1760

De cochlea Archimedis

Leonhard Euler

DE COCHLEA ARCHIMEDIS.

AVCTORE

LEON. EVLERO.

Cochlea Archimedis cum ob inuentionis antiquitatem, tum ob eius frequentiſſimum vſum in aquis hauriendis, tantopere celebrata, atque in vulgus cognita, vt vix vllus Hydraulicarum Machinarum ſcriptor reperiatur, qui eius conſtructionem atque vtilitatem non abunde explicuerit. Quodſi vero ad cauſam ſpectemus, cur haec machina ad aquam eleuandam ſit apta, et quomodo eius actio ſecundum mechanica principia abſoluatur, apud vetuſtiores quidem auctores nihil plane inuenimus, quod rationem ſaltem probabilem in ſe contineat, recentiores vero hanc inueſtigationem vel prorſus praeterierunt, vel leuiter ſaltem ac minus accurate ſunt perſecuti. Ita quamuis haec machina ſit notiſſima, eiusque praxis frequentiſſima, tamen fateri cogimur, eius Theoriam maxime adhuc eſſe abſconditam, atque tam modum, quo aqua per eam eleuatur, quam vires ad eius actionem requiſitas etiam nunc fere penitus latere. Atque hoc eo magis mirum videri debet, cum non ſolum ceterae Machinae ab antiquitate ad nos transmiſſae, felici cum ſucceſſu ad leges mechanicas ſint reuocatae, ſed etiam ipſa ſcientia mechanica eousque exculta ſit, vt ad omnis generis machinas explicandas ſufficiens videatur. Quin etiam a plerisque omne studium, quod a Geometris ope Analyseos sublimioris in Mechanica vlterius excolenda consumitur, subtile magis quam vtile censeri solet.

Verum si rationem cochleae Archimedis diligentius contemplemur, vulgaria mechanicae principia ei explicandae minime sufficientia deprehendemus: propterea quod ea manifeste ad Theoriam motus aquae per tubos mobiles pertineat quod argumentum a nemine fere adhuc est tractatum. Quod enim ad motum aquae in genere attinet, non dudum admodum est, ex quo is studiosius inuestigari atque ad principia mechanica inuestigari est coeptus, de motu autem aquae per tubos mobiles vix quisquam reperitur, qui aliquid in medium attulerit, vel tantum cogitauerit. Quam, obrem cum nunc quidem principia, quibus omnis aquae motus innititur, satis sint euoluta, operam dabo, vt ea quoque ad motum aquae, quo per cochleam hanc Archimedis fertur, accommodem, indeque omnia phaenomena, quae in hoc motu consideranda occurrant, clare ac distincte explanem. Quae igitur hac de re sum meditatus, sequentibus propositionibus sum complexurus; et quoniam cochleae Archimedeae duplicis generis construi solent, quarum alterae helices suas circa cylindrum, alterae vero circa conum habent circumvolutas, a cochlea cylindrica exordiar; eiusque Theoria stabilita ad cochleas quoque conicas perscrutandas non difficulter progredi licebit.

PRO-

PROBLEMA. I.

1. Dato motu, quo cylindrus circumagitur, et aquae celeritate per cochleam seu helicem cylindro circumductam, determinare verum cuiusque aquae particulae motum, hoc est eum motum, qui ex motu gyratorio cylindri et motu aquae progressiuo per helicem componitur.

SOLVTIO.

Sit circulus ACB basis cylindri, cuius superficiei helix est circumducta, recta CD ad basin in centro C perpendicularis axis cylindri, circa quem cylindrus cum helice in gyrum agitur. Ponatur basis semidiameter $CA = CB = a$, et sit EZ portio helicis in superficie cylindri, quae cum peripheria basis faciat angulum $ZEY = \zeta$; et a puncto helicis quocunque Z ad basin ducatur axi parallela ZY; voceturque arcus $EY = s$; est $YZ = s\,\mathrm{tang.}\,\zeta$, quae cum helice faciet angulum $EZY = 90^\circ - \zeta$; et longitudo helicis erit $E = Z\frac{s}{\mathrm{cof.}\,\zeta}$.

Iam aquae per helicem transfluentis celeritas sit debita altitudini v; helicem enim EZ vbique eiusdem amplitudinis assumo, ita vt eodem temporis instanti omnis aquae in helice contentae eadem sit celeritas $= \sqrt{v}$. Deinde quia tota helix circa axem CD gyratur, sit puncti E celeritas gyratoria circa punctum C debita altitudini u. Recta autem AB sit fixa, quae scilicet non cum cylindro moueatur: atque initio quidem punctum E fuerit in A; inde autem tempore elapso $= t$ motu angulari peruenerit in E, sitque arcus $AE = p$; erit ob motum angularem $dp = dt\sqrt{u}$.

Nunc consideretur primo motus aquae per helicem quasi quiescentem, ac celeritas particulae aquae in Z erit $= \sqrt{v}$ eiusque directio erit Zz, qui motus resoluatur in duos, quorum alterius directio sit secundum YZ, alterius secundum Zv seu Yy, atque celeritas secundum YZ erit $= \sqrt{v}$, sin ζ celeritas vero secundum Zv seu Yy erit $= \sqrt{v}.\ \text{cos}.\ \zeta$.

Ad hunc posteriorem motum adiungi nunc debet motus gyratorius, quippe qui in eandem directionem tendit, ex quo prodit tota celeritas puncti Z secundum directionem $Yy = \sqrt{u} + \sqrt{v}.\ \text{cos}.\ \zeta$.

Quoniam vero directio Yy est variabilis, reducatur ea ad directiones constantes; quem in finem ex Y ad rectam fixam AB ducatur perpendicularis YX, ac vocentur tres coordinatae locum puncti Z determinantes $CX = x$, $XY = y$, et $YZ = z$, erit primo $z = s$ tang. ζ; tum vero ob arcum $AY = p + s$, et angulum $A = CY \frac{p+s}{a}$, erit $CX = x = a\,\text{cos}.\ \frac{p+s}{a}$ et $XY = y = a$ sin $\frac{p+s}{a}$. Tum ducta Yu rectae AB parallela erit angulus $Yyu = \frac{p+s}{a}$. Hinc motus secundum Yy resoluetur in binos alios, alterum secundum Yu seu AC cuius celeritas $= (\sqrt{u} + \sqrt{v}\,\text{cos}.\ \zeta)$ sin. $\frac{p+s}{a}$, alterum vero secundum XY cuius celeritas $= (\sqrt{u} + \sqrt{v}\,\text{cos}.\ \zeta)\,\text{cos}.\ \frac{p+s}{a}$; celeritate secundum YZ existente $= \sqrt{v}.\ \text{sin}.\ \zeta$.

Quare loco puncti Z ad ternas coordinatas fixas reducto, quae sunt:

$CX = x = a\,\text{cos}.\ \frac{p+s}{a}$, $XY = y = a\,\text{sin}.\ \frac{p+s}{a}$, et $YZ = s\,\text{tang}.\ \zeta$

verus

versus particulae in Z versantis motus pariter secundum has ternas directiones fixas resoluetur, eritque

Celeritas motus secundum $CX = -(\sqrt{u} + \sqrt{v}\,\text{cos}.\zeta)\,\text{sin}.\frac{p+s}{a}$

Celeritas motus secundum $XY = +(\sqrt{u} + \sqrt{v}\,\text{cos}.\zeta)\,\text{cos}.\frac{p+s}{a}$

Celeritas motus secundum $YZ = \sqrt{v}.\ \text{sin}.\zeta.$

COROLL. 1.

2. Hinc iam facile reperitur vera celeritas particulae aquae in Z versantis, cum enim hae ternae directiones sint inter se normales, erit vera celeritas aequalis radici quadratae ex summa quadratorum harum trium celeritatum, ex quo vera celeritas erit $= \sqrt{(u + v + 2\sqrt{uv}.\,\text{cos}.\zeta)}$.

COROLL. 2.

3. Cum particula aquae in Z tempusculo dt perueniat in helicis punctum z, existente $Zz = \frac{ds}{\text{cos}.\zeta}$, et $Yy.\ Zv = ds$, celeritas autem in helice sit $= \sqrt{v}$, erit $Zz: \frac{ds}{\text{cos}.\zeta} = dt\sqrt{v}$, vnde fit $ds = dt\sqrt{v}.\,\text{cos}.\zeta$, praeterea vero iam vidimus esse $dp = dt\sqrt{u}$.

COROLL. 3.

4. Celeritates quoque particulae aquae Z secundum ternas directiones fixas exprimentur per differentialia coordinatarum x, y, z ad elementum temporis dt applicatas.

Erit scilicet ex natura resolutionis motus:

Celeritas secundum $CX = \frac{dx}{dt} = -(\sqrt{u} + \sqrt{v}.\,\text{cos}.\zeta)\,\text{sin}.\frac{p+s}{a}$

Celeritas secundum $XY = \frac{dy}{dt} = (\sqrt{u} + \sqrt{v}.\,\text{cos}.\zeta)\,\text{cos}.\frac{p+s}{a}$

Celeritas secundum $YZ = \frac{dz}{dt} = \sqrt{v}.\ \ \text{sin}.\zeta.$

Qua-

Quarum formularum identitas intelligitur ex valoribus differentialibus $dp = dt\sqrt{u}$ et $ds = dt\sqrt{v}.\,\text{cof.}\,\zeta$.

PROBLEMA. 2.

5. Datis tam celeritate, qua aqua per helicem promouetur, quam celeritate, qua cylindrus cum helice circa axem CD in gyrum agitur, inuenire vires, quibus quamque aquae particulam Z follicitari oportet, vt hunc motum profequi queat.

SOLVTIO.

Sit celeritas qua aqua praefenti temporis momento per helicem EZ promouetur $= \sqrt{v}$, celeritas autem gyratoria cylindri $= \sqrt{u}$. Tum initium helicis iam fit in E vt fit $AE = p$, et particula aquae, quam confideramus, in Z, vt ducta ZY axi CD parallela, fit arcus $EY = s$, exiftente angulo helicis $YEZ = \zeta$. Porro locus puncti Z reducatur ad ternas coordinatas fixas $CX = x$, $XY = y$ et $YZ = z$; erit vti vidimus:

$$x = a\ \text{cof.}\,\tfrac{p+s}{a};\ y = a\ \text{fin.}\,\tfrac{p+s}{a}\ \text{et}\ z = s\ \text{tang}\ \zeta$$

denotante a femidiametrum $CA = CB$ bafis cylindri. Pofito vero elemento temporis $= dt$, vt fit $dp = dt\sqrt{u}$ et $ds = dt\sqrt{v}.\ \text{cof.}\ \zeta$, fumtoque hoc differentiali dt conftanti, ex principiis mechanicis conftat, particulam aquae in Z a tribus viribus acceleratricibus vrgeri debere, quae fint:

fecundum directionem $CX = \frac{2ddx}{dt^2}$

fecundum directionem $XY = \frac{2ddy}{dt^2}$

fecundum directionem $YZ = \frac{2ddz}{dt^2}$

Verum

Verum cum ex ſupra oſtenſis ſit

$$\frac{dx}{dt} = -(\sqrt{u} + \sqrt{v} \text{ coſ. } \zeta) \text{ ſin. } \frac{p+s}{a}$$

$$\frac{dy}{dt} = (\sqrt{u} + \sqrt{v} \text{ coſ. } \zeta) \text{ coſ. } \frac{(p+s)}{a}$$

et $-\frac{dz}{dt} = \sqrt{v} \text{ ſin. } \zeta$

erit denuo differentiando

$$\frac{ddx}{dt^2} = -(\frac{du}{2dt\sqrt{u}} + \frac{dv \text{ coſ. } \zeta}{2dt\sqrt{v}}) \text{ ſin. } \frac{p+s}{a} - \frac{1}{a}(\sqrt{u} + \sqrt{v} \text{ coſ. } \zeta)^2 \text{ coſ. } \frac{p+s}{a}$$

$$\frac{ddy}{dt^2} = (\frac{du}{2dt\sqrt{u}} + \frac{dv \text{ coſ. } \zeta}{2dt\sqrt{v}}) \text{ coſ. } \frac{p+s}{a} - \frac{1}{a}(\sqrt{u} + \sqrt{v} \text{ coſ. } \zeta)^2 \text{ ſin. } \frac{p+s}{a}$$

$$\frac{ddz}{dt^2} = \frac{dv}{2dt\sqrt{v}} \text{ ſin. } \zeta$$

Tres ergo vires acceleratrices quaeſitae ſunt

I. ſec. $CX = -\frac{1}{dt}(\frac{du}{\sqrt{u}} + \frac{dv \text{ coſ. } \zeta}{\sqrt{v}}) \text{ ſin. } \frac{p+s}{a} - \frac{2}{a}(\sqrt{u} + \sqrt{v} \text{ coſ. } \zeta)^2 \text{ coſ. } \frac{p+s}{a}$

II. ſec. $XY = +\frac{1}{dt}(\frac{du}{\sqrt{u}} + \frac{dv \text{ coſ. } \zeta}{\sqrt{v}}) \text{ coſ. } \frac{p+s}{a} - \frac{2}{a}(\sqrt{u} + \sqrt{v} \text{ coſ. } \zeta)^2 \text{ ſin. } \frac{p+s}{a}$

III ſec. $YZ = \frac{dv}{dt\sqrt{u}} \text{ ſin. } \zeta$.

COROLL. I.

6. Transferantur duae priores vires primum in punctum Y, ita vt hoc punctum a duabus viribus acceleratricibus vrgeatur, ſecundum directiones YM et YN, Tab. II. Fig. 2. qae ſunt

Vis ſec. $YM = -\frac{1}{dt}(\frac{du}{\sqrt{u}} + \frac{dv \text{ coſ. } \zeta}{\sqrt{v}}) \text{ ſin. } \frac{p+s}{a} - \frac{2}{a}(\sqrt{u} + \sqrt{v} \text{ coſ. } \zeta)^2 \text{ coſ. } \frac{p+s}{a}$

Vis ſec. $YN = +\frac{1}{dt}(\frac{du}{\sqrt{u}} + \frac{dv \text{ coſ. } \zeta}{\sqrt{v}}) \text{ coſ. } \frac{p+s}{a} - \frac{2}{a}(\sqrt{u} + \sqrt{v} \text{ coſ. } \zeta)^2 \text{ ſin. } \frac{p+s}{a}$

COROLL. 2.

7. Nunc hae duae vires in duas alias transformari poterunt, quae agant ſecundum directiones Yy, et YO, quarum haec ſit ad ſuperficiem cylindri normalis; atque ob angulum MYO ACY $= \frac{p+s}{a}$, ex his duabus viribus reſultabit

I Vis fecundum Yy = Vis YN cof. $\frac{p+s}{a}$ − Vis YM fin. $\frac{p+s}{a}$

II Vis fecundum YO = Vis YN fin. $\frac{p+s}{a}$ + Vis YM cof. $\frac{p+s}{a}$

COROLL. 3.

8. Hinc ergo loco duarum virium, quae follicitabant fecundum directiones CX et XY, vel YM et YN, in calculum introducentur duae hae aliae fecundum directiones Yy et YO, quae erunt

Vis fecundum $Yy = + \frac{1}{dt}\left(\frac{du}{\sqrt{v}} + \frac{dv \cos\zeta}{\sqrt{v}}\right)$

Vis fecundum $YO = -\frac{2}{a}(\sqrt{u} + \sqrt{v}\cos\zeta)^2$

ficque angulus $p + s$ non amplius in calculo reperitur.

PROBLEMA. 3.

9. Tres vires ante inuentas ad tres alias reducere, quarum vna fit fecundum directionem helicis Zz directa, duae reliquae vero fint ad ipfam helicem normales.

SOLUTIO.

Tab. II. Fig. 3. Sit Zz elementum helicis, vbi nunc particula aquae, quae vires inuentas fuftinet, verfatur: fitque Zo non folum ad helicem Zz, fed etiam ad ipfius cylindri fuperficiem in Z normalis, deinde fit recta Zr in ipfa fuperficie cylindri fita, atque ad Zz normalis. Tres igitur vires inuentae ad tres alias reduci debent, quae particulam aquae follicitent fecundum directiones Zz, Zo et Zr. Ac primo quidem vis inuenta fecundum YZ agens $= \frac{dv}{dt\sqrt{v}}$ fin ζ, ob angulum helicis $YEZ = \zeta$, dabit

I vim

I vim ſecundum $Zr = -\frac{dv}{dt\sqrt{v}}$ ſin. ζ coſ. ζ

II vim ſecundum $Zz = +\frac{dv}{dt\sqrt{v}}$ ſin. ζ ſin. ζ

Deinde vis, quae ſecundum directionem Yy ſeu Zv agere inuenta eſt $= \frac{1}{dt}\left(\frac{du}{\sqrt{u}} + \frac{dv\,\text{coſ.}\,\zeta}{\sqrt{v}}\right)$, dabit vires

I ſecundum $Zz = \frac{du}{dt\sqrt{u}}$ coſ. $\zeta + \frac{dv}{dt\sqrt{v}}$ coſ. ζ^2

II ſecundum $Zr = \frac{du}{dt\sqrt{u}}$ ſin. $\zeta + \frac{dv}{dt\sqrt{v}}$ ſin. ζ coſ. ζ

Tertio vis, quae ſecundum directionem YO agere eſt inuenta, dabit nunc ſola

vim ſecundum $Zo = -\frac{2}{a}(\sqrt{u} + \sqrt{v}.\,\text{coſ.}\,\zeta)^2$

Quare tres vires acceleratrices, quibus particula aquae in Z ſollicitari debet, vt motum propoſitum perſequatur, erunt:

I ſecundum directionem $Zz = \frac{du}{dt\sqrt{u}}$ coſ. $\zeta + \frac{dv}{dt\sqrt{v}}$,

II ſecundum directionem $Zr = \frac{du}{dt\sqrt{u}}$ ſin. ζ

III ſecundum directionem $Zo = -\frac{2}{a}(\sqrt{u} + \sqrt{v}\,\text{coſ.}\,\zeta)^2$

SCHOLION.

10. Habemus ergo vires, quibus ſingulae aquae particulae ſollicitatae eſſe debent, vt motus, quem aſſumſimus, ſubſiſtere poſſit. Iſtas autem vires hic ideo ad tres directiones Zz, Zr, et Zo reuocaui, quo facilius cum viribus, quibus aqua in tubo actu ſollicitatur, comparari poſſint; vt enim quantitates v et u verum aquae et cylindri motum exhibeant, neceſſe eſt, vt tres illae vires inuentae conueniant cum viribus, quibus aqua reuera vrgetur. Hae autém vires ſunt primo ſtatus compreſſionis aquae in tubo, deinde appreſſio aquae ad latera

latera tubi, quae ſecundum ambas directiones Zr et $Z\varrho$ ad directionem tubi normales exhiberi ſolet. Tertio vero grauitas, qua ſingulae aquae particulae deorſum nituntur, imprimis examini eſt ſubiicienda, quod ſequenti problemate inſtituemus.

PROBLEMA 4.

11. Si cylindrus fuerit vtcunque ad horizontem inclinatus, definire vires ſecundum ternas praedictas directiones, quibus ſingulae aquae particulae Z in helice ob grauitatem ſollicitantur.

SOLUTIO.

Tab. II. Fig. 4. Exprimat angulus θ inclinationem baſis cylindri ad horizontem, ſitque in plano baſis punctum fixum A ſummum, punctum B vero imum, ita vt recta AB cum axe cylindri CD in plano verticali ſit conſtituta. In hoc plano per centrum baſis C ducatur horizontalis CH, eritque angulus $ACH = \theta$, ſeu ſi ex puncto B erigatur recta verticalis BG axem in G interſecans, erit quoque angulus $BGC = \theta$, atque ob grauitatem ſingulae aquae particulae ſollicitabuntur deorſum ſecundum directiones ipſi GB parallelas, et vis acceleratrix haec vbique erit $= 1$. Fig. 1. Iam in prima figura ducatur quoque recta BG cum axe CD conſtituens angulum $BGC = \theta$, ac particula aquae in Z vrgebitur vi acceleratrice $= 1$ ſecundum directionem rectae BG parallelam. Reſoluatur haec vis ſecundum directiones GC et CB, prodibitque

Vis

Vis secundum $GC = 1 \cos.\theta$, et vis secundum $CB = 1 \sin.\theta$. Ex priori habebimus pro particula aquae Z vim secundum $ZY = \cos.\theta$, ex posteriori vero vim secundum $YM = -\sin.\theta$, vnde ob angulum $MYO = \frac{p+s}{a}$, oritur vis secundum $YO = -\sin.\theta \cos.\frac{p+s}{a}$ et vim secundum $Yy = +\sin.\theta \sin.\frac{p+s}{a}$. Hinc ergo punctum Z sollicitabitur ab his tribus viribus acceleratricibus: Tab. II. Fig. 2. Fig. 3.

I secundum directionem ZY vi $= \cos.\theta$
II secundum directionem Zo vi $= -\sin.\theta \cos.\frac{p+s}{a}$
III secundum directionem Zv vi $= +\sin.\theta \sin.\frac{p+s}{a}$

Ex his porro ob angulum $zZv = \zeta$ orientur:
Primo vis secundum $Zz =$ vi $Zv \cos.\zeta -$ vi $ZY \sin.\zeta$
Tum vis secundum $Zr =$ vi $Zv \sin.\zeta +$ vi $ZY \cos.\zeta$

Quare pro tribus directionibus Zz, Zr et Zo obtinebimus sequentes vires acceleratrices ex grauitate oriundas:

I Vim secundum $Zz = \cos.\zeta \sin.\theta \sin.\frac{p+s}{a} - \sin\zeta \cos.\theta$
II Vim secundum $Zr = \sin.\zeta \sin.\theta \sin.\frac{p+s}{a} + \cos.\zeta \cos.\theta$
III Vim secundum $Zo = -\sin.\theta \cos.\frac{p+s}{a}$.

PROBLEMA 5.

12. Dato, vt hactenus, tam cylindri, quam aquae per helicem motu, definire statum compressionis aquae in singulis helicis punctis. Fig. 3.

SOLVTIO.

Praesenti temporis instanti, quo initium helicis est in E, existente arcu $AE = p$, consideremus helicis punctum Z, vt sit $EY = s$, et $YZ = s \operatorname{tang} \zeta$ existente helicis angulo $YEZ = \zeta$, sitque status compressionis aquae in puncto $Z = q$, seu denotet q profunditatem, ad quam aqua quiescens in pari statu compressionis existat, eritque pro hoc momento q functio quaepiam ipsius s, et in puncto proximo z, existente $Yy = ds$, status compressionis erit $= q + dq$. Sit iam amplitudo helicis $= bb$, erit particula aquae in portiuncula Zz contenta $= \frac{bbds}{\operatorname{cof.} \zeta}$; quae ergo in Z propelletur vi motrice $= bbq$, in z vero repelletur vi $= bb(q + dq)$; vnde existit vis motrix repellens, seu secundum zZ vrgens $= bbdq$, quae praebet vim acceleratricem $= \frac{dq \operatorname{cof.} \zeta}{ds}$. Quare ob statum compressionis particula aquae in elemento helicis Zz contenta secundum directionem Zz sollicitabitur vi acceleratrice $= -\frac{dq \operatorname{cof.} \zeta}{ds}$. Praeterea vero ob grauitatem eadem particula, vti vidimus, sollicitatur secundum Zz vi acceleratrice $= \operatorname{cof.} \zeta \operatorname{fin.} \theta \operatorname{fin.} \frac{p+s}{a} - \operatorname{fin.} \zeta \operatorname{cof.} \theta$, vnde coniunctim tam ob grauitatem, quam ob statum compressionis aquae, particula aquae in helicis puncto Z contenta vrgebitur secundum directionem Zz vi acceleratrice, quae erit

$$\operatorname{cof.} \zeta \operatorname{fin.} \theta \operatorname{fin.} \frac{p+s}{a} - \operatorname{fin.} \zeta \operatorname{cof.} \theta - \frac{dq \operatorname{cof.} \zeta}{ds}$$

haec-

haecque eſt vis, qua iſta particula actu vrgetur, ſecundum directionem Zz; ex quo neceſſe eſt, vt ea aequalis ſit illi vi, qua ſupra punctum Z ad motus conſeruationem ſollicitari debere inuenimus, ſecundum eandem directionem Zz. Quae cum ſit inuenta $= \frac{du}{dt\sqrt{u}}$ coſ. $\zeta + \frac{dv}{dt\sqrt{v}}$ habebimus hanc aequationem:

dq coſ. $\zeta = ds$ coſ. ζ ſin. θ ſin. $\frac{p+s}{a} - ds$ ſin. ζ coſ. $\theta - \frac{du}{dt\sqrt{u}} ds$ coſ. $\zeta - \frac{dv}{dt\sqrt{u}} ds$, vbi, quoniam ad praeſens tantum temporis momentum reſpicimus, quantitates a tempore t pendentes, quae ſunt p, u, v, itemque $\frac{du}{dt}$ et $\frac{dv}{dt}$, tanquam conſtantes ſunt ſpectandae, ex quo integratione inſtituta habebimus q coſ. $\zeta = C - a$ coſ. ζ ſin. θ coſ. $\frac{p+s}{c} - s$ ſin. ζ coſ. $\theta - \frac{s\,du\,\text{coſ.}\,\zeta}{dt\sqrt{v}} - \frac{s\,dv}{ds\sqrt{v}}$ vnde ſtatus compreſſionis aquae in ſingulis helicis punctis pro praeſenti temporis momento innoteſcit.

PROBLEMA 6.

13. Si data aquae portio in helice reperiatur, atque cylindrus datam ad horizontem inclinationem tenens motu quocunque in gyrum agatur, inuenire motum quo iſta aquae portio per helicem promouebitur.

SOLVTIO.

Sit baſis cylindri ſemidiameter $CA = CB = a$, et angulus, quem helix EF cum baſi cylindri conſtituit $BEF = \zeta$. Axis autem cylindri PQ cum recta verticali QR conſtituat angulum $PQR = \theta$, quo eodem angulo Tab. II. Fig. 5.

gulo baſis cylindri ad horizontem erit inclinata. In baſi autem ſit A punctum ſummum et B infimum. Praeſenti autem temporis momento ſit initium helicis in E, exiſtente eius a puncto ſummo interuallo ſeu arcu $AE = p$: et cylindrus in plagam AEB gyretur, ita vt puncti E celeritas ſit $= \sqrt{u}$, erit $dp = dt\sqrt{u}$. Occupet nunc portio aquae in helice contenta ſpatium MN, cuius longitudo ſit $MN = f$, ac ductis axi parallelis MS et NT ſit aquae ab initio helicis diſtantia $EM = x$, erit $EN = x + f$, et $ES = x \text{ coſ.}\,\zeta$, atque $ET = (x+f)\text{coſ.}\,\zeta$; celeritas vero, qua haec aquae portio praeſenti momento per helicem promouetur, ſit $= \sqrt{v}$. His poſitis, ſi in portione aquae MN punctum quodpiam medium Z conſideretur, et arcus EY ponatur $= s$, erit ſtatus compreſſionis aquae in Z, qui per altitudinem q exprimatur, vti in problemate praecedente eſt erutus;

$$q \text{ coſ.}\,\zeta = C - a \text{ coſ.}\,\zeta \text{ ſin }\theta \text{ coſ. } \frac{p+s}{a} - s \text{ ſin }\zeta \text{ coſ.}\,\theta - \frac{s\,du\,\text{coſ.}\,\zeta}{dt\sqrt{u}} - \frac{s\,dv}{dt\sqrt{v}}$$

Iam vero conſtat in vtroque termino M et N ſtatum compreſſionis euaneſcere debere; ſiue ergo ponatur $s = x \text{ coſ.}\,\zeta$ ſiue $s = (x+f)\text{ coſ.}\,\zeta$, fieri debet $q = 0$: vnde duplex naſcitur aequatio

$$0 = C - a \text{ coſ.}\,\zeta \text{ ſin.}\,\theta \text{ coſ. } \frac{p + x \text{ coſ.}\,\zeta}{a} - x \text{ ſin.}\,\zeta \text{ coſ.}\,\zeta \text{ coſ.}\,\theta - x \text{ coſ.}\,\zeta \left(\frac{du \text{ coſ.}\,\zeta}{dt\sqrt{u}} + \frac{dv}{dt\sqrt{u}}\right)$$

$$0 = C - a \text{ coſ.}\,\zeta \text{ ſin.}\,\theta \text{ coſ.} \frac{p + (x+f)\text{coſ.}\,\zeta}{a} - (x+f)\text{coſ.}\,\zeta \left(\text{ſin.}\,\zeta \text{ coſ.}\,\theta + \frac{du \text{ coſ}\,\zeta}{dt\sqrt{u}} + \frac{dv}{dt\sqrt{v}}\right)$$

vnde, conſtantem C eliminando, obtinebitur, diuidendo per coſ. ζ, haec aequatio.

o ſin.

$$a\,\text{fin.}\,\theta\,\text{cof.}\,\tfrac{p+x\,\text{cof.}\,\zeta}{a} = a\,\text{fin.}\,\theta\,\text{cof.}\,\tfrac{p+(x+f)\,\text{cof.}\,\zeta}{a} + f\,\text{fin.}\,\zeta\,\text{cof.}\,\theta + \tfrac{f\,du\,\text{cof.}\,\zeta}{dt\,\sqrt{u}} + \tfrac{f\,dv}{dt\sqrt{v}}$$

vnde motus aquae per helicem definiri debet, vti enim eſt $dp = dt\sqrt{u}$, ita erit $dx = dt\sqrt{v}$.
Multiplicetur ergo haec aequatio per $dp + dx\,\text{cof.}\,\zeta = dt\sqrt{u} + dt\,\text{cof.}\,\zeta.\sqrt{v}$, eritque integrando

$$a^2\,\text{fin.}\,\theta\,\text{fin.}\,\tfrac{p+x\,\text{cof.}\,\zeta}{a} = a^2\,\text{fin.}\,\theta\,\text{fin.}\,\tfrac{p+(x+f)\,\text{cof.}\,\zeta}{a} + f(p+x\,\text{cof.}\,\zeta)\,\text{fin.}\,\zeta\,\text{cof.}\,\theta + f\int\left(\tfrac{du\,\text{cof.}\,\zeta}{\sqrt{u}} + \tfrac{dv}{\sqrt{v}}\right)(\sqrt{u} + \text{cof.}\,\zeta.\sqrt{v})$$

COROLL. 1.

14. Si igitur motus gyrationis cylindri fuerit vniformis, ſeu u conſtans, ponatur $u = k$, ob $du = 0$ erit

$$a^2\,\text{fin.}\,\theta\,\text{fin.}\,\tfrac{p+x\,\text{cof.}\,\zeta}{a} = a^2\,\text{fin.}\,\theta\,\text{fin.}\,\tfrac{p+(x+f)\,\text{cof.}\,\zeta}{a} + f(p+x\,\text{cof.}\,\zeta)\,\text{fin.}\,\zeta\,\text{cof.}\,\theta + 2f\sqrt{kv} + fv\,\text{cof.}\,\zeta + \text{Conſt.}$$

Vbi eſt $p = t\sqrt{k}$, ita vt haec aequatio ob $\sqrt{v} = \frac{dx}{dt}$ duas tantum variabiles t et x inuoluat. Conſtans autem ex ſtatu initiali debet definiri.

COROLL. 2.

15. Si portio aquae in tubo MN fuerit infinite parua ſeu $f = 0$, erit $\text{fin.}\,\frac{p+(x+f)\,\text{cof.}\,\zeta}{a} = \text{fin.}\,\frac{p+x\,\text{cof.}\,\zeta}{a} + \frac{f\,\text{cof.}\,\zeta}{a}\,\text{cof.}\,\frac{p+x\,\text{cof.}\,\zeta}{a}$
hoc ergo caſu motus definietur hac aequatione:
$\text{Conſt.} = a\,\text{cof.}\,\zeta\,\text{fin.}\,\theta\,\text{cof.}\,\frac{p+x\,\text{cof.}\,\zeta}{a} + (p+x\,\text{cof.}\,\zeta)\,\text{fin.}\,\zeta\,\text{cof.}\,\theta + 2\sqrt{kv} + v\,\text{cof.}\,\zeta$. Quodſi ergo haec particula initio quieuerit in E, punctumque E fuerit in A, ita vt poſito $x = 0$, ſit $p = 0$ et $v = 0$ erit $a\,\text{cof.}\,\zeta\,\text{fin.}\,\theta\left(1 - \text{cof.}\,\frac{p+x\,\text{cof.}\,\zeta}{a}\right) = (p+x\,\text{cof.}\,\zeta)\,\text{fin.}\,\zeta\,\text{cof.}\,\theta + 2\sqrt{kv} + v\,\text{cof.}\,\zeta$.

COROLL. 3.

16. Si in casu corollarii praecedentis ponatur angulus $\frac{p + x\,\text{cos.}\,\zeta}{a} = \Phi$, vt sit $dt = \frac{a\,d\Phi}{\sqrt{k} + \text{cos.}\,\zeta\sqrt{v}}$, ob $dp = dt\sqrt{k}$ et $dx = dt\sqrt{v}$, relatio inter Φ et v hac exprimetur aequatione:

$$a\,\text{cos.}\,\zeta\,\text{fin.}\,\theta(1 - \text{cof.}\,\Phi) = a\Phi\,\text{fin.}\,\zeta\,\text{cof.}\,\theta + 2\sqrt{kv} + v\,\text{cof.}\,\zeta$$

ex qua fit $\sqrt{k} + \text{cof.}\,\zeta$, $\sqrt{v} = \sqrt{(k - a\Phi\,\text{fin.}\,\zeta\,\text{cof.}\,\zeta\,\text{cof.}\,\theta + a\,\text{cof.}\,\zeta^2\,\text{fin}\,\theta(1 - \text{cof}\,\Phi))}$

ideoque $$dt = \frac{a\,d\Phi}{\sqrt{(k - a\Phi\,\text{fin.}\,\zeta\,\text{cof.}\,\zeta\,\text{cof.}\,\theta + a\,\text{cof.}\,\zeta^2\,\text{fin.}\,\theta(1 - \text{cof.}\,\Phi))}}$$

COROLL. 4.

17. Simili modo si generaliter, posito tamen motu gyratorio constante, seu $u = k$, ponatur $\frac{p + x\,\text{cof.}\,\zeta}{a} = \Phi$ et $\frac{f\,\text{cof.}\,\zeta}{a} = \gamma$, erit quoque $dt = \frac{a\,d\Phi}{\sqrt{k} + \text{co.}\,\zeta.\sqrt{v}}$ et $\frac{a\,\text{cof.}\,\zeta\,\text{fin.}\,\theta}{\gamma}\,\text{fin.}\,\Phi = \frac{a\,\text{cof}\,\zeta\,\text{fin.}\,\theta}{\gamma}\,\text{fin.}(\gamma + \Phi) + a\Phi\,\text{fin.}\,\zeta\,\text{cof.}\,\theta + 2\sqrt{kv} + v\,\text{cof.}\,\zeta \pm C.$
ideoque

$$\sqrt{k} + \text{cof.}\,\zeta\sqrt{v} = \sqrt{(C + \tfrac{a}{\gamma}\,\text{cof.}\,\zeta^2\,\text{fin.}\,\theta\,(\text{fin.}\,\Phi - \text{fin}(\gamma + \Phi)) - a\Phi\,\text{fin.}\,\zeta\,\text{cof.}\,\zeta\,\text{cof.}\,\theta)}$$

vnde fit

$$dt = \frac{a\,d\Phi}{\sqrt{(C + \frac{a}{\gamma}\,\text{cof.}\,\zeta^2\,\text{fin.}\,\theta(\text{fin.}\,\Phi\,\text{fin.}(\gamma + \Phi)) \cdot a\Phi\,\text{fin}\,\zeta.\text{cof.}\,\zeta\,\text{cof.}\,\theta)}}$$

vbi Φ denotat angulum ACS, et γ angulum SCT, qui est constans.

COROLL. 5.

18 Si cylindrus in partem contrariam celeritate $= \sqrt{k}$ circumagatur, pro $\sqrt{k}$ scribi debet $-\sqrt{k}$, arcusque p negatiue erit accipiendus, ita vt sit $\Phi = \frac{x\,\text{cof.}\,\zeta - p}{a}$.

Quare

Quare cum ſit $p > \frac{x \operatorname{cof.} \zeta}{a}$, etiam angulus Φ nega-
tiue accipiatur, habebimus ergo pro hoc motu:
$\Phi = \frac{p - x \operatorname{cof.} \zeta}{a}$; et $dt = \frac{a\, d\Phi}{\sqrt{k} - \operatorname{cof.} \zeta . \sqrt{v}}$ atque $\sqrt{k} - \operatorname{cof.} \zeta, \sqrt{v}$
$= \sqrt{(C - \frac{a}{\gamma} \operatorname{cof.} \zeta^2 \operatorname{fin.} \theta (\operatorname{fin.} \Phi - \operatorname{fin.} (\Phi - \gamma)) + a \Phi \operatorname{fin.} \zeta \operatorname{cof.} \zeta \operatorname{cof.} \theta)}$.

COROLL. 6.

19. Si hoc caſu initio $t = 0$, quo erat $p = 0$, et $\sqrt{v} = 0$, fuerit $x = EM = g$; ideoque $\Phi = -\frac{g \operatorname{cof.} \zeta}{a}$; ponamus hunc angulum initialem $ECS = \varepsilon$, vt fuerit initio $\Phi = -\varepsilon$, erit $\sqrt{k} - \operatorname{cof.} \zeta . \sqrt{v} = \sqrt{(k + \frac{a}{\gamma} \operatorname{cof.} \zeta^2 \operatorname{fin.} \theta}$ $(\operatorname{fin.} (\varepsilon + \gamma) - \operatorname{fin.} \varepsilon - \operatorname{fin.} \Phi + \operatorname{fin.} (\Phi - \gamma)) + a(\varepsilon + \Phi) \operatorname{fin.} \zeta \operatorname{cof.} \zeta \operatorname{cof.} \theta)$.

PROBLEMA 7.

20. Si, dum cylindrus data celeritate vniformiter in plagam BEA gyratur, helici in C particula aquae ſeu globulus inſeratur, qui deinde a motu cylindri abripiatur, determinare motum globuli per helicem. Tab. II. Fig. 5.

SOLVTIO.

Sit $\sqrt{k}$ celeritas, qua punctum cylindri E in gyrum agitur, in ſenſum EA; fueritque eo momento, quo globulus in orificium helicis E immittitur, angulus $ACE = \alpha$, et $t = 0$. Fieri autem nequit, vt celeritas globuli initialis ſit $= 0$; ſi enim celeritas eius reſpectu tubi ſecundum EM ponatur $= \sqrt{v}$, eius celeritas vera erit $= \sqrt{(k + v - 2 \operatorname{cof.} \zeta . \sqrt{kv})}$, quae non poteſt euaneſcere. Ponamus ergo hanc celeritatem initio fuiſſe minimam, ac reperimus $\sqrt{v} = \operatorname{cof.} \zeta . \sqrt{k}$, ita vt celeritas vera fuerit $= \operatorname{fin.} \zeta . \sqrt{k}$, cuius directio ad

EM erat normalis. Iam elapſo tempore t, ſit vt ſupra AE $= p$; globulus vero reperiatur in M exiſtente EM $= x$, cuius celeritas relatiue in tubo ſecundum MN ſit $= \sqrt{v}$, erit $dp = -dt\sqrt{k}$ et $dx = dt\sqrt{v}$: et per §. 15. motus definietur hac aequatione, ſumta ſcilicet celeritate $\sqrt{k}$ negatiua.

$$\text{Conſt} = a\,\text{cof.}\,\zeta\,\text{ſin.}\,\theta\,\text{cof.}\,\frac{p + x\,\text{cof.}\,\zeta}{a} + (p + x\,\text{cof.}\,\zeta)\,\text{ſin.}\,\zeta\,\text{cof}\,\theta - 2\sqrt{kv} + v\,\text{cof.}\,\zeta.$$

Conſtans autem ita eſt definienda, vt poſito $t = 0$, ſeu $\frac{p}{a} = \alpha$, fiat $x = 0$ et $\sqrt{v} = \text{cof.}\,\zeta.\sqrt{k}$, ſicque erit

$$\text{Conſt} = a\,\text{cof.}\,\zeta\,\text{ſin.}\,\theta\,\text{cof.}\,\alpha + \alpha a\,\text{ſin.}\,\zeta\,\text{cof.}\,\theta - 2k\,\text{cof.}\,\zeta + k\,\text{cof.}\,\zeta^2$$

Ponatur angulus ACS $= \Phi$, erit $\Phi = \frac{p + x\,\text{cof.}\,\zeta}{a}$ et $d\Phi = -\frac{dt\sqrt{k} + dt\,\text{cof.}\,\zeta.\sqrt{v}}{a}$. Confecerit autem cylindrus motu angulari tempore t angulum $= \omega$, in plagam BEA, erit $d\omega = \frac{dt\sqrt{k}}{a}$, et $\omega = \frac{t\sqrt{k}}{a}$, quem angulum loco temporis, tamquam eius menſuram in calculum introducamus, erit $\frac{p}{a} = \alpha - \omega$, $\Phi = \alpha - \omega + \frac{x\,\text{cof.}\,\zeta}{a}$; et ob $dx = dt\sqrt{v} = \frac{a\,d\omega\sqrt{v}}{\sqrt{k}}$ habebimus $d\Phi = -d\omega + \frac{d\omega\,\text{cof.}\,\zeta.\sqrt{v}}{\sqrt{k}}$ ſeu $d\omega = \frac{d\Phi\sqrt{k}}{-\sqrt{k} + \text{cof.}\,\zeta.\sqrt{v}}$. Noſtra autem aequatio erit

$$a\,\text{cof.}\,\zeta\,\text{ſin.}\,\theta\,\text{cof.}\,\alpha + \alpha a\,\text{ſin.}\,\zeta\,\text{cof.}\,\theta - 2k\,\text{cof.}\,\zeta + k\,\text{cof.}\,\zeta^2 = a\,\text{cof.}\,\zeta\,\text{ſin.}\,\theta\,\text{cof.}\,\Phi + a\Phi\,\text{ſin.}\,\zeta\,\text{cof.}\,\theta - 2\sqrt{kv} + v\,\text{cof.}\,\zeta$$

ex qua obtinemus:

$$\text{cof.}\,\zeta\sqrt{v} - \sqrt{k} = -\sqrt{(k\,\text{ſin.}\,\zeta^4 + a\,\text{cof.}\,\zeta^2\,\text{ſin.}\,\theta(\text{cof.}\,\alpha - \text{cof.}\,\Phi) + a(\alpha - \Phi)\,\text{ſin.}\,\zeta\,\text{cof.}\,\zeta\,\text{cof}\,\theta)}$$

vnde ad datum valorem ipſius Φ elicimus valorem ipſius $\sqrt{v}$, quo inuento erit

$$d\omega = \frac{-d\Phi\sqrt{k}}{\sqrt{(k\,\text{ſin.}\,\zeta^4 + a\,\text{cof.}\,\zeta^2\,\text{ſin.}\,\theta\,(\text{cof.}\,\alpha - \text{cof.}\,\Phi) + a\,(\alpha - \Phi)\,\text{ſin.}\,\zeta\,\text{cof.}\,\zeta\,\text{cof.}\,\theta}}$$

cuius integrale ita debet capi, vt poſito $\omega = 0$, fiat $\Phi = \alpha$.

$\Phi = \alpha$. Ex hac ergo aequatione integrali viciſſim ad datum tempus angulo ω expreſſum, reperitur angulus Φ, ex eoque porro locus globuli in helice, ſeu portio $EM = x = \frac{a(\Phi - \alpha + \omega)}{\text{coſ.}\,\zeta}$, eiusque inſuper celeritas relatiua in helice $\surd v$ ſcilicet

$$\surd v = \frac{\surd k}{\text{coſ.}\,\zeta} - \surd(k\,\text{ſin.}\,\zeta^2\,\text{tang.}\,\zeta^2 + a\,\text{ſin.}\,\theta(\text{coſ.}\,\alpha - \text{coſ.}\,\Phi) + a\,(\alpha - \Phi)\,\text{tang.}\,\zeta\,\text{coſ.}\,\theta)$$

COROLL 1.

21. Expreſſio coſ. $\zeta\,\surd v - \surd k$ deſignat celeritatem veram puncti S in baſi, quod globulo in M reſpondet. Cum enim globulus velocitate $\surd v$ in helice ſecundum MN progredi ponatur, erit eius celeritas angularis circa axem $= \text{coſ}\,\zeta . \surd v$, reſpectu helicis; quia autem helix ipſa in plagam oppoſitam conuertitur celeritate $= \surd k$, erit vera globuli celeritas rotatoria, ſeu motus quo punctum S a ſummitate A recedit $= \text{coſ.}\,\zeta\,\surd v - \surd k$.

COROLL 2.

22. Ipſo autem motus initio, quo $\surd v = \text{coſ.}\,\zeta \surd k$; haec celeritas erat negatiua, ſcilicet $= (\text{coſ.}\,\zeta^2 - 1)\surd k = -\text{ſin.}\,\zeta^2\,\surd k$, ſtatim ergo ab initio etiam nunc erit negatiua: ſeu angulus $ACS = \Phi$ diminuetur, quae eſt ratio, cur calculus pro coſ. $\zeta\,\surd v - \surd k$ valorem praebuerit negatiuum

$$\text{coſ.}\,\zeta\surd v - \surd k = -\surd(k\,\text{ſin}\,\zeta^4 + a\,\text{coſ.}\,\zeta^2\,\text{ſin.}\,\theta\,(\text{coſ.}\,\alpha - \text{coſ.}\,\Phi) + a\,(\alpha - \Phi)\,\text{ſin.}\,\zeta\,\text{coſ.}\,\zeta\,\text{coſ.}\,\theta)$$

Hic ergo valor in affirmatiuum abire, ſeu angulus $ACS = \Phi$ augmenta capere nequit, niſi poſtquam fuerit quantitas illa radicalis $= 0$. Poſtquam autem hoc eue-

euenerit, tum ſigni illius radicalis valor affirmatiue erit accipiendus.

COROLL 3.

23. Quoniam autem ab initio angulus Φ decreſcit tam diu, donec valor quantitatis illius radicalis euaneſcit, eousque Φ vltra α diminuetur, ſeu erit $\Phi < \alpha$: Ponatur ergo $\Phi = \alpha - \psi$, vt ſit

$$\operatorname{coſ.}\zeta\sqrt{v}-\sqrt{k} = -\sqrt{(k\operatorname{ſin.}\zeta^4 + a\operatorname{coſ.}\zeta^2\theta(\operatorname{coſ.}\alpha - \operatorname{coſ.}(\alpha-\psi)) + a\,\psi\operatorname{ſin.}\zeta\operatorname{coſ.}\zeta\operatorname{coſ.}\theta)}$$

ſicque quamdiu augendo valorem ipſius ψ, iſta quantitas radicalis realem retinet valorem, tamdiu globulus a motu cylindri in plagam BεA abripietur; neque prius in plagam contrariam motum ſuum vertet, quam vbi ψ eousque increuerit, vt ſit

$$k\operatorname{ſin.}\zeta^4 + a\operatorname{coſ.}\zeta^2\operatorname{ſin.}\theta(\operatorname{coſ.}\alpha - \operatorname{coſ.}(\alpha-\psi)) + a\psi\operatorname{ſin.}\zeta\operatorname{coſ.}\zeta\operatorname{coſ.}\theta = 0$$

COROLL 4.

24. Quia autem augendo ψ extremus terminus continuo creſcit, medius vero qui eſt negatiuus $-a\operatorname{coſ.}\zeta^2$ $\operatorname{ſin.}\theta(\operatorname{coſ.}(\alpha-\psi)-\operatorname{coſ.}\alpha)$ tamdiu tantum creſcit, quoad fiat $\psi = \alpha$, ſeu $\Phi = 0$, manifeſtum eſt, niſi formula illa in nihilum abeat, antequam fiat $\psi = \alpha$, eam nunquam eſſe euanituram; globulumque continuo celerius ſecundum motum cylindri gyratorium abreptum iri. Hoc ergo caſu punctum S continuo celerius in plagam BEA conuertetur.

COROLL 5.

25. Si ergo quantitas iſta radicalis ponatur $= V$, vt ſit $\operatorname{coſ.}\zeta\sqrt{v} - \sqrt{k} = -V$, ſeu $\sqrt{v} = \frac{\sqrt{k}-V}{\operatorname{coſ.}\zeta}$, ob valorem ipſius V hoc caſu continuo creſcentem, celeritas

ritas globuli progreſſiua in helice ſecundum directionem eius EMN tandem euaneſcet, poſteaque adeo fiet negatiua, quod vbi acciderit, globulus per helicem reuertetur, ac per orificium E iterum erumpet; ſiquidem cylindrus fuerit longus, vt globulus in ſuperiori helicis termino K non erumpat, antequam reuertatur.

SCHOLION.

26. Cum poſito $\Phi = \alpha - \psi$, et

$$V = \sqrt{}(k \text{ ſin. } \zeta^4 - a \text{ coſ. } \zeta^2 \text{ ſin. } \theta (\text{coſ. } (\alpha - \psi) - \text{coſ. } \alpha) + a \psi \text{ ſin. } \zeta \text{ coſ. } \zeta \text{ coſ. } \theta)$$

quantitas V tamdiu negatiue ſit accipienda, ſeu habeatur coſ. $\zeta \sqrt{}v - \sqrt{}k = -V$, quamdiu augendo angulum ψ quantitas V realem obtinet valorem; ſtatim autem atque haec quantitas V euaſerit $= o$, inde angulus ψ iterum decreſcat, ſignumque contrarium ipſi V tribui debeat, vt ſit coſ. $\zeta \sqrt{}v - \sqrt{}k = + V$; duos habebimus caſus principales euoluendos, quorum altero vspiam augendo ψ ab initio ſit $V = o$, altero vero hoc nunquam euenit. Statim autem ab initio fiet $V = o$, ſi ſit vel $k = o$ vel $\zeta = o$: tum aliquo tempore poſt initium hoc euenire ponamus, denique vero nunquam; vnde ſequentes caſus diligentius euoluamus.

CASVS I.

27. Ponamus ergo primo motum cylindri rotatorium penitus euaneſcere, ſeu eſſe $k = o$. Cum igitur in ipſo initio fiat $V = o$, ſtatim ab initio ipſi V contrarium ſigno tribui debet, vt ſit coſ. $\zeta \sqrt{}v = + V$, ſeu $\sqrt{}v = \frac{V}{\text{coſ. } \zeta}$, atque angulus ψ inde iam erit negatiuus, ſeu angulus Φ continuo creſcet, vt ſit

$V = \sqrt{}$

$V = \sqrt{}(a \operatorname{cof.} \zeta^2 \operatorname{fin.} \theta (\operatorname{cof.} \alpha - \operatorname{cof.} \Phi) + a(\alpha - \Phi) \operatorname{fin.} \zeta \operatorname{cof.} \zeta \operatorname{cof.} \theta)$ et $dt = \frac{a\, d\omega}{\sqrt{k}} = \frac{a\, d\Phi}{V}$, atque $EM = x = \frac{a(\Phi - \alpha)}{\operatorname{cof.} \zeta}$.
Quia ergo initio erat $\Phi = \alpha$, et $\sqrt{v} = 0$, ponamus tempore elapſo t, eſſe $\Phi = \alpha + \psi$, vt ſit

$V = \sqrt{}(a \operatorname{cof.} \zeta^2 \operatorname{fin.} \theta (\operatorname{cof.} \alpha - \operatorname{cof.} (\alpha + \psi)) - a \psi \operatorname{fin.} \zeta \operatorname{cof.} \zeta \operatorname{cof.} \theta)$

et $x = \frac{a \psi}{\operatorname{cof.} \zeta}$ atque $dt = \frac{a\, d\psi}{V}$.

Hic iam perſpicuum eſt, fieri omnino non poſſe, vt angulus ψ continuo creſcat, niſi ſit $\operatorname{fin} \zeta \operatorname{cof.} \zeta \operatorname{cof.} \theta = 0$, quem caſum ſeorſim euoluere conueniet. Quodſi vero ψ creſcere ceſſet, quo eueniet, vbi $V = 0$, ibi globulus ad ſtatum quietis redigetur, ac in helice regredi incipiet, a quo ergo momento valor ipſius V negatiue capi debebit, angulusque ψ iterum decreſcet, donec fiat $\psi = 0$. et tum corpus rurſus in E, ſicuti initio, haerebit; vnde eundem motum denuo inchoabit.

At euenire poteſt, vt haec globuli reuerſio in ipſum quaſi initium motus incidat, atque angulus ψ ne mininum quidem augeri queat, quin angulus Φ maneat nullus, vel adeo fiat negatiuus.

Prior caſus locum habebit, ſi poſito ψ infinite paruo, valor ipſius V nihilominus maneat $= 0$; id quod vſu veniet $\operatorname{fin.} a \operatorname{cof.} \zeta^2 \operatorname{fin.} \theta \operatorname{fin.} \alpha = a \operatorname{fin.} \zeta \operatorname{cof.} \zeta \operatorname{cof.} \theta$ ſeu $\operatorname{fin.} \alpha = \frac{\operatorname{tang} \zeta}{\operatorname{tang} \theta}$ ac tum corpus perpetuo in puncto E quieſcet; hic enim directio helicis erit horizontalis.

Poſterior caſus autem locum habebit, ſi $\operatorname{fin.} \alpha < \frac{\operatorname{tang} \zeta}{\operatorname{tang} \theta}$ quo globulus ne in helicem quidem ingredietur, ſed ſtatim inde delabetur; vel ſi cylindrus deorſum eſſet continuatus, mutata directione globulus per helicis partem interiorem deſcenſurus eſſet; ita vt angulus ψ tum fieret negatiuus perinde ac valor ipſius x, et V.

Hi

Hi autem casus locum non inueniunt, nisi sit $\theta > \zeta$, seu inclinatio basis cylindri ad horizontem maior, quam angulus BEF, quem helix cum basi cylindri constituit. Hunc autem motum in helice quiescente fusius non persequor, cum nihil habeat difficultatis.

CASVS II.

28. Ponamus motum gyratorium cylindri ita esse comparatum, vt motus gyratorius globuli circa axem, qui angulo ψ indicatur, et initio cum motu gyratorio cylindri in eandem plagam fuerat directus, post aliquod tempus in plagam oppositam reflectatur.

Angulus ergo ψ eo vsque augeri poterit, vt fiat
$k\,\text{fin}.\zeta^4 = a\,\text{cof}.\zeta^2\,\text{fin}.\theta(\text{cof}.(\alpha-\psi)-\text{cof}.\alpha) - a\psi\,\text{fin}.\zeta\,\text{cof}.\zeta\,\text{cof}.\theta$
seu $V=o$; hoc autem fieri nequit, nisi sit

$$\text{cof}.\,(\alpha-\psi) - \text{cof}.\,\alpha > \frac{\text{tang}.\,\zeta}{\text{tang}.\,\theta}.\,\psi.$$

cum igitur ab initio fuisset $\psi = o$, necesse est, vt posito ψ euanescente, sit $\text{fin}.\,\alpha > \frac{\text{tang}\,\zeta}{\text{tang}.\,\theta}$. Deinde valor ipsius $\text{cof}.\,(\alpha-\psi)-\text{cof}.\,\alpha - \frac{\text{tang}\,\zeta}{\text{tang}.\,\theta}\,\psi$ erit maximus, si

$$\text{fin}.\,(\alpha-\psi) = \frac{\text{tang}.\,\zeta}{\text{tang}.\,\theta}.$$

Concipiamus hoc pro ψ valore substituto fieri

$$\text{cof}.\,(\alpha-\psi) - \text{cof}.\,\alpha - \frac{\text{tang}.\,\zeta}{\text{tang}.\,\theta} : \psi = M$$

atque vt valor ipsius V augendo ψ tandem euanescere queat, necesse est, vt sit $k\,\text{fin}.\,\zeta^4 < a\,M\,\text{cof}.\,\zeta^2\,\text{fin}.\,\theta$. Quare, vt hic casus locum habere possit, sequentes tres conditiones requiruntur.

I. vt sit $\text{tang}.\,\theta > \text{tang}.\,\zeta$ seu $\theta > \zeta$; ita vt fractio $\frac{\text{tang}.\,\zeta}{\text{tang}.\,\theta}$ vnitatem non excedat.

II. vt sit $\text{fin}.\,\alpha > \frac{\text{tang}.\,\zeta}{\text{tang}.\,\theta}$: ac denique

III. vt sit $k < a\,M\,\frac{\text{cof}.\,\zeta^2\,\text{fin}.\,\theta}{\text{fin}.\,\zeta^4}$.

 Quo-

Quoties ergo hae tres conditiones locum inueniunt, globulus in helice in ſenſum BEA circa axem cylindri circumferetur, donec deſcripſerit angulum ψ, vt fiat:

$$V = \sqrt{}(k\,\text{ſin.}\,\zeta^4 - a\,\text{coſ.}\,\zeta^2\,\text{ſin.}\,\theta(\text{coſ.}(\alpha - \psi) - \text{coſ.}\,\alpha) + a\psi\,\text{ſin.}\,\zeta\,\text{coſ.}\,\zeta\,\text{coſ.}\,\theta) = 0;$$

tumque erit $\text{coſ.}\,\zeta\,\sqrt{}v - \sqrt{}k = 0$, ſeu globuli celeritas relatiua per helicem $\sqrt{}v = \frac{\sqrt{}k}{\text{coſ.}\,\zeta}$; cum antequam ad hunc locum perueniat, ſit $\sqrt{}v = \frac{\sqrt{}k - V}{\text{coſ.}\,\zeta}$; exiſtente $x = \frac{a\omega - a\psi}{\text{coſ.}\,\zeta}$ et $d\omega = \frac{dt\sqrt{}k}{a} - \frac{d\psi\sqrt{}k}{V}$. Poſtquam autem hunc locum attigerit, angulus ψ continuo decreſcet, ſeu motus angularis globuli fiet contrarius motui cylindri, et tribuendo ipſi V ſignum contrarium, habebitur $\sqrt{}v = \frac{\sqrt{}k + V}{\text{coſ.}\,\zeta}$, et quando fiet $\psi = 0$, erit $V = \text{ſin.}\,\zeta^2\sqrt{}k$; hincque $\sqrt{}v = \frac{(1 + \text{ſin.}\,\zeta^2)}{\text{coſ.}\,\zeta}\sqrt{}k$ et $x = \frac{a\omega}{\text{coſ.}\,\zeta}$. Inde fiet ψ negatiuum, et diſtantia x adhuc magis creſcet, dum poſito ψ negatiuo fiet $x = \frac{a\omega + a\psi}{\text{coſ.}\,\zeta}$, donec fiat

$$V = \sqrt{}(k\,\text{ſin.}\,\zeta^4 + a\,\text{coſ.}\,\zeta^2\,\text{ſin}\,\theta(\text{coſ.}\,\alpha - \text{coſ}(\alpha + \psi)) - a\psi\,\text{ſin.}\,\zeta\,\text{coſ.}\,\zeta\,\text{coſ.}\,\theta) = 0$$

et eo vsque erit $\sqrt{}v = \frac{\sqrt{}k + V}{\text{coſ.}\,\zeta}$: vbi autem fuerit $V = 0$, euadet $\sqrt{}v = \frac{\sqrt{}k}{\text{coſ.}\,\zeta}$; qui ergo valor ante hoc tempus maximus fuit, vbi erat $\text{ſin.}(\alpha + \psi) = \frac{\text{tang.}\,\zeta}{\text{tang.}\,\theta}$. Poſtquam autem fuerit $V = 0$, angulus ψ iterum decreſcet, indeque etiam diſtantia x minora capiet incrementa, eritque $\sqrt{}v = \frac{\sqrt{}k - V}{\text{coſ.}\,\zeta}$, donec euadat $\psi = 0$; tumque erit $V = \text{ſin.}\,\zeta^2\sqrt{}k$ et $\sqrt{}v = \text{coſ.}\,\zeta\sqrt{}k$, atque $x = \frac{a\omega}{\text{coſ.}\,\zeta}$. Hoc ergo tempore celeritas $\sqrt{}v$ eadem erit, quae erat initio, indeque motus ſimili modo propagabitur.

Motus

Motus ergo per helicem continuo erit progressiuus, si perpetuo fuerit $V < \sqrt{k}$: sin autem inter eas motus partes, vbi $\sqrt{v} = \frac{\sqrt{k} - V}{\cos. \zeta}$, eueniat, vt fiat $V > \sqrt{k}$, tum globulus ibi per helicem regredietur, donec $\sqrt{v}$ iterum fiat affirmatiuum. Valores autem affirmatiui praeualebunt; vidimus enim post primam periodum, qua celeritas ad initialem redit, globulum spatium absoluisse in helice $x = \frac{a\omega}{\cos. \zeta}$, et post n huiusmodi periodos promouebitur per spatium helicis $x = \frac{n a \omega}{\cos. \zeta}$, sicque continuo altius eleuabitur, donec tandem per superius orificium K eiiciatur.

CASVS III.

29. Ponamus motum ita esse comparatum, vt postquam ab initio angulus $\psi = \alpha - \phi$ increscere coepit, nunquam euadat

$$V = \sqrt{(k \sin. \zeta^4 - a \cos. \zeta^2 \sin. \theta (\cos. (\alpha - \psi) - \cos. \alpha) + a \psi \sin. \zeta \cos. \zeta \cos. \theta)} = 0$$

vnde hic angulus ψ continuo magis augebitur, valorque ipsius V increscet. Tum autem prodibit $\sqrt{v} = \frac{\sqrt{k} - V}{\cos. \zeta}$, ex quo sequitur, celeritatem $\sqrt{v}$ tandem euanescere, globulumque inde ad inferiorem cylindri partem reuerti, donec in E iterum elabatur. Hoc etiam intelligitur ex formula $x = \frac{a(\omega - \psi)}{\cos. \zeta}$; distantia enim x diminuetur, si fuerit $d\omega < d\psi$ seu, $\frac{d\psi \sqrt{k}}{V} < d\psi$, quod vtique euenit, quando $\sqrt{k} < V$ seu $\sqrt{v}$ negatiuum.

Hic ergo casus, quo globulum non vltra datum terminum in helice promouere licet, in sequentibus casibus locum habet:

1° Si tang. $\theta < \text{tang}\, \zeta$ seu angulus PQR < BEF, quomodocunque reliquae quantitates se habeant.

2° Si fuerit sin. $\alpha < \frac{\text{tang.}\, \zeta}{\text{tang.}\, \theta}$, ita vt etiamsi sit $\zeta < \theta$ tamen hoc casu globulus reuertatur in helice.

3° Etiamsi sit $\zeta < \theta$ et sin. $\alpha > \frac{\text{tang.}\, \zeta}{\text{tang.}\, \theta}$, tamen casus tertius locum inuenit, si fuerit $k > \frac{a \cos.\, \zeta^2 \sin.\, \theta}{\sin.\, \zeta^4} \cdot \mathrm{M}$ denotante M maximum valorem, quem expressio $\cos.\, (\alpha - \psi) - \cos.\, \alpha - \frac{\text{tang.}\, \zeta}{\text{tang.}\, \theta} \cdot \psi$ induere valet.

Hinc ergo patet, gyrationis motum nimis celerem non esse aptum ad globulum ad datam quamuis altitudinem eleuandum, cum motus tardior hunc effectum praestare valeat. Fieri ergo potest, vt ob gyrationem nimium velocem effectu frustremur, quem tamen tardiore motu consequi possemus.

EXEMPLUM.

30. Sit $\frac{\text{tang.}\, \zeta}{\text{tang.}\, \theta} = \frac{1}{2}$ et angulus initialis ACE rectus, seu $\alpha = 90°$, atque $\psi = 90° - \Phi$, sicque ψ denotabit angulum, quo globulus circa axem versus punctum summum A ab E est translatum tempore t, quo cylindrus per angulum $= \omega$ est conuersus, ita vt sit $d\omega = \frac{dt \sqrt{k}}{a}$. Habebimus ergo $\mathrm{V} = \sqrt{} (k \sin.\, \zeta^4 - a \cos.\, \zeta^2 \sin.\, \theta\, (\sin.\, \psi - \frac{1}{2} \psi))$, et $d\omega = \frac{d\psi \sqrt{k}}{\mathrm{V}}$ atque $\sqrt{} v = \frac{\sqrt{k} - \mathrm{V}}{\cos.\, \zeta}$; nec non $x = \frac{a(\omega - \psi)}{\cos.\, \zeta}$. Quamdiu ergo motus gyratorius globuli in sensum BEA dirigitur, valor ipsius V in his formulis affirmatiue accipi debet, contra vero negatiue.

Ab initio ergo crescente ψ, decrescit valor ipsius V ob sin. $\psi > \frac{1}{2}\psi$: quamdiu manet $k \sin.\, \zeta^4 > a \cos.\, \zeta^2$ sin.

$\text{fin.}\,\theta(\text{fin.}\,\psi - \frac{1}{2}\psi)$. Cum igitur ipſius $\text{fin.}\,\psi - \frac{1}{2}\psi$ valor maximus ſit, $\text{fin.}\,\psi = 60^\circ = \frac{1}{3}\pi$, denotante π angulum duobus rectis aequalem, fiatque hic valor maximus $= \frac{1}{2}\sqrt{3} - \frac{1}{6}\pi = 0{,}3424267$. Quod ſi ergo fuerit $k\,\text{fin.}\,\zeta^4 < 0{,}3424267\; a\,\text{coſ.}\,\zeta^2\,\text{fin.}\,\theta$, caſus ſecundus locum habebit, caſus vero tertius ſi $k\,\text{fin.}\,\zeta^4 > 0{,}3424267\; a\,\text{coſ.}\,\zeta^2\,\text{fin.}\,\theta$. Illo ſcilicet globulus motu angulari tandem reuertetur, hoc vero nunquam. Sit breuitatis ergo $\frac{\text{coſ.}\,\zeta^2\,\text{ſin.}\,\theta}{\text{ſin.}\,\zeta^4} = n$, vt ſit

$$V = \text{fin.}\,\zeta^2 \sqrt{(k - na(\text{fin.}\,\psi - \tfrac{1}{2}\psi))}$$

I. Ac ponamus primo eſſe $k > 0{,}3424267\,na$; atque angulus ψ continuo creſcet, valor autem ipſius V initio decreſcet, donec fiat $\psi = 60^\circ$, vbi valor ipſius V erit minimus, ſcilicet $= \text{fin.}\,\zeta^2\sqrt{(k - 0{,}3424267\,na)}$, ideoqne celeritas globuli progreſſiua per helicem maxima. Inde vero valor ipſius V iterum augebitur, tandemque quando $\text{fin.}\,\psi = \frac{1}{2}\psi$, quod euenit ſi $\psi = 108^\circ, 36^{I}\, 13^{II}, 56^{III}, 22^{IV}$ fiet $V = \text{fin.}\,\zeta^2.\sqrt{k}$ et $\sqrt{v} = \text{coſ.}\,\zeta.\sqrt{k}$, quae celeritati initiali eſt aequalis. Poſtea vero creſcente vlterius angulo ψ, valor ipſius V magis augebitur, fietque tandem $V = \sqrt{k}$, ſeu $k(1 - \text{fin.}\,\zeta^4) = a\,\text{coſ.}\,\zeta^2\,\text{fin.}\,\theta$ $(\frac{1}{2}\psi - \text{fin.}\,\psi)$ ſeu $\frac{1}{2}\psi - \text{fin.}\,\psi = \frac{k(1 + \text{ſin.}\,\zeta^2)}{a\,\text{ſin.}\,\theta}$; hicque celeritas globuli in helice euaneſcet, ex quo ω reuerti incipiet, et quidem motu accelerato, quoniam, creſcente ψ vltra hunc terminum, quantitas V eo maiora capit augmenta. Definito autem ψ ex aequatione $\frac{1}{2}\psi - \text{fin.}\,\psi = \frac{k(1 + \text{ſin.}\,\zeta^2)}{a\,\text{ſin.}\,\theta}$, quantitas $x = \frac{a(\omega - \psi)}{\text{coſ.}\,\zeta}$ dabit ſpatium in helice, ad quod globulus penetrauerit, et vnde deinceps reuertitur. Tempus autem, quo huc vsque pertingit, seu angulus ω, a cylindro interea motu gyratorio confectus, definitur hac aequatione:

$$\omega = \int \frac{d\psi \sqrt{k}}{\text{sin.}\,\zeta^2 \sqrt{(k - na\,\text{sin.}\,\psi + \tfrac{1}{2} na \psi)}}$$

quo inuento simul vera via x in helice percursa innotescit.

II. Ponamus esse $k < 0{,}3424267\, na$, angulusque ψ eo vsque crescet, donec fiat $k = na(\text{sin.}\,\psi - \tfrac{1}{2}\psi)$, quod euenit antequam euadet $\psi = 60^\circ$; tumque erit $\sqrt{v} = \frac{\sqrt{k}}{\text{cos}.\,\zeta}$ ob $V = o$, hactenus ergo celeritas $\sqrt{v}$ augendo increuit; hicque constituamus primam partem motus globuli per helicem.

2^do^. Ab hoc autem momento angulus ψ iterum diminuetur, et valor $V = \text{sin.}\,\zeta^2 \sqrt{(k - na(\text{sin.}\,\psi - \tfrac{1}{2}\psi))}$ negatiue capi debet, vt sit $\sqrt{v} = \frac{\sqrt{k} + V}{\text{cos}.\,\zeta}$, sicque labente tempore valor ipsius V iterum increscet, donec euadente $\psi = o$, fiat $V = \text{sin.}\,\zeta^2 \sqrt{k}$ et $\sqrt{v} = \frac{(1 + \text{sin}.\,\zeta^2)\sqrt{k}}{\text{cos}.\,\zeta}$: hicque secundam motus partem terminemus, in cuius fine $\psi = o$, et celeritas globuli $\sqrt{v}$, maior existit, quam adhuc fuit.

3°. Nunc igitur angulus ψ negatiuus esse incipit; posito ergo $-\psi$ loco ψ, habebimus $V = \text{sin.}\,\zeta^2 \sqrt{(k + na(\text{sin.}\,\psi - \tfrac{1}{2}\psi))}$, manente $\sqrt{v} = \frac{\sqrt{k} + V}{\text{cos}.\,\zeta}$: et quia $\text{sin.}\,\psi - \tfrac{1}{2}\psi$ crescit, quamdiu ψ est $< 60^\circ$, ad hunc vsque terminum $\psi = 60^\circ$, valor ipsius V, hincque celeritas $\sqrt{v}$ augebitur; et facto $\psi = 60^\circ$, celeritas globuli in helice progressiua erit maxima, scilicet

$$\sqrt{v} = \frac{\sqrt{k} + \text{sin.}\,\zeta^2 \sqrt{(k + 0{,}3424267\, na)}}{\text{cos}.\,\zeta},$$

4^to^.

4to. Deinde vlterius crescente hoc angulo ψ, qui nunc est $= \Phi - \alpha$, valor ipsius V iterum decrescet, et quando fit $\psi = 108^{\circ}, 36^{\mathrm{I}}, 13^{\mathrm{II}}, 56^{\mathrm{III}}, 22^{\mathrm{IV}}$, erit $V = \sin.\zeta^2 \sqrt{k}$ et $\sqrt{v} = \frac{(1 + \sin.\zeta^2)\sqrt{k}}{\cos.\zeta}$.

5to. Angulus autem ψ vltra hunc terminum crescere perget, et quia tum $\frac{1}{2}\psi > \sin \zeta$, erit $V = \sin.\zeta^2 \sqrt{(k - na(\frac{1}{2}\psi - \sin.\psi))}$, et $\sqrt{v} = \frac{\sqrt{k} + V}{\cos.\zeta}$. Valor ergo ipsius V continuo fiet minor, indeque etiam celeritas $\sqrt{v}$, donec fiat $\frac{1}{2}\psi - \sin.\psi = \frac{k}{na}$, quo casu erit $\sqrt{v} = \frac{\sqrt{k}}{\cos.\zeta}$.

6to. Tum autem hic angulus ψ, qui maior est quam $108^{\circ}, 36^{\mathrm{I}}$, iterum decrescet, fietque $\sqrt{v} = \frac{\sqrt{k} - V}{\cos.\zeta}$ existente $V = \sin.\zeta^2 \sqrt{(k - na(\frac{1}{2}\psi - \sin.\psi))}$; sicque celeritas $\sqrt{v}$ decrescet, et quando fit $\psi = 108^{\circ}\ 36^{\mathrm{I}}$, prodibit $V = \sin.\zeta^2 \sqrt{k}$ et $\sqrt{v} = \cos.\zeta \sqrt{k}$, quae aequalis est celeritati initiali.

7mo. Porro angulus ψ infra hunc terminum decrescet, et ob $\sin.\psi > \frac{1}{2}\psi$, erit $V = \sin.\zeta^2 \sqrt{(k + na(\sin.\psi - \frac{1}{2}\psi))}$ et $\sqrt{v} = \frac{\sqrt{k} - V}{\cos.\zeta}$: et quando fit $\psi = 60^{\circ}$, quo casu valor ipsius V erit maximus $= \sin.\zeta^2 \sqrt{(k + 0,3424267\, na)}$, et celeritas globuli minima $\sqrt{v} = \frac{\sqrt{k} - \sin.\zeta^2 \sqrt{(k + 0,3424267\, na)}}{\cos.\zeta}$. Nisi ergo sit $\sqrt{k} > \sin.\zeta^2 \sqrt{(k + 0,3424267\, na)}$ seu $k > \frac{0,3424267\, a \sin.\theta^2}{1 + \sin.\zeta^2}$, cum sit $k < \frac{0,3424267\, a \cos.\zeta^2 \sin.\theta}{\sin.\zeta^4}$, globulus antequam ad hunc terminum peruenit, regredietur in helice, propterea quod eius celeritas $\sqrt{v}$ fit negatiua. Reuertitur ergo globulus, si sit $k < \frac{0,3424267\, a \sin.\theta}{1 + \sin.\zeta^2}$, non autem reuertetur, sed perpetuo per cochleam progredi perget, si sit $k > \frac{0,3424267\, a \sin.\theta}{1 + \sin.\zeta^2}$. Quia autem esse debet $k < \frac{0,3424267\, a \cos.\zeta^2 \sin.\theta}{\sin.\zeta^4}$, manifestum est, hunc

casum

caſum locum obtinere non poſſe, niſi ſit $1 > 2\,\text{fin.}\,\zeta^4$, ſeu $\text{fin.}\,\zeta < \sqrt[4]{\tfrac{1}{2}}$; hoc eſt: niſi angulus helicis ζ minor ſit quam $57°, 14^{I}$.

8^{vo}. Poſtquam autem angulus ψ vltra $60°$ fuerit diminutus, etiam vlterius decreſcet, eritque adhuc

$$\mathrm{V} = \text{fin.}\,\zeta^2 \sqrt{(k + na(\text{fin.}\,\psi - \tfrac{1}{2}\psi))} \text{ et } \sqrt{v} = \frac{\sqrt{k} - \mathrm{V}}{\text{cof.}\,\zeta}$$

valorque ipſius V continuo fiet maior, vt et celeritas $\sqrt{v}$, quae mox affirmatiua reddetur, et facto $\psi = 0$ redibit ea, vti erat initio, $\sqrt{v} = \text{cof.}\,\zeta.\sqrt{k}$.

Cum globulus huc peruenerit, angulus ψ iterum negatiuus euadet, ſeu motus angularis globuli motum cylindri ſequetur, ſeu erit iam $\Phi < \alpha$, ſeu $\Phi < 90°$; vel globulus in ſuperiorem cylindri medietatem eleuabitur, cum a Nro. 3^{tio} in inferiore eſſet verſatus: atque nunc pari modo motum ſuum proſequetur, atque ab initio fecerat; ita vt iam eaedem motus partes, quas deſcripſimus, ſint rediturae.

Quod vero ad tempora attinet, quibus quaeque motus huius pars abſoluitur, ea nonniſi per quadraturas definiri poterunt ope formulae $d\omega = \frac{d\psi\sqrt{k}}{\mathrm{V}}$; quippe cuius integratio exhiberi nequit.

PROBLEMA 8.

31. Si vna integra helicis circumuolutio EFGe aqua fuerit repleta, atque cylindrus ſubito in gyrum agi incipiat celeritate vniformi, quae in puncto E ſit $= \sqrt{k}$, idque in ſenſum helici contrarium BEA, inuenire motum, quo iſta aquae portio per helicem promouebitur.

SOLV-

SOLVTIO.

Positis basis cylindri radio $CA = a$, angulo helicis $BEF = \zeta$, angulo, quem axis cylindri PQ cum verticali constituit, $PQR = \theta$; sit ipso motus initio angulus $ACE = \alpha$; quo tempore aqua in helice spatium $EFGe = f$ occupet, quod cum vni integrae reuolutioni sit aequale, posito $\frac{f\,cos.\zeta}{a} = \gamma$, erit γ angulus quatuor rectis aequalis, seu denotante $1 : \pi$ rationem radii ad semicircumferentiam, erit $\gamma = 2\pi$ et $f = \frac{2\pi a}{cos.\zeta}$, et ipsa aquae copia $= \frac{2\pi a b b}{cos.\zeta}$, siquidem $b b$ designet amplitudinem helicis.

Iam elapso tempore t, quo ipse cylindrus circa axem conuersus erit angulo $= \omega$, vt sit $d\omega = \frac{dt\sqrt{k}}{a}$, seu $\omega = \frac{t\sqrt{k}}{a}$, ideoque $t = \frac{a\omega}{\sqrt{k}}$, peruenerit aqua in helice in situm $MFGem$; ponatur ergo spatium $EM = x$ et celeritas, qua aqua per helicem promouetur $= \sqrt{v}$; vt sit $dx = dt\sqrt{v} = \frac{a\,d\omega\sqrt{v}}{\sqrt{k}}$. Ponatur angulus $ACS = \Phi$, et ob angulum $ECS = \frac{x\,cos.\zeta}{a}$, quia punctum E angulo ω ad A accessit, erit $\Phi = \alpha - \omega + \frac{x\,cos.\zeta}{a}$, ideoque $\frac{x\,cos.\zeta}{a} = \omega + \Phi - \alpha$: et hinc $\frac{dx\,cos.\zeta}{a} = \frac{d\omega\,cos.\zeta\sqrt{v}}{\sqrt{k}} = d\omega + d\Phi$, ita vt sit $d\omega = \frac{d\Phi\sqrt{k}}{cos.\zeta.\sqrt{v} - \sqrt{k}}$. At ex §. 17 habebitur haec aequatio ob $\gamma = 2\pi$ et $fin.(\gamma + \Phi) = fin.\Phi$:

$$cos.\zeta.\sqrt{v} - \sqrt{k} = \sqrt{(C - a\Phi\,fin.\zeta\,cos.\zeta\,cos.\theta)}.$$

Ipso autem motus initio aquae in tubo helicis eiusmodi motus imprimitur, vt sit $\sqrt{v} = cos.\zeta.\sqrt{k}$, quo casu cum sit $\Phi = \alpha$, erit

$$cos.\zeta.\sqrt{v} - \sqrt{k} = -\sqrt{(k\,fin.\zeta^2 + a(\alpha - \Phi)\,fin.\zeta\,cos.\zeta\,cos.\theta)}.$$

Ab initio ergo angulus ϕ, qui ipso initio erat $= \alpha$, decrescit, seu terminus aquae M propius ad lineam supremam A a eleuatur, quam fuerat initio. Ponamus tempore t hanc approprinquationem factam esse per angulum ψ, vt sit $\phi = \alpha - \psi$, erit $\frac{x \cos. \zeta}{a} = \omega - \psi$ et $x = \frac{a(\omega - \psi)}{\cos. \zeta}$ tum vero $\cos. \zeta \sqrt{v} - \sqrt{k} = -\sqrt{(k \sin. \zeta^4 + a\psi \sin. \zeta \cos. \zeta \cos. \theta)}$ seu $\sqrt{v} = \frac{\sqrt{k} - \sqrt{(k \sin. \zeta^4 + a\psi \sin. \zeta \cos. \zeta \cos. \theta)}}{\cos. \zeta}$ eritque $d\omega = \frac{d\psi \sqrt{k}}{\sqrt{(k \sin. \zeta^4 + a\psi \sin. \zeta \cos. \zeta \cos. \theta)}}$

Hinc cum initio, quo $\omega = 0$, sit quoque $\psi = 0$, erit integrando:

$$\frac{a\omega \sin. \zeta \cos. \zeta \cos. \theta}{2\sqrt{k}} = \sqrt{(k \sin. \zeta^4 + a\psi \sin. \zeta \cos. \zeta \cos. \theta)} - \sin. \zeta^2 \sqrt{k}$$

hincque porro $\psi = \omega \sin. \zeta^2 + \frac{a\omega\omega}{4k} \sin. \zeta \cos. \zeta \cos. \theta$

Ex quo obtinemus pro tempore per angulum ω indicato:

$$\sqrt{v} = \cos. \zeta \sqrt{k} - \frac{a\omega \sin. \zeta \cos. \theta}{2\sqrt{k}}$$

et $x = a\omega \cos. \zeta - \frac{aa\omega\omega}{4k} \sin. \zeta \cos. \theta$

elapso autem tempore t est $\omega = \frac{t\sqrt{k}}{a}$; ita vt sit

$$\sqrt{v} = \cos. \zeta \sqrt{k} - \tfrac{1}{2} t \sin. \zeta \cos. \theta$$

et $x = t \cos. \zeta \sqrt{k} - \frac{1}{4} tt \sin. \zeta \cos. \theta$

spatium ergo SM, per quod aqua iam secundum directionem axis cylindri erit promota, erit

$$x \sin. \zeta = t \sin. \zeta \cos. \zeta \sqrt{k} - \tfrac{1}{4} tt \sin. \zeta^2 \cos. \theta$$

vnde spatium, per quod verticaliter iam erit eleuata aqua concluditur

$$x \sin. \zeta \cos. \theta = t \sin. \zeta \cos. \zeta \cos. \theta \sqrt{k} - \tfrac{1}{4} tt \sin. \zeta^2 \cos. \theta^2$$

COROLL. 1.

32. Si cylindrus plane non in gyrum ageretur, sed in quiete relinqueretur, vt esset $k = 0$, tunc elapso tempore

tempore t eſſet $\sqrt{v} = -\frac{1}{2} t \,\text{ſin.}\, \zeta \,\text{coſ.}\, \theta$ et $x = -\frac{1}{4} tt \,\text{ſin.}\, \zeta \,\text{coſ.}\, \theta$. Aqua ergo, ſiquidem cochlea deorſum vltra E eſſet continuata, motu vniformiter accelerato, per cylindrum deſcenderet, eiusque motus ſimilis foret deſcenſui corporis ſuper plano inclinato, cuius anguli inclinationis ad horizontem ſinus eſſet $= \text{ſin.}\, \zeta^2 \,\text{coſ.}\, \theta$

COROLL. 2.

33. Cylindro autem in gyrum acto in ſenſum BEA celeritate $= \sqrt{k}$, aqua quidem ab initio motus ſecundum cylindrum aſcendet, quamdiu fuerit $k > \frac{1}{2} a \omega \,\text{tang}\, \zeta \,\text{coſ.}\, \theta$ ſeu $\sqrt{k} > \frac{1}{2} t \,\text{tang}\, \zeta \,\text{coſ.}\, \theta$: Elapſo autem tempore $t = \frac{2\sqrt{k}}{\text{tang}\, \zeta \,\text{coſ.}\, \theta}$, motus aſcenſus ceſſabit, poſteaque aqua per cylindrum deſcendere incipiet.

COROLL. 3.

34. Poſito ergo $t = \frac{2\sqrt{k}}{\text{tang}\, \zeta \,\text{coſ.}\, \theta}$, maximum ſpatium x per quod aqua in cochlea fuerit promota, erit $x = \frac{k \,\text{coſ.}\, \zeta^2}{\text{ſin.}\, \zeta \,\text{coſ.}\, \theta}$; ideoque ſecundum longitudinem cylindri confecit ſpatium $x \,\text{ſin}\, \zeta = \frac{k \,\text{coſ.}\, \zeta^2}{\text{coſ.}\, \theta}$; et perpendiculariter reperietur eleuata ad altitudinem $x \,\text{ſin.}\, \zeta \,\text{coſ.}\, \theta = k \,\text{coſ.}\, \zeta^2$.

COROLL. 4.

35. Portio ergo aquae, quae integram ſpiralis reuolutionem implet, ope cochleae archimedeae ad maiorem altitudinem eleuari nequit, quam quae ſit $= k \,\text{coſ.}\, \zeta^2$. Quo celerius ergo cylindrus in gyrum agitur, eo altius haec aquae portio eleuari poteſt, et haec quidem altitudo proportionalis erit quadrato celeritatis gyrationis.

COROLL. 5.

36. Sit altitudo, ad quam aqua ope cochleae Archimedis eleuari debeat, $= c$; praestabiturque hoc tempore t vt sit

$$c = t \sin.\zeta \cos.\zeta \cos.\theta \sqrt{k} - \tfrac{1}{4} tt \sin.\zeta^2 \cos.\theta^2$$

seu $$t = \frac{2\cos.\zeta \sqrt{k} - 2\sqrt{(k\cos\zeta^2 - c)}}{\sin.\zeta \cos.\theta}.$$

Vt iam hoc tempus sit omnium minimum, angulus ζ ita esse debet comparatus, vt sit $\operatorname{tang}\zeta^4 = 1 - \frac{c}{k}$

seu $\operatorname{tang}\zeta = \sqrt[4]{(1 - \frac{c}{k})}$.

COROLL. 6.

37 Posito autem $\operatorname{tang}\zeta = \sqrt[4]{(1 - \frac{c}{k})}$, erit tempus illud minimum, quo aqua per altitudinem c eleuatur: $$t = \frac{2\sqrt{k}}{\cos.\theta}(\cos.\zeta - \operatorname{tang}.\zeta) = \frac{2\sqrt{k} - 2\sqrt{(k-c)}}{\cos\theta, \sqrt[4]{(1 - \frac{c}{k})}}$$

quod fit infinitum si $k = c$, at vero nullum si $k = \infty$. Quo maior ergo capiatur celeritas gyratoria $\sqrt{k}$, et quo minor simul statuatur angulus $PQR = \theta$, eo breuiori tempore aqua ad altitudinem c eleuabitur.

COROLL. 7.

38. Patet ergo etiamsi cochlea Archimedis situm obtineat verticalem, eius tamen ope aquam ad quamvis altitudinem eleuari posse, dummodo cochlea satis celeriter in gyrum agatur. Hoc autem casu ob $\theta = 0$, perinde est siue aqua integram helicis reuolutionem impleat, siue secus. Ac tempus quidem eleuationis hoc

casu

casu minus erit, quam si cylindrus ad horizontem esset inclinatus.

SCHOLION.

39. Patet ergo insignem esse differentiam inter eleuationem aquae per cochleam Archimedis, prout aqua eleuanda vel integram spirae reuolutionem impleat, vel tantum minimam eius portionem occupet, si enim aqua integram spiram adimplet, ea non vltra certam altitudinem eleuari potest, quantumuis celeriter cochlea in gyrum agatur; contra autem vidimus, si minima aquae portio tantum cochleae immittatur, fieri posse, vt ea ad quamuis altitudinem eleuetur, atque hoc quidem motu gyrationis non admodum celeri: nam ex praecedentibus perspicitur, motum nimis celerem ascensui aduersari, et aquam iterum deorsum ferre, quae tamen a motu tardiore continuo ascendere perrexisset. Vt enim particula aquae cochleae in E initio immissa continuo ascendere pergat, primum requiritur vt sit $\theta > \zeta$ seu ang. PQR > ang. BEF. Deinde vt sit sin. α seu sin. ACE $> \frac{\tang \zeta}{\tang \theta}$: tertio autem requiritur, vt, denotante M maximum valorem positiuum, quem expressio $\cos.(\alpha - \psi) - \cos.\alpha - \frac{\tang \zeta}{\tang \theta}.\psi$ recipere valet, quod euenit casu $\sin.(\alpha - \psi) = \frac{\tang \zeta}{\tang \theta}$, sit $k > a\,M. \frac{\cos.\zeta^2 \sin.\theta}{\sin.\zeta^4}$. Si ergo altitudo celeritati gyrationis debita k superaret hanc quantitatem, aqua, postquam ad certam altitudinem peruenisset, iterum delaberetur. Verum neuter horum casuum in praxi communi, vbi cochlea Archimedis ad aquas eleuandas adhibetur, locum habet: quodsi enim tota cylindri basis inferior AB aquae est submersa, tota helix

helix ſemper eſt aqua repleta, vnde quaeſtio, quanta celeritate et ad quantam altitudinem cochlea in gyrum acta aquam ſit eleuatura, ab his binis, quas tractauimus, penitus eſt diuerſa, propterea quod aqua in E continuo influit, in K vero iterum egeritur. Hanc igitur quaeſtionem difficillimam in ſequente problemate enodare conabor.

PROBLEMA 9.

40. Si tota baſis cylindri aquae ſit ſubmerſa, isque motu vniformi in gyrum agatur, definire motum aquae per cochleam.

SOLVTIO.

Poſitis, vt hactenus, radio baſis $CA = a$, angulo helicis $BEF = \zeta$, et inclinationis $PQR = \theta$: ſit altitudo aquae ſupra centrum baſis $C = c$, longitudo totius cylindri $Aa = Bb = b$, et EFGHIK repraeſentet totam helicem, cuius propterea longitudo eſt $= \frac{b}{\sin.\zeta}$; ac ſi eius amplitudo dicatur $= bb$, erit quantitas aquae in helice contentae $= \frac{bbb}{\sin.\zeta}$; tum vero ſumma ſpirarum ad baſin relatarum praebebit in eius peripheria arcum $= \frac{b\cos.\zeta}{\sin.\zeta}$. Scilicet ſi a puncto helicis quocunque Z ad baſin ducatur recta axi parallela ZY, arcusque EY ponatur $= s$, poſito $s = 0$, habebitur terminus helicis inferior E, at poſito $s = \frac{a\cos.\zeta}{\sin.\zeta}$ prodibit terminus helicis ſuperior K. Gyretur nunc cylindrus in ſenſum BEA, ita vt celeritas puncti E ſit $= \sqrt{k}$: poſitoque arcu $EA = p$, elapſo tempuſculo dt erit $dp = -dt\sqrt{k}$. Praeſenti autem temporis momento ſit aquae per helicem rem aſcendentis celeritas $= \sqrt{v}$: quod ſi iam ſtatus compreſſionis aquae in helicis loco quocunque Z ponatur $= q$, exiſtente arcu EY $= s$, hanc ſupra inuenimus aequationem

$$q \,\text{coſ.}\, \zeta = C - a \,\text{coſ.}\, \zeta \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p+s}{a} - s \,\text{ſin.}\, \zeta \,\text{coſ.}\, \theta - \frac{s\, dv}{dt \sqrt{v}}.$$

Quando autem aqua in K libere effluit, poſito $s = \frac{b \,\text{coſ.}\, \zeta}{\text{ſin.}\, \zeta}$, ſtatus compreſſionis in K euaneſcere debet, erit ergo $C = a \,\text{coſ.}\, \zeta \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p \,\text{ſin.}\, \zeta + b \,\text{coſ.}\, \zeta}{a \,\text{ſin.}\, \zeta} + b \,\text{coſ.}\, \zeta \,\text{coſ.}\, \theta + \frac{b\, dv \,\text{coſ.}\, \zeta}{dt \,\text{ſin.}\, \zeta \sqrt{v}}$. Exprimat g ſtatum compreſſionis in altero termino E, vbi $s = 0$ erit

$$g \,\text{coſ.}\, \zeta = a \,\text{coſ.}\, \zeta \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p \,\text{ſin.}\, \zeta + b \,\text{coſ.}\, \zeta}{a \,\text{ſin.}\, \zeta} + b \,\text{coſ.}\, \zeta \,\text{coſ.}\, \theta + \frac{b\, dv \,\text{coſ.}\, \zeta}{dt \,\text{ſin.}\, \zeta \sqrt{v}}$$
$$- a \,\text{coſ.}\, \zeta \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p}{a}$$

ſiue per coſ. ζ diuidendo:

$$g = a \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p \,\text{ſin.}\, \zeta + b \,\text{coſ.}\, \zeta}{a \,\text{ſin.}\, \zeta} - a \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p}{a} + b \,\text{coſ.}\, \theta + \frac{b\, dv}{dt \,\text{ſin.}\, \zeta \sqrt{v}}.$$

Totum ergo negotium huc redit, vt ſtatus compreſſionis aquae in termino E definiatur, qui cum a profunditate orificii E, ſub aqua pendeat, reperitur puncti E altitudo ſuper centro C $= a \,\text{coſ.}\, \frac{p}{a} \,\text{ſin.}\, \theta$, ideoque profunditas orificii E ſub aqua erit $= c - a \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p}{a}$. Cum igitur celeritas aquae in helicem influentis ſit debita altitudini v, ſtatus compreſſionis aquae in E aeſtimari debet per altitudinem $c - a \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p}{a} - v$, vnde habemus:

$$c = a \,\text{ſin.}\, \theta \,\text{coſ.}\, \frac{p \,\text{ſin.}\, \zeta + b \,\text{coſ.}\, \zeta}{a \,\text{ſin.}\, \zeta} + b \,\text{coſ.}\, \theta + \frac{b\, dv}{dt \,\text{ſin.}\, \zeta \sqrt{v}} + v$$

Ponatur angulus ACE $= \Phi$, vt ſit $p = a\Phi$ et $dt = \frac{-a\, d\Phi}{\sqrt{k}}$ tum vero ſit angulus $\frac{b \,\text{coſ}\, \zeta}{a \,\text{ſin.}\, \zeta} = \gamma$, ſeu $b = a\gamma \,\text{tang.}\, \zeta$, erit $c = a \,\text{ſin.}\, \theta \,\text{coſ.}\, (\Phi + \gamma) + a\gamma \,\text{tang.}\, \zeta \,\text{coſ.}\, \theta - \frac{\gamma\, dv \sqrt{k}}{d\Phi \,\text{coſ.}\, \zeta \sqrt{v}} + v$

Ponamus $2\sqrt{kv} = z$ vt ſit $v = \frac{zz}{4k}$, habemus:

$-\gamma dt$

$$-\gamma dz + \frac{zzd\Phi\,\mathrm{cof.}\,\zeta}{4k} + ad\Phi\,\mathrm{cof.}\,\zeta\,\mathrm{fin.}\,\theta\,\mathrm{cof.}\,(\Phi+\gamma) = d\Phi\,(c\,\mathrm{cof}\,\zeta - a\gamma\,\mathrm{fin.}\,\zeta\,\mathrm{cof.}\,\theta)$$

Ex qua aequatione valor ipſius z definiri debet.

Quod autem ad preſſionem aquae ad latera tubi attinet, quatenus inde motui gyrationis reſiſtitur, ſupra vidimus a grauitate aquae oriri vim ſecundum $Zr = \mathrm{fin.}\,\zeta\,\mathrm{fin.}\,\theta\,\mathrm{fin.}$ Tab. II. Fig. 3. $\frac{p+s}{a} + \mathrm{cof.}\,\zeta\,\mathrm{cof.}\,\theta$, vnde oritur vis ſecundum $Zv = \mathrm{fin.}\,\zeta^2\,\mathrm{fin.}\,\theta\,\mathrm{fin.}\,\frac{p+c}{a} + \mathrm{fin.}\,\zeta\,\mathrm{cof.}\,\zeta\,\mathrm{cof.}\,\theta$, quae per elementum aquae $= \frac{bbds}{\mathrm{cof.}\,\zeta}$ et radium a multiplicata dat momentum elementare motui reſiſtens, vnde totum momentum erit

$$abb\,(b\,\mathrm{cof.}\,\zeta\,\mathrm{cof.}\,\theta + \frac{a\,\mathrm{fin.}\,\zeta^2\,\mathrm{fin.}\,\theta}{\mathrm{cof.}\,\zeta}\,(\mathrm{cof.}\,\Phi - \mathrm{cof.}\,(\Phi+\gamma)))$$

tantum ergo momentum a vi gyrante ſuperari debet.

COROLL. 1.

41. Pendet ergo determinatio motus aquae per cochleam Archimedis a reſolutione huius aequationis differentialis:

$$-\gamma dz + \frac{zzd\Phi\,\mathrm{cof.}\,\zeta}{4k} + ad\Phi\,\mathrm{cof.}\,\zeta\,\mathrm{fin.}\,\theta\,\mathrm{cof.}\,(\Phi+\gamma) = d\Phi\,(\mathrm{cof.}\,\zeta - a\gamma\,\mathrm{fin.}\,\zeta\,\mathrm{cof.}\,\theta)$$

vel ob $\gamma = \frac{b\,\mathrm{cof.}\,\zeta}{a\,\mathrm{fin.}\,\zeta}$ iſtius aequationis

$$-\frac{bdz}{a\,\mathrm{fin.}\,\zeta} + \frac{zzd\Phi}{4k} + ad\Phi\,\mathrm{fin.}\,\theta\,\mathrm{cof.}\,(\Phi+\gamma) = d\Phi\,(c - b\,\mathrm{cof.}\,\theta)$$

quae cum pluribus difficultatibus ſit obnoxia, patet theoriam Cochleae Archimedis maxime eſſe arduam.

COR.

COROLL. 2.

42. Si cochlea in quiete relinquitur, vt fit $k = o$, loco elementi $d\Phi$ expedit in calculo relinqui elementum temporis dt et ob angulum Φ constantem habebitur:

$$\frac{b\,dv}{dt\,\text{sin.}\,\zeta\,\sqrt{v}} + v = c - b\,\text{cos.}\,\theta - a\,\text{sin.}\,\theta\,\text{cos.}(\Phi+\gamma)$$

vnde mox nascetur motus vniformis, $v = c - b\,\text{cos}\,\theta - a\,\text{sin.}\,\theta\,\text{cos.}(\Phi+\gamma)$ quo aqua per cochleam fluet, siquidem sit $c > b\,\text{cos.}\,\theta + a\,\text{sin.}\,\theta\,\text{cos.}(\Phi+\gamma)$

COROLL. 3.

43. Si cylindrus in situ verticali sit positus ob $\theta = o$ erit $-\frac{b\,dz}{a\,\text{sin.}\,\zeta} + \frac{zz\,d\Phi}{4k} = d\Phi(c-b)$; vnde fit $d\Phi = \frac{4bk\,dz}{(4k(b-c)+zz)a\,\text{sin.}\,\zeta}$ et integrando $\frac{a\Phi\,\text{sin.}\,\zeta}{4bk}\sqrt{4k(b-c)} = \text{A tang.}\,\frac{z}{\sqrt{4k(b-c)}}$, vbi est $\sqrt{v} = \frac{z}{\sqrt{4k}}$. Cum autem, si initio fuerit $\Phi = o$ et $z = o$, labente tempore angulus Φ euadat negatiuus, perspicuum est, valorem quoque ipsius z prodire negatiuum; ideoque hoc casu aqua non ascendet, sed descendet, quod quidem per se est euidens.

COROLL. 4.

44. In casu autem coroll. praec. quo $b > c$, eiusmodi constantem addi oportet, vt posito $\Phi = o$ fiat $\sqrt{v} = \frac{z}{\sqrt{4k}} = \text{cos.}\,\zeta\sqrt{k}$, sicque erit $\frac{\sqrt{v}}{\sqrt{(b-c)}} = \text{tang.}\left(\frac{\text{cos.}\,\zeta.\sqrt{k}}{\sqrt{(b-c)}} + \frac{a\Phi\,\text{sin.}\,\zeta}{2b}\sqrt{\frac{b-c}{k}}\right)$; progressu autem temporis fit Φ negatiuum, ideoque ascensus penitus cessat, cum sit $-\Phi = \frac{2bk\,\text{cos.}\,\zeta}{a(b-c)\,\text{sin.}\,\zeta}$.

SCHOLION.

45. Assumsi in huius casus integratione, cochleam initio fuisse aqua repletam, subitoque rotari incepisse; sic

ſic enim vtique celeritas initialis aquae progreſſiua per cochleam fit $= \text{coſ.}\, \zeta \sqrt{k}$ Sin autem ſtatus initialis ita concipiatur, vt obturato inferiori orificio cochlea in gyrum agatur, tum vero ſubito orificium iterum aperiri, aqua hoc momento ſeſe iam ad motum tubi accommodauerit neceſſe eſt, ita vt tum pro motus initio futurum ſit $v = o$. Hanc ergo ob rem aqua ſtatim deſcendere incipiet, neque vlla eius gutta ſupra eiicietur, ſiquidem ſit $b > c$. Quanquam autem hunc caſum quo $\theta = o$ feliciter expedire licuit, tamen pro ſitu cochleae inclinato, nihil admodum ex aequatione inuenta elicere licet, ſed natura motus aquae his caſibus nobis abſcondita manet, propterea quod haec aequatio ad formulam Riccatianam referenda commode tractari nequit. Ex quo inſigne Analyſeos defectus exemplum agnoſcimus, quod machinae frequentiſſimo vſu maxime peruulgatae effectus pendeat a reſolutione huiusmodi aequationis, cui artificia in Analyſi adhuc detecta non ſufficiant, qui caſus mihi adeo mirabilis eſt viſus, vt etiamſi in hac inueſtigatione ſcopum, quem mihi propoſueram, non attigerim, tamen hoc argumentum digniſſimum exiſtimauerim, quo Geometrarum vires ad id penitus expediendum incitarem, quo labore non ſolum maxima commoda in Mechanicam redundabunt, ſed etiam Analyſeos limites haud mediocriter promouebuntur.

DE

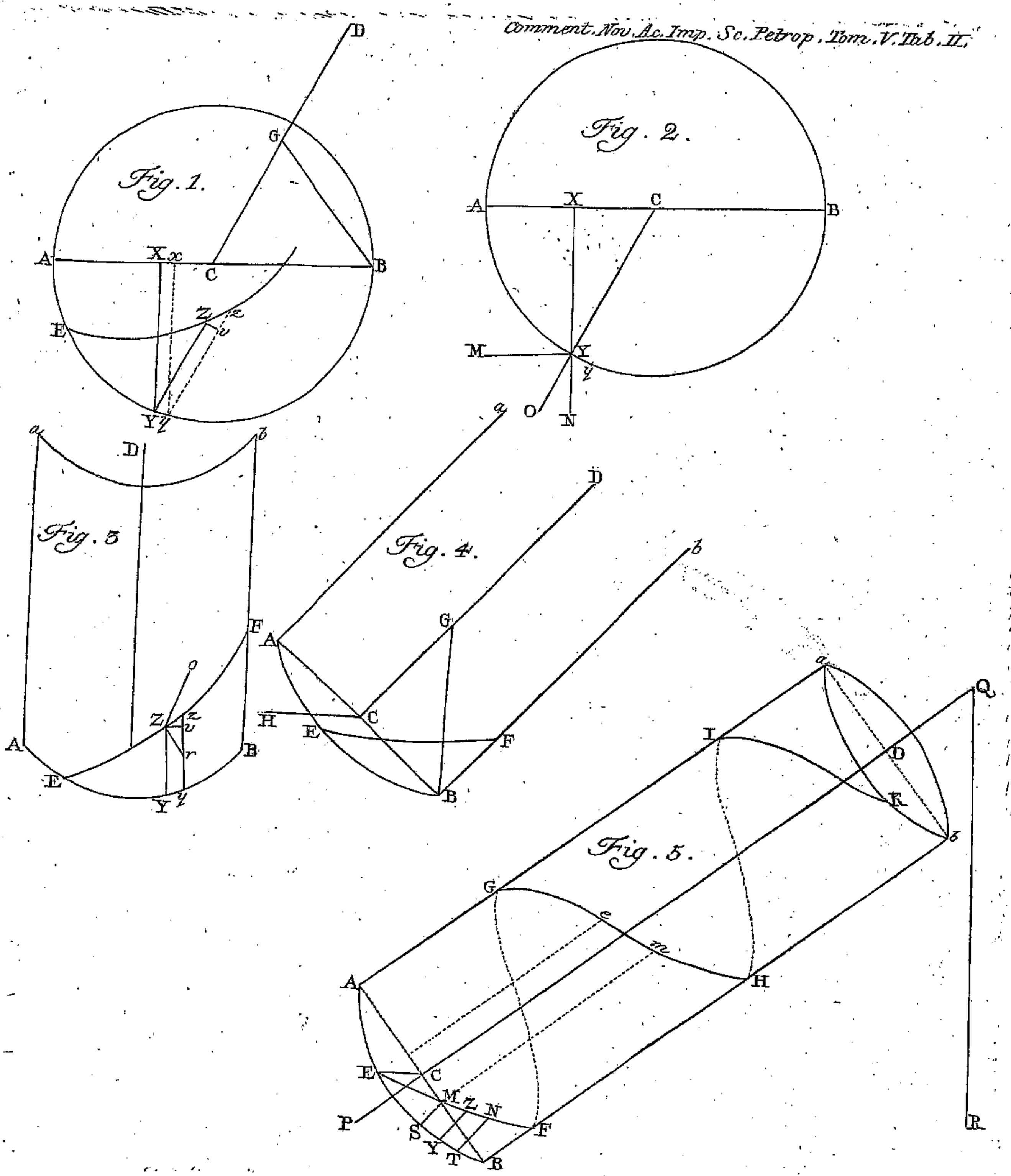
Fig. 1.
Fig. 2.
Fig. 3
Fig. 4.
Fig. 5.