1768

De telescopiis quatuor lentibus instructis eorumque perfectione

Leonhard Euler

DE TELESCOPIIS QVATVOR LENTIBVS INSTRVCTIS EORVMQVE PERFECTIONE.

Auctore

L. EVLERO.

1.

Cum telescopia vulgaria binis lentibus, altera conuexa altera concaua, instructa hoc incommodo laborent, quod campum nimis arctum ostendant, neque ad maiores multiplicationes extendi queant; telescopia autem astronomica, binis lentibus conuexis instructa, in situ inuerso obiecta exhibeant, quo contemplatio obiectorum terrestrium non mediocriter perturbatur; vt huic duplici incommodo occurratur, telescopia ex quatuor lentibus conuexis componere coeperunt. Tubo scilicet astronomico adiunxerunt insuper binas lentes conuexas, quibus repraesentatio inuersa noua inuersione erecta redderetur. Primo quidem has nouas lentes adiectas inter se aequales fecerunt, tum vero inaequalitatem admittendo animaduerterunt, campum conspicuum aliquantum augeri, atque adeo confusionem colorum iridis imminui posse, quo pacto hoc telescopiorum genus haud mediocriter est perfectum.

2. Nuper autem in Anglia his telescopiis ex quatuor lentibus compositis quintam lentem atque adeo mox sextam adiecerunt, hacque lentium multiplicatione insignia commoda obtineri posse animaduerterunt, dum non solum campus conspicuus multo magis ampliari, sed etiam repraesentatio a coloribus iridis penitus liberari poterat. Cum autem sola experientia ducti hunc perfectionis gradum sint adepti, neque vlla Theoria in subsidium vocata videatur, nullum est dubium, quin haec eadem commoda beneficio Theoriae augeri, atque adeo ad maximum gradum, quem quidem natura admittit, euehi queant.

3. Veram autem Theoriam, cui omnium instrumentorum dioptricorum tam telescopiorum quam Microscopiorum constructio innititur, in XIII. Volumine Actorum Academiae Regiae Borussicae equidem mihi tradidisse videor, vnde omnia, quae ad horum instrumentorum perfectionem spectant, hauriri debere videntur. Quanta igitur incrementa hinc tam pro quatuor lentibus, quam iis insuper quintam ac sextam adiiciendo, assequi liceat, hic diligentius inuestigare constitui. In quo negotio ne in nimis longas ambages incidam, ex loco allegato subsidia, quae tam ad campum amplificandum, quam ad colores iridis tollendos faciunt, depromam, neque principiis, quibus innituntur, commemorandis immorabor. De lente etiam obiectiua perficienda hic non ero sollicitus, cum iam saepius docuerim, quomodo

modo eius duplicatione vel etiam triplicatione confusio ab eius apertura oriunda non solum imminui, sed etiam ad nihilum redigi debeat.

De Telescopiis quatuor lentibus instructis.

Tab. III. Fig. 3. 4. 4. Quatuor lentibus super communi axe in A, B, C, D constitutis, ante omnia ad binas imagines est respiciendum, quarum altera in foco lentis obiectiuae A, qui sit in α, est sita, atque inuersa, altera autem in γ focum lentis ocularis D cadere debet, eaque vt est erecta, oculo in O posito obiecta situ erecto spectanda offert. Binae igitur lentes mediae B et C imaginem ex α in γ transferre debent, ac vulgo quidem eae ita constitui solent, vt lens B focum suum in puncto α habens, radios parallelos ad lentem C mittat, quae deinde eos in γ iterum colligat, ita vt huius lentis focus in γ incidat.

Fig. 3. 5. Idem autem effectus praeterea duplici modo obtineri potest. Primo enim si interuallum $B\alpha$ minus sit, quam distantia foci lentis B, haec imaginem antrorsum proiiciet in $\mathfrak{b}$, vnde lens C eam demum colliget in γ. Hoc casu in denominationibus a me vsitatis erit numerus $B=\frac{-B\mathfrak{b}}{B\alpha}$, quem propterea ponam $B=\frac{-b}{b-1}$ existente $b>1$, vt sit $\mathfrak{B}=\frac{B}{B+1}=b$; deinde pro lente C erit numerus $C=\frac{C\gamma}{C\mathfrak{b}}=\frac{c}{1-c}$ existente $c<1$, vt sit $\mathfrak{C}=\frac{C}{1+C}=c$; vbi notandum in casu vulgari fieri $b=1$ et $c=0$.

Tab. III. Fig. 4. Secundus casus est, quo interuallum $B\alpha$ superat distantiam foci lentis B, tum ista lens imaginem ex α retrorsum proiicit in $\mathfrak{b}$, vnde lens C eam reducit in γ. Hoc ergo casu ob numerum $B=\frac{+B\mathfrak{b}}{B\alpha}$ ponam $B=\frac{b}{1-b}$ existente $b<1$, vt sit $\mathfrak{B}=\frac{B}{B+1}=b$ et pro lente C ponam numerum $C=\frac{-C\gamma}{C\mathfrak{b}}=\frac{-c}{1+c}$ vt sit $\mathfrak{C}=-c$ vnde iterum casus vulgaris resultat si $b=1$ et $c=0$; posterior autem ex priori oritur sumendo c negatiuum.

6. Sit nunc distantia foci lentis obiectiuae $p=\alpha$; lentis $B=p'$, lentis $C=p''$ et lentis $D=p'''$; ac semidiametri aperturae harum trium posteriorum lentium statuantur $\theta'p'$; $\theta''p''$ et $\theta'''p'''$. Tum posita multiplicationis ratione $m:1$ et semidiametro campi apparentis $=\Phi$, habemus statim $\theta'''+\theta''+\theta'-\Phi=-m\Phi$ ideoque $\Phi=\frac{-\theta'''-\theta''-\theta'}{m-1}$. Deinde ob

$$B=\frac{-b}{b-1};\ \mathfrak{B}=b;\ C=\frac{c}{1-c};\ \mathfrak{C}=c;\ D=\infty \text{ et } \mathfrak{D}=1$$

ponamus porro:

$$\Pi'=b\theta'-\Phi;\ \Pi''=c\theta''+\theta'-\Phi;\ \Pi'''=\theta'''+\theta''+\theta'-\Phi$$

eruntque distantiae focales lentium:

$$p'=\frac{b\Phi}{\Pi'}\alpha;\ p''=\frac{bc}{b-1}\cdot\frac{\Phi}{\Pi''}\alpha;\ p'''=\frac{-bc}{(b-1)(1-c)}\cdot\frac{\Phi}{\Pi'''}\alpha$$

et

et interualla lentium:

$$AB = \frac{b\theta'}{\Pi'}\alpha;\ BC = \frac{-b}{b-1}\cdot\frac{\Phi(c\theta''-(b-1)\theta')}{(b\theta'-\Phi)(c\theta''+\theta'-\Phi)}\alpha;$$

$$CD = \frac{-bc}{(b-1)(1-c)}\cdot\frac{\Phi(-\theta''+(c-1)\theta''')}{\Pi''\Pi'''}\alpha \text{ et pro loco oculis } \frac{\theta'''p'''}{\Pi'''} = DO.$$

7. Cum haec interualla esse debeant positiua et $AB > \alpha$ patet primo tam θ' quam Π' positiua esse debere. Deinde vt campum apparentem quam maximum reddamus, statuamus $\theta' = q\omega$, $\theta'' = -\omega$, et $\theta''' = -\omega$, ita vt sit $q < 1$, et $\Phi = \frac{2-q}{m-1}\omega$, vbi si hae lentes sunt aeque conuexae vtrinque, sumi potest $\omega = \frac{1}{4}$. Sit quoque breuitatis gratia $\Phi = M\omega$ seu $M = \frac{2-q}{m-1}$, et ob

$$\Pi' = (bq-M)\omega;\ \Pi'' = (-c+q-M)\omega;\ \Pi''' = -(2-q+M)\omega$$

habebimus:

$$p' = \frac{bM}{bq-M}\alpha;\ p'' = \frac{bc}{b-1}\cdot\frac{M}{q-c-M}\alpha;\ p''' = \frac{bc}{(b-1)(1-c)}\cdot\frac{M}{2-q+M}\alpha$$

$$AB = \frac{bq}{bq-M}\alpha;\ BC = \frac{b((b-1)q+cM)}{(b-1)(bq-M)(q-c-M)}\alpha;\ CD = \frac{bc(2-c)M}{(b-1)(1-c)(q-c-M)(2-q+M)}\alpha$$

et pro loco oculi $DO = \frac{1}{2-q+M}p'''$. Vbi notandum est si $b > 1$, litteram c capi debere positiue, negatiue vero si $b < 1$.

8. His praemissis, quibus campo iam maximam amplitudinem conciliauimus, dummodo fractio q tam exigua accipiatur, quam reliquae circumstantiae permittunt, consideremus conditionem, quam

quam destructio colorum iridis postulat, in hac aequatione contentam:

$$\frac{\theta'}{\Pi'} + \frac{\theta''}{\Pi''} + \frac{\theta'''}{\Pi'''} = 0 \text{ seu } \frac{q}{bq-M} - \frac{1}{q-c-M} + \frac{1}{2-q+M} = 0$$

ideoque $\frac{q}{bq-M} = \frac{2-2q+c+2M}{(q-c-M)(2-q+M)}$, vnde inuenitur

$$b = \frac{M}{q} + \frac{(q-c-M)(2-q+M)}{2-2q+c+2M} = \frac{(2-q)MM+(2-q)(2-2q+c)M+(2-q)q(q-c)}{q(2-2q+c+2M)}$$

quem valorem oportet esse maiorem vnitate, si quidem c positiue accipiatur. Euidens autem est fractionem q notabiliter maiorem capi debere, quam $c+M$, quia alioquin fractio $\frac{1}{q-c-M}$ fieret nimis magna, vnde etiam fractio $\frac{q}{bq-M}$ ac propterea interuallum AB enormiter augeretur.

9. Ne igitur hoc eueniat, atque vt ob minutam fractionem q etiam campus amplietur, conueniet fractioni c valorem tribui negatiuum simulque sumi $b < 1$, ex quo formulae nostrae erunt: Fig. 4.

$$p' = \frac{bM}{bq-M}\alpha;\ p'' = \frac{bc}{1-b}\cdot\frac{M}{q+c-M}\alpha;\ p''' = \frac{bc}{(1-b)(1+c)}\cdot\frac{M}{2-q+M}\alpha$$

$$AB = \frac{bq}{bq-M}\alpha;\ BC = \frac{b((1-b)q+cM)}{(1-b)(bq-M)(q+c-M)}\alpha;\ CD = \frac{bc(2+c)M}{(1-b)(1+c)(q+c-M)(2-q+M)}\alpha$$

et pro loco oculi $DO = \frac{1}{2-q+M}\cdot p'''$. At vt iridis colores euanescant, fieri debet $b = \frac{M}{q} + \frac{(q+c-M)(2-q+M)}{2-2q-c+2M}$, quem valorem vnitate minorem esse oportet, quod in assumtione numeri c probe est obseruandum; simul vero etiam vt $bq-M$ positiuum prodeat, quorum vtrumque postulat, vt c notabiliter minus quam $2-2q-1-2M$ accipiatur. Interim tamen ob $b < 1$ sumi oportet $q > M$, quod ob M fractionem eo eo minorem, quo maior eſt multiplicatio, non impediet, quo minus campus apparens Φ ſatis prope ad $\frac{2\omega}{m-1}$ accedat, dum caſu vulgari q maius ſemiſſi vnitatis accipi debeat.

10. Statuamus ergo $q = nM$, vt ſit $n > 1$, eritque

$$p' = \frac{b}{bn-1}\alpha;\ p'' = \frac{bc}{1-b}\cdot\frac{M}{c+(n-1)M}\alpha;\ p''' = \frac{bc}{(1-b)(1+c)}\cdot\frac{M}{2-(n-1)M}\alpha$$

$$AB = \frac{bn}{bn-1}\alpha;\ BC = \frac{b(c+(1-b)nM)}{(1-b)(bn-1)(c+(n-1)M)}\alpha;\ CD = \frac{bc(2+c)M}{(1-b)(1+c)(c+(n-1)M)(2-(n-1)M)}\alpha$$

et $DO = \frac{1}{2-(n-1)M}\cdot p'''$. Tum vero ob. $M = \frac{2-nM}{m-1}$ erit $M = \frac{2}{m+n-1}$ et $\Phi = \frac{2}{m+n-1}\omega$; et colorum deſtructio praebet:

$b = \frac{1}{n} + \frac{(c+(n-1)M)(2-(n-1)M)}{2-c-2(n-1)M}$, vnde cum eſſe debeat $b < 1$ erit $\frac{(c+(n-1)M)(2-(n-1)M)}{2-c-2(n-1)M} < \frac{n-1}{n}$ hincque.

$$c\left(\frac{3n-1}{n}-(n-1)M\right) < \frac{2(n-1)}{n} - \frac{2(n-1)(2n-1)}{n}M + (n-1)^2 MM.$$

Ergo quia M eſt fractio valde parua erit $c < \frac{2(n-1)}{3n-1}$ et propius $c < \frac{2(n-1)}{3n-1} - \frac{2(n-1)(5nn-4n+1)}{(3n-1)^2}M$.

11. *Exemplum I.* Sit $n = 2$, hincque $M = \frac{2}{m+1}$ et $\Phi = \frac{2\omega}{m+1}$, atque capiatur $c = \frac{1}{4}$, fiet $b = \frac{1}{2} + \frac{2+7M-4MM}{7-8M}$, qui valor cum ex loco oculi, vnde colores irides euaneſcunt, deductus ſummi rigoris non ſit capax, ſufficit valorem prope verum aſſumſiſſe. At ſi multiplicationem ſumamus $m = 15$, fit $M = \frac{1}{8}$, et $b = \frac{1}{2} + \frac{15}{32}$, qui valor tam prope ad vnitatem accedit, vt interualla lentium pro-

prodirent nimis magna. Quare capiamus $c = \frac{1}{8}$, et pro caſu $M = \frac{1}{8}$ reperimus $b = \frac{1}{2} + \frac{15}{104}$; ex quo ſtatuamus $b = \frac{9}{14}$, ac determinationes noſtrae erunt

$$p' = \frac{9}{4}\alpha;\ p'' = \frac{9}{5}\cdot\frac{M\alpha}{1+8M};\ p''' = \frac{1}{5}\cdot\frac{M\alpha}{2-M}$$

$$AB = \frac{9}{5}\alpha;\ BC = \frac{9}{5}\cdot\frac{7-40M}{2(1+8M)}\alpha;\ CD = \frac{17}{5}\cdot\frac{M\alpha}{(1+8M)(2-M)}$$

vnde Teleſcopium oritur vehementer longum. Ratio huius incommodi in hoc ſita eſt quod $bn = 1$ adipiſcitur valorem nimis paruum.

12. *Exemplum II.* Sit $n = 3$, ideoque $M = \frac{2}{m+2}$ et $\Phi = \frac{2\omega}{m+2}$; debet ergo eſſe $c < \frac{1}{2} - \frac{5}{2}M$ et $b = \frac{1}{3} + \frac{(c+2M)(2-2M)}{2-c-4M}$; vbi ne interuallum AB fiat nimis magnum, numero b maximus valor tribui debet, at ob $b < 1$ fiet tamen ſemper $AB > \frac{3}{2}\alpha$. At ne interuallum $BC = \frac{b(c+3(1-b)M)}{(1-b)(3b-1)(c+2M)}\alpha$ fiat nimis magnum fractio $\frac{c}{1-b}$ diminui debet quantum fieri poteſt. Quod ſi ponamus $c = \frac{1}{4}$ fit $b = \frac{1}{3} + \frac{2(1-M)(1+8M)}{7-16M}$, et $b = \frac{31}{36}$ caſu $M = \frac{1}{8}$ ſit ergo $c = \frac{1}{8}$ fit $b = \frac{1}{3} + \frac{2(1-M)(1+16M)}{15-32M}$, et $b = \frac{107}{132}$ caſu $M = \frac{1}{8}$. Sattuamus ergo $b = \frac{3}{4}$, et $c = \frac{1}{8}$, habebimusque has determinationes:

$$p' = \frac{3}{5}\alpha;\ p'' = \frac{3M\alpha}{1+16M};\ p''' = \frac{M\alpha}{6(1-M)}$$

$$AB = \frac{9}{5}\alpha;\ BC = \frac{12(1+6M)\alpha}{5(1+16M)};\ CD = \frac{17M\alpha}{6(1+16M)(1-M)}$$

qui valores pro multiplicatione $m = 20$ circiter efunt iuſti.

13. Ex-

13. *Exempl III.* Sit $n=5$, ideoque $M=\frac{2}{m+4}$ et $\Phi=\frac{2\alpha}{m+4}$, atque quo facilius omnes variationes ex variis valoribus c oriundas perfpicere queamus, calculum ad datam multiplicationem $m=28$ accommodemus, vt fit $M=\frac{1}{16}$, eruntque formulae noftrae

$$p'=\frac{5b\alpha}{5b-1};\quad p''=\frac{bc}{1-b}\cdot\frac{\alpha}{4(1+4c)};\quad p'''=\frac{bc}{(1-b)(1+c)}\cdot\frac{\alpha}{28}$$

$$AB=\frac{5b\alpha}{5b-1};\quad BC=\frac{b(16c+5(1-b))\alpha}{4(1-b)(5b-1)(1+4c)};\quad CD=\frac{bc(2+c)\alpha}{7(1-b)(1+c)(1+4c)}$$

at effe oportet $b=\frac{1}{5}+\frac{7(1+4c)}{8(2-2c)}$, et $c<\frac{61}{264}$; Hinc fi fit; erit

$c=\frac{1}{4}$; $b=\frac{9}{10}$ et $\begin{cases} p'=\frac{9}{35}\alpha; & p''=\frac{1}{30}\alpha; & p'''=\frac{9}{143}\alpha \\ AB=\frac{9}{7}\alpha; & BC=\frac{81}{56}\alpha; & CD=\frac{81}{280}\alpha \therefore AD=3\frac{1}{140}\alpha \end{cases}$

$c=\frac{1}{5}$; $b=\frac{4}{5}$ et $\begin{cases} p'=\frac{4}{15}\alpha; & p''=\frac{1}{9}\alpha; & p''=\frac{1}{42}\alpha \\ AB=\frac{4}{3}\alpha; & BC=\frac{7}{9}\alpha; & CD=\frac{22}{189}\alpha;\ AD=2\frac{43}{189}\alpha \end{cases}$

$c=\frac{1}{6}$; $b=\frac{3}{4}$ et $\begin{cases} p'=\frac{3}{11}\alpha; & p''=\frac{3}{45}\alpha; & p'''=\frac{3}{196}\alpha \\ AB=\frac{15}{11}\alpha; & BC=\frac{141}{220}\alpha; & CD=\frac{39}{490}\alpha;\ AD=2\frac{907}{10780}\alpha \end{cases}$

$c=\frac{1}{7}$; $b=\frac{2}{3}$ et $\begin{cases} p'=\frac{2}{7}\alpha; & p''=\frac{1}{24}\alpha; & p'''=\frac{1}{168}\alpha \\ AB=\frac{10}{7}\alpha; & BC=\frac{11}{21}\alpha; & CD=\frac{17}{378}\alpha;\ AD=(2-\frac{1}{378})\alpha \end{cases}$

vlterius fractionem c diminuere non licet, quia lens ocularis nimis fieret exigua.

14. *Exempl. IV.* Minuamus aliquanto magis campum fitque $n=9$ ideoque $M=\frac{2}{m+8}$; maneat $m=28$, et fit $M=\frac{1}{18}$, vnde

$$p'=\frac{b}{9b-1}\alpha;\quad p''=\frac{bc}{2(1-b)(4+9c)}\alpha;\quad p'''=\frac{bc}{28(1-b)(1+c)}\alpha$$

$$AB=\frac{9b}{9b-1}\alpha;\quad BC=\frac{9b(2c+1-b)}{2(1-b)(9b-1)(4+9c)}\alpha;\quad CD=\frac{9bc(5+c)}{28(1-b)(1+c)(4+9c)}\alpha$$

tum

tum vero $b=\frac{1}{9}+\frac{14(4+9c)}{9(10-9c)}$ et $c<\frac{4}{33}$. Hinc fi fit; erit

$c=\frac{1}{9}$; $b=\frac{39}{45}$ et $\begin{cases} p'=\frac{39}{311}\alpha; & p''=\frac{13}{30}\alpha; & p'''=\frac{19}{560}\alpha \\ AB=\frac{351}{311}\alpha; & BC=\frac{3471}{3110}\alpha; & CD=\frac{741}{1400}\alpha \end{cases}$

$c=\frac{1}{10}$; $b=\frac{19}{25}$ et $\begin{cases} p'=\frac{19}{151}\alpha; & p''=\frac{19}{98}\alpha; & p'''=\frac{19}{308}\alpha \\ AB=\frac{171}{151}\alpha; & BC=\frac{4275}{7399}\alpha; & CD=\frac{518}{4158}\alpha \end{cases}$

$c=\frac{1}{11}$; $b=\frac{10}{11}$ et $\begin{cases} p'=\frac{10}{79}\alpha; & p''=\frac{5}{17}\alpha; & p'''=\frac{5}{182}\alpha \\ AB=\frac{90}{79}\alpha; & BC=\frac{510}{1101}\alpha; & CD=\frac{875}{3458}\alpha;\ AD=1,587\alpha \end{cases}$

$c=\frac{1}{13}$; $b=\frac{7}{8}$ et $\begin{cases} p'=\frac{7}{55}\alpha; & p''=\frac{7}{138}\alpha; & p'''=\frac{1}{84}\alpha \\ AB=\frac{63}{55}\alpha; & BC=\frac{651}{6330}\alpha; & CD=\frac{98}{1476}\alpha. \end{cases}$

15. Poftremus hic cafus imprimis ad praxin accommodatus videtur, quia lentis obiectiuae diftantia focalis α fit pro hac multiplicatione circiter 45 dig. ficque tota tubi longitudo erit 65 dig. et campus confpicuus maior erit eo, quem huiusmodi tubus more vulgari conftructus oftendit. Nollem hic c minus accipere, quia lentes p'' et p''' nimis fierent paruae, hocque ipfo confufionem augerent. Verum fi campo plus detrahere velimus tribuendo ipfi n maiores valores, breuiora telescopia huius generis obtineri poterunt, quae vulgaria cum ratione campi, tum puritate repraefentationis fuperabunt. Videamus autem quantum lucri impetrare valeamus, fi quintam infuper lentem adiungamus, quam primo ante focum obiectiuae lentis conftitui conueniet.

De

De Telescopiis quinque lentibus instructis.

16. Secunda lente B ante focum obiectiuae constituta, erit pro ea $B=\frac{-b}{1+b}$ et $\mathfrak{B}=-b$, reliquae lentes sint dispositae vt ante ideoque $C=\frac{c}{1-c}$; $\mathfrak{C}=c$; $D=\frac{-d}{1+d}$; $\mathfrak{D}=-d$; $E=\infty$ et $\mathfrak{E}=1$. Hinc adperturas in computum ducendo ponatur breuitatis ergo $\Pi'=-b\theta'-\Phi$; $\Pi''=c\theta''+\theta'-\Phi$; $\Pi'''=-d\theta'''+\theta''+\theta'-\Phi$ ac denique $\Pi''''=\theta''''+\theta'''+\theta''+\theta'-\Phi$, denotante Φ semidiametrum campi conspicui: ac si multiplicatio statuatur $=m$, erit $\Phi=\frac{-\theta''''-\theta'''-\theta''-\theta'}{m-1}$, vnde patet si singulis litteris $\theta', \theta'', \theta'''$ et θ'''' valores negatiuos tribuere liceret eosque maximos, campum apparentem vlterius augeri posse. Sunt autem hae litterae θ eiusmodi fractiones, vt si p fuerit distantia foci lentis, ad quam refertur, θp sit semidiameter aperturae, vnde patet valores harum litterarum non vltra $\frac{1}{4}$ augeri posse.

17. His positis habemus in genere pro lentium singularum distantiis foci earumque interuallis sequentes formulas, denotante α distantiam foci lentis obiectiuae:

$$p'=\frac{+\mathfrak{B}\Phi}{\Pi'}\alpha; \qquad AB=\left(1+\frac{\Phi}{\Pi'}\right)\alpha=\frac{\Phi+\Pi'}{\Pi'}\alpha$$

$$p''=\frac{-B\mathfrak{C}\Phi}{\Pi''}\alpha; \qquad BC=\frac{-B\Phi(\Pi'-\Pi'')}{\Pi'\Pi''}\alpha$$

$$p'''=\frac{+BC\mathfrak{D}\Phi}{\Pi'''}\alpha; \qquad CD=\frac{+BC\Phi(\Pi''-\Pi''')}{\Pi''\Pi'''}\alpha$$

$$p''''=\frac{-BCD\mathfrak{E}\Phi}{\Pi''''}\alpha; \qquad DE=\frac{-BCD\Phi(\Pi'''-\Pi'''')}{\Pi'''\Pi''''}\alpha$$

ac

ac pro loco oculi $EO=\frac{\theta''''}{\Pi''''}p''''$.

Tum vero vt colores iridis euanescant, huic satisfieri oportet aequationi:

$$\frac{\theta'}{\Pi'}+\frac{\theta''}{\Pi''}+\frac{\theta'''}{\Pi'''}+\frac{\theta''''}{\Pi''''}=0.$$

18. Sit ω maximus valor, quem fractiones θ assequi possunt, si quidem lens fuerit vtrinque aeque conuexa, ac pro campo amplificando ponamus;

$$\theta''''=-\omega;\ \theta'''=-\omega;\ \theta''=-r\omega \text{ et } \theta'=q\omega$$

vt fiat $\Phi=\frac{2+r-q}{m-1}\omega$, seu $\Phi=M\omega$ posito $M=\frac{2+r-q}{m-1}$, Tum ob $\Pi'=-(bq+M)\omega$; $\Pi''=-(cr-q+M)\omega$; $\Pi'''=(d-r+q-M)\omega$ et $\Pi''''=-(2+r-q+M)\omega$, erit

$$p'=\frac{bM}{bq+M}\alpha; \qquad AB=\frac{bq}{bq+M}\alpha$$

$$p''=\frac{bc}{1+b}\cdot\frac{M}{q-cr-M}\alpha; \qquad BC=\frac{b}{1+b}\cdot\frac{M((b+1)q-cr)}{(bq+M)(q-cr-M)}\alpha$$

$$p'''=\frac{bcd}{(1+b)(1-c)}\cdot\frac{M}{d-r+q-M}\alpha; \qquad CD=\frac{bc}{(1+b)(1-c)}\cdot\frac{M(d-(1-c)r)}{(q-cr-M)(d-r+q-M)}\alpha$$

$$p''''=\frac{bcd}{(1+b)(1-c)(1+d)}\cdot\frac{M}{2+r-q+M}\alpha; \qquad DE=\frac{bcd}{(1+b)(1-c)(1+d)}\cdot\frac{M(1+d)}{(d-r+q-M)(2+r-q+M)}\alpha$$

et pro loco oculi $EO=\frac{p''''}{2+r-q+M}$.

19. Cum igitur destructio colorum hanc requirat aequationem:

$$\frac{q}{bq+M}+\frac{r}{q-cr-M}+\frac{1}{d-r+q-M}=\frac{1}{2+r-q+M}$$

vbi per hypothesin q debet esse quantitas positiua, ideoque quam minima accipienda; ante omnia videndum est, num litterae r valor positiuus tribui

possit,

poſſit, quo pacto augmentum in campum apparentem inferretur. Hunc in finem ponamus primo $r=0$, ſicque fiat neceſſe eſt

$$\frac{q}{bq+M}+\frac{1}{d+q-M}=\frac{1}{2-q+M} \text{ exiſtente } q>M$$

vbi obſeruo, quodſi ſecunda lens B in ipſum focum lentis obiectiuae incidat, fore $b=\infty$, ideoque $d=2-2q+2M$, qui caſus vtique meretur, vt ſtudioſius expendatur.

20. *Caſus I.* quo $b=\infty$ et $r=0$. Ob $d=2-2q+2M$ habebimus ſequentes determinationes: Primo $\Phi=\frac{2-q}{m-1}\omega$ et $M=\frac{2-q}{m-1}$. Deinde:

$p'=\frac{M}{q}\alpha$; $\quad AB=\alpha$

$p''=c.\frac{M}{q-M}\alpha$; $\quad BC=\frac{M}{q-M}\alpha$

$p'''=\frac{cd}{1-c}.\frac{M}{2-q+M}\alpha$; $\quad CD=\frac{c}{1-c}.\frac{Md}{(q-M)(1-q+M)}\alpha$

$p''''=\frac{cd}{(1-c)(1+d)}.\frac{M}{2-q+M}\alpha$; $DE=\frac{2cd}{(1-c)(1+d)}.\frac{M}{2-q+M}\alpha$

et pro loco oculi $EO=\frac{p''''}{2-q+M}$,

vbi ob quantitatem d iam definitam, quantitates q et c determinandae ſuperſunt, ita tamen vt ſit $q>M$ et $c<1$, tum vero videndum eſt, ne diſtantia focalis p'' nimis fiat exigua; vnde c vix minus quam $\frac{1}{4}$ accipiendum videtur, ſiquidem lens obiectiua parum confuſionis pariat; at ne tubus nimis fiat longus fractionem q maiorem quam $2M$ capi conueniet.

21. Quod-

21. Quodſi ergo ponamus $c=\frac{1}{4}$, $q=(n+1)M$ erit $d=2(1-nM)$ et $\Phi=\frac{2-(n+1)M}{m-1}\omega$ exiſtente $M=\frac{2-(n+1)M}{m-1}$ ſeu $M=\frac{2}{m+n}$ et $\Phi=M\omega$, tum vero

$p'=\frac{1}{n+1}\alpha$ $\quad AB=\alpha$

$p''=\frac{1}{4n}\alpha$ $\quad BC=\frac{1}{n}\alpha$

$p'''=\frac{6(1-nM)}{2-nM}M\alpha$ $\quad CD=\frac{6(1-nM)}{n(2-nM)}\alpha$

$p''''=\frac{6(1-nM)}{(4-nM)(3-2nM)}M\alpha$; $DE=\frac{12(1-nM)}{(2-nM)(3-2nM)}M\alpha$.

et pro loco oculi $FO=\frac{p''''}{2-nM}$, quibus formulis cum praecedentibus comparandis patet pro eadem multiplicatione eodemque campo aliquid lucri in longitudine tubi obtineri poſſe. Poſito enim vt ſupra $m=28$, $n=8$ vt ſit $M=\frac{1}{18}$, reperitur

$p'=\frac{1}{9}\alpha$; $p''=\frac{1}{32}\alpha$; $p'''=\frac{5}{48}\alpha$; $p''''=\frac{25}{888}\alpha$

$AB=\alpha$; $BC=\frac{1}{8}\alpha$; $CD=\frac{15}{38}\alpha$; $DE=\frac{15}{133}\alpha$.

22. Quando autem lens obiectiua eſt ſimplex, tum α tantum valorem obtinet, vt liceat ponere $c=\frac{1}{2}$, vnde poſito $q=(n+1)M$, vt ſit $d=2(1-nM)$ et $M=\frac{2}{m+n}$, habebimus

$p'=\frac{1}{n+1}\alpha$; $\quad AB=\alpha$

$p''=\frac{1}{2n}\alpha$ $\quad BC=\frac{1}{n}\alpha$

$p'''=\frac{2(1-nM)}{2-nM}M\alpha$; $\quad CD=\frac{2(1-nM)}{n(2-nM)}\alpha$

$p''''=\frac{2(1-nM)}{(2-nM)(3-2nM)}M\alpha$; $DE=\frac{4(1-nM)}{(2-nM)(3-2nM)}M\alpha$.

Quod-

Quodsi iam ponamus $m = 28$, $n = 8$, vt sit $\mathrm{M} =$ [illegible] erit

$$p' = \frac{1}{9}\alpha;\quad p'' = \frac{1}{18}\alpha;\quad p''' = \frac{5}{126}\alpha;\quad p'''' = \frac{5}{266}\alpha$$
$$\mathrm{AB} = \alpha;\quad \mathrm{BC} = \frac{1}{8}\alpha;\quad \mathrm{CD} = \frac{5}{56}\alpha;\quad \mathrm{DE} = \frac{9}{133}\alpha.$$

vnde sumto $\alpha = 45$ dig. longitudo tantum fit $56\frac{1}{3}$ dig. cum ante esset 65 dig. Poterat ergo n minus accipi, ideoque campus ampliari.

23. Hactenus sumsi $r = 0$, neque etiam hoc lentium statu huic litterae notabilem valorem positiuum dare licet, vt inde maior campus obtineatur, cum tamen vnam lentem superaddendo iure haud mediocriter maiorem campum expectare queamus. Quare secundam lentem vltra focum primae remoueri conueniet, neque parum campum augere licebit, si etiam litterae d valor negatiuus tribuatur, tum enim ipsi r maximus adeo valor scilicet $= 1$ tribui poterit. Posito autem $b = -b$, $d = -d$ et $r = 1$ habebimus: $\Phi = \frac{3-q}{m-1}\omega$ et $\mathrm{M} = \frac{1-q}{m-1}$ tum vero

$$p' = \frac{bM}{bq-M}\alpha;\qquad \mathrm{AB} = \frac{bq}{bq-M}\alpha$$
$$p'' = \frac{bc}{b-1}\cdot\frac{M}{q-c-M}\alpha;\qquad \mathrm{BC} = \frac{b}{b-1}\cdot\frac{M((b-1)q+c)}{(bq-M)(q-c-M)}\alpha$$
$$p''' = \frac{bcd}{(b-1)(1-c)}\cdot\frac{M}{d+1-q+M};\qquad \mathrm{CD} = \frac{bc}{(b-1)(1-c)}\cdot\frac{M(d+1-c)}{(q-c-M)(d+1-q+M)}\alpha$$
$$p'''' = \frac{bcd}{(b-1)(1-c)(d-1)}\cdot\frac{M}{3-q+M}\alpha;\qquad \mathrm{DE} = \frac{bcd}{(b-1)(1-c)(d-1)}\cdot\frac{M(d-2)}{(d+1-q+M)(3-q+M)}\alpha$$

item $\mathrm{EO} = \frac{p''''}{3-q+M}$ et

$$\frac{q}{bq-M} - \frac{1}{q-c-M} + \frac{1}{d+1-q+M} + \frac{1}{3-q+M} = 0.$$

24. Hic primum obseruandae sunt istae conditiones, vt sit $b > 1$, $c < 1$, $bq > \mathrm{M}$ et $q > c + \mathrm{M}$, tum vero in id est incumbendum, vt ipsi q minimus valor concilietur. Hunc in finem ponamus $c = 0$, et $b = 1$ ita vt $\frac{c}{b-1} = f$, debebitque esse

$$\frac{1-q}{q-M} = \frac{1}{d+1-q+M} + \frac{1}{3-q+M},$$

vnde q eo minus prodit, quo minus accipiatur d, at $d > 2$ statuamus ergo $d + 1 - q + \mathrm{M} = \frac{4}{3}(3 - q + \mathrm{M})$ seu $d = 3 - \frac{1}{3}(q - \mathrm{M})$ vt prodeat $\frac{1-q}{q-M} = \frac{7}{4(3-q+M)}$. Sit iam breuitatis gratia $q - \mathrm{M} = z$ ideoque $\frac{1-q}{z} = \frac{7}{12-4z}$ seu $12 - 12q - 11z + 4qz = 0$. Verum ob $\mathrm{M} = \frac{1-q}{m-1}$ est $q - \mathrm{M} = z = \frac{mq-1}{m-1}$, quo valore substituto prodit $12(m-1)(1-q) = (11-4q)(mq-3)$ et radice extracta

$$q = \frac{23m - \sqrt{(337mm - 352m)}}{8m} = \frac{23-\sqrt{337}}{8} + \frac{21}{m\sqrt{337}} \text{ proxime}$$

seu $q = 0,58 + \frac{8}{7m}$. Patet ergo q superare debere $\frac{1}{2}$, nisi multiplicatio m sit praegrandis, vnde campi apparentis semidiameter fit $\Phi = \frac{2\frac{5}{12}}{m-1}\omega$, qui notabiliter maior est, quam casu praecedente.

25. Operae pretium videtur hunc casum penitus euoluere, et cum sit $b = 1$, $c = 0$, $\frac{c}{b-1} = f$, quia postremae aequationi proxime satisfecisse sufficit, sit $q = \frac{7}{12}$ et $d = 3$, habebimusque sequentes determinationes; vbi $\mathrm{M} = \frac{5}{12(m-1)}$

$$p' = \frac{5M}{3-5M}\alpha \qquad AB = \frac{3}{3-5M}\alpha$$
$$p'' = \frac{5fM}{3-5M}\alpha \qquad BC = \frac{5M(3+5f)}{(3-5M)^2}\alpha$$
$$p''' = \frac{15fM}{17+5M}\alpha \qquad CD = \frac{100fM}{(3-5M)(17+5M)}\alpha$$
$$p'''' = \frac{15fM}{24+10M}\alpha \qquad DE = \frac{75fM}{2(17+5M)(12+5M)}\alpha$$

vbi numerus f arbitrio noſtro relinquitur, quem ergo ita paruum accipi conueniet, vt poſtrema lens diſtantiam focalem obtineat quaſi vnius digiti: quo minor enim haec admittitur, eo magis tubus contrahitur.

26. Vel ſumatur $q-M=\frac{1}{2}$, qui caſus ad praecedentem redit, ſi multiplicatio ſit $m=25$, et maneat $d=3$; cum ergo ſit $\frac{mq-3}{m-1}=\frac{1}{2}$ erit $q=\frac{m+5}{2m}$ et $M=\frac{5}{2m}$, ideoque $\Phi=\frac{5}{2m}\omega$, et reliquae determinationes erunt:

$$p' = 2M\alpha; \qquad AB = (1+2M)\alpha$$
$$p'' = 2fM\alpha; \qquad BC = 2(1+2f+2M)M\alpha$$
$$p''' = \tfrac{6}{7}fM\alpha; \qquad CD = \tfrac{16}{7}fM\alpha$$
$$p'''' = \tfrac{3}{5}fM\alpha; \qquad DE = \tfrac{6}{35}fM\alpha$$
$$\text{et } EO = \tfrac{2}{5}p'''' = \tfrac{6}{25}fM\alpha.$$

27. Si potuiſſemus ipſi d minorem tribuere valorem, maius lucrum in campo aſſequi liceret; quod vt fieri queat, quantitatem c negatiue capiamus, prodibunt hae formulae

p

$$p' = \frac{b}{bq-M}M\alpha \qquad AB = \frac{bq}{bq-M}\alpha$$
$$p'' = \frac{bc}{1-b}\cdot\frac{M\alpha}{q+c-M} \qquad BC = \frac{b}{1-b}\cdot\frac{(c+(1-b)q)M\alpha}{(bq-M)(c+q-M)}$$
$$p''' = \frac{bcd}{(1-b)(1+c)}\cdot\frac{M\alpha}{d+1-q+M} \quad CD = \frac{bc}{(1-b)(1+c)}\cdot\frac{(d+1+c)M\alpha}{(c+q-M)(d+1-q+M)}$$
$$p'''' = \frac{bcd}{(1-b)(1+c)(d-1)}\cdot\frac{M\alpha}{2-q+M}; \quad DE = \frac{bcd}{(1-b)(1+c)(d-1)}\cdot\frac{(d-2)M\alpha}{(d+1-q+M)(2-q+M)}$$

exiſtente $M=\frac{m-q}{m-1}$; $\Phi=M\omega$ et $EO=\frac{p''''}{2-q+M}$.

At pro coloribus iridis tollendis ſtatui oportet

$$\frac{1}{c+q-M} = \frac{q}{bq-M} + \frac{c}{d+1-q+M} + \frac{1}{2-q+M}.$$

Euidens autem eſt eſſe debere $b<1$, $bq>M$, et $d>2$.

Sit $m=10$; et $q=\frac{3}{4}$ erit $M=\frac{1}{4}$ et $\Phi=\frac{\omega}{4}$ hincque ſumto $\omega=\frac{1}{4}$ erit $\Phi=3°, 35'$ et diameter campi apparentis $=7°, 10'$. Tum vero pro coloribus deſtruendis erit

$$\frac{1}{c+\frac{1}{2}} = \frac{3}{3b-1} + \frac{1}{d+\frac{1}{2}} + \frac{2}{3}.$$ Sit iam $b=1$, $c=0$, at $\frac{c}{1-b}=f$, erit

$$2 = \tfrac{3}{2} + \tfrac{2}{3} - \frac{1}{d+\frac{1}{2}}$$ ſeu $\frac{1}{10}=\frac{1}{d+\frac{1}{2}}$ hincque $d=9\frac{1}{2}$;

vnde teleſcopium ita erit comparatum;

$$p' = \tfrac{1}{2}\alpha; \qquad AB = \tfrac{3}{2}\alpha$$
$$p'' = \tfrac{f}{2}\alpha; \qquad BC = (\tfrac{3}{4}+f)\alpha$$
$$p''' = \tfrac{19}{40}f\alpha; \qquad CD = \tfrac{21}{40}f\alpha$$
$$p'''' = \tfrac{19}{170}f\alpha; \qquad DE = \tfrac{57}{170}f\alpha$$

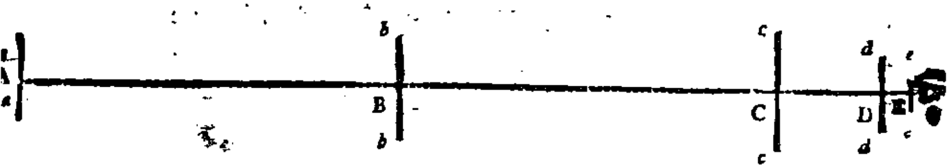
et pro oculo $EO=\frac{2}{3}\cdot p''''$.

vbi lentis obiectiuae diſtantia focalis ſumi poteſt $\alpha=12$ dig. hinc cum ſit $p''''=\frac{114}{85}f$, vt haec diſtantia focalis fiat fere vnius digiti, ſumi debet $f=\frac{3}{4}$.

Lenti autem obiectiuae apertura tribui debet, cuius semidiameter $= \frac{1}{4}$ dig. et diameter $= \frac{1}{2}$ dig. ex quo constructio huius telescopii ita se habebit:

Lentis	distantia focalis	diam. aperturae	Interualla lentium
in A	12 dig.	$\frac{1}{2}$ dig.	$AB = 18$ dig.
in B	6	$2\frac{1}{4}$	$BC = 18$
in C	$4\frac{1}{2} = 4,5000$	$2\frac{1}{4}$	$CD = 4\frac{29}{40} = 4,725$
in D	$2\frac{11}{80} = 2,1375$	1,07	$DE = \frac{512}{685} = 0,754$
in E	$1\frac{1}{170} = 1,0059$	0,50	$EO = \frac{1}{3}p'''' = 0,402$

Tota ergo longitudo huius tubi erit fere 42 dig. seu $3\frac{1}{2}$ pedum et figura huius instrumenti erit:



28. Vt vero isti aequationi in genere satisfaciam, ponam:

$b - \frac{M}{q} = \frac{3-q+M}{3+\mu}$ et $d + 1 - q + M = \frac{1-q+M}{1-\nu}$, vnde fit $b = \frac{M}{q} + \frac{3-q+M}{3+\mu}$ et $d = \frac{2+\nu}{1-\nu} - \frac{\nu}{1-\nu}(q-M)$

tum vero erit

$\frac{1}{c+q-M} = \frac{3+\mu-\nu}{3-q+M}$, hincque $c = \frac{3}{3+\mu-\nu} - \frac{(6+\mu-\nu)(q-M)}{3+\mu-\nu}$.

At quia $b < 1$ oportet sit $\frac{M}{q} < \frac{\mu+q-M}{3+\mu}$, ideoque $q > M$. Ponamus ergo $q = (1+n)M$, vt sit $M = \frac{3}{m+n}$, et postrema conditio dat $\frac{3}{1+n} < \frac{\mu+nM}{3+\mu}$ seu $\mu n + n(n+1)M > 3$ et

et quia numerus n, quantum fieri potest, minui debet, numerum μ ita accipi oportet vt fiat $\mu n > 3$; tum ergo habebimus:

$$b = \frac{6+3n+\mu}{(1+n)(3+\mu)} - \frac{nM}{3+\mu};\quad c = \frac{3-(6+\mu-\nu)nM}{3+\mu-\nu};\quad d = \frac{2+\nu-\nu nM}{1-\nu}$$

$$1-b = \frac{\mu n - 3}{(1+n)(3+\mu)} + \frac{nM}{3+\mu};\quad 1+c = \frac{6+\mu-\nu-(6+\mu-\nu)nM}{3+\mu-\nu};\quad d-1 = \frac{1+2\nu-\nu nM}{1-\nu}$$

$$\text{et } d-2 = \frac{3\nu-\nu nM}{1-\nu}.$$

29. Maximum ergo campum obtinebimus, si sumamus $n = 0$, vnde valor ipsius μ ita debet esse infinitus, vt fiat $\mu n > 3$. Sit igitur $\mu n = 3 + f$, eritque $b = 1$, $c = 0$, at ob $1 - b = \frac{f}{\mu}$ et $c = \frac{3-(3+f)M}{\mu}$ fiet $\frac{c}{1-b} = \frac{3-(3+f)M}{f}$ tum vero $d = \frac{2+\nu}{1-\nu}$; $d - 1 = \frac{1+2\nu}{1-\nu}$ et $d - 2 = \frac{3\nu}{1-\nu}$. Hoc modo obtinebimus telescopium, in quo ob $M = \frac{3}{m}$, erit semidiameter campi conspicui $\Phi = \frac{3\omega}{m}$, ideoque triplo maior quam in tubis vulgaribus astronomicis. Verum hinc altera lens in B focum sortitur infinite remotum, et distantia AB fit infinite longa, ita vt in praxi hunc casum assequi nequeamus. Quare pro n valores aliquanto maiores accipiamus.

30. *Casus I.* Sit $n = 1$, ideoque $q = 2M$, $M = \frac{3}{m+1}$ et $\Phi = \frac{3\omega}{m+1}$. Deinde ob $\mu n > 3$ ponatur $\mu = 3 + f$, eritque

$$b = \frac{12+f-2M}{2(6+f)};\ c = \frac{3-(9+f-\nu)M}{6+f-\nu};\ \text{et } d = \frac{2+\nu-\nu M}{1-\nu}$$

et telescopii determinationes erunt:

p

$p' = \frac{b}{2b-1}\alpha;$ $\qquad AB = \frac{2b}{2b-1}\alpha$

$p'' = \frac{bc}{1-b}\cdot\frac{M\alpha}{c+M};$ $\qquad BC = \frac{b}{1-b}\cdot\frac{(2(1-b)M+c)\alpha}{(2b-1)(c+M)}$

$p''' = \frac{bcd}{(1-b)(1+c)}\cdot\frac{M\alpha}{d+1-M};$ $CD = \frac{bc}{(1-b)(1+c)}\cdot\frac{(d+1+c)M\alpha}{(c+M)(d+1-M)}$

$p'''' = \frac{bcd}{(1-b)(1+c)(d-1)}\cdot\frac{M\alpha}{1-M};$ $DE = \frac{bcd}{(1-b)(1+c)(d-1)}\cdot\frac{(d-1)M\alpha}{(d+1-M)(1-M)}$

et pro loco oculi $EO = \frac{p''''}{1-M}$.

Fit ergo $AB = \frac{12+f-2M}{6-2M}\alpha$, ideoque $AB > 2\alpha$, et ob $1-b = \frac{f+2M}{2(6+f)}$ et $c+M = \frac{2-M}{6+f-\nu}$ erit $p'' = \frac{1+f-1M}{f+2M}\cdot\frac{2-(8+f-\nu)M}{1-M}M\alpha$.

quam ergo lentem, vt et reliquas, pro lubitu ad praxin accommodare licet; at vero multiplicatio m maior esse debet quam $8+f-\nu$.

31. *Exempl.* Ob enormen tubi longitudinem hic casus tantum ad exiguas multiplicationes accommodari potest. Sumatur ergo $f=0$; $\nu=1$ et multiplicatio $m=8$, vnde fit $M=\frac{1}{9}$ et $q=\frac{1}{3}$, atque $\Phi=\frac{7}{3}\omega$. Deinde ob $b=\frac{17}{18}$, $c=\frac{1}{27}$ et $d=\infty$ sequentes habebimus determinationes:

$p' = \frac{17}{16}\alpha;$ $\theta'p' = \frac{17}{24}\alpha\omega;$ $AB = \frac{17}{8}\alpha$

$p'' = \frac{17}{24}\alpha;$ $\theta''p'' = \frac{17}{24}\alpha\omega;$ $BC = \frac{17}{4}\alpha$

$p''' = \frac{17}{88}\alpha;$ $\theta'''p''' = \frac{17}{88}\alpha\omega;$ $CD = \frac{119}{176}\alpha$

$p'''' = \frac{17}{176}\alpha;$ $\theta''''p'''' = \frac{17}{176}\alpha\omega;$ $DE = \frac{17}{176}\alpha$

et pro loco oculi $EO = \frac{9}{8}p'''' = \frac{51}{1408}\alpha$

tota vero tubi longitudo $= \frac{79}{11}\alpha$. Quodsi ergo capiatur $\alpha = 10$ dig. tubus habebitur 72 dig. seu 6 pedum,

dum, qui quidem tantum octies auget, sed triplo maiorem campum patefacit, quam tubi ordinarii.

32. *Casus II.* Sit $n=2$, seu $q=3M$, et $M=\frac{1}{m+1}$; iam ob $\mu n > 3$ ponatur $\mu = \frac{3}{2}+f$ eritque

$b = \frac{15+2f-12M}{2(9+2f)};$ $c = \frac{6-2(15+2f-2\nu)M}{15+2f-3\nu};$ $d = \frac{1+\nu-2\nu M}{1-\nu}$

hinc $1-b = \frac{3+2f+12M}{2(9+2f)}$

ideoque $M < \frac{1}{15+2f-2\nu}$ seu $m > 13+2f-2\nu$. Determinationes vero erunt

$p' = \frac{b}{3b-1}\alpha;$ $\qquad AB = \frac{4b}{3b-1}\alpha$

$p'' = \frac{bc}{1-b}\cdot\frac{M\alpha}{c+2M};$ $\qquad BC = \frac{b}{1-b}\cdot\frac{(3(1-b)M+c)\alpha}{(3b-1)(c+2M)}$

$p''' = \frac{bcd}{(1-b)(1+c)}\cdot\frac{M\alpha}{d+1-2M};$ $CD = \frac{bc}{(1-b)(1+c)}\cdot\frac{(c+1+d)M\alpha}{(c+2M)(d+1-2M)}$

$p'''' = \frac{bcd}{(1-b)(1+c)(d-1)}\cdot\frac{M\alpha}{1-2M};$ $DE = \frac{bcd}{(1-b)(1+c)(d-1)}\cdot\frac{(d-1)M\alpha}{(d+1-2M)(1-2M)}$

vbi notetur esse

$3b-1 = \frac{6(1-2M)}{9+2f};$ $c+2M = \frac{2(3-2M)}{15+2f-3\nu};$ et $d+1-2M = \frac{2-2M}{1-\nu}$.

33. *Exempl.* Applicemus hunc casum ad multiplicationem aliquanto maiorem, sitque $m=16$, hinc $M=\frac{1}{17}$ et $q=\frac{3}{17}$, tum vero capiatur $\nu=1$, vt sit $d=\infty$, quia hoc commode fieri licet, atque habebimus:

$b = \frac{25+2f}{3(9+2f)};$ $1-b = \frac{2+4f}{3(9+2f)};$ $3b-1 = \frac{16}{9+2f}$

$c = \frac{5-2f}{2(11+2f)};$ $1+c = \frac{27+4f}{2(11+2f)};$ $c+2M = \frac{169}{34(11+2f)}$

 hincque

hincque sequentes mensuras:

$$p' = \frac{25+2f}{48}\alpha \qquad AB = \frac{25+2f}{16}\alpha$$
$$p'' = \frac{25+2f}{2+4f}\cdot\frac{5-2f}{96}\alpha \qquad BC = \frac{25+2f}{2+4f}\cdot\frac{7+2f}{32}\alpha$$
$$p''' = \frac{25+2f}{2+4f}\cdot\frac{5-2f}{38+4f}\cdot\frac{\alpha}{6} \qquad CD = \frac{25+2f}{2+4f}\cdot\frac{5-2f}{38+4f}\cdot\frac{11+2f}{32}\alpha$$
$$p'''' = \frac{25+2f}{2+4f}\cdot\frac{5-2f}{38+4f}\cdot\frac{\alpha}{16} \qquad DE = \frac{25+2f}{2+4f}\cdot\frac{5-2f}{38+4f}\cdot\frac{\alpha}{16}$$

et pro loco oculi $EO = \frac{5}{8}p''''$; vbi notandum pro lente obiectiua, si fuerit simplex capi, oportere fere $\alpha = 24$. dig.

34. Ne hic lens ocularis nimis fiat exigua ponere debemus $f = 1$ vel adhuc minus, vt $f = \frac{1}{2}$, vnde pro hac duplici hypothesi habebimus has determinationes:

I. $f = 1$		II. $f = \frac{1}{2}$	
$p' = \frac{9}{16}\alpha$;	$AB = 1\frac{11}{16}\alpha$	$p' = \frac{13}{24}\alpha$;	$AB = \frac{13}{8}\alpha$
$p'' = \frac{9}{64}\alpha$;	$BC = 1\frac{17}{64}\alpha$	$p'' = \frac{13}{48}\alpha$;	$BC = 1\frac{5}{8}\alpha$
$p''' = \frac{3}{56}\alpha$;	$CD = \frac{117}{896}\alpha$	$p''' = \frac{13}{120}\alpha$;	$CD = \frac{39}{160}\alpha$
$p'''' = \frac{9}{448}\alpha$;	$DE = \frac{9}{448}\alpha$	$p'''' = \frac{13}{320}\alpha$;	$DE = \frac{13}{320}\alpha$
	$AE = 3\frac{93}{896}\alpha$.		$AE = 3\frac{171}{320}\alpha$

Si lens obiectiua composita adhibeatur, vt α vnius tantum pedis accipi queat, hypothesis $f = \frac{1}{2}$ egregium telescopium suppeditare videtur.

35. *Casus III.* Sit $n = 3$ seu $q = 4M$ et $M = \frac{3}{m+3}$; ob $\mu n > 3$ ponatur $\mu = 1 + f$ eritque

$$b = \frac{16+f-12M}{4(4+f)};\quad c = \frac{8-8(7+f-\nu)M}{6+f-\nu};\quad d = \frac{2+\nu-2\nu M}{1-\nu}$$
$$4b-1 = \frac{12(1-M)}{4+f};\quad c+q-M = \frac{8(1-M)}{6+f-\nu};\quad d+1-q+M = \frac{2(1-M)}{1-\nu}$$

vbi

vbi esse debet $m > 18 + 3f - 3\nu$. Sumamus ergo $\nu = 1$, ac reperimus

$$p' = \frac{16+f-12M}{48(1-M)}\alpha;\qquad AB = \frac{16+f-12M}{12(1-M)}\alpha$$
$$p'' = \frac{16+f-12M}{3(f+4M)}\cdot\frac{1-6M-fM}{1-M}\cdot M\alpha;\qquad BC = \frac{(16+f-12M)(6+f-20M-6fM)}{36(1-M)(f+4M)}\alpha$$
$$p''' = \frac{16+f-12M}{f+4M}\cdot\frac{1-6M-fM}{8+f-3(6+f)M}\cdot M\alpha;\qquad CD = \frac{(16+f-12M)(1-6M-fM)(5+f)}{3(f+4M)(8+f-3(6+f)M)(1-M)}M\alpha$$
$$p'''' = \frac{16+f-12M}{3(f+4M)}\cdot\frac{1-6M-fM}{8+f-3(6+f)M}\cdot\frac{M\alpha}{1-M};\qquad DE = \frac{(16+f-12M)(1-6M-fM)}{8(f+4M)(8+f-3(6+f)M)}\cdot\frac{M\alpha}{1-M}$$

et pro loco oculi $EO = \frac{p''''}{8(1-M)}$; vnde pro quauis multiplicatione huiusmodi telescopia confici possunt.

36. Exempl. Sit multiplicatio $m = 33$, ideoque $M = \frac{1}{12}$ ac pro lente obiectiua simplici statui oportet $\alpha = 96$ dig. circiter, erit

$$p' = \frac{15+f}{44}\alpha;\qquad AB = \frac{15+f}{11}\alpha$$
$$p'' = \frac{15+f}{3f+1}\cdot\frac{6-f}{132}\alpha;\qquad BC = \frac{15+f}{3f+1}\cdot\frac{7+2f}{11}\alpha$$
$$p''' = \frac{15+f}{3f+1}\cdot\frac{6-f}{12(26+3f)}\alpha;\qquad CD = \frac{15+f}{3f+1}\cdot\frac{6-f}{26+3f}\cdot\frac{5+f}{12}\alpha$$
$$p'''' = \frac{15+f}{3f+1}\cdot\frac{6-f}{33(26+3f)}\alpha;\qquad DE = \frac{15+f}{3f+1}\cdot\frac{6-f}{26+3f}\cdot\frac{\alpha}{33}$$

et pro oculo $EO = \frac{4}{11}p''''$.

Sumatur ergo $f = 2$ ac prodibit

	dig.		dig.
$p' = \frac{17}{44}\alpha =$	$37,091$	$AB = \frac{17}{11}\alpha =$	$148,364$
$p'' = \frac{17}{231}\alpha =$	$7,065$	$BC = \frac{17}{21}\alpha =$	$77,714$
$p''' = \frac{17}{1344}\alpha =$	$1,214$	$CD = \frac{17}{264}\alpha =$	$6,182$
$p'''' = \frac{17}{1848}\alpha =$	$0,883$	$DE = \frac{17}{1848}\alpha =$	$0,883$
		longitudo $EO =$	$233,143$

37. Po-

37. Potuiſſemus quoque in formulis §. 27. quantitatem d negatiue accipere, vt lens penultima ante focum lentis ocularis incidiſſet, ſed quia commode ſtatui poteſt $d=\infty$, inde eosdem caſus eliciuiſſimus. Interim in hoc telescopiorum genere nimia longitudo imprimis ad praxin eſt inepta, vnde in hoc nobis erit incumbendum, vt ea ad minorem longitudinem contrahamus, id quod ſine notabili campi apparentis iactura fieri nequit. Verum lentium numerum augendo non ſolum idem campus ſeruari ſed etiam vltra augeri poterit. Tum autem binas lentes B et C ita conſtitui oportet, vt ob eas campus adeo coarctetur, adiectione autem ſextae lentis iterum dilatetur. Ex quo ſex lentes adhibendo non maiorem campum, ſed breuiores tubos aſſequi conabimur.

De Telescopiis ſex lentibus inſtructis.

38. Pro quinque ergo lentibus poſt obiectiuam collocandis ponamus:

$$\mathrm{B}=\frac{-b}{1+b};\ \mathrm{C}=\frac{c}{1-c};\ \mathrm{D}=\frac{-d}{1+d};\ \mathrm{E}=\frac{-e}{e-1};\ \mathrm{F}=\infty$$

$$\mathfrak{B}=-b;\ \mathfrak{C}=c;\ \mathfrak{D}=-d;\ \mathfrak{E}=e;\ \mathfrak{F}=1.$$

tum vero

$$\Pi'=-b\theta'-\Phi;\ \Pi''=c\theta''+\theta'-\Phi;\ \Pi'''=-d\theta'''+\theta''+\theta'-\Phi$$

$$\Pi''''=e\theta''''+\theta'''+\theta''+\theta'-\Phi;\ \Pi^{V}=\theta^{V}+\theta''''+\theta'''+\theta''+\theta'-\Phi.$$

Cum

Cum iam ſit pro campo apparente $\Phi=\frac{-\theta'''''-\theta''''-\theta'''-\theta''-\theta'}{m-1}$ ſtatuamus:

$$\theta'=q\omega;\ \theta''=-r\omega;\ \theta'''=-\omega;\ \theta''''=-\omega;\ \theta^{V}=-\omega$$

vt fiat $\Phi=\frac{1+r-q}{m-1}\omega$, et breuitatis ergo $\Phi=\mathrm{M}\omega$ poſito $\mathrm{M}=\frac{1+r-q}{m-1}$ ſicque habebimus

$$\Pi'=-(bq+\mathrm{M})\omega;\ \Pi''=(q-cr-\mathrm{M})\omega;\ \Pi'''=+(d-r+q-\mathrm{M})\omega$$

$$\Pi''''=-(e+1+r-q+\mathrm{M})\omega;\ \Pi'''''=-(3+r-q+\mathrm{M})\omega$$

ſatisfacereque debemus huic aequationi:

$$\frac{-q}{bq+\mathrm{M}}-\frac{r}{q-cr-\mathrm{M}}-\frac{1}{d-r+q-\mathrm{M}}+\frac{1}{e+1+r-q+\mathrm{M}}+\frac{1}{1+r-q+\mathrm{M}}=0.$$

39. Hinc autem pro determinatione Teleſcopii ſequentes oriuntur formulae:

$$p'=\frac{b\mathrm{M}}{bq+\mathrm{M}}\alpha \qquad \mathrm{AB}=\frac{bq}{bq+\mathrm{M}}\alpha$$

$$p''=\frac{bc}{1+b}\cdot\frac{\mathrm{M}\alpha}{q-cr-\mathrm{M}} \qquad \mathrm{BC}=\frac{b}{1+b}\cdot\frac{((1+b)q-cr)\mathrm{M}\alpha}{(bq+\mathrm{M})(q-cr-\mathrm{M})}$$

$$p'''=\frac{bcd}{(1+b)(1-c)}\cdot\frac{\mathrm{M}\alpha}{d-r+q-\mathrm{M}} \qquad \mathrm{CD}=\frac{bc}{(1+b)(1-c)}\cdot\frac{(d-(1-c)r)\mathrm{M}\alpha}{(q-cr-\mathrm{M})(d-r+q-\mathrm{M})}$$

$$p''''=\frac{bcde}{(1+b)(1-c)(1+d)}\cdot\frac{\mathrm{M}\alpha}{e+1+r-q+\mathrm{M}} \qquad \mathrm{DE}=\frac{bcd}{(1+b)(1-c)(1+d)}\cdot\frac{(d+e+1)\mathrm{M}\alpha}{(d-r+q-\mathrm{M})(e+1+r-q+\mathrm{M})}$$

$$p^{V}=\frac{bcde}{(1+b)(1-c)(1+d)(e-1)}\cdot\frac{\mathrm{M}\alpha}{1+r-q+\mathrm{M}} \qquad \mathrm{EF}=\frac{bcde}{(1+b)(1-c)(1+d)(e-1)}\cdot\frac{(e-1)\mathrm{M}\alpha}{(e+1+r-q+\mathrm{M})(1+r-q+\mathrm{M})}$$

et pro loco oculi $\mathrm{FO}=\frac{1}{1+r-q+\mathrm{M}}p^{V}$

vnde patet has conditiones obſeruari oportere, vt ſit

$$(1+b)q>cr;\ d>(1-c)r;\ \text{et}\ e>2.$$

40. *Caſus I.* Conſideremus caſum facillimum, quo

$$b=\infty;\ e=\infty\ \text{et}\ r=0$$

et aequatio adimplenda eſt $\frac{-1}{d-r+q-\mathrm{M}}+\frac{1}{1+r-q+\mathrm{M}}=0$

vnde

vnde fit $d=3-2(q-M)$. Statuatur ergo $q=(1+n)M$ vt fit $d=3-2nM$, et formulae nostrae abibunt in has:

$p' = \frac{\alpha}{1+n}$	$AB=\alpha$
$p'' = c.\frac{\alpha}{n}$	$BC=\frac{\alpha}{n}$
$p''' = \frac{c(3-2nM)}{1-c}.\frac{M\alpha}{3-nM}$	$CD=\frac{c}{1-c}.\frac{d\alpha}{n(3-nM)}$
$p'''' = \frac{c(3-2nM)}{1-c}.\frac{M\alpha}{4-2nM}$	$DE=\frac{c(3-2nM)}{(1-c)(4-2nM)}.\frac{M\alpha}{3-nM}$
$p''''' = \frac{c(3-2nM)}{(1-c)(4-2nM)}.\frac{M\alpha}{3-nM}$	$EF=\frac{c(3-2nM)}{(1-c)(4-2nM)}.\frac{M\alpha}{3-nM}$

cum ergo fit $M=\frac{3-q}{m-1}=\frac{3-M-nM}{m-1}$ erit $M=\frac{3}{m+n}$ et semidiameter campi apparentis $\Phi=\frac{2\omega}{m+n}$.

41. *Exempl. I.* Sit multiplicatio $m=10$, ac sumatur $n=2$ et $c=\frac{1}{3}$, tum vero sit lentis obiectiuae distantia foci $\alpha=16$ dig. erit ergo $M=\frac{1}{4}$, $q=\frac{3}{4}$ et $\Phi=\frac{\omega}{4}$ seu $\Phi=3°, 35'$ et mensurae pro telescopio construendo erunt

$p' = 5\frac{1}{3}$ dig.	$AB=16$
$p'' = 4$	$BC= 8$
$p''' = 3\frac{1}{3}$	$CD= 6\frac{2}{3}$
$p'''' = 2\frac{2}{3}$	$DE= 1\frac{1}{13}$
$p''''' = 1\frac{8}{13}$	$EF= 1\frac{1}{13}$
$FO=\frac{2}{3}p'''''$	$AF=32\frac{8}{13}$

Hic ergo tubus circiter 33 dig. longus obiecta in ratione decupla multiplicabit, campum autem aperiet, cuius diameter $2\frac{1}{2}$ vicibus maior est quam in tubis consuetis astronomicis.

42. *Exempl. II.* Sit multiplicatio maior $m=25$, maneatque $n=2$, at capiatur $c=\frac{2}{3}$, erit $M=\frac{1}{9}$, et $q=\frac{1}{3}$; lentis autem obiectiuae distantia focalis sumi debet $\alpha=54$ dig. vnde pro campo apparente $\Phi=\frac{\omega}{9}$ seu $\Phi=1°, 35'$. Mensurae autem pro costructione huius telescopii sunt:

$p' = 18$ dig.	$AB=54$
$p'' = 10\frac{4}{5}$	$BC=27$
$p''' = 3\frac{17}{35}$	$CD=16\frac{14}{35}$
$p'''' = 2\frac{7}{8}$	$DE= 1\frac{7}{188}$
$p''''' = 1\frac{7}{235}$	$EF= 1\frac{7}{205}$
$FO=\frac{9}{21}p'''''$	$AF=99\frac{116}{205}$

ita vt hic tubus futurus sit 100 dig. seu $8\frac{1}{3}$ pedum.

43. *Exempl. III.* Vt tubus breuior reddatur sumatur $n=3$, sitque multiplicatio $m=24$, et $\alpha=52$ dig. erit $M=\frac{1}{9}$ et $\Phi=\frac{\omega}{9}$ vt ante. Mensurae autem pro constructione telescopii sunt

$p' = 13$ dig.	$AB= 52$
$p'' = 6\frac{14}{15}$	$BC= 17\frac{1}{3}$
$p''' = 3\frac{12}{17}$	$CD= 10\frac{1}{9}$
$p'''' = 2\frac{94}{115}$	$DE= 1\frac{1}{95}$
$p''''' = 1\frac{1}{5}$	$EF= 1\frac{1}{95}$
et $FO=\frac{1}{8}p'''''$	$AF= 81\frac{7}{15}$

hic ergo tubus sesquipede breuior est quam praecedens, et tamen eundem campum complectitur, sed tantillo minus multiplicat.

44. *Casus II.* Propositus nunc sit iste casus $b=\infty$ et $d=\infty$ et quia $e>2$ ponamus $e+1+r-q$

$-q+M=\frac{3+r-q+M}{1-v}$ vt habeamus $e=\frac{2+v}{1-v}-\frac{v(q-r-M)}{1-v}$ atque

$$\frac{r}{q-cr-M}=\frac{2-v}{3+r-q+M} \text{ seu } \frac{q-M}{r}-c=\frac{3+r}{2-v}-\frac{(q-M)}{2-v}$$

vnde fit $q=M+\frac{r(3+2c+r-vc)}{2+r-v}$. At eſt $c<1$ et $q<1$, quare cum ſit $(m-1)M+q=3+r$ erit

$$Mm=3-\frac{r(1+2c+v-vc)}{2+r-v} \text{ ſeu } M=\frac{(2-v)(3+r-cr)}{(2+r-v)m}.$$

Statuatur breuitatis gratia $n=\frac{r(3+2c+r-vc)}{M(2+r-v)}$ vt ſit $q=(1+n)M$ eritque $q-cr-M=\frac{r(3+r-cr)}{2+r-v}$ et $q-r-M=\frac{r(1+v+2c-vc)}{2+r-v}$ atque

$p'=\frac{\alpha}{1+n}$;	$AB=\alpha$
$p''=\frac{cM\alpha}{q-cr-M}$;	$BC=\frac{M\alpha}{q-cr-M}$
$p'''=\frac{c}{1-c}\cdot M\alpha$;	$CD=\frac{c}{1-c}\cdot\frac{M\alpha}{q-cr-M}$
$p''''=\frac{ce}{1-c}\cdot\frac{(1-v)M\alpha}{3-q+r+M}$;	$DE=\frac{c}{1-c}\cdot\frac{(1-v)M\alpha}{3-q+r+M}$
$p^{V}=\frac{ce}{(1-c)(e-1)}\cdot\frac{M\alpha}{3-q+r+M}$;	$EF=\frac{ce}{(1-c)(e-1)}\cdot\frac{(1-v)(e-2)M\alpha}{(3-q+r+M)^2}$
	et $FO=\frac{p''''}{3-q+r+M}$.

45. Quo clarius indolem huius caſus perſpiciamus, primum obſeruo ad tubum contrahendum neceſſe eſſe, vt $q-cr-M>M$. Statuamus ergo $q-cr-M=kM$, ac pro M introducto valore reperimus $r=\frac{k(2-v)}{m}$, hincque $M=\frac{3m+k(2-v)(1-c)}{m(m+k)}$ et $q=(1+k)M+cr$, ac $q-r-M=\frac{k(1+v+2c-vc)}{m+k}$.
Ex his propoſita multiplicatione m, ſumtoque numero k ſicut pro tubo contrahendo videbitur, adhuc litterae c et v arbitrio noſtro relinquuntur, quarum

rum haec v ad neutrum ſuorum limitum o et 1 nimis prope accedere debet, ne interualla lentium DE et EF minora euadant, quam earum craſſities patitur; tum vero litteram c ita ſumi conueniet, vt poſtrema lens non fiat nimis exigua, quod imprimis eſt cauendum, quia hinc noua confuſio inveheretur.

46. Ponamus ergo $k=2$, vt fiat $BC=\frac{1}{2}AB$, tum vero $c=\frac{1}{2}$ vt fiat quoque $CD=\frac{1}{2}BC$; eritque $r=\frac{2(2-v)}{m}$; $M=\frac{3m+\frac{4}{3}(2-v)}{m(m+2)}$ et $q-r-M=\frac{2(5+2v)}{3(m+2)}$, hinc $q=\frac{(31-2v)m+16(2-v)}{3m(m+2)}$ et $n+1=\frac{q}{M}=\frac{(31-2v)m+16(2-v)}{9m+4(2-v)}$; tum vero $e=\frac{2+v}{1-v}-\frac{2}{3}\frac{v(5+2v)}{(1-v)(m+2)}$. Ponatur breuitatis ergo $\frac{2(5+2v)}{3(m+2)}=z$, ſeu $q-r-M=z$ erit:

$p'=\frac{\alpha}{1+n}$	$AB=\alpha$
$p''=\frac{1}{4}\alpha$	$BC=\frac{1}{2}\alpha$
$p'''=\frac{1}{2}M\alpha$	$CD=\frac{1}{4}\alpha$
$p''''=\frac{1}{2}e.\frac{(1-v)M\alpha}{3-z}$	$DE=\frac{1}{2}.\frac{(1-v)M\alpha}{3-z}$
$p^{V}=\frac{e}{2(e-1)}.\frac{M\alpha}{3-z}$	$EF=\frac{e}{2(e-1)}.\frac{(1-v)(e-2)M\alpha}{(3-z)^2}$
	et $FO=\frac{1}{3-z}p^{V}$.

47. Si ponamus inſuper $v=\frac{1}{2}$ habebimus:
$M=\frac{3m+2}{m(m+2)}$; $z=\frac{4}{m+2}$; $e=5-\frac{4}{m+2}$; $n+1=\frac{10m+8}{3m+2}$ tum vero $q=\frac{10m+8}{m(m+2)}$ et $r=\frac{3}{m}$.
Sit exempli gratia multiplicatio $m=30$, eritque $M=\frac{23}{240}$; $n+1=\frac{77}{23}$; $z=\frac{1}{8}$; $e=\frac{39}{8}$; $q=\frac{77}{240}$ et $r=\frac{1}{10}$; hincque

$$\begin{array}{ll|l} p' & = \frac{23}{77}\alpha & AB = \alpha \\ p'' & = \frac{1}{8}\alpha & BC = \frac{1}{2}\alpha \\ p''' & = \frac{23}{480}\alpha & CD = \frac{1}{4}\alpha \\ p'''' & = \frac{13}{320}\alpha & DE = \frac{1}{160}\alpha \\ p^{V} & = \frac{13}{880}\alpha & EF = \frac{13}{1440}\alpha \end{array}$$

Longitudo $AF = (1 + \frac{148}{186})\alpha$,

et pro loco oculi $FO = \frac{8}{23} \cdot p^{V}$

vbi ob $\alpha = 72$ dig. lens ocularis non fit nimis parva, atque adeo duplicata vt fit $\alpha = 48$ dig.

48. Vt magis contrahamus tubum, ponamus $k = 3$, $c = \frac{3}{4}$ et $v = \frac{1}{2}$ et pro multiplicatione m in genere $r = \frac{9}{2m}$ et $q = \frac{15(7m+9)}{8m(m+3)}$ hincque porro has determinationes:

$$\begin{array}{ll|l} p' & = \frac{8m+9}{5(7m+9)}\alpha & AB = \alpha \\ p'' & = \frac{1}{15}\alpha & BC = \frac{1}{3}\alpha \\ p''' & = \frac{8m+9}{8m(m+3)}\alpha & CD = \frac{1}{9}\alpha \\ p'''' & = \frac{5(8m+15)}{24m(m+3)}\alpha & DE = \frac{1}{6m}\alpha \\ p^{V} & = \frac{5(8m+15)}{3m(32m+51)}\alpha & EF = \frac{5(8m+15)}{6m(32m+51)}\alpha \end{array}$$

et pro loco oculi $FO = \frac{8(m+1)}{3(8m+9)} p^{V}$. Tota autem tubi longitudo fit quasi $= 1\frac{1}{2}\alpha$, et semidiameter campi $\Phi = \frac{3(8m+9)}{8m(m+3)}\omega$.

De Telescopiis septem lentibus instructis.

49. Vt formulas nimis complicatas hic euitemus, ac statim insignem campum cum modica longi-

longitudine adipiscamur; lentem secundam bBb in ipso foco α lentis obiectiuae constituamus, tertiam vero cCc ibi, vbi obiectum in A positum per lentem B suam imaginem esset proiecturum. Quatuor autem reliquarum binas D, E ante imaginem erectam I postremas vero F et G pone eam collacari sumamus. Habebimus ergo pro his lentibus:

$B = -1$; $C = \frac{c}{1-c}$; $D = \frac{-d}{1+d}$; $E = \frac{-e}{1+e}$; $F = \frac{-f}{f-1}$; $G = \infty$

$\mathfrak{B} = \infty$; $\mathfrak{C} = c$; $\mathfrak{D} = -d$; $\mathfrak{E} = -e$; $\mathfrak{F} = f$; $\mathfrak{G} = 1$

$\theta' = q\omega$; $\theta'' = 0\omega$; $\theta''' = \omega$; $\theta'''' = -\omega$; $\theta^{V} = -\omega$; $\theta^{VI} = -\omega$

hincque semidiameter campi $\Phi = \frac{4-q}{m-1}\omega$. Ponatur $\frac{4-q}{m-1} = M$ vt fit $\Phi = M\omega$ erit

$\Pi' = \infty\omega$; $\Pi'' = (q-M)\omega$; $\Pi''' = +(d+q-M)\omega$
$\Pi'''' + (e-1+q-M)\omega$; $\Pi^{V} = -(f+2-q+M)\omega$ et
$\Pi^{VI} = -(4-q+M)\omega$.

50. His positis determinationes telescopii sunt sequentes:

$$\begin{array}{ll|l} p' & = \frac{M\alpha}{q}; & AB = \alpha \\ p'' & = \frac{cM\alpha}{q-M}; & BC = \frac{M\alpha}{q-M} \\ p''' & = \frac{cd}{1-c} \cdot \frac{M\alpha}{d+q-M}; & CD = \frac{cd}{(1-c)} \cdot \frac{M\alpha}{(q-M)(d+q-M)} \\ p'''' & = \frac{cde}{(1-c)(1+d)} \cdot \frac{M\alpha}{e-1+q-M}; & DE = \frac{cd}{(1-c)(1+d)} \cdot \frac{(e-1-d)M\alpha}{(d+q-M)(e-1+q-M)} \\ p^{V} & = \frac{cdef}{(1-c)(1+d)(1+e)} \cdot \frac{M\alpha}{f+2-q+M}; & EF = \frac{cde}{(1-c)(1+d)(1+e)} \cdot \frac{(f+e+1)M\alpha}{(e-1+q-M)(f+2-q+M)} \\ p^{VI} & = \frac{cdef}{(1-c)(1+d)(1+e)(f-1)} \cdot \frac{M\alpha}{4-q+M} & FG = \frac{cdef}{(1-c)(1+d)(1+e)(f-1)} \cdot \frac{(f-2)M\alpha}{(f+2-q+M)(4-q+M)}. \\ & & \text{et } GO = \frac{p^{VI}}{4-q+M}. \end{array}$$

51. Destructio autem colorum iridis postulat vt sit:

$$\frac{-1}{d+q-M}-\frac{1}{e-1+q-M}+\frac{1}{f+2-q+M}+\frac{1}{4-q+M}=0$$

vbi cum sit $e-1>d$ et $f>2$ statuamus

$$e-1+q-M=\frac{d+q-M}{1-\mu} \text{ et } f+2-q+M=\frac{4-q+M}{1-\nu}$$

fietque $d+q-M=\frac{2-\mu}{2-\nu}(4-q+M)$ et $e-1+q-M=\frac{(2-\mu)(4-q+M)}{(1-\mu)(2-\nu)}$ vnde quantitates d, e et f definiuntur, litterae autem μ et ν intra limites 0 et 1 sumendae arbitrio nostra relinquuntur.

Ab est $4-q+M=Mm$, hinc $q-M=4-Mm$, et pro numeratoribus $e-1-d=\frac{\mu(2-\mu)Mm}{(1-\mu)(2-\nu)}$; $f+e+1=\frac{(4-3\mu-3\nu+2\mu\nu)Mm}{(1-\mu)(1-\nu)(2-\nu)}$ et $f-2=\frac{\nu Mm}{1-\nu}$ porro autem $d=\frac{4-\mu-\nu}{2-\nu}Mm-4$; $e=\frac{4-3\mu-\nu+\mu\nu}{(1-\mu)(2-\nu)}Mm-3$ et $f=\frac{\nu}{1-\nu}Mm+2$, si insuper ponatur $q=(1+n)M$ erit $q-M=nM$ et $M=\frac{4}{m+n}$.

52. His valoribus substitutis pro constructione telescopii hae habentur mensurae:

$p'=\frac{\alpha}{1+1}$	$AB=\alpha$
$p''=\frac{c\alpha}{n}$	$BC=\frac{\alpha}{n}$
$p'''=\frac{c}{1-c}\cdot\frac{d(2-\nu)\alpha}{(2-\mu)m}$	$CD=\frac{c}{1-c}\cdot\frac{d(2-\nu)\alpha}{(2-\mu)mnM}=\frac{c}{1-c}\cdot\frac{(2-\mu)m-(2-\nu)n}{(2-\mu)mn}\alpha$
$p''''=\frac{cd}{(1-c)(1+d)}\cdot\frac{e(1-\mu)(2-\nu)\alpha}{(2-\mu)m}$	$DE=\frac{cd}{(1-c)(1+d)}\cdot\frac{\mu(2-\nu)\alpha}{(2-\mu)m}$
$p^{V}=\frac{cde}{(1-c)(1+d)(1+e)}\cdot\frac{f(1-\nu)\alpha}{m}$	$EF=\frac{cde}{(1-c)(1+d)(1+e)}\cdot\frac{(4-3\mu-3\nu+2\mu\nu)\alpha}{(2-\mu)m}$
$p^{VI}=\frac{cdef}{(1-c)(1+d)(1+e)(f-1)}\cdot\frac{\alpha}{m}$	$FG=\frac{cdef}{(1-c)(1+d)(1+e)(f-1)}\cdot\frac{\nu\alpha}{m}$

et pro loco oculi erit $GO=\frac{1}{Mm}p^{VI}=\frac{m+n}{4m}p^{VI}$.

Hic

Hic ergo etiam littera c arbitrio nostro permittitur, qua effici potest vt vltima lens ad vsum sit maxime accommodata.

53. Exemplum afferamus sumendo $\mu=\frac{16}{17}$; $\nu=\frac{4}{11}$; $n=2$ sitque multiplicatio $m=10$, ac sit $M=\frac{1}{3}$, tum vero

$d=\frac{127}{51}$; $e=38$; $f=\frac{12}{11}$ vnde hae mensurae

$p'=\frac{\alpha}{2}$	$AB=\alpha$
$p''=c.\frac{\alpha}{6}$	$BC=\frac{1}{6}\alpha$
$p'''=\frac{c}{1-c}\cdot\frac{127\alpha}{2.3.5.11}$	$CD=\frac{c}{1-c}\cdot\frac{127}{4.5.11}\alpha$
$p''''=\frac{c}{1-c}\cdot\frac{19.127\alpha}{2.5.11.89}$	$DE=\frac{c}{1-c}\cdot\frac{4.127}{5.11.89}\alpha$
$p^{V}=\frac{c}{1-c}\cdot\frac{19.41.127\alpha}{9.5.11.13.89}$	$EF=\frac{c}{1-c}\cdot\frac{4.19.127}{3.5.13.89}\alpha$
$p^{VI}=\frac{c}{1-c}\cdot\frac{19.41.127\alpha}{3.5.13.61.89}$	$FG=\frac{c}{1-c}\cdot\frac{4.19.41.127}{8.5.11.13.61.89}\alpha$
	et $GO=\frac{1}{15}p^{VI}$

quae ad praxin satis videntur idoneae, atque adeo capere licet $c=\frac{2}{1}$, quo pacto tubus magis contrahitur.

54. Sequentes positiones $\mu=\frac{4}{7}$ et $\nu=\frac{1}{6}$ ad praxin vtiles videntur, quas ergo in genere prosequar; primo autem praebent

$f=Mm+2$; $e=5Mm-3$; $d=\frac{9}{7}Mm-4$ existente $M=\frac{4}{m+n}$

tum vero prodeunt sequentes mensurae:

p'

$p' = \frac{\alpha}{1+n}$ | $AB = \alpha$

$p'' = \frac{c\alpha}{n}$ | $BC = \frac{\alpha}{n}$

$p''' = \frac{c}{1-c} \cdot \frac{5d\alpha}{4m}$ | $CD = \frac{c}{1-c} \cdot \frac{4m-5n}{4mn}\alpha$

$p'''' = \frac{cd}{(1-c)(1+d)} \cdot \frac{e\alpha}{4m}$ | $DE = \frac{cd}{(1-c)(1+d)} \cdot \frac{\alpha}{m}$

$p^{V} = \frac{cde}{(1-c)(1+d)(1+e)} \cdot \frac{f\alpha}{2m}$ | $EF = \frac{cde}{(1-c)(1+d)(1+e)} \cdot \frac{5\alpha}{4m}$

$p^{VI} = \frac{cdef}{(1-c)(1+d)(1+e)(f-1)} \cdot \frac{\alpha}{m}$ | $FG = \frac{cdef}{(1-c)(1+d)(1+e)(f-1)} \cdot \frac{\alpha}{2m}$

et $GO = \frac{m+n}{4m} p^{VI}$.

Deinde campi apparentis ſemidiameter eſt $\Phi = \frac{4\omega}{m+n}$, circiter quadruplo maior quam in tubis vſitatis; at eſt $q = \frac{4(1+n)}{m+n}$, vnde apertura ſecundae lentis B ſumi debet.

55. Circa numerum n euidens eſt, quo maior is accipiatur, eo minorem prodire longitudinem, tum autem ſimul campus apparens imminuitur. Cum enim per formulas primum inuentas ſit $\Phi = \frac{4-q}{m-1}\omega$, hic autem poſuerimus $q = \frac{4(1+n)}{m+n}$, vt ſit $n = \frac{qm-4}{4-q}$ patet aucto numero hoc n, etiam q augeri, ideoque campum minui in ratione numeri $4-q$. Poſito autem $n=1$, ſit $q = \frac{8}{m+1}$ et $\Phi = \frac{4}{m+1}\omega$; tubus vero aſtronomicus vulgaris pro eadem multiplicatione praebet $\Phi = \frac{1}{m+1}\omega$, vnde hoc caſu noſter tubus quadruplo ampliorem campum detegit, ſed ob $n=1$, plus quam duplo ſit longior. Si ergo campus non 4 ſed $3\frac{1}{2}$ partibus amplior eſſe debeat, ſumi oportet $n = \frac{m+8}{7}$, ſin autem $3\frac{3}{4}$ partibus, ſit $n = \frac{m+16}{15}$, vnde patet admodum exigua campi

campi imminutione longitudinem vehementer contrahi, idque eo magis, quo maior fuerit multiplicatio.

Exemplum I.

56. Sit multiplicatio $m=10$, et ſumatur $n=1$, fietque $M = \frac{4}{11}$ et $q = \frac{8}{11}$, hinc ſemidiameter campi $\Phi = \frac{4\omega}{11}$, qui ſumto $\omega = \frac{1}{4}$ erit $\Phi = 5°, 12'$, et diameter $10°, 24'$. Tum ob $Mm = \frac{40}{11} = 3{,}6364$, ſit $d = 2{,}5455$; $e = 15{,}1818$; $f = 5{,}6364$ vnde ſequentes prodeunt menſurae:

$p' = 0{,}5\,\alpha$ | $AB = \alpha$

$p'' = c.\,\alpha$ | $BC = \alpha$

$p''' = \frac{c}{1-c} \cdot 0{,}3182\,\alpha$ | $CD = \frac{c}{1-c} \cdot 0{,}8750\,\alpha$

$p'''' = \frac{c}{1-c} \cdot 0{,}2725\,\alpha$ | $DE = \frac{c}{1-c} \cdot 0{,}0718\,\alpha$

$p^{V} = \frac{c}{1-c} \cdot 0{,}1898\,\alpha$ | $EF = \frac{c}{1-c} \cdot 0{,}0505\,\alpha$

$p^{VI} = \frac{c}{1-c} \cdot 0{,}0819\,\alpha$ | $FG = \frac{c}{1-c} \cdot 0{,}0409\,\alpha$

et $GO = \frac{c}{1-c} \cdot 0{,}0225\,\alpha$

Hinc tota longitudo $AO = 2\alpha + \frac{c}{1-c} \cdot 1{,}0607\,\alpha$ ſumto hic $\alpha = 15$ dig. accipi poterit $c = \frac{5}{11}$, vt prodeat $p^{VI} = 1$ dig.

Exemplum II.

57. Sit multiplicatio $m=15$, et capiatur $n=2$, vt fiat $M = \frac{4}{17}$ et $q = \frac{12}{17}$, ideoque $\Phi = \frac{4\omega}{17}$ ſeu $\Phi = 3°, 22'$. Iam ob $Mm = \frac{60}{17} = 3{,}5294$, erit $d = 2{,}3530$; $e = 14{,}6470$; $f = 5{,}5294$ vnde ſequentes menſurae obtinentur:

p'

$p' = 0,3333\,\alpha$	$AB = \alpha$
$p'' = c \cdot 0,500\,\alpha$	$BC = 0,500\,\alpha$
$p''' = \frac{c}{1-c} \cdot 0,1961\,\alpha$	$CD = \frac{c}{1-c} \cdot 0,4167\,\alpha$
$p'''' = \frac{c}{1-c} \cdot 0,1713\,\alpha$	$DE = \frac{c}{1-c} \cdot 0,0468\,\alpha$
$p^{V} = \frac{c}{1-c} \cdot 0,1211\,\alpha$	$EF = \frac{c}{1-c} \cdot 0,0328\,\alpha$
$p^{VI} = \frac{c}{1-c} \cdot 0,0535\,\alpha$	$FG = \frac{c}{1-c} \cdot 0,0267\,\alpha$
et	$GO = \frac{c}{1-c} \cdot 0,0152\,\alpha$

Hinc tota longitudo $AO = 1,5\,\alpha + \frac{c}{1-c} \cdot 0,5382\,\alpha$.

58. Litterae f, e, d eatenus tantum variantur, quatenus valor Mm mutatur, hic autem commode idem retineri poteſt. Quod ſi ergo ſecundum exemplum prius in genere ſtatuamus $n = \frac{m}{15}$ fit $M = \frac{42}{11m}$ et $Mm = \frac{42}{11}$ et $q = \frac{4(m+121)}{11m}$ atque $\Phi = \frac{42}{11m}\,\omega$ vnde ſequentes determinationes naſcuntur:

$p' = \frac{10}{m+10}\,\alpha$	$AB = \alpha$
$p'' = c \cdot \frac{10}{m}\,\alpha$	$BC = 10 \cdot \frac{\alpha}{m}$
$p''' = \frac{c}{1-c} \cdot 3,1819 \cdot \frac{\alpha}{m}$	$CD = \frac{c}{1-c} \cdot 8,7500 \cdot \frac{\alpha}{m}$
$p'''' = \frac{c}{1-c} \cdot 2,7249 \cdot \frac{\alpha}{m}$	$DE = \frac{c}{1-c} \cdot 0,7179 \cdot \frac{\alpha}{m}$
$p^{V} = \frac{c}{1-c} \cdot 1,8983 \cdot \frac{\alpha}{m}$	$EF = \frac{c}{1-c} \cdot 0,5052 \cdot \frac{\alpha}{m}$
$p^{VI} = \frac{c}{1-c} \cdot 0,8189 \cdot \frac{\alpha}{m}$	$FG = \frac{c}{1-c} \cdot 0,4095 \cdot \frac{\alpha}{m}$
et	$GO = \frac{c}{1-c} \cdot 0,2252 \cdot \frac{\alpha}{m}$

Hinc tota longitudo $AO = \alpha + \frac{10\alpha}{m} + \frac{c}{1-c} \cdot 10,6078 \cdot \frac{\alpha}{m}$. Hic debet ſumi pro aperturis $Bb = \frac{10}{11} \cdot \frac{\alpha}{m}$ dig. et $Cc = \frac{10}{m} \cdot Aa$.

59. Sin autem in genere ſecundum alterum exemplum ponatur $n = \frac{[illegible]}{17}$, vnde fit $M = \frac{6}{[illegible]}$ et $\Phi = \frac{6}{1\,m}\,\omega$ atque $q = \frac{4(2m+1)}{17m}$ determinationes pro conſtructione Teleſcopii erunt:

$p' = \frac{15}{2m+15}\,\alpha$	$AB = \alpha$
$p'' = c \cdot 7,5\,\frac{\alpha}{m}$	$BC = 7,5 \cdot \frac{\alpha}{m}$
$p''' = \frac{c}{1-c} \cdot 2,9412 \cdot \frac{\alpha}{m}$	$CD = \frac{c}{1-c} \cdot 6,2500 \cdot \frac{\alpha}{m}$
$p'''' = \frac{c}{1-c} \cdot 2,5695 \cdot \frac{\alpha}{m}$	$DE = \frac{c}{1-c} \cdot 0,7017 \cdot \frac{\alpha}{m}$
$p^{V} = \frac{c}{1-c} \cdot 1,8162 \cdot \frac{\alpha}{m}$	$EF = \frac{c}{1-c} \cdot 0,4927 \cdot \frac{\alpha}{m}$
$p^{VI} = \frac{c}{1-c} \cdot 0,8019 \cdot \frac{\alpha}{m}$	$FG = \frac{c}{1-c} \cdot 0,4010 \cdot \frac{\alpha}{m}$
et	$GO = \frac{c}{1-c} \cdot 0,2272 \cdot \frac{\alpha}{m}$

Hinc tota longitudo $AO = \alpha + \frac{15\alpha}{2m} + \frac{c}{1-c} \cdot 8,0726 \cdot \frac{\alpha}{m}$ quae notabiliter minor eſt quam caſu praecedente, ſed campus apparens aliquanto eſt minor in ratione $34 : 33$. Sumto $\omega = \frac{1}{4}$, lentis in B ſemidiameter aperturae eſt $Bb = \frac{15}{17} \cdot \frac{\alpha}{m}$ dig. at lentis in C capi debet $= \frac{15}{2m} \cdot Aa$, nempe ſemper $= \frac{BC}{AB} \cdot Aa$.

60. Adiungamus tertium caſum, quo $n = \frac{m}{5}$, et qui in maioribus multiplicationibus inſignem vſum praeſtabit. Erit ergo $M = \frac{10}{3m}$ hinc $\Phi = \frac{10}{3m}\,\omega$, qui valor ad praecedentem eſt vt 17 ad 18; tum $q = \frac{2(m+5)}{3m}$, et reliquae determinationes:

$p' = \frac{5}{m+5}\,\alpha$	$AB = \alpha$
$p'' = c \cdot 5\,\frac{\alpha}{m}$	$BC = 5\,\frac{\alpha}{m}$
$p''' = \frac{c}{1-c} \cdot 2,5000\,\frac{\alpha}{m}$	$CD = \frac{c}{1-c} \cdot 3,7500 \cdot \frac{\alpha}{m}$
$p'''' = \frac{c}{1-c} \cdot 2,2778\,\frac{\alpha}{m}$	$DE = \frac{c}{1-c} \cdot 0,6667 \cdot \frac{\alpha}{m}$
$p^{V} = \frac{c}{1-c} \cdot 1,6566\,\frac{\alpha}{m}$	$EF = \frac{c}{1-c} \cdot 0,4659 \cdot \frac{\alpha}{m}$
$p^{VI} = \frac{c}{1-c} \cdot 0,7645\,\frac{\alpha}{m}$	$FG = \frac{c}{1-c} \cdot 0,3823 \cdot \frac{\alpha}{m}$
et	$GO = \frac{c}{1-c} \cdot 0,2293 \cdot \frac{\alpha}{m}$

Longitudo tubi $= \alpha + 5\frac{\alpha}{m} + \frac{c}{1-c} \cdot 5,4942\,\frac{\alpha}{m}$ poſito $Bb = \frac{5}{6} \cdot \frac{\alpha}{m}$, et $Cc = \frac{5}{m} \cdot Aa$.

 61.

61. Quod ad numerum c attinet, cum in genere ita definire possumus, vt [illegible] aequetur quantitati datae veluti vni digito, eritque tum $c = \frac{m}{z+m}$, at tum pro tribus hypothesibus habebimus sequentes mensuras in digitis expressas.

Hyp. I. $\Phi = \frac{40}{11m}\omega$
$p = \frac{10}{m+10}\alpha$; $AB = \alpha$
$p' = \frac{10}{m+\alpha}\alpha$; $BC = \frac{10}{m}\alpha$
$p'' = 3{,}1819$; $CD = 8{,}7500$
$p''' = 2{,}7249$; $DE = 0{,}7179$
$p^{V} = 1{,}8983$; $EF = 0{,}5052$
$p^{VI} = 0{,}8159$; $FG = 0{,}4095$
et $GO = 0{,}2252$
$AO = $ [illegible] $\alpha + 10{,}6078$

Hyp. II. Φ $\frac{62}{1 \cdot m}\omega$
$p' = $ [illegible] α; $AB = \alpha$
$p'' = $ [illegible] α; $BC = \frac{15}{2m}\alpha$
$p''' = 2{,}9412$; $CD = 6{,}2500$
$p'''' = 2{,}5695$; $DE = 0{,}7017$
$p^{V} = 1{,}8162$; $EF = 0{,}4927$
$p^{VI} = 0{,}5019$; $FG = 0{,}4010$
et $GO = 0{,}2272$
$AO = $ [illegible] $\alpha + 8{,}0726$

Hyp. III. Φ $\frac{10}{3m}\omega$
$p' = \frac{5}{m+5}\alpha$; $AB = \frac{4}{5}\alpha$
$p'' = \frac{5}{m+\alpha}\alpha$; $BC = \frac{5}{m}\alpha$
$p''' = 2{,}5000$; $CD = 3{,}7500$
$p'''' = 2{,}2778$; $DE = 0{,}6667$
$p^{V} = 1{,}6566$; $EF = 0{,}4659$
$p^{VI} = 0{,}7645$; $FG = 0{,}3823$
et $GO = 0{,}2293$
$AO = \frac{m+5}{m}\alpha + 5{,}4942$

sicque quatuor postremae lentes in qualibet hypothesi manent eaedem vna cum suis internallis pro omnibus multiplicationibus.

Con-

Conclusio.

62. Telescopia haec omnino singulare genus constituunt, quod prorsus discrepat a telescopiis vulgaribus quatuor lentibus instructis; horumque character potissimum in tertia lente C consistit, quae tam exiguam aperturam postulat, cuiusque propterea ope radii peregrini commode excluduntur. Haec autem Telescopia septem lentium isto fundamento nituntur, quod sit $b = \infty$ et $r = 0$, seu quod secunda lens B in ipso foco lentis obiectiuae, tertia vero C in eo loco, vbi ipsius obiectiuae imago per lentem B proiecta cadit, collocetur. Qua positione cum etiam in casibus quinque ac sex lentium vti licuerit, ea, quae quinque lentibus sunt instructa, speciem quasi principalem huius generis telescopiorum exhibere est censenda, vnde reliqua sex et septem lentium multiplicatione lentium ocularium ad campum dilatandum facta sint natae. Quare has species ad idem genus referendas simili forma aspectui exposuisse iuuabit.

Species principalis V lentibus constans.

63 Si ponamus $n = \frac{m}{k}$, vt sit $\Phi = \frac{2k\omega}{m(k+1)}$, tum vero $\frac{c\alpha}{(1-c)m} = z$, ideoque $c = \frac{mz}{mz+\alpha}$, mensurae pro constructione horum telescopiorum ita se habebunt:

Lentis

Lentis	dist. foci	semid. apert.	Interuallum.
in A . . .	α;	$Aa = x$	$AB = \alpha$
in B . .	$\frac{k}{m+k}\alpha$;	$Bb = \frac{2k\omega}{m(k+1)}\alpha$,	$BC = \frac{k}{m}\alpha$
in C . .	$\frac{kz}{mz+\alpha}\alpha$;	$Cc = \frac{k}{m}x$;	$CD = (k-1)z$
in D . .	$\frac{2(k-1)}{k+1}z$;	$Dd = \frac{2(k-1)\omega}{k+1}z$;	$DE = \frac{4(k-1)}{3k-1}z$
in E . .	$\frac{2(k-1)}{3k-1}z$;	$Ee = \frac{2(k-1)\omega}{3k-1}z$;	$EO = \frac{kk-1}{k(3k-1)}z$.

Hypoth. I. $m > 4$	Hyp. II. $m > 3$
$k=2$; $z=\frac{5}{3}$; $Aa=\frac{m}{50}$; $AB=\alpha$	$k=3$; $z=2$; $Aa=\frac{m}{50}$; $AB=\alpha$
$p' = \frac{2}{m+2}\alpha$; $Bb=\frac{1}{3}\cdot\frac{\alpha}{m}$; $BC=\frac{2}{m}\alpha$	$p' = \frac{3}{m+3}\alpha$; $Bb=\frac{3}{8}\frac{\alpha}{m}$; $BC=\frac{3}{m}\alpha$
$p'' = \frac{10}{5m+2\alpha}\alpha$; $Cc=\frac{5}{50}$; $CD=\frac{5}{3}$	$p'' = \frac{6}{2m+\alpha}\alpha$; $Cc=\frac{3}{50}$; $CD=4$
$p''' = \frac{5}{3}$; $Dd=\frac{5}{12}$; $DE=2$	$p''' = 2$; $Dd=\frac{1}{8}$; $DE=2$
$p'''' = 1$; $Ee=\frac{1}{4}$; $EO=\frac{3}{4}$	$p'''' = 1$; $Ee=\frac{1}{4}$; $EO=\frac{2}{3}$
$\Phi=\frac{1}{3m}$ seu $\Phi=\frac{1145}{m}$ min.	$\Phi=\frac{3}{8m}$ seu $\Phi=\frac{1289}{m}$ min.

Hypoth. III. $m > 3\frac{1}{2}$	Hyp. IV. $m > 2\frac{1}{4}$
$k=4$; $z=\frac{11}{8}$; $Aa=\frac{m}{50}$; $AB=\alpha$	$k=5$; $z=\frac{7}{4}$; $Aa=\frac{m}{50}$; $AB=\alpha$
$p' = \frac{4}{m+4}\alpha$; $Bb=\frac{2}{5}\cdot\frac{\alpha}{m}$; $BC=\frac{4}{m}\alpha$	$p' = \frac{5}{m+5}\alpha$; $Bb=\frac{5}{18}\cdot\frac{\alpha}{m}$; $BC=\frac{5}{m}\alpha$
$p'' = \frac{44}{11m+8\alpha}\alpha$; $Cc=\frac{4}{50}$; $CD=\frac{11}{2}$	$p'' = \frac{35}{7m+4\alpha}\alpha$; $Cc=\frac{5}{50}$; $CD=7$
$p''' = \frac{11}{5}$; $Dd=\frac{11}{80}$; $DE=2$	$p''' = \frac{7}{3}$; $Dd=\frac{7}{12}$; $DE=2$
$p'''' = 1$; $Ee=\frac{1}{4}$; $EO=\frac{5}{8}$	$p'''' = 1$; $Ee=\frac{1}{4}$; $EO=\frac{3}{5}$
$\Phi=\frac{2}{5m}$ seu $\Phi=\frac{1375}{m}$ min.	$\Phi=\frac{5}{12m}$ seu $\Phi=\frac{1432}{m}$ min.

Hypoth. V. $m > 2\frac{2}{5}$	Hyp. VI. $m > 2\frac{1}{3}$
$k=6$; $z=\frac{17}{10}$; $Aa=\frac{m}{50}$; $AB=\alpha$	$k=7$; $z=\frac{5}{3}$; $Aa=\frac{m}{50}$; $AB=\alpha$
$p' = \frac{6}{m+6}\alpha$; $Bb=\frac{3}{7}\cdot\frac{\alpha}{m}$; $BC=\frac{6}{m}\alpha$	$p' = \frac{7}{m+7}\alpha$; $Bb=\frac{7}{16}\cdot\frac{\alpha}{m}$; $BC=\frac{7}{m}\alpha$
$p'' = \frac{102}{17m+10\alpha}\alpha$; $Cc=\frac{6}{50}$; $CD=\frac{17}{2}$	$p'' = \frac{35}{5m+3\alpha}\alpha$; $Cc=\frac{7}{50}$; $CD=10$
$p''' = \frac{17}{7}$; $Dd=\frac{17}{18}$; $DE=2$	$p''' = \frac{5}{3}$; $Dd=\frac{5}{8}$; $DE=2$
$p'''' = 1$; $Ee=\frac{1}{4}$; $EO=\frac{7}{12}$	$p'''' = 1$; $Ee=\frac{1}{4}$; $EO=\frac{4}{7}$
$\Phi=\frac{3}{7m}$ seu $\Phi=\frac{1473}{m}$ min.	$\Phi=\frac{7}{16m}$ seu $\Phi=\frac{1503}{m}$ min.

I.

I. AE = $\frac{m+[illegible]}{m}\alpha + 4$	III. AE = $\frac{m+[illegible]}{m}\alpha + 7\frac{1}{2}$	V. AE = $\frac{m+6}{m}\alpha + 10\frac{1}{2}$ dig.
II. AE = $\frac{m+[illegible]}{m}\alpha + 6$	IV. AE = $\frac{m+[illegible]}{m}\alpha + 9$	VI. AE = [illegible] $\alpha + 12$ dig.

Species secundaria VI lentibus constans.

64. Factis iisdem denominationibus, primo erit semidiameter campi apparentis $\Phi =$ [illegible]; reliquae vero mensurae.

Lentis dist. foci.		semid. apert.	Interuallum
in A	α;	$Aa = x$;	$AB = \alpha$
in B . .	$\frac{k}{m+k}\alpha$;	$Bb = \frac{2k\omega}{m(k+1)}\alpha$;	$BC = \frac{k}{m}\alpha$
in C . .	$\frac{kz}{mz+\alpha}\alpha$;	$Cc = \frac{k}{m}x$;	$CD = (k-1)z$
in D . .	$\frac{2(k-1)}{k+1}z$;	$Dd = \frac{2(k-1)\omega}{k+1}z$;	$DE = \frac{3(k-1)}{2(3k-1)}z$
in E . .	[illegible] z;	$Ee = \omega$ Dist. foci	EF = [illegible] z
in F . .	[illegible] z;	$Ff = \omega$ dist. foci	$FO = \frac{kk-1}{2k(2k-1)}z$

Hypoth. I. $m > \infty$.	Hyp. II. $m > 9$
$k=2$; $z=2$; $Aa=$ [illegible]; $AB=\alpha$	$k=3$; $z=$ [illegible]; $Aa=$ [illegible]; $AB=\alpha$
$p' = \frac{[illegible]}{m+2}\alpha$; $Bb=$ [illegible] $\frac{\alpha}{m}$; $BC=\frac{2}{m}\alpha$	$p' = \frac{3}{m+3}\alpha$; $Bb=$ [illegible] $\frac{\alpha}{m}$; $BC=\frac{3}{m}\alpha$
$p'' =$ [illegible] α; $Cc=$ [illegible]; $CD=2$	$p'' =$ [illegible] α; $Cc=$ [illegible]; $CD=3\frac{1}{3}$
$p''' = 2$; $Dd=\frac{1}{2}$; $DE=1$	$p''' =$ [illegible]; $Dd=$ [illegible]; $DE=1$
$p'''' = 2$; $Ee=\frac{1}{2}$; $EF=1$	$p'''' = \frac{9}{4}$; $Ee=$ [illegible]; $EF=1$
$p^{V} = 1$; $Ff=\frac{1}{4}$; $FO=\frac{1}{2}$	$p^{V} = 1$; $Ff=\frac{1}{4}$; $FO=\frac{9}{4}$
$\Phi=$ [illegible] min. $AF=\frac{m+2}{m}\alpha+4$, imposs.	$\Phi=$ [illegible] min. $AF=\frac{m+1}{5}\alpha+5\frac{1}{3}$

Hypoth. III. $m > 6$

$k = 4$; $z = \frac{14}{9}$; $Aa = \frac{m}{53}$; $AB = \alpha$

$p' = \frac{4}{m+4}\alpha$; $Bb = \frac{3}{5}\cdot\frac{\alpha}{m}$; $BC = \frac{4}{m}\alpha$

$p'' = \frac{56}{14m+9\alpha}\alpha$; $Cc = \frac{4}{18}$; $CD = 4\frac{2}{3}$

$p''' = \frac{14}{5}$; $Dd = \frac{7}{33}$; $DE = 1$

$p'''' = \frac{12}{5}$; $Ee = \frac{8}{5}$; $EF = 1$

$p^{V} = 1$; $Ff = \frac{1}{4}$; $FO = \frac{5}{12}$

$\Phi = \frac{2061}{m}$ min. $AF = \frac{m+4}{m}\alpha + 6\frac{2}{3}$

Hyp. IV. $m > 5$

$k = 5$; $z = \frac{3}{2}$; $Aa = \frac{m}{5}$; $AB = \alpha$

$p' = \frac{5}{m+5}\alpha$; $Bb = \frac{5}{8}\cdot\frac{\alpha}{m}$; $BC = \frac{5}{m}\alpha$

$p'' = \frac{15}{3m+1\alpha}\alpha$; $Cc = \frac{5}{33}$; $CD = 6$

$p''' = 3$; $Dd = \frac{3}{4}$; $DE = 1$

$p'''' = \frac{5}{2}$; $Ee = \frac{5}{8}$; $EF = 1$

$p^{V} = 1$; $Ff = \frac{1}{4}$; $FO = \frac{3}{5}$

$\Phi = \frac{2149}{m}$ min. $AF = \frac{m+5}{m}\alpha + 8$

Hypoth. V. $m > 4\frac{1}{2}$

$k = 6$; $z = \frac{22}{15}$; $Aa = \frac{m}{30}$; $AB = \alpha$

$p' = \frac{6}{m+6}\alpha$; $Bb = \frac{9}{14}\cdot\frac{\alpha}{m}$; $BC = \frac{6}{m}\alpha$

$p'' = \frac{112}{22m+15\alpha}\alpha$; $Cc = \frac{6}{38}$; $CD = 7\frac{1}{3}$

$p''' = \frac{21}{7}$; $Dd = \frac{11}{14}$; $DE = 1$

$p'''' = \frac{18}{7}$; $Ee = \frac{9}{14}$; $EF = 1$

$p^{V} = 1$; $Ff = \frac{1}{4}$; $FO = \frac{7}{18}$

$\Phi = \frac{2210}{m}$ min. $AF = \frac{m+6}{m}\alpha + 9\frac{1}{3}$

Hyp VI. $m > 4\frac{1}{5}$

$k = 7$; $z = \frac{13}{9}$; $Aa = \frac{m}{33}$; $AB = \alpha$

$p' = \frac{7}{m+7}\alpha$; $Bb = \frac{21}{32}\cdot\frac{\alpha}{m}$; $BC = \frac{7}{m}\alpha$

$p'' = \frac{91}{15m+9\alpha}\alpha$; $Cc = \frac{7}{33}$; $CD = 8\frac{2}{3}$

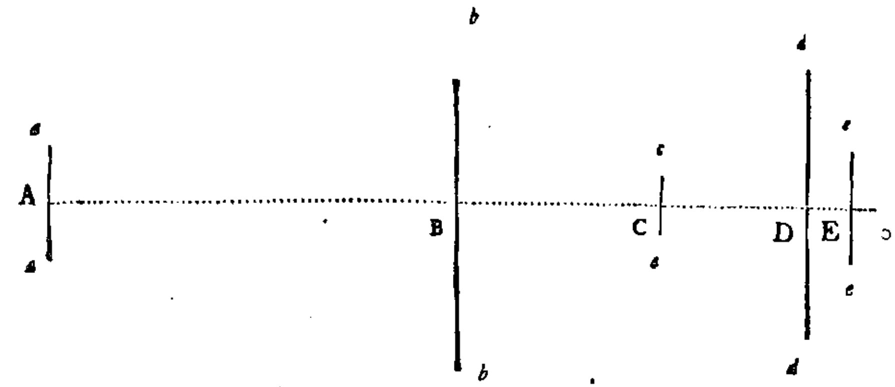
$p''' = \frac{13}{4}$; $Dd = \frac{13}{16}$; $DE = 1$

$p'''' = \frac{21}{8}$; $Ee = \frac{21}{32}$; $EF = 1$

$p^{V} = 1$; $Ff = \frac{1}{4}$; $FO = \frac{8}{21}$

$\Phi = \frac{2256}{m}$ min. et $AF = \frac{m+7}{m}\alpha + 10\frac{2}{3}$.

Tabula

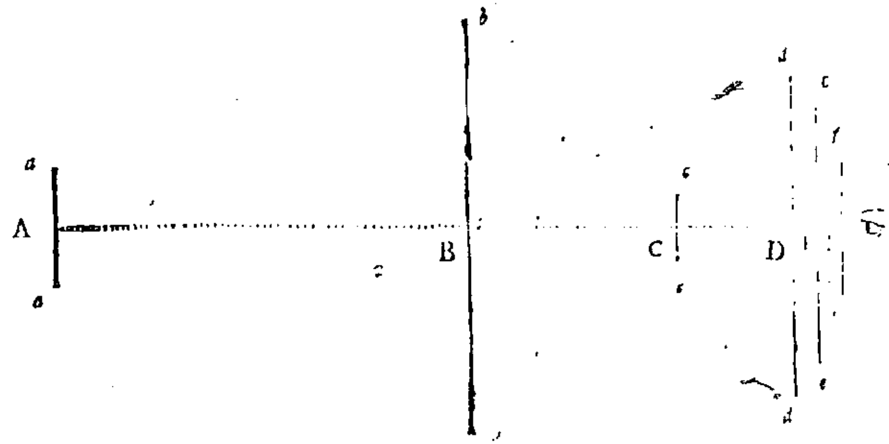


Tabula Telescopiorum ex quinque lentibus compositorum.

Multiplicatio	Lentis A focus.	A a	Dist A B	Lentis B focus.	B b	Dist. BC	Lentis C focus.	C c	Dist. C D	Lentis D focus.	D d	Dist. D E	Longit. tubi.	semid. campi.
5	8	0,1	8	4,00	0,67	8,00	4,18	0,1	7	2,33	0,58	2	25,	4°.46'
10	18	0,2	18	6,00	0,75	9,00	4,44	0,1	7	2,33	0,58	2	36,	2. 23
15	30	0,3	30	7,50	0,83	10,00	4,66	0,1	7	2,33	0,58	2	49,	1. 35
20	42	0,4	42	8,40	0,87	0,50	4,77	0,1	7	2,33	0,58	2	61, 5	1. 11
25	56	0,5	56	9,33	0,93	11,20	4,91	0,1	7	2,33	0,58	2	76, 2	0. 57
30	70	0,6	70	10,00	0,97	11,67	5,00	0,1	7	2,33	0,58	2	90,67	0. 47
35	85	0,7	85	10,62	1,01	12,14	5,09	0,1	7	2,33	0,58	2	106, 14	0. 41
40	100	0,8	100	11,11	1,04	12,50	5,15	0,1	7	2,33	0,58	2	121, 50	0. 36
45	120	0,9	120	12,00	1,11	13,33	5,22	0,1	7	2,33	0,58	2	142, 33	0. 32
50	140	1,0	140	12,73	1,17	14,00	5,38	0,1	7	2,33	0,58	2	163,	0. 28
60	180	1,2	180	13,85	1,25	15,00	5,53	0,1	7	2,33	0,58	2	204,	0. 24

Hic lentis vltimae E distantia foci est 1 dig. semid. aperturae $Ee = \frac{1}{4}$ et post eam distantia oculi $EO = \frac{1}{3}$ dig.

Tabula

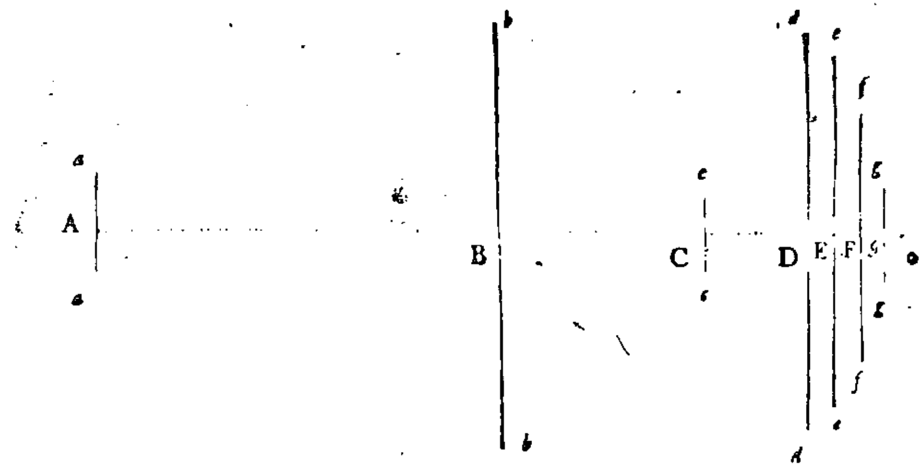


Tabula Teleſcopiorum ſex lentibus conſtantium.

Multiplicatio	Lentis A focus	A *a*	Diſt AB	Lentis B focus	B *b*	Diſt. BC	Lentis C focus	Longitudo tubi AF	ſemid. campi
5	8	0,1	8	4,00	1,00	8,00	3,67	24,00	7. [illegible]
10	18	0,2	18	6,00	1,14	9,00	4,09	35,00	3. 35
15	30	0,3	30	7,50	1,25	10,00	4,28	45,00	2. 23
20	42	0,4	42	8,40	1,31	10,50	4,37	60,50	1. 47
25	56	0,5	56	9,33	1,40	11,20	4,44	75,20	1. 26
30	70	0,6	70	10,00	1,46	11,67	4,57	89,67	1. 11
35	85	0,7	85	10,62	1,51	12,14	4,64	105,14	1. 1
40	100	0,8	100	11,11	1,56	12,50	4,69	120,50	0. 54
45	120	0,9	120	12,00	1,67	13,33	4,52	141,67	0. 48
50	140	1,0	140	12,73	1,76	14,00	4.58	162,00	0. 43

Lentis C aperturae ſemidiameter eſt [illegible] dig. et diſtantia CD . . 6 dig.
Lentis D focus eſt 3 dig. ſemid. apert. D*d* . [illegible] dig. et diſt. DE . 1 dig.
Lentis E focus eſt 2[illegible] dig. ſemid. apert. E*e* . [illegible] dig. et diſt. EF . 1 dig.
Lentis F focus eſt 1 dig. ſemid. apert. F*f* . [illegible] dig. et oculi diſt. FO . [illegible] dig.

Tabula



Tabula Telescopiorum ex ſeptem lentibus compoſitorum.

Multi plicat.	Lentis A focus.	Aa	Diſt. AB	Lentis B focus.	Bb	Diſt. BC	Lentis C focus.	Longid. tubi AF	ſemid. campi.
5	8	0,1	8	4,00	1,33	8,00	3,63	23,33	9°,32′
10	18	0,2	18	6,00	1,50	9,00	3,83	34,33	4, 46
15	30	0,3	30	7,50	1,66	10,00	4,00	47,33	3, 10
20	42	0,4	42	8,40	1,74	10,50	4,08	59,80	2, 22
25	56	0,5	56	9,33	1,86	11,20	4,18	74,53	1, 54
30	70	0,6	70	10,00	1,94	11,67	4,24	89,00	1, 34
35	85	0,7	85	10,62	2,02	12,14	4,30	104,47	1, 22
40	100	0,8	100	11,11	2,08	12,50	4,35	119,83	1, 12
45	120	0,9	120	12,00	2,22	13,33	4,44	140,67	1, 4
50	140	1,0	140	12,73	2,31	14,00	4,51	161,33	0, 56

Lentis C apert. ſemid. eſt conſtanter Cc. $\frac{1}{10}$ dig. et diſt CD . . 5 dig.
Lentis D focus eſt $3\frac{1}{8}$ dig ſemid. ap. Dd. $\frac{5}{8}$ dig. et diſt. DE . 0,89 dig.
Lentis E focus eſt 3,04 dig. ſem. ap. Ee. 0,76. et diſt. EF . 0,62 dig.
Lentis F focus eſt 2,21 dig. ſem. ap Ff. 0,55 et diſt. FG . 0,51 dig.
Lentis G focus eſt 1,02 dig. ſemid. apert. Gg . 0,26 et diſt. oculi GO . . 0,30 dig.

DE