



1763

# Annotationes in locum quendam Cartesii ad circuli quadraturam spectantem

Leonhard Euler

Follow this and additional works at: <https://scholarlycommons.pacific.edu/euler-works>

 Part of the [Mathematics Commons](#)

Record Created:

2018-09-25

## Recommended Citation

Euler, Leonhard, "Annotationes in locum quendam Cartesii ad circuli quadraturam spectantem" (1763). *Euler Archive - All Works*. 275. <https://scholarlycommons.pacific.edu/euler-works/275>

This Article is brought to you for free and open access by the Euler Archive at Scholarly Commons. It has been accepted for inclusion in Euler Archive - All Works by an authorized administrator of Scholarly Commons. For more information, please contact [mgibney@pacific.edu](mailto:mgibney@pacific.edu).

ANNOTATIONES  
 IN LOCVM QVENDAM CARTESII AD  
 CIRCVLII QVADRATVRAM  
 SPECTANTEM.

Auctore

L. EVLERO.

**I**n excerptis ex Manuscriptis *Cartesii* paucis quidem verbis refertur constructio quaedam geometrica promptissime ad circuli veram dimensionem appropinquans, sed quae siue *Cartesius* ipse eam inuenerit, siue ab alio habuerit communicatam, acutissimum inuentoris ingenium, illo praesertim tempore, luculenter declarat. Qui deinceps hoc idem argumentum pertractarunt, quantum equidem memini, nullam huius eximiae constructionis mentionem faciunt, ut periculum sit, ne tandem penitus obliuione obruatur. Demonstratio quidem, quae non adiuncta reperitur, haud difficulter suppletur; verum non solum elegantia constructionis vberiore explicationem meretur, sed etiam tam insignes conclusiones inde deduci possunt, quae per se omni attentione dignae videntur. Pulcherrima autem haec constructio ipsis *Cartesii* verbis ita est proposita:

„Ad quadrandum circulum nihil aptius inuenio, Tab. I.  
 „quam si dato quadrato *bf* adiungatur rectangulum *cg* Fig. 1.  
 „comprehensum sub lineis *ac* et *bc*, quod sit aequale  
 „quartae parti quadrati *bf*: item rectangulum *db* si-  
 „cum ex lineis *da, dc*, aequale quartae parti praecedentis;

dentis; et eodem modo rectangulum  $ei$ , atque alia  
 infinita vsque ad  $x$ : et erit haec linea  $ax$  diameter  
 circuli, cuius circumferentia aequalis est circumferen-  
 tia quadrati  $bf$ .

Vis igitur huius constructionis in hoc consistit,  
 vt continua appositione istiusmodi rectangulorum  $cg$ ,  
 $db$ ,  $ei$ , etc. quorum anguli superiores dextri in diago-  
 nalem quadrati productam cadunt, tandem ad punctum  
 $x$  perueniatur, quo terminatur diameter circuli  $ax$ ,  
 cuius peripheria aequalis est perimetro quadrati  $bf$ , seu  
 quadruplo rectae  $ab$ .

Cum horum rectangulorum quodque aequetur  
 parti quartae praecedentis, iam ipse *Cartesius* obseruat,  
 summam omnium horum rectangulorum aequalem fore  
 parti tertiae quadrati  $bf$ ; quod quidem manifestum est,  
 cum huius seriei  $\frac{1}{2} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} +$  etc. in infinitum  
 continuatae summa sit  $= \frac{1}{2}$ .

Praeterea etiam *Cartesius* indicat rationem, cui  
 haec constructio innititur; concipit scilicet polygona re-  
 gularia 8, 16, 32, 64 etc. laterum, quorum perime-  
 tri sint inter se aequales simulque perimetro quadrati  $bf$ .  
 Iam cum  $ab$  sit diameter circuli quadrato inscripti, ita  
 affirmat fore  $ac$  diametrum circuli octogono inscripti,  
 tum vero  $ad$  diametrum circuli 16gono,  $ae$  32gono  
 inscripti, et ita porro. Vnde liquet  $ax$  fore diametrum  
 circuli polygono infinitorum laterum regulari inscripti,  
 ideoque eius peripheriam aequari perimetro quadrati.

Quo facilius demonstrationem huius constructionis  
 adornem, obseruo, quae hic de diametris circulorum  
 dicuntur, etiam valere pro radiis, ita vt  $ab$ ,  $ac$ ,  $ad$ ,

$ae$

etc. spectari possit tanquam radii circulorum, quibus si circumscribantur polygona regularia 4, 8, 16, 32 etc. laterum, eorum perimetri futurae sint inter se aequales.

**Problema.**

Dato circulo, cui polygonum regulare circumscriptum, inuenire circulum alium, cui si polygonum regulare duplo plurium laterum circumscribatur, perimeter huius polygoni aequalis sit futurae perimetro illius polygoni.

**Solutio.**

Sit ENM circulus datus et EP semilatus poly- Fig. 2.  
goni ipsi circumscripti, centro existente in C; CF autem sit radius circuli quaesiti, et FQ semilatus polygoni ipsi circumscribendi. Necessesse ergo est, vt sit FQ semissis ipsius EP, et angulus FCQ semissis anguli ECP. Quare recta CQ angulum ECP, et recta QO ipsi CE parallela lineam EP bisecabit. Cum nunc

$$\text{sit } EV : CE = FQ : CF$$

$$\text{et } EV : CE = EP : CE + CP$$

$$\text{erit } FQ : CF = EP : CE + CP$$

sed quia  $FQ = \frac{1}{2}EP$ , erit etiam  $CF = \frac{1}{2}(CE + CP)$ . Hinc auferatur CF, et habebitur  $EF = \frac{1}{2}(CP - CE)$  ex quo erit  $\text{rectangulum } CF \cdot EF = \frac{1}{4}(CP^2 - CE^2) = \frac{1}{4}EP^2$  ideoque punctum F ita definiri debet, vt sit rectangulum, sub CF et EF comprehensum, aequale parti quartae quadrati rectae EP, seu ipsi quadrato rectae FQ.

Coroll. 1

## Coroll. 1.

Cum sit  $CF \cdot EF = FQ^2$  erit  $CF : FQ = FQ : EF$ , unde ducta recta  $QE$ , fiet triangulum  $FQE$  simile triangulo  $FCQ$ , vel  $ECV$ , ideoque angulus  $FQE$  aequalis angulo  $ECV$ .

## Coroll. 2.

Cum sit  $CE : EV = EO : EF$ , punctum  $F$  etiam ita definiri poterit: ex  $O$  ducatur recta ad  $CV$  productam normalis, eaque basi  $CE$  in  $F$  occurret.

## Coroll. 3.

Si polygonum circulo  $ENM$  circumscriptum sit  $n$  laterum, erit angulus  $ECP = \frac{\pi}{n}$ , denotante  $\pi$  mensuram duorum angulorum rectorum; et angulus  $FCQ = \frac{\pi}{2n}$ . Hinc si radius  $CE = r$ , erit  $EP = r \operatorname{tang} \frac{\pi}{2n}$  et  $FQ = \frac{1}{2} r \operatorname{tang} \frac{\pi}{n}$ .

## Coroll. 4.

Iam quia angulus  $FQE = \frac{\pi}{2n}$  erit  $EF = FQ \operatorname{tang} \frac{\pi}{2n} = \frac{1}{2} r \operatorname{tang} \frac{\pi}{n} \operatorname{tang} \frac{\pi}{2n}$ . Verum si vocemus  $CF = s$ , erit  $FQ = s \operatorname{tang} \frac{\pi}{2n}$ , unde ob  $FQ = \frac{1}{2} r \operatorname{tang} \frac{\pi}{n}$  fiet  $s = \frac{1}{2} r \operatorname{tang} \frac{\pi}{n} \cot \frac{\pi}{2n}$ .

### Demonstratio Constructionis Cartesianae.

Fig. 3. Sit iam  $CE$  radius circuli quadrato inscripti,  $CF$  octogono inscripti,  $CG$  polygono regulari 16 laterum,  $CH$

CH polygono 32 laterum et ita porro. Sit porro EP semilatus quadrati, FQ semilatus octogoni, GR polygona 16, HS polygona 32 laterum, etc. et quia haec polygona eiusdem perimetri assumuntur, erit  $FQ = \frac{1}{2}EP$ ;  $GR = \frac{1}{2}FQ = \frac{1}{4}EP$ ;  $HS = \frac{1}{2}GR = \frac{1}{8}FQ = \frac{1}{8}EP$ , etc. Iam ex problemate praemisso est CF. EF =  $\frac{1}{2}EP^2 = FQ^2$ ; tum vero ex eodem simili modo

$$CG. FG = \frac{1}{2}FQ^2 = \frac{1}{2}CF. EF = GR^2$$

$$CH. GH = \frac{1}{2}GR^2 = \frac{1}{2}CG. FG = HS^2 \text{ etc.}$$

sicque puncta F, G, H etc. eodem plane modo determinantur, uti habet constructio *Cartesiana*; et quia interualla EF, FG, GH etc. continuo fiunt minora, satis promte ad punctum vltimum  $x$  appropinquatur, eritque  $Cx$  radius circuli, cuius peripheria aequatur perimetro polygonorum praecedentium, ideoque rectae EP octies sumtae. Q. E. D.

### Coroll. 1.

Si ponatur  $CE = a$ ,  $CF = b$ ,  $CG = c$ ,  $CH = d$ , etc. progressio harum quantitatum ita est comparata, ut sit ob  $EP = a$

$b(b-a) = \frac{1}{2}aa$ ;  $c(c-b) = \frac{1}{2}b(b-a)$ ;  $d(d-c) = \frac{1}{2}c(c-b)$  etc. ideoque

$$b = \frac{a + \sqrt{2aa}}{2}; c = \frac{b + \sqrt{2bb - ab}}{2}; d = \frac{c + \sqrt{2cc - bc}}{2} \text{ etc.}$$

et harum quantitatum infinitesima est radius circuli cuius peripheria est  $= 8a$ .

## Coroll. 2.

Cum fit angulus ECP semirectus, seu  $ECP = \frac{\pi}{4}$ ,  
erunt anguli  $FCQ = \frac{\pi}{8}$ ;  $GCR = \frac{\pi}{16}$ ;  $HCS = \frac{\pi}{32}$ , etc.  
Quare ob  $EP = a$ ;  $FQ = \frac{1}{2}a$ ;  $GR = \frac{1}{4}a$ ;  $HS = \frac{1}{8}a$  etc.  
erit per cotangentes:

$$CE = a \cot. \frac{\pi}{4}; \quad CF = \frac{1}{2}a \cot. \frac{\pi}{8}; \quad CG = \frac{1}{4}a \cot. \frac{\pi}{16}; \quad CH = \frac{1}{8}a \cot. \frac{\pi}{32} \text{ etc.}$$

Vnde denotante  $n$  numerum infinitum, fit harum li-  
nearum ultima  $= \frac{1}{2n}a \cot. \frac{\pi}{4n}$ .

## Coroll. 3.

Sed  $\cot. \frac{\pi}{4n} = 1$ ;  $\tan. \frac{\pi}{4n}$ ; et quia angulus  $\frac{\pi}{4n}$  est  
infinite parvus, erit  $\tan. \frac{\pi}{4n} = \frac{\pi}{4n}$ , ideoque  $\cot. \frac{\pi}{4n} = \frac{4n}{\pi}$ .  
Quare linearum illarum ultima fit  $= \frac{1}{2}a$ , quo radio si  
circulus describatur, erit eius peripheria  $= 2\pi \cdot \frac{1}{2}a = \pi a$ .

## Coroll. 4.

Deinde quia ex coroll. 4. praec. probl. est  $EF$   
 $= FQ \tan. FCQ$ , erit ob eandem rationem:

$FG = GR \tan. GCR$ ;  $GH = HS \tan. HCS$  etc.  
vnde haec intervalla sequenti modo exprimentur:

$EF = \frac{1}{2}a \tan. \frac{\pi}{8}$ ;  $FG = \frac{1}{4}a \tan. \frac{\pi}{16}$ ;  $GH = \frac{1}{8}a \tan. \frac{\pi}{32}$  etc.  
antecedens vero ad analogiam  $CE = a \tan. \frac{\pi}{4} = a$ .

## Coroll. 5.

His cum praecedentibus collatis nanciscemur:

$$CF = a (\tan. \frac{\pi}{4} + \frac{1}{2} \tan. \frac{\pi}{8}) = \frac{1}{2}a \cot. \frac{\pi}{8}$$

$$CG = a (\tan. \frac{\pi}{4} + \frac{1}{2} \tan. \frac{\pi}{8} + \frac{1}{4} \tan. \frac{\pi}{16}) = \frac{1}{4}a \cot. \frac{\pi}{16}$$

$$CH = a (\tan. \frac{\pi}{4} + \frac{1}{2} \tan. \frac{\pi}{8} + \frac{1}{4} \tan. \frac{\pi}{16} + \frac{1}{8} \tan. \frac{\pi}{32}) = \frac{1}{8}a \cot. \frac{\pi}{32}$$

etc.

siquae

sicque omnium huiusmodi progressionum summae expedite assignari possunt.

Coroll. 6.

In infinitum ergo progrediendo obtinebimus summationem huius seriei:

$\text{tang. } \frac{\pi}{4} + \frac{1}{2} \text{tang. } \frac{\pi}{8} + \frac{1}{4} \text{tang. } \frac{\pi}{16} + \frac{1}{8} \text{tang. } \frac{\pi}{32} + \text{etc.} = \frac{1}{2} \frac{\pi}{\pi}$   
 quae ergo per quadraturam circuli determinatur. Hinc occasionem arripio sequens problema soluendi.

Problema.

Denotante  $\Phi$  arcum quemcunque circuli cuius radius = 1, inuenire summam huius seriei infinitae:

$\text{tang. } \Phi + \frac{1}{2} \text{tang. } \frac{1}{2}\Phi + \frac{1}{4} \text{tang. } \frac{1}{4}\Phi + \frac{1}{8} \text{tang. } \frac{1}{8}\Phi + \frac{1}{16} \text{tang. } \frac{1}{16}\Phi \text{ etc.}$

Solutio.

Si in fig. 2. vti supra est constructa, ponatur angulus  $ECP = \Phi$ , erit  $FCQ = \frac{1}{2}\Phi$ : iam posito  $FQ = 1$  erit  $EP = 2$ , hincque  $CE = 2 \cot. \Phi$ ;  $CF = \cot. \frac{1}{2}\Phi$  et  $EF = \text{tang. } \frac{1}{2}\Phi$ , ex quo habetur:

$2 \cot. \Phi = \cot. \frac{1}{2}\Phi - \text{tang. } \frac{1}{2}\Phi$  et  $\text{tang. } \frac{1}{2}\Phi = \cot. \frac{1}{2}\Phi - 2 \cot. \Phi$   
 eodemque modo  $\text{tang. } \Phi = \cot. \Phi - 2 \cot. 2\Phi$ . Collocentur hi valores tangentium per cotangentes expressi in serie proposita

$\text{tang. } \Phi = \cot. \Phi - 2 \cot. 2\Phi$

$\frac{1}{2} \text{tang. } \frac{1}{2}\Phi = \frac{1}{2} \cot. \frac{1}{2}\Phi - \cot. \Phi$

$\frac{1}{4} \text{tang. } \frac{1}{4}\Phi = \frac{1}{4} \cot. \frac{1}{4}\Phi - \frac{1}{2} \cot. \frac{1}{2}\Phi$

$\frac{1}{8} \text{tang. } \frac{1}{8}\Phi = \frac{1}{8} \cot. \frac{1}{8}\Phi - \frac{1}{4} \cot. \frac{1}{4}\Phi$

etc.

X 2

et

Tab. I.  
Fig. 2.



et colligendo consequemur :

$$\begin{aligned} \text{tang. } \Phi &= \text{cot. } \Phi - 2 \text{cot. } 2\Phi \\ \text{tang. } \Phi + \frac{1}{2} \text{tang. } \frac{1}{2}\Phi &= \frac{1}{2} \text{cot. } \frac{1}{2}\Phi - 2 \text{cot. } 2\Phi \\ \text{tang. } \Phi + \frac{1}{2} \text{tang. } \frac{1}{2}\Phi + \frac{1}{4} \text{tang. } \frac{1}{4}\Phi &= \frac{1}{4} \text{cot. } \frac{1}{4}\Phi - 2 \text{cot. } 2\Phi \\ \text{tang. } \Phi + \frac{1}{2} \text{tang. } \frac{1}{2}\Phi + \frac{1}{4} \text{tang. } \frac{1}{4}\Phi + \frac{1}{8} \text{tang. } \frac{1}{8}\Phi &= \frac{1}{8} \text{cot. } \frac{1}{8}\Phi - 2 \text{cot. } 2\Phi \\ &\text{etc.} \end{aligned}$$

unde in infinitum progrediendo, si  $n$  denotet numerum infinitum, quia  $\text{tang. } \frac{1}{n}\Phi = \frac{\Phi}{n}$ , hincque  $\text{cot. } \frac{1}{n}\Phi = \frac{n}{\Phi}$ , erit:  $\frac{1}{n} \text{cot. } \frac{1}{n}\Phi = \frac{1}{\Phi}$ , ideoque summa seriei propositae:

$$\text{tang. } \Phi + \frac{1}{2} \text{tang. } \frac{1}{2}\Phi + \frac{1}{4} \text{tang. } \frac{1}{4}\Phi + \frac{1}{8} \text{tang. } \frac{1}{8}\Phi + \text{etc.} = \frac{1}{\Phi} - 2 \text{cot. } 2\Phi$$

Unde si  $2\Phi$  est angulus rectus, seu  $\Phi = \frac{\pi}{2}$ , ob  $\text{cot. } \frac{\pi}{2} = 0$  fit seriei summa  $= \frac{1}{\Phi} = \frac{2}{\pi}$ , qui est casus supra tractatus.

Ex hac serie plures aliae deriuari possunt non minus notatu dignae.

I. Ex eius differentiatione adipiscimur :

$$\begin{aligned} \frac{1}{\text{cos. } \Phi^2} + \frac{1}{4 \text{cos. } \frac{1}{2}\Phi^2} + \frac{1}{4^2 \text{cos. } \frac{1}{4}\Phi^2} + \frac{1}{4^3 \text{cos. } \frac{1}{8}\Phi^2} + \frac{1}{4^4 \text{cos. } \frac{1}{16}\Phi^2} \\ + \text{etc.} = -\frac{1}{\Phi \sin. \Phi} + \frac{4}{\sin. 2\Phi^2} \end{aligned}$$

vel cum sit  $\frac{1}{\text{cos. } \Phi} = \text{sec. } \Phi$  erit quoque

$$(\text{sec. } \Phi)^2 + \frac{1}{2} (\text{sec. } \frac{1}{2}\Phi)^2 + \frac{1}{4^2} (\text{sec. } \frac{1}{4}\Phi)^2 + \frac{1}{4^3} (\text{sec. } \frac{1}{8}\Phi)^2 + \text{etc.}$$

II. Deinde ob  $\text{cos. } \Phi^2 = \frac{1 + \text{cos. } 2\Phi}{2}$ , et  $\text{sin. } 2\Phi^2 = \frac{\text{sin. } \Phi^2 \text{cos. } \Phi^2}{\Phi}$  erit vbique per 2 diuidendo :

$$\begin{aligned} \frac{1}{1 + \text{cos. } 2\Phi} + \frac{1}{4(1 + \text{cos. } \Phi)} + \frac{1}{4^2(1 + \text{cos. } \frac{1}{2}\Phi)} \\ + \frac{1}{4^3(1 + \text{cos. } \frac{1}{4}\Phi)} + \text{etc.} = \frac{2}{1 - \text{cos. } 4\Phi} - \frac{1}{2\Phi \sin. \Phi} \end{aligned}$$

seu

seu pro  $\Phi$  scribendo  $\frac{1}{2}\Phi$

$$\frac{1}{1 + \text{col. } \Phi} + \frac{1}{4(1 + \text{col. } \frac{1}{2}\Phi)} + \frac{1}{4^2(1 + \text{col. } \frac{1}{4}\Phi)} + \dots + \frac{1}{4^n(1 + \text{col. } \frac{1}{2^n}\Phi)} + \text{etc.} = \frac{2}{1 - \text{col. } 2\Phi - \Phi\Phi}$$

III. Si series inuenta per  $d\Phi$  multiplicetur et integretur, ob  $\int d\Phi \text{ tang. } \Phi = \int \frac{d \sin \Phi}{\cos \Phi} = -l \text{ col. } \Phi$ , et  $\int 2 d\Phi \text{ cot. } 2\Phi = l \text{ sin. } 2\Phi$ , habebitur

$$-l \text{ col. } \Phi - l \text{ col. } \frac{1}{2}\Phi - l \text{ col. } \frac{1}{4}\Phi - l \text{ col. } \frac{1}{8}\Phi - l \text{ col. } \frac{1}{16}\Phi - \text{etc.} = l\Phi - l \text{ sin. } 2\Phi + \text{Const.}$$

ad quam constantem definiendam ponamus  $\Phi = 0$ , et quia  $l \text{ col. } 0 = l 1 = 0$ , ex priori parte habemus 0, ex posteriori vero ob  $\text{sin. } 2\Phi = 2\Phi$ , habemus  $l\Phi - l 2\Phi + \text{Const.} = -l 2 + \text{Const.}$  vnde  $\text{Const.} = l 2$ . Hinc ad numeros progrediendo erit:

$$\frac{1}{\text{col. } \Phi \text{ col. } \frac{1}{2}\Phi \text{ col. } \frac{1}{4}\Phi \text{ col. } \frac{1}{8}\Phi \text{ col. } \frac{1}{16}\Phi \text{ etc.}} = \frac{2\Phi}{\text{sin. } 2\Phi}$$

IV. Cum sit  $\frac{1}{\text{col. } \Phi} = \text{sec. } \Phi$ , habebitur etiam hoc Theorema pro secantibus:

$$\text{sec. } \Phi \text{ sec. } \frac{1}{2}\Phi \text{ sec. } \frac{1}{4}\Phi \text{ sec. } \frac{1}{8}\Phi \text{ sec. } \frac{1}{16}\Phi \text{ etc.} = \frac{2\Phi}{\text{sin. } 2\Phi}$$

vnde si ratio diametri ad peripheriam ponatur = 1:  $\pi$  et  $q$  denotet angulum rectum, si statuamus  $2\Phi = q$ , =  $\frac{\pi}{2}$  erit:

$$\text{sec. } \frac{1}{2}q \text{ sec. } \frac{1}{4}q \text{ sec. } \frac{1}{8}q \text{ sec. } \frac{1}{16}q \text{ sec. } \frac{1}{32}q \text{ etc.} = \frac{\pi}{2}$$

## Problema.

Invenire seriem quantitatum:  $a, b, c, d, e, f$ , etc. cuius haec sit proprietas, ut sit:

$$c(c-b) = \frac{1}{2}b(b-a); d(d-c) = \frac{1}{2}c(c-b); e(e-d) = \frac{1}{2}d(d-c) \text{ etc.}$$

seu ut quantitates inde derivatae

$$b(b-a); c(c-b); d(d-c); e(e-d); f(f-e), \text{ etc.}$$

decrefant secundum rationem quadruplam.

## Solutio.

Cum sit  $\text{tang. } \frac{1}{2}\Phi = \cot. \frac{1}{2}\Phi - 2 \cot. \Phi$ , si multiplicemus utrinque per  $\cot. \frac{1}{2}\Phi$ , ob  $\text{tang. } \frac{1}{2}\Phi \cot. \frac{1}{2}\Phi = 1$  erit  $\cot. \frac{1}{2}\Phi (\cot. \frac{1}{2}\Phi - 2 \cot. \Phi) = 1$ . Statuatur ergo  $a = r \cot. \Phi; b = \frac{1}{2}r \cot. \frac{1}{2}\Phi; c = \frac{1}{4}r \cot. \frac{1}{4}\Phi; d = \frac{1}{8}r \cot. \frac{1}{8}\Phi$ , etc. eritque

$$\frac{\frac{1}{2}b}{r} \left( \frac{\frac{1}{2}b}{r} - \frac{\frac{1}{2}a}{r} \right) = 1 \quad \text{hinc } b(b-a) = \frac{r^2}{4}$$

$$\frac{\frac{1}{4}c}{r} \left( \frac{\frac{1}{4}c}{r} - \frac{\frac{1}{4}b}{r} \right) = 1 \quad \text{hinc } c(c-b) = \frac{r^2}{16}$$

$$\frac{\frac{1}{8}d}{r} \left( \frac{\frac{1}{8}d}{r} - \frac{\frac{1}{8}c}{r} \right) = 1 \quad \text{hinc } d(d-c) = \frac{r^2}{64}$$

etc.

Quare haec series

$a = r \cot. \Phi; b = \frac{1}{2}r \cot. \frac{1}{2}\Phi; c = \frac{1}{4}r \cot. \frac{1}{4}\Phi; d = \frac{1}{8}r \cot. \frac{1}{8}\Phi$ ; etc. hanc habet proprietatem, ut quantitates inde formatae

$$b(b-a); c(c-b); d(d-c); e(e-d); \text{ etc.}$$

in ratione quadrupla decrefant.

Coroll. 1.

Coroll. 1.

Datis duobus terminis primis  $a$  et  $b$  reliqui  $c, d, e, f$  inde successive ita determinantur, ut sit  
 $c = \frac{b + \sqrt{(2bb - ab)}}{2}$ ;  $d = \frac{c + \sqrt{(2cc - bc)}}{2}$ ;  $e = \frac{d + \sqrt{(2dd - cd)}}{2}$  etc.  
 ideoque binis terminis initialibus pro lubitu assumtis, tota series ope harum formularum exhiberi potest.

Coroll. 2.

Datis autem terminis  $a$  et  $b$ , inde angulus  $\Phi$  cum quantitate  $r$  ita definitur, ut sit:

$$\text{tang. } \Phi = \frac{2\sqrt{(bb - ab)}}{a} \text{ et } r = 2\sqrt{(bb - ab)}$$

unde inuento angulo  $\Phi$  reliqui termini etiam ita exprimentur, ut sit:

$$c = \frac{1}{2}r \cot. \frac{1}{2}\Phi; d = \frac{1}{2}r \cot. \frac{1}{4}\Phi; e = \frac{1}{4}r \cot. \frac{1}{8}\Phi \text{ etc.}$$

Coroll. 3.

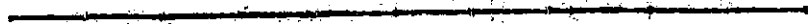
Hinc istius seriei termini infinitesimi fient  $= \frac{r}{\Phi}$  ad quem valorem termini seriei satis cito conuergunt. Quærat in circulo radii  $= r$ , arcus cuius tangens  $= \frac{2\sqrt{(bb - ab)}}{a}$ , qui arcus sit  $= \Phi$ , et seriei nostræ termini infinitesimi erunt  $= \frac{2\sqrt{(bb - ab)}}{\Phi}$ .

Scholion.

Caeterum hic monuisse iuuabit puncta  $P, Q$ , Tab. I.  $R, S, x$  sita esse in curua quadratrice veterum, Fig. 3. propterea quod applicatae  $EP, FQ, GR, HS$  eandem inter se rationem tenent, quam anguli  $ECP, FCQ$ .

168 ANNOT. IN LOCVM QVENDAM CART.

FCQ, GCR, HCS etc. Et quoniam  $x$ , vbi haec curua in basin incidit, iam olim circuli quadraturam indicare est inuentum, vnde et istud nomen est inditum, constructio Cartesii cum hac veterum quadratura egregie quidem conuenit; sed multo commodius et accuratius puncta E, F, G, H etc. successive praebet, quam a continua bisectione angulorum expectari queat.



DEMON-