1780

Investigatio perturbationum quae in motu terrae ab actione Veneris producunter: cum tabula perturbationum istarum

Leonhard Euler

INVESTIGATIO PERTVRBATIONVM, QVAE IN MOTV TERRAE AB ACTIONE VENERIS PRODVCVNTVR.

Auctore
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§. 1.

Tab. XIII. Fig. 2. Existente Sole in S sit A T orbita Terrae, B V Veneris, ambae in plano eclipticae sitae. Sumamus autem initio, vnde tempora metimur, ambos Planetas fuisse in coniunctione i. e. in A et B; nunc vero elapso tempore t, cui motus Terrae medius respondeat $= \theta$, Terram versari in T, venerem vero in V, vocemusque angulos A S T $= \Phi$ et B S V $= \psi$; tum vero sit angulus T S V $= \eta$, ita vt sit $\eta = \psi - \Phi$, et iam η designet elongationem Veneris a Terra, ex Sole visam. Praeterea vocetur distantia Terrae a Sole S T $= v$, Veneris autem distantia S V vt constans spectetur, sitque S V $= a$. Denique statuatur distantia Veneris a Terra T V $= w$, ita vt $w = \sqrt{vv + aa - 2av \cos. \eta}$.

§. 2. Exprimatur jam massa Solis per vnitatem sitque massa Terrae $= m$, quam ex Parallaxi Solis conclusiclusimus $= \frac{1}{1000000}$, eique massam Veneris aequalem supponamus. His positis Terra ad Solem sollicitabitur in directione T S, vi $= \frac{m+1}{vv}$ et a Venere sollicitabitur in directione T V, vi $= \frac{m}{ww}$. Denique quia etiam Sol, a Venere vrgetur vi $= \frac{m}{aa}$, haec vis contrario modo, secundum directionem V S, Terrae est applicanda. Has autem ternas vires ad duas revocare licet, complendo parallologrammum S T O V; tum enim vis T V $= \frac{m}{w \cdot w}$ resoluetur in vim secundum T S $= \frac{mv}{w^3}$ et in vim secundum T O $= \frac{ma}{w^3}$, cuius directio conuenit cum directione S V. Hinc ergo omnino Terra sollicitabitur in directione T S,

$$vi = \frac{1+m}{vv} + \frac{mv}{w^3};$$

tum vero etiam in directione V S,

$$vi = \frac{m}{aa} - \frac{ma}{w^3}.$$

§. 3. Inuentis his viribus ex T ad axem S A demittatur perpendiculum T X, et vocentur binae coordinatae S X $= x$ et X T $= y$, secundum quas ambae vires sollicitantes, resoluantur, vnde orietur vis secundum S X

$$= -\frac{(1+m)\, cos.\, \Phi}{vv} - \frac{mv\, cos.\, \Phi}{w^3} - \frac{m\, cos.\, \psi}{aa} + \frac{ma\, cos.\, \psi}{w^3}$$

et vis secundum X T

$$= -\frac{(1+m)\, sin.\, \Phi}{vv} - \frac{mv\, sin.\, \Phi}{w^3} - \frac{m\, sin.\, \psi}{aa} + \frac{ma\, sin.\, \psi}{w^3}$$

quibus viribus cum accelerationes debeant esse aequales, quae sunt secundum easdem directiones $\frac{ddx}{d\theta^2}$ & $\frac{ddy}{d\theta^2}$, habebuntur hae duae aequationes:

$$\frac{ddx}{d\theta^2} = -\frac{(1+m)\, cos.\, \Phi}{vv} - \frac{mv\, cos.\, \Phi}{w^3} - \frac{m\, cos.\, \psi}{aa} + \frac{ma\, cos.\, \psi}{w^3}$$

$$\frac{ddy}{d\theta^2} = -\frac{(1+m)\, sin.\, \Phi}{vv} - \frac{mv\, sin.\, \Phi}{w^3} - \frac{m\, sin.\, \psi}{aa} + \frac{ma\, sin\, \psi}{w^3}$$

ex quibus aequationibus omnia repeti debent, quae ad institutum nostrum desiderantur.

§. 4. Cum iam sit $x = v \operatorname{cos.} \phi$ et $y = v \operatorname{fin.} \phi$ erit $dx = dv \operatorname{cos.} \phi - v\, d\phi \operatorname{fin.} \phi$ et $dy = dv \operatorname{fin.} \phi + v\, d\phi \operatorname{cos.} \phi$; porro vero

$$\text{I. } ddx = ddv \operatorname{cos.} \phi - 2dv\, d\phi \operatorname{fin.} \phi - v\, d\phi^2 \operatorname{cos.} \phi - v\, dd\phi \operatorname{fin.} \phi$$

$$\text{II. } ddy = ddv \operatorname{fin.} \phi + 2dv\, d\phi \operatorname{cos.} \phi - v\, d\phi^2 \operatorname{fin.} \phi + v\, dd\phi \operatorname{cos.} \phi$$

ex quibus formulis per combinationem colliguntur sequentes:

$$\text{I. } ddy \operatorname{cos.} \phi - ddx \operatorname{fin.} \phi = 2\, dv\, d\phi + v\, dd\phi$$

$$\text{II. } ddx \operatorname{cos.} \phi + ddy \operatorname{fin.} \phi = ddv - v\, d\phi^2.$$

Hic iam loco ddx et ddy valores ex primis aequationibus, ex actione virium ortis, substituantur, prodibitque

$$\frac{2\, dv\, d\phi + v\, dd\phi}{d\theta^2} = -\frac{m}{aa} (\operatorname{fin.} \psi \operatorname{cos.} \phi - \operatorname{cos.} \psi \operatorname{fin.} \phi) + \frac{ma}{w^3} (\operatorname{fin.} \psi \operatorname{cos.} \phi - \operatorname{cos.} \psi \operatorname{fin.} \phi)$$

$$\frac{ddv - v\, d\phi^2}{d\theta^2} = -\frac{(1+m)}{vv} - \frac{mv}{w^3} - \frac{m}{aa} (\operatorname{cos.} \psi \operatorname{cos.} \phi + \operatorname{fin.} \psi \operatorname{fin.} \phi) + \frac{ma}{w^3} (\operatorname{cos.} \psi \operatorname{cos.} \phi + \operatorname{fin.} \psi \operatorname{fin.} \phi$$

siue ob $\psi - \phi = \eta$ erit

$$\frac{2\, dv\, d\phi + v\, dd\phi}{d\theta^2} = \frac{ma}{w^3} \operatorname{fin.} \eta - \frac{m}{aa} \operatorname{fin.} \eta$$

$$\frac{ddv - v\, d\phi^2}{d\theta^2} = -\frac{(1+m)}{vv} - \frac{mv}{w^3} - \frac{m}{aa} \operatorname{cos.} \eta + \frac{ma}{w^3} \operatorname{cos.} \eta.$$

§. 5. Hic totum negotium pendet ab idonea euolutione membrorum per w^3 diuisorum, vnde reliquas aequationum partes ad sinistram transponamus, vt aequationes nanciscamur huius formae:

$$\frac{2\, dv\, d\phi + v\, dd\phi}{d\theta^2} + \frac{m}{aa} \operatorname{fin.} \eta = \frac{ma}{w^3} \operatorname{fin.} \eta$$

$$\frac{ddv - v\, d\phi^2}{d\theta^2} + \frac{1+m}{vv} + \frac{m \operatorname{cos.} \eta}{aa} = \frac{m}{w^3} (a \operatorname{cos.} \eta - v)$$

Vidi-

Vidimus autem initio, esse $w = \sqrt{vv + aa - 2av \operatorname{cof.} \eta}$, vbi, quia hi termini, vtpote littera m affecti, per se sunt quam minimi, etiam distantiam v tanquam constantem spectare licebit, siquidem ab excentricitate orbitae Terrae mentem abstrahamus, quippe quae non solum est satis parua, sed etiam in praesenti negotio nihil in actione Veneris mutare est censenda; quam ob caussam loco v scribamus distantiam mediam Terrae a Sole, quam ponimus $= 1$, sicque erit $w = \sqrt{1 + aa - 2a \operatorname{cof.} \eta}$, ideoque

$$w = \sqrt{1 + aa} \cdot \sqrt{1 - \frac{2a}{1+aa} \operatorname{cof.} \eta},$$

vbi loco $\frac{2a}{1+aa}$ scribamus litteram n, cuius valor, ob distantiam mediam Veneris a Sole $a = 0,72344$, erit $n = 0,94979$. Erit autem nunc

$$\frac{m}{w^3} = \frac{m}{(1+aa)^{\frac{3}{2}}} (1 - n \operatorname{cof.} \eta)^{-\frac{3}{2}}$$

siue, si breuitatis gratia ponatur

$$\frac{m}{(1+aa)^{\frac{3}{2}}} = \mu, \text{ erit } \frac{m}{w^3} = \mu (1 - n \operatorname{cof.} \eta)^{-\frac{3}{2}},$$

vbi notetur esse $\mu = 0,0000015\, \Phi$.

§. 6. Alio autem loco hanc formulam irrationalem pro hoc ipso casu iam euolui, atque inueni esse

$$(1 - n \operatorname{cof.} \eta)^{-\frac{3}{2}} = A + B \operatorname{cof.} \eta + C \operatorname{cof.} 2\eta + D \operatorname{cof.} 3\eta + \text{etc.}$$

et pro his litteris A, B, C, etc. sequentes exactissimos, methodo prorsus singulari, adeptus sum valores:

$A = 9,39852$; $B = 16,68153$; $C = 13,87191$
$D = 11,17685$; $E = 8,80776$; $F = 6,85206$
$G = 5,26990$; $H = 4,04433$; $I = 3,08789$.

Ho-

Horum autem valorum numericorum loco in calculo retineamus litteras A, B, C, etc.

§. 7. Quoniam igitur in nostra priore aequatione continetur membrum

$$\frac{m\,a\,\text{fin.}\,\eta}{w^3} = \mu\,a\,\text{fin.}\,(A + B\,\text{cof.}\,\eta + C\,\text{cof.}\,2\eta + D\,\text{cof.}\,3\eta + \text{etc.})$$

facta euolutione hoc membrum ita erit expressum

$$\mu\,a\left(\begin{array}{l} A\,\text{fin.}\,\eta + \tfrac{1}{2}B\,\text{fin.}\,2\,\eta + \tfrac{1}{2}C\,\text{fin.}\,3\,\eta + \tfrac{1}{2}D\,\text{fin.}\,4\,\eta + \text{etc.} \\ -\tfrac{1}{2}C\,\text{fin.}\,\eta - \tfrac{1}{2}D\,\text{fin.}\,2\,\eta - \tfrac{1}{2}E\,\text{fin.}\,3\,\eta - \tfrac{1}{2}F\,\text{fin.}\,4\,\eta - \text{etc.}\end{array}\right)$$

Pro alterius vero aequationis membro dextro erit primo

$$\frac{m\,a\,\text{cof.}\,\eta}{w^3} = \mu\,a\,\text{cof.}\,\eta\,(A + B\,\text{cof.}\,\eta + C\,\text{cof.}\,2\eta + D\,\text{cof.}\,3\eta + \text{etc.})$$

fiue facta euolutione

$$\frac{m\,a\,\text{cof.}\,\eta}{w^3} = \mu\left(\begin{array}{l} \tfrac{1}{2}B + A\,\text{cof.}\,\eta + \tfrac{1}{2}B\,\text{cof.}\,2\,\eta + \tfrac{1}{2}C\,\text{cof.}\,3\,\eta + \text{etc.} \\ \quad + \tfrac{1}{2}C\,\text{cof.}\,\eta + \tfrac{1}{2}D\,\text{cof.}\,2\,\eta + \tfrac{1}{2}E\,\text{cof.}\,3\,\eta + \text{etc.}\end{array}\right)$$

Pro altera vero eiusdem membri parte, quae est $-\frac{m v}{w^3}$, tuto assumere licet $v = 1$, quoniam supponimus, actione Veneris sublata, Terram in circulo esse progressuram; sicque ista pars dabit,

$$-\mu\,(A + B\,\text{cof.}\,\eta + C\,\text{cof.}\,2\,\eta + D\,\text{cof.}\,3\,\eta + \text{etc.})$$

Hanc ob rem si pro vtraque parte iunctim sumta ponamus hanc seriem:

$$\mu\,(A' + B'\,\text{cof.}\,\eta + C'\,\text{cof.}\,2\,\eta + D'\,\text{cof.}\,3\,\eta + \text{etc.}) \text{ erit.}$$

$$A' = \tfrac{1}{2}aB - A;\; B' = \tfrac{1}{2}a(2A + C) - B;\; C' = \tfrac{1}{2}a(B + D) - C$$
$$D' = \tfrac{1}{2}a(C + E) - D;\; E' = \tfrac{1}{2}a(D + F) - E;\; \text{etc.}$$

cui ergo expressioni: $\mu\,(A' + B'\,\text{cof.}\,\eta + C'\,\text{cof.}\,2\,\eta + \text{etc.})$ aequale esse debet membrum finistrum

$$\frac{d\,d\,v - v\,d\,\Phi^2}{d\,\phi^2} + \frac{1 + m}{v\,v} + \frac{m\,\text{cof.}\,\eta}{a\,a}.$$

§. 8. Incipiamus nunc ab euolutione primae aequationis, et quoniam assumimus Terram fine actione Veneris

neris in circulo motu vniformi esse processuram in distantia media $= 1$, ita vt etiam foret $\Phi = \theta$, ideoque $\frac{d\Phi}{d\theta} = 1$; nunc accedente actione Veneris hae quantitates quasi infinite parum immutabuntur. Statuamus ergo tum fore

$$v = 1 + \mu p \text{ ac } \frac{d\Phi}{d\theta} = 1 + \mu q;$$

vnde in compositione membra, quae continerent μ^2, tuto omitti poterunt. Cum igitur sit

$$\frac{dv}{d\theta} = \frac{\mu dp}{d\theta} \text{ et } \frac{dd\Phi}{d\theta^2} = \frac{\mu dq}{d\theta},$$

oritur hinc sequens aequatio:

$$\frac{2 dv d\Phi + v dd\Phi}{d\theta^2} + \frac{m}{aa} \sin.\eta = \frac{2\mu dp + \mu dq}{d\theta} + \frac{m}{aa} \sin.\eta$$
$$= \frac{ma}{w^3} \sin.\eta.$$

Pro cuius parte dextra scribamus hanc seriem:

$$\mu(\mathfrak{B} \sin.\eta + \mathfrak{C} \sin.2\eta + \mathfrak{D} \sin.3\eta + \mathfrak{E} \sin.4\eta + \text{etc.}$$

ita vt ob resolutionem huius membri iam supra traditam sit

$$\mathfrak{B} = \tfrac{1}{2} a(2A - C);\ \mathfrak{C} = \tfrac{1}{2} a(B - D);\ \mathfrak{D} = \tfrac{1}{2} a(C - E);$$
$$\mathfrak{E} = \tfrac{1}{2} a(D - F);\ \mathfrak{F} = \tfrac{1}{2} a(E - G);\ \text{etc.}$$

atque hinc aequatio resoluenda erit

$$\frac{2dp + dq}{d\theta} + \frac{m}{\mu aa} \sin.\eta = \mathfrak{B} \sin.\eta + \mathfrak{C} \sin.2\eta + \mathfrak{D} \sin.3\eta + \text{etc.}$$

vbi notetur esse $\frac{m}{\mu aa} = \frac{(1 + aa)^{\frac{3}{2}}}{aa}$, quem numerum breuitatis gr. per litteram k designemus, ita vt sit $k = 3,592551$, et nostra aeqnatio nunc erit

$$\frac{2dp + dq}{d\theta} + k \sin.\eta = \mathfrak{B} \sin.\eta + \mathfrak{C} \sin.2\eta + \mathfrak{D} \sin.3\eta + \text{etc.}$$

quam igitur integrari oportet.

§. 9. Quoniam hic duo anguli η et θ insunt, nosse oportet relationem $d\eta$ et $d\theta$. Erat autem

$$\eta = \psi - \Phi, \text{ unde fit } \frac{d\eta}{d\theta} = \frac{d\psi}{d\theta} - \frac{d\Phi}{d\theta}.$$

Hoc autem loco vtrumque motum Terrae ac Veneris vt vniformem ſpectare licet, ita vt ſit $\frac{d\Phi}{d\theta} = 1$. Pro Venere autem, eius motus diurnus in tabulis exhibetur $= 1^\circ, 36', 9'' = 5769''$, dum pro Terra eſt $59', 8'' = 3548''$. Quocirca habemus

$$\frac{d\psi}{d\theta} = \frac{5769}{3548}, \text{ vnde fit } \frac{d\eta}{d\theta} = \frac{2221}{3548}.$$

Ponamus autem

$$d\theta = i\, d\eta, \text{ eritque } i = \frac{3548}{2221} = 1,597479.$$

Nunc igitur manifeſtum eſt, aequationem noſtram, per $d\theta = i\, d\eta$ multiplicatam, euadere integrabilem; reperietur enim

$$2p + q - ik \operatorname{cof.}\eta = \Delta - i\mathfrak{B}\operatorname{cof.}\eta - \tfrac{1}{2} i\mathfrak{C}\operatorname{cof.}2\eta - \tfrac{1}{3} i\mathfrak{D}\operatorname{cof.}3\eta - \text{etc.}$$

ex qua propterea fit

$$q = \Delta - 2p + i(k - \mathfrak{B})\operatorname{cof.}\eta - \tfrac{1}{2} i\mathfrak{C}\operatorname{cof.}2\eta - \tfrac{1}{3} i\mathfrak{D}\operatorname{cof.}3\eta - \text{etc.}$$

§. 10. Aggrediamur iam poſteriorem aequationem, pro qua notetur fore $\frac{ddv}{d\theta^2} = \frac{\mu\, ddp}{d\theta^2}$, tum vero

$$\frac{v\, d\Phi^2}{d\theta^2} = 1 + \mu(2q + p), \text{ et } \frac{1+m}{vv} = \frac{1+m}{1+2\mu p}$$

ſiue ſupra et infra per $1 - 2\mu p$ multiplicando erit

$$\frac{1+m}{vv} = 1 + m - 2\mu p$$

quibus valoribus ſubſtitutis aequationis noſtrae membrum ſiniſtrum erit

$$\frac{\mu\, ddp}{d\theta^2} - \mu(3p + 2q) + m + \frac{m \operatorname{cof.}\eta}{aa}.$$

Quod ſi iam per μ dividamus, et loco $\frac{m}{\mu aa} = 3,592551$ ſcribamus k, loco $\frac{m}{\mu} = 1,880217$ vero ſcribamus l, poſterior aequatio hanc induet formam:

$\frac{ddp}{d\theta^2}$

$\frac{ddp}{d\theta^2} - 3p - 2q + l + k\,\text{cof.}\,\eta = A' + B'\text{cof.}\,\eta + C'\text{cof.}\,2\eta + D'\text{cof.}\,3\eta\ \text{etc.}$
in qua fi loco q valor ante inuentus fubftituatur, fiet
$\frac{ddp}{d\theta^2} - 3p + l + k\,\text{cof.}\,\eta = A' + B'\text{cof.}\,\eta + C'\text{cof.}\,2\eta + D'\text{cof.}\,3\eta + \text{etc.}$
$+ 4p - 2\Delta + 2i(k - \mathfrak{B})\,\text{cof.}\,\eta - \frac{1}{2}i\mathfrak{C}\,\text{cof.}\,2\eta - \frac{2}{3}i\mathfrak{D}\,\text{cof.}\,3\eta - \text{etc.}$
fiue $\frac{ddp}{d\theta^2} + p = 2\Delta - l + A' + (2i(k - \mathfrak{B}) - k + B')\,\text{cof.}\,\eta + (C' - \frac{1}{2}i\mathfrak{C})\,\text{cof.}\,2\eta$
$+ (D' - \frac{2}{3}i\mathfrak{D})\,\text{cof.}\,2\eta + (E' - \frac{2}{4}i\mathfrak{E})\,\text{cof.}\,3\eta + \text{etc.}$
cuius loco breuitatis gratia fcribamus

$$\frac{ddp}{d\theta^2} + p = \mathfrak{A}' + \mathfrak{B}'\,\text{cof.}\,\eta + \mathfrak{C}'\,\text{cof.}\,2\eta + \mathfrak{D}'\,\text{cof.}\,3\eta + \text{etc.}$$

ita vt fit

$$\mathfrak{A}' = 2\Delta - l + A';\ \mathfrak{B}' = 2i(k - \mathfrak{B}) - k + B';\ \mathfrak{C}' = C' - i\mathfrak{C}$$
$$\mathfrak{D}' = D' - \tfrac{2}{3}i\mathfrak{D};\ \mathfrak{E}' = E' - \tfrac{2}{4}i\mathfrak{E};\ \text{etc.}$$

§. 11. Manifeftum autem eft, huic aequationi fatisfieri, ftatuendo

$$p = \alpha + \beta\,\text{cof.}\,\eta + \gamma\,\text{cof.}\,2\eta + \delta\,\text{cof.}\,3\eta + \text{etc.}$$

vnde ob $\frac{d\eta}{d\theta} = \frac{1}{i}$ membrum finiftrum refoluitur in has duas feries:

$$\frac{ddp}{d\theta^2} = -\frac{\beta}{ii}\,\text{cof.}\,\eta - \frac{4\gamma}{ii}\,\text{cof.}\,2\eta - \frac{9\delta}{ii}\,\text{cof.}\,3\eta - \frac{16\varepsilon}{ii}\,\text{cof.}\,4\eta - \text{etc.}$$
$$+ p = \alpha + \beta\,\text{cof.}\,\eta + \gamma\,\text{cof.}\,2\eta + \delta\,\text{cof.}\,3\eta + \varepsilon\,\text{cof.}\,4\eta + \text{etc.}$$

ita vt, fingulis ambarum partium membris feorfim aequatis, fe prodeant fponte fequentes determinationes:

$$\alpha = \mathfrak{A}';\ \beta(1 - \tfrac{1}{ii}) = \mathfrak{B}';\ \gamma(1 - \tfrac{4}{ii}) = \mathfrak{C}';\ \delta(1 - \tfrac{9}{ii}) = \mathfrak{D}';\ \text{etc.}$$

ideoque

$$\alpha = \mathfrak{A}';\ \beta = \frac{\mathfrak{B}'}{1 - \frac{1}{ii}};\ \gamma = \frac{\mathfrak{C}'}{1 - \frac{4}{ii}};\ \delta = \frac{\mathfrak{D}'}{1 - \frac{9}{ii}};\ \varepsilon = \frac{\mathfrak{E}'}{1 - \frac{16}{ii}}.$$

§. 12. Cum igitur ex valoribus litterarum A, B, C, D, etc. fupra §. 6. inuentis facile colligi queant va-

 lores

lores deriuati A', B', C', D', etc. tum vero $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, etc. ac denique $\mathfrak{A}'$, $\mathfrak{B}'$, $\mathfrak{C}'$, $\mathfrak{D}'$, etc. ex iis iam deduci possunt α, β, γ, δ, etc. vnde porro innotescunt valores p & q, quarum prior praebet exiguam illam mutationem quam actio Veneris in distantia Terrae et Sole producit, cum sit $v = + \mu p$. Denique ex valore p deriuatur valor ipsius q quem breu. gr. statuamus:

$$q = \alpha' + \beta' \text{cos.}\, \eta + \gamma' \text{cos.}\, 2\delta + \delta' \text{cos.}\, 3\eta + \text{etc.}$$

ita vt sit

$$\alpha' = \Delta - 2\alpha;\ \beta' = i(k - \mathfrak{B}) - 2\beta;\ \gamma' = -2\gamma - \tfrac{1}{2} i \mathfrak{C},$$
$$\delta' = -2\delta - \tfrac{1}{3} i \mathfrak{D};\ \varepsilon' = -2\varepsilon - \tfrac{1}{4} i \mathfrak{E};\ \text{etc.}$$

Inuento autem valore q inde colligitur series

$$\frac{d\Phi}{d\theta} = 1 + \mu\alpha' + \mu\beta' \text{cos.}\, \eta + \mu\gamma' \text{cos.}\, 2\eta + \text{etc.}$$

ex qua pro quouis tempore vera Solis longitudo concluditur fore

$$\Phi = (1 + \mu\alpha')\theta + \mu i \beta' \text{sin.}\,\eta + \tfrac{1}{2}\mu i \gamma' \text{sin.}\,2\eta + \tfrac{1}{3}\mu i \delta' \text{sin.}\,3\eta + \text{etc.}$$

vbi pars prima $(1 + \mu\alpha')\theta$ exhibet longitudinem mediam Terrae, quam quia supponimus esse exacte $= \theta$, sequitur esse debere $\alpha' = 0$. Reliquae autem partes continent inaequalitates motus periodici, quae ergo pendent a sinibus angulorum η, 2η, 3η, 4η, etc. Hoc modo sequens tabula perturbationem est facta.

VLTE-

Tabula Perturbationum in diftantia et motu Terrae, ab actione Veneris, in eam agente, ortarum.

Argumentum
Elongatio Veneris a Terra.

Signa	0.		I.		II.		III.		IV.		V.		Signa
Grad.	Long. −	Dift. +	Long. −	Dift. +	Long. −	Dift. −	Long. +	Dift. −	Long. +	Dift. −	Long. +	Dift. +	Grad.
0	0, 0	20	4, 0	5	0, 3	14	6, 9	16	11, 1	1	8, 4	18	30
1	0, 2	20	4, 0	4	0, 1	14	7, 1	16	11, 1	+	8, 1	19	29
2	0, 4	20	4, 0	3	+	14	7, 3	15	11, 2	0	7, 9	19	28
3	0, 6	20	3, 9	3	0, 3	15	7, 6	15	11, 2	1	7, 7	20	27
4	0, 8	20	3, 9	2	0, 5	15	7, 8	15	11, 2	2	7, 5	20	26
5	1, 0	20	3, 9	1	0, 8	15	8, 0	14	11, 2	2	7, 2	21	25
6	1, 2	20	3, 8	0	1, 0	16	8, 2	14	11, 1	3	7, 0	21	24
7	1, 4	19	3, 8	—	1, 3	16	8, 4	14	11, 1	4	6, 7	22	23
8	1, 6	19	3, 7	1	1, 5	16	8, 6	13	11, 1	4	6, 4	22	22
9	1, 8	19	3, 6	1	1, 7	16	8, 8	13	11, 0	5	6, 2	23	21
10	2, 0	18	3, 5	2	2, 0	17	9, 0	12	10, 9	6	5, 9	23	20
11	2, 2	18	3, 4	3	2, 2	17	9, 1	12	10, 9	6	5, 7	23	19
12	2, 4	17	3, 3	4	2, 5	17	9, 3	11	10, 8	7	5, 4	24	18
13	2, 5	17	3, 2	4	2, 7	17	9, 4	11	10, 7	8	5, 1	24	17
14	2, 7	16	3, 1	5	3, 0	17	9, 6	10	10, 7	8	4, 9	24	16
15	2, 8	16	3, 0	6	3, 2	17	9, 8	10	10, 5	9	4, 6	25	15
16	3, 0	15	2, 9	6	3, 5	17	9, 9	9	10, 4	10	4, 3	25	14
17	3, 1	15	2, 7	7	3, 7	17	10, 0	9	10, 3	10	4, 0	25	13
18	3, 2	14	2, 6	8	4, 0	17	10, 1	8	10, 2	11	3, 7	25	12
19	3, 3	13	2, 4	8	4, 2	17	10, 3	8	10, 1	12	3, 4	26	11
20	3, 5	13	2, 3	9	4, 5	17	10, 4	7	9, 9	12	3, 1	26	10
21	3, 6	12	2, 1	9	4, 7	17	10, 5	7	9, 8	13	2, 8	26	9
22	3, 6	11	1, 9	10	5, 0	27	10, 6	6	9, 6	14	2, 5	26	8
23	3, 7	11	1, 7	10	5, 2	17	10, 7	5	9, 5	14	2, 2	26	7
24	3, 8	10	1, 6	11	5, 5	17	10, 8	5	9, 3	15	1, 9	27	6
25	3, 8	9	1, 4	11	5, 7	17	10, 9	4	9, 1	15	1, 6	27	5
26	3, 9	8	1, 2	12	6, 0	17	10, 9	3	8, 9	16	1, 2	27	4
27	3, 9	8	1, 0	12	6, 2	17	11, 0	3	8, 7	17	0, 9	27	3
28	3, 9	7	0, 8	13	6, 5	16	11, 0	2	8, 6	17	0, 6	27	2
29	4, 0	6	0, 6	13	6, 7	16	11, 1	2	8, 5	18	0, 3	27	1
30	4, 0	5	0, 3	14	6, 9	16	11, 1	1	8, 4	18	0, 0	27	0
Grad.	+ Long.	− Dift.	+ Long.	+ Dift.	− Long.	+ Dift.	− Long.	+ Dift.	− Long.	− Dift.	− Long.	− Dift.	Grad.
Signa	XI.		X.		IX.		VIII.		VII.		VI.		Signa