit $B = 0$, qui autem caſus ad problema prius reuolueretur; tum vero erit

$$m + n = \frac{a a A (B + C) + b b C (A + B)}{A C a b},$$

quae quantitas in infinitum augeri poteſt, ſi fiat vel $a = 0$ vel $b = 0$, minima autem euadet caſu quo $\frac{a}{b} = \sqrt{\frac{c (A + B)}{A (B + C)}}$, tum autem eius valor minimus erit $= 2 \sqrt{\frac{(A + B)(B + C)}{A C}}$, qui ergo ſemper binario eſt maior.

At ſi ſumere velimus tam $\alpha = 0$ quam $\beta = 0$, peculiarem hic caſus euolutionem poſtulat, cum inde ſit $4 m n - 4 \operatorname{cof.} \Phi^2 = 0$; inde enim ſit $m n = \operatorname{cof.} \Phi^2$, quod

autem

autem ob $m n > 1$ nunquam fieri poteſt, niſi ſit $B = 0$, hoc eſt niſi corpus B abſit, quo caſu fieret $\Phi = 0$ vel $\Phi = 180°$, hincque $u = \frac{m - n}{m + n \pm 2}$; foret autem $m + n > 2$. Hoc igitur caſu nullus plane motus ſequeretur, ſed omnia tria corpora in ſtatu quietis perpetuo perſeuerarent.

Poſtquam autem motum trium corporum A, B, C feliciter determinare nobis contigit, operae quoque pretium erit tenſionem vtriusque fili inueſtigare, quam ex ipſis primis aequationibus elici oportet, vbi

I. coſ. p + II. ſin. p dat $P = \frac{A (d d x \cos. p + d d y \sin. p)}{2 g d t^2}$.

Cum igitur ſit

$$x = \frac{-(B + C) a \cos. p - C b \cos. q}{A + B + C} \text{ et}$$

$$y = \frac{-(B + C) a \sin. p - C b \sin. q}{A + B + C}$$

erit pro tenſione

$$\frac{P}{A} = \frac{-a (B+C)(\cos. p . d d \cos. p + \sin. p d d. \sin. p) - b C (\cos. q . d d. \cos. p + \sin. q d d. \sin. p)}{(A + B + C) 2 g d t^2}$$

quae aequatio euoluta praebet

$$\frac{P}{A} = \frac{a (B + C) d p^2 - b C (d d p \sin. (q - p) - d p^2 \cos. (q - p))}{(A + B + C) 2 g d t^2}.$$

Statuamus vt ſupra breuitatis gratia $\frac{(B + C) a}{C b} = m$, fietque

$$\frac{P}{A C b} = \frac{m d p^2 - d d p \sin. (q - p) + d p^2 \cos. (q - p)}{(A + B + C) 2 g d t^2}.$$

Vtamur hic porro ſuperioribus valoribus introductis ſcil. $q - p = \Phi$, $d p = \frac{1}{2}(u - 1) d \Phi$, $d q = \frac{1}{2}(u + 1) d \Phi$, et $d t = \theta d \Phi$, eritque $\frac{d p}{d t} = \frac{u - 1}{2 \theta}$, hinc $\frac{d d p}{d t} = \frac{1}{2} d. \frac{u - 1}{\theta}$ et

$$\frac{d d p}{d t^2} = \frac{1}{2 \theta d \Phi} d. \frac{u - 1}{\theta},$$

quibus valoribus ſubſtitutis habebimus:

P

$$\frac{P}{ACb} = \frac{m(u-1)^2 - \frac{2\theta}{d\Phi} d.\frac{u-1}{\theta} \text{ſin.}\,\Phi + (u-1)^2 \text{coſ.}\,\Phi}{8 g \theta \theta (A + B + C)}$$

vnde tenſio quaeſita erit:

$$P = \frac{b\,A\,C}{8 g (A + B + C)\theta\theta}\left(m(u-1)^2 - \frac{2\theta}{d\Phi} d.\frac{u-1}{\theta} \text{ſin.}\Phi + (u-1)^2 \text{coſ.}\Phi\right)$$

vbi, quia literas u et θ per angulum Φ determinauimus, tota haec expreſſio ad quantitates finitas reducetur. Eodem autem modo etiam altera tenſio Q definiri poterit, neque vero opus erit has ſubſtitutiones actu euoluere, cum inde nullae formulae concinnae expectari queant.

Caſus ſpecialioris euolutio.

§. 3. Illuſtremus ſolutionem noſtri Problematis caſu ſimpliciſſimo, quo tria corpora A, B, C ſunt inter ſe aequalia; tum vero ſint etiam ambo fila A et B eiusdem longitudinis, ac primo pro ſingulis coordinatis habebimus ſequentes valores:

$$x = -\tfrac{1}{3} a (2 \text{coſ.} p + \text{coſ.} q);\quad y = -\tfrac{1}{3} a (2 \text{ſin.} p + \text{ſin.} q);$$
$$x' = \tfrac{1}{3} a (\text{coſ.} p - \text{coſ.} q);\quad y' = \tfrac{1}{3} a (\text{ſin.} p - \text{ſin.} q);$$
$$x'' = \tfrac{1}{3} a (\text{coſ.} p + 2 \text{coſ.} q);\quad y'' = \tfrac{1}{3} a (\text{ſin.} p + 2 \text{ſin.} q);$$

vnde vtique ſequitur fore

$$x + x' + x'' = 0 \text{ et } y + y' + y'' = 0,$$

quemadmodum ſcilicet hypotheſis noſtra poſtulat, qua commune centrum grauitatis trium corporum in puncto O ad quietem reduximus, ita vt tota determinatio ad ambos angulos p et q ſit perducta; pro quibus inueniendis, ob numeros $n = m = 2$, ſolutio generalis ſupra data ita omnia ad angulum Φ accommodat, vt ſit

$$1^{\circ}.\ dt = \frac{2\, d\Phi \sqrt{(4 - \text{coſ.}\,\Phi^2)}}{\sqrt{(\beta(4 + 2\,\text{coſ.}\,\Phi) - \alpha\alpha)}};$$

tum vero, ſumto

$$u = \frac{\alpha \surd (4 - cof. \Phi^2)}{2 + cof. \Phi \surd (\beta (4 + 2 cof. \Phi) - \alpha \alpha)},$$

ex hoc valore nanciſcimur:

$$p = \tfrac{1}{2} \int u \, d\Phi - \tfrac{1}{2} \Phi \text{ et } q = \tfrac{1}{2} \int u \, d\Phi + \tfrac{1}{2} \Phi.$$

§. 4. Quod ſi eadem methodo motum plurium corporum, filis connexorum inueſtigare velimus, nullum eſt dubium, quin ſimilibus artificiis in ſubſidium vocandis tota ſolutio ad ternas aequationes differentiales primi gradus reduci queat, in quibus ſcilicet inſint terni anguli p, q et r, ſub quibus terna fila ad axem inclinantur. Verum vtcunque labor iſte ſucceſſerit, ſemper ad formulas vehementer intricatas perueniri neceſſe eſt, quam ob cauſam iſtam inueſtigationem vlterius non proſequor.