1753

Theoria motus lunae exhibens omnes eius inaequalitates

Leonhard Euler

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THEORIA
MOTUS LUNAE
EXHIBENS
OMNES EIVS INAEQUALITATES

IN
ADDITAMENTO.
HOC IDEM ARGUMENTUM ALITER TRACTATUR
SIMULQUE OSTENDITUR
QUEMADMODUM MOTUS LUNAE CUM OMNIBUS
INAEQUALITATIBUS
INNUMERIS ALIIS MODIS
REPRÆSENTARI
ATQUE AD CALCULUM REVOCARI POSSIT
AUCTORE
L. E U L E R O

IMPENSIS
ACADEMIAE IMPERIALIS SCIENTIARUM
PETROPOLITANAE
ANNO 1753.

Academiam Scientiarum Petro-
politanam triennio abhinc
omnes, qui ingenii viribus
confisi, ad examinandam
Neutonianam de motu Lunae Theoriam,
animum applicare vellent, inuitasse, atque
ei, qui in hac parte tenuisset primas, prae-
mii loco proposuisse nummos aureos cen-
tum; postmodum Celeberrimum Clairau-
tum huius certaminis exstitisse victorem,
publico Academicorum, qui Petropoli
agunt, iudicio divulgatum.

* 2

Cele-

Celeberrimus Eulerus, Academiae Petropolitanae membrum honorarium, officii sui esse existimavit, ferre vna cum ceteris de illa Clairauti dissertatione iudicium. Transmisit ergo huc ad nos una cum sua sententia amplissimam de eodem argumento dissertationem; quae quo celerius innotesceret, visum est Academiae Praefidi, minoris Russiae Hetmano Illustrissimo Cyrillo Gregoridæ, Comiti Rasumouio, eam tradere Academicis in solenni conuentu examinandam, ea fini, vt si suffragio Academicorum comprobata, dignaque iudicata foret, quae orbis eruditi proponeretur theatro, ea praelo quam maturrime subliceretur; quandoquidem ille mos iam indè a principio obtinuit, vt, quae in publico coetu praeleguntur dissertationes, eae vel ante solennem actum, vel paruulo intermisso spatio typis diuulgentur postea.

Cete-

Ceterum dissertatio illa constat magnam partem calculis, veris quidem illis et omnibus numeris absolutis, sed propter nimiam sui molem atque difficultatem auditu molestissimis: quae si recitarentur publice, periculum erat, ne auditorum animi auscultandis iis deficerent, neue Academia Summorum patientia. Virorum abuti, caeterosque enicare odio videretur. Praeuidit hoc incommodum, prouiditque Illustrissimi Praefidis sapientia. Mandavit Astronomiae Professori V. C. Nicetae Popouio, cuius id temporis dicendi erat prouincia, vt prolixam illam Euleri dissertationem, omisis calculis, redigeret in compendium, et quae inde excepisset, auditorum causa recitaret publice. Quo quidem facto et auribus hominum consuluit, et tamen rerum capitum participes eos facere aequabili temperamento instituit: at-

que dissertationem, ne quid forte naeuorum obreperet, ipso auctore coram typis excudi aequum censuit.

Iam qui illo tempore interfuerant conuentui Academicorum solenni auditores, aequis nimis auribusque ausculta-
runt excerpta recitantem Euleriana Po-
pouium: est ergo quod sperare liceat, et
iis, quorum interest, vt qui hoc genere
studiorum maxime delectantur, factum iri
satis ipsa dissertatione. Datum Petropoli
Nov. 1752.

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INTRO-

INTRODUCTIO.

Cum nullum sit dubium, quin summi Newtoni Theoria, qua motum Planetarum felicissimo cum successu certis legibus adstrinxit, plurimum ad motum Lunae accuratius determinandum contulerit, maxime sane in Astronomia momenti est quaestio: vtrum haec Newtoni Theoria omnino sit sufficiens omnibus motus inaequalitatibus, quae in Luna observantur, exactissime explicandis nec ne? Quanquam autem a Newtoni affectis plerumque affirmari solet, nullam in motu Lunae observari inaequalitatem, cuius ratio in ista Theoria non contineatur; tamen tantum abest, ut hic consensus a quoquam perspicue sit monstratus, ut potius applicatio huius Theoriae ad Lunam tantis implicetur calculi difficultatibus, quibus penitus evolvendis vires ingenii humani vix sufficere videntur. Plurimae quidem adhuc prodierunt Tabulae Lunares, quae ex Theoria Newtoniana deductae perhibentur, sed praeterquam quod saepius ultra 5' ab observationibus discrepent, earum convenientia cum Theoria ipsa nequaquam est evicta; quin potius pleraeque Tabulae inaequalitatum non tam Theoriae quam observationibus sunt superstructae. Huiusmodi ergo tabularum siue consensus siue dissensus cum observationibus neque ad Theoriam Newtonianam plenissime confirmandam, neque ad eam infringendam allegari

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TRO-

gari potest: nam quatenus istae tabulae observationibus satisfaciunt, hoc non soli Theoriae est tribuendum; quatenus autem cum observationibus minus conveniunt, hoc ne Theoriae quidem imputari poterit, propterea quod istae Tabulae non soli Theoriae innituntur.

Quaestio itaque, cujus mentionem feci, recte enodari nequit, nisi ante eiusmodi Tabulae exhibeantur, quae ex sola Theoria, nullis observationibus in subsidium vocatis, sint formatae; tum enim demum ex huiusmodi Tabularum collatione cum ingenti observationum summo studio institutarum copia diiudicare licebit, vtrum Theoria omnibus observationibus respondeat, an vero correctione quapiam indigeat. Non difficile quidem est ex principiis mechanicis motum Lunae aequationibus analyticis complecti; quoniam autem hae aequationes plures variables inter se permixtas continent, atque adeo differentialibus secundi ordinis implicantur, earum resolutio maximis difficultatibus est obnoxia; et quoniam alio modo nisi per approximationem suscipi non potest, vtcunque instituat, semper non leue dubium remanet, vtrum partes, quae in calculo sunt neglectae et praetermissae, nihil, quod in motu Lunae esset notabile, efficere poterint. Hoc modo explicatio motuum Lunae tota ad solam Analysis transfertur, ac difficultates, quibus premittitur, inde oriuntur, quod Analysis nondum satis est exculca.

Cum igitur Theoria Newtoniana hoc principio latissime patente innitatur, quod omnia corpora coelestia se mutuo attrahant in ratione reciproca duplicata distantiarum,

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rum, si motum Lunae secundum hanc Theoriam definire velimus, vires erunt spectandae omnes, quibus Luna sollicitatur. Atque inter has vires primaria est ea, qua ad terram vrgetur, quae si sola adesset, terraque quiesceret, Luna in ellipsi perfecta secundum regulas Keplerianas motum suum circa terram absolueret. At cum Luna praeterea aequae ac terra ipsa etiam ad solem trahatur, hae vi motus ille regularis non mediocriter perturbabitur: atque haec vis a sole profecta omnium difficultatum, quae in determinatione motus Lunae offenduntur, causa est existimanda. Reliquae enim vires, quibus forte Luna secundum Theoriam Newtoni ad reliquos planetas vrgeri deberet, tam sunt exiguae, vt effectus inde oriundus merito pro nihilo haberi queat.

Solas ergo vires solis ac terrae in computum duci oportet, si motum Lunae secundum Theoriam definire velimus, atque cum ex his viribus formulae analyticae fuerint erutae, quae motum Lunae complectantur, omne studium in his formulis ita evoluendis erit impendendum, vt inde ad quoduis tempus propositum locus Lunae assignari, ac more apud Astronomos solito secundum longitudinem et latitudinem definiri queat. Hinc porro Tabulae Astronomicae pro motu Lunae erunt condendae, quibus omnes inaequalitates tam in longitudine quam in latitudine exhibeantur, ex quibus si pro cuiusvis observationis momento locus Lunae computetur, consensus vel dissensus calculi ab observationibus Theoriam vel confirmabit, vel ejus defectum declarabit. Neque tamen Theoria sola huiusmodi Tabulis contruendis sufficit, sed quaedam ele-

menta extrinsecus ab observationibus assumi oportet, quae sunt 1°. Excentricitas orbitae Lunarise, quae salua theoria vel major vel minor esse potuisset; pender enim a motu Lunae primitus impresso, quem Theoria non determinat, sed tanquam cognitum assumis. 2°. Locus Lunae medius pro quapiam Epocha proposita, qui pariter ex observationibus est concludendus. 3°. Locus Apogei orbitae Lunarise pro Epocha quadam data. 4°. Tempus periodicum Lunae secundum motum medium, quod pender a distantia Lunae media a Terra, ideoque ex sola Theoria definiri nequit. 5°. Locus nodorum Lunae pro Epocha quadam data: et 6° denique inclinatio media orbitae Lunarise ad planum Eclipticae.

His autem sex elementis per observationes definitis reliqua omnia, quibus ad locum Lunae pro quouis tempore assignandum opus est, ex sola Theoria sunt petenda; quae primo ad quoduis tempus locum Apogei eiusque ideo motum verum praebere debet, ut inde ex loco Lunae medio eius anomalia media colligi queat. Deinde Theoria quoque omnes correctiones seu Prosthaphaereses, quae loco Lunae medio vel additae vel sublatae eius locum verum exhibeant, suppeditare debet; atque istae correctiones, quae motus inaequalitates appellari solent, partim ab Anomalia media Lunae, partim ab eius Phasi seu elongatione a sole, partimque ab Anomalia solis media pendent, ex quo triplici fonte numerus inaequalitatum in immensum augetur. Porro etiam Theoria motum nodi eiusque omnes inaequalitates indicare tenetur, ac denique etiam pro quouis tempore orbitae Lunarise veram inclinationem

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ad eclipticam, ut inde eius latitudinem veram eruere liceat.

Cum autem, ut iam innui, nemo adhuc omnes inaequalitates, quae in motu Lunae reperiuntur, ex Theoria elicuerit, ut ex iis iudicium ferri possit, quantum haec Theoria cum observationibus conveniat; etsi nullum est dubium quin discrimen, si quod deprehenderetur, admodum paruum sit futurum: iam pridem haec quaestio ex solo motu Apogei dirimi est coepta, dum aliis motus Apogei ex observationibus cognitus magnopere a Theoria discrepare est visus, alii autem etiam hoc loco pulcherrimum consensum Theoriae et veritatis iactauerunt. Mirum autem est, ipsum Newtonum nihil circa motum Apogei ex Theoria statuisse, sed eum ex solis observationibus in calculum transtulisse, cum tamen motum nodorum summa sagacitate ex Theoria eliquisset, atque veritati consentaneum ostendisset. Cur igitur motum Apogei plane silentio praeterierit, nulla alia ratio subesse videtur, nisi quod animadverterit hunc motum, prouti ex Theoria prodiret, observationibus parum fore conformem. Ex iis enim, quae Newtonus in suo immortalis opere de motu absidum in genere tradidit, non admodum difficile videtur motum apogei Lunae definire: verum hic praeter expectationem evenit, ut motus apogei annuus vix 20° superans reperiatur, cum tamen ex observationibus constet, Apogeeum Lunae intervallo unius anni ultra 40° promoueri.

Sive autem ista motus Apogei quantitas 20° legitime sit ex Theoria derivata, sive minus; consideratio Apogei tutissimum praebet remedium quaestionem de sufficientia

A 3

Theo-

Theoriae Newtonianae decidendi. Quamvis enim ex Theoria inaequalitas quaequam in ipso motu Lunae aliquot minutis secundis vel etiam primis maior minorue prodiret, quam experientia monstraret, tamen tantilla differentia merito vel leui culpiam errori in observationibus, vel vicio in approximatione commissio tribueretur; quandoquidem aliunde certum est Theoriam Newtonianam non admodum a veritate recedere. At longe aliter est comparata ratio motus Apogei: quodsi enim vires Lunam sollicitantes tantillum a Theoria Newtoniana discrepent, ut ex iis in ipso motu Lunae vix perceptibile discrimen nasceretur, tamen inde in motu Apogei annuo differentia plurimum graduum oriri poterit. Quae tanta differentia cum nulli errori vel observationum vel ipsius calculi, siquidem omni cura instituat, tribui queat, inuestigatio motus apogei certissimum suppeditat criterium iudicandi, vtrum quaequam Theoria veritati sit consentanea nec ne?

Quodsi ergo calculo rite administrato Theoria Newtoni reperiatur tantum Apogei Lunaris motum exhibere, quantus per observationes deprehenditur, scilicet ultra 40° quotannis; fortius certe argumentum, quo veritas huius Theoriae indubie demonstratur, desiderari nequit. Sin autem contra eueniat, ut progressio Apogei annua ex Theoria rite derivata notabiliter a 40° deficiat, hinc certo erit concludendum Theoriam Newtonianam correctione quapiam indigere, neque vires, quibus Luna reuera sollicitatur, exactissime huic Theoriae esse conformes.

Verum haec ipsa quaestio, vtrum Theoria Newtoniana ad verum apogei Lunae motum perducatur nec ne? est pro-

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fundissimae indaginis, atque summam in calculo circumspeditionem ac sollertiam requirit. Quanquam enim ex principiis generalibus, unde vulgo motus absidum definiri solet, satis luculenter semissis tantum pro motu Apogei Lunae elicitur; tamen quoniam in calculo plures termini, qui in determinationem motus Lunae ingrediuntur, ob paruitatem sunt reiecti, merito dubitatio suboritur, num hi ipsi termini, si eorum ratio esset habita, non istum defectum compensare valuerint? Quin etiam non defuere Geometrae, qui consensum huius Theoriae cum vero Apogei motu demonstrare sunt conati: verum plerumque non difficile erat paralogismum in ipsorum ratiociniis deprehendere. Maximam autem hoc loco attentionem meretur iudicium profundissimi Geometrae Clairaut, qui cum primum validissimis argumentis statuisset, Newtoni Theoriam non ultra dimidium veri apogei Lunaris motus suppeditare, subito in contrariam abiit sententiam statuens hanc Theoriam elegantissime cum veritate conspirare; neque certe tantae perspicaciae Viri pristina sententia, quam omni studio propugnauerat, recessisse est putandus, nisi firmissimis argumentis eo esset adactus.

Cum autem omnes rationes, quae Ipsum ad hanc retractionem impulserint, nondum publice exposuerit, liceat mihi quidem, qui semper contrariae sententiae fui addictus, tantisper arduam hanc quaestionem tanquam nondum decisam spectare, donec per propriam inuestigationem inueniero, quid de ea sit statuendum. Postquam enim iam a longo temporis intervallo plurimum studii in indagatione motus Lunae consumissem, ac variis methodis insistens semper

semper conclusionem Theoriae Newtonianae minus fauentem essem adeptus; quam tamen pro rite demonstrata venditare non eram ausus, propterea quod approximatione essem usus, ac semper suspicio quaedam ratione terminorum praetermissorum remaneret: nuper in aliam in-cidi viam hanc inuestigationem suscipiendi, quae mihi multo certior videtur, ita ut per eam nulla dubitatione interiecta ad veritatem penetrare confidam. Ne autem si forte Theoriam Newtonianam minus sufficientem in-ueniō, calculum secundum aliam Theoriam de nouo in-stituere cogar, statim meam inuestigationem in latiori sen-su exordiar, viresque quibus Luna ad terram sollicitatur, non exacte sed proxime tantum quadratis distantiarum re-ciproce proportionales assumam: deinceps scilicet inno-tescet, virum haec aberratio a regula Newtoniana locum habeat nec ne? Calculum autem ita adornabo, ut quicquid euenerit, non solum verum apogei motum assequar, sed etiam omnes Lunae inaequalitates inde elicere valeam, quas deinceps Astronomorum more tabulis complecti licebit.

Primum ergo problema in latissimo significato con-cipiam, ut corporis a viribus quibuscunque sollicitati mo-tum sim inuestigaturus: deinde vires, quibus Luna actu virgeri censenda est, in calculum introducam, et aequatio-nes Lunae motum determinantes exhibebo. Has porro aequationes variis modis in alias formas transmutabo, do-nec eas eo perduxero, ubi ad finem propositum maxime accommodatae videbuntur: quo cum peruenero, tandem tam motum Apogei, quam cunctas Lunae inaequalitates motus ex calculo derivare studebo.

CAPUT

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APUT

CAPUT I.

DE MOTU CORPORIS A VIRIBUS QUIBUS-
CUNQUE SOLLICITATI.

§. 1.

Quoniam corpus a viribus quibuscunque sollicitari ponimus, fieri potest, ut eius motus non in eo-
dem plano absoluat. Hinc ad ejus motum per calculum ita repraesentandum, ut ad quoduis tem-
pus verus locus, in quo corpus versabitur, assignari queat, conueniet corporis motum ad planum quoddam fixum pro lubitu assumtum referri. Exhibeat igitur Ta-
bula hoc planum, atque corpus iam versetur extra hoc planum in puncto L, unde ad planum demittatur per-
pendiculum LM; eritque punctum M locus corporis ad planum relatus. Quod si ergo ad quoduis tempus propositum hunc corporis locum relatum M, simulque eius a plano distantiam LM indicare valeamus, verus corporis locus L ad hoc tempus innotescet.

Fig. 2.

§. 2. Ad locum autem puncti M commodius de-terminandum, assumamus in plano rectam quandam fixam CQ pro axe, ita ut ducta ex M ad hanc rectam perpendiculari MP, locus puncti M more apud Geo-
metras recepto per coordinatas orthogonales definiatur. Assumpto ergo porro in axe puncto quodam fixo C, unde abscissae CQ computentur, erit PM applicata puncto M respondens, & ipsum punctum L determina-
bitur per tres coordinatas inter se normales CQ, PM &

B

M L.

ML. Cum igitur praesentis temporis momento corpus in L versari ponatur, vocentur istae tres coordinatae eo spectantes:

$$CP = p; PM = q \text{ et } ML = r$$

elapso autem temporis elemento, quod per dt indicemus, coordinatae ternae tum locum corporis indicantes erunt:

$$p + dp; q + dq; \text{ et } r + dr.$$

§. 3. Quaecunque nunc fuerint vires, quibus corpus sollicitatur, eae semper per cognitam virium resolutionem reduci poterunt ad ternas vires secundum directiones ternarum coordinatarum vrgentes. Ponamus ergo vim $= P$, qua corpus in L secundum directionem ipsi P C parallelam trahitur: eam porro vim $= Q$, qua corpus secundum directionem ipsi M P parallelam trahitur: eamque denique vim $= R$, qua corpus secundum directionem L M sollicitatur. Has scilicet vires ita directas concipio, vt si corpus earum actioni libere obediret, valores coordinatarum p, q, r inde diminuerentur. His positis, ex principiis Mechanicae constat, si elementum temporis dt pro constanti assumatur, motum corporis his tribus formulis differentialibus determinari

$$I. ddp = -\frac{1}{2} P dt^2; II. ddq = -\frac{1}{2} Q dt^2; III. ddr = -\frac{1}{2} R dt^2.$$

§. 4. Verum hae coordinatae ad vsum astronomicum, ad quem hic potissimum respicimus, non satis sunt accommodatae. Nam si spectatorem in C constitutum assumamus, lectis L; vbi corpus cernetur, commodissime

me per quantitatem rectae CM, et angulum QCM vna cum angulo MCL repraesentatur: atque si tabula planum eclipticae referat, rectaque CQ ad principium arietis sit directae, angulus QCM in Astronomia vocari solet sideris longitudo, angulus MCL vero eius latitudo, et recta CM eius distantia curtata. Vocemus ergo porro:

I. Distantiam curtatam seu rectam $CM = x$

II. Longitudinem seu angulum $QCM = \phi$

III. Latitudinem seu angulum $MCL = \psi$

ac posito constanter sinu toto $= 1$, erunt valores coordinatarum ante adhibitarum:

$$CP = p = x \cos \phi; PM = q = x \sin \phi \& ML = r = x \tan \psi$$

atque distantia corporis vera a puncto C erit $CL = x \sec \psi = \frac{x}{\cos \psi}.$

§. 5. Sumtis nunc differentialibus more consueto obtinebimus:

$$dp = dx \cos \phi - x d\phi \sin \phi; dq = dx \sin \phi + x d\phi \cos \phi$$

$$\text{et } dr = dx \tan \psi + \frac{x d\psi}{\cos^2 \psi}$$

atque hinc denuo differentialibus sumendis reperietur,

$$ddp = ddx \cos \phi - 2dx d\phi \sin \phi - x dd\phi \sin \phi - x d\phi^2 \cos \phi$$

$$ddq = ddx \sin \phi + 2dx d\phi \cos \phi + x dd\phi \cos \phi - x d\phi^2 \sin \phi$$

$$ddr = ddx \tan \psi + \frac{2dx d\psi}{\cos^2 \psi} + \frac{x dd\psi}{\cos^2 \psi} + \frac{2x d\psi^2 \sin \psi}{\cos^3 \psi}$$

Binae priores formulae rite combinatae suppeditabunt sequentes multo concinniores

$$ddp \cos \phi + ddq \sin \phi = ddx - x d\phi^2$$

$$ddq \cos \phi - ddp \sin \phi = 2dx d\phi + x dd\phi$$

sicque habebitur:

$$ddx - x d\phi^2 = -\frac{1}{2} dt^2 (P \cos \phi + Q \sin \phi)$$

$$2dx d\phi + x dd\phi = -\frac{1}{2} dt^2 (Q \cos \phi - P \sin \phi)$$

Tertiam vero aequationem deinceps magis tractabilem efficiemus.

§. 6 Manifestum autem est formulam $P \cos \phi + Q \sin \phi$ praebere vim ex viribus P et Q compositam, qua corpus in L secundum directionem rectae MC vrgetur, formulam vero alteram $Q \cos \phi - P \sin \phi$ exprimere vim ex eadem resolutione secundum directionem MQ ad MC normalem directam. Cum igitur hae duae vires assumptis binis P et Q aequivaleant, ponamus esse

I. Vim corpus L secundum MC trahentem $= V$

II. Vim corpus L secundum MQ trahentem $= T$
manente tertia, vi corpus ad planum normaliter secundum LM vrgente $= R$. Atque sequentes habebimus aequationes:

$$I. 2dx d\phi + x dd\phi = -\frac{1}{2} T dt^2$$

$$II. ddx - x d\phi^2 = -\frac{1}{2} V dt^2$$

$$III. ddx \tan \psi + \frac{2dx d\psi}{\cos \psi^2} + \frac{x dd\psi}{\cos \psi^2} + \frac{2x d\psi^2 \sin \psi}{\cos \psi^3} = -\frac{1}{2} R dt^2$$

§. 7. Quo autem effectum tertiae vis R commodius ad calculum revocemus, more apud Astronomos recepto contemplemur planum, in quo corpus durante elemento

elemento temporis dt mouetur, et quod simul per punctum C transeat. Hoc igitur planum cum plano assumpto intersectionem alicubi formabit, quae sit recta $C\Omega$, ac linea nodorum appellari solet; sicque erit ΩCL planum orbitae, in qua corpus L praesenti instanti mouetur, et angulus, quo hoc planum ΩCL ad planum fixum QCM inclinatur, vocatur inclinatio orbitae ad eclipticam pro tempore praesenti. Cum igitur ex his duabus rebus latitudo sideris definiri solet, ponamus.

Longitudinem nodi ascendentis seu angulum $Q\Omega = \pi$ ac inclinationem orbitae ΩCL ad eclipticam $= \varphi$ atque loco latitudinis ψ has duas quantitates, π et φ definire oportebit.

§. 8. Tertia ergo aequatio in duas dispertietur, ad quas inveniendas ex M et L ad lineam nodorum $C\Omega$ ducantur normales MN et LN , eritque angulus $LN\dot{M}$ [mensura inclinationis orbitae ad eclipticam]; ideoque $LN\dot{M} = \varphi$. Tum vero ob angulum $\Omega CM = \phi - \pi$ et $CM = x$ erit:

$$CN = x \cos(\phi - \pi) \text{ et } MN = x \sin(\phi - \pi)$$

hinc elicietur $ML = x \tan \varphi \sin(\phi - \pi)$, unde prodit $\tan MCL = \tan \psi = \tan \varphi \sin(\phi - \pi)$, quae formula inseruit latitudini ψ ex cognita inclinatione φ et loco nodi eiusue longitudine π inveniendae, si quidem iam cognita fuerit longitudo sideris ϕ . Quoniam autem sidus elemento temporis dt in eodem plano manet, in differentiatione formulae $\tan \psi = \tan \varphi \sin(\phi - \pi)$,

B 3

quanti-

quantitates π et φ tanquam constantes spectari poterunt, eritque idcirco.

$$\frac{d\psi}{\cos\psi^2} = d\varphi \tan\varphi \cos(\varphi - \pi)$$

§. 9. Interim tamen nihil impedit, quominus in eadem differentiatione quantitates π et φ tanquam variables tractemus, quales reuera esse possunt successu temporis; vnde orietur haec aequatio:

$$\frac{d\psi}{\cos\psi^2} = \frac{d\varphi}{\cos\varphi^2} \sin(\varphi - \pi) + (d\varphi - d\pi) \tan\varphi \cos(\varphi - \pi)$$

hicque valor ipsius $\frac{d\psi}{\cos\psi^2}$ collatus cum praecedente praebit hanc aequalitatem:

$$\frac{d\varphi}{\cos\varphi^2} \sin(\varphi - \pi) = d\pi \tan\varphi \cos(\varphi - \pi)$$

$$\text{vnde obtinemus } \frac{d\varphi}{\sin\varphi \cos\varphi} = \frac{d\pi \cos(\varphi - \pi)}{\sin(\varphi - \pi)} = \frac{d\pi}{\tan(\varphi - \pi)}$$

Cum iam fit $\frac{d\varphi}{\sin\varphi \cos\varphi} = \frac{d. \tan\varphi}{\tan\varphi} = d. \frac{1}{\tan\varphi}$, erit

$$d. \frac{1}{\tan\varphi} = \frac{d\pi}{\tan(\varphi - \pi)}$$

ex quo, si longitudo nodi iam fuerit reperta, sine labore inclinatio ad eclipticam φ investigari poterit.

§. 10. Differentiemus formulam primo inuentam

$$\frac{d\psi}{\cos\psi^2} = d\varphi \tan\varphi \cos(\varphi - \pi)$$

$$\text{iterum, et cum fit } d. \tan\varphi = \frac{d\pi \tan\varphi \cos(\varphi - \pi)}{\sin(\varphi - \pi)}$$

erit:

erit:

$$\frac{d d\psi}{\cos\psi^2} + \frac{2d\psi^2 \sin\psi}{\cos\psi^3} = d d\varphi \tan\varphi \cos(\varphi - \pi) + \frac{d\varphi d\pi \tan\varphi \cos(\varphi - \pi)^2}{\sin(\varphi - \pi)} - d\varphi(d\varphi - d\pi) \tan\varphi \sin(\varphi - \pi)$$

$$\text{feu } \frac{d d\psi}{\cos\psi^2} + \frac{2d\psi^2 \sin\psi}{\cos\psi^3} = d d\varphi \tan\varphi \cos(\varphi - \pi) + \frac{d\varphi d\pi \tan\varphi}{\sin(\varphi - \pi)} - d\varphi^2 \tan\varphi \sin(\varphi - \pi)$$

qui valores pro ψ in tertia aequatione superiori substituti suppeditabunt:

$$d d x \tan\varphi \sin(\varphi - \pi) + 2 d x d\varphi \tan\varphi \cos(\varphi - \pi) + x d d\varphi \tan\varphi \cos(\varphi - \pi) + \frac{x d\varphi d\pi \tan\varphi}{\sin(\varphi - \pi)} - x d\varphi^2 \tan\varphi \sin(\varphi - \pi) = -\frac{1}{2} R d t^2$$

quae transmutatur in hanc:

$$(d d x - x d d\varphi) \tan\varphi \sin(\varphi - \pi) + (2 d x d\varphi + x d d\varphi) \tan\varphi \cos(\varphi - \pi) + \frac{x d\varphi d\pi \tan\varphi}{\sin(\varphi - \pi)} = -\frac{1}{2} R d t^2$$

§. 11. Commode, hic evenit ut in ista formula illae ipsae expressiones differentio-differentiales $d d x - x d d\varphi$ et $2 d x d\varphi + x d d\varphi$ occurrant, quae ex actione duarum reliquarum virium sunt enatae; vnde si formularum harum valores aequivalentes $-\frac{1}{2} V d t^2$ et $-\frac{1}{2} T d t^2$ substituamus, impetrabimus

$$-\frac{1}{2} V d t^2 \tan\varphi \sin(\varphi - \pi) - \frac{1}{2} T d t^2 \tan\varphi \cos(\varphi - \pi) + \frac{x d\varphi d\pi \tan\varphi}{\sin(\varphi - \pi)} = -\frac{1}{2} R d t^2$$

qua differentiale $d\pi$, quo promotio elementaris lineae nodorum indicatur, ita determinabitur, ut sit

$$d\pi = \frac{1}{2} d t^2 \cdot \frac{\sin(\varphi - \pi)}{x d\varphi} (V \sin(\varphi - \pi) + T \cos(\varphi - \pi) - \frac{R}{\tan\varphi})$$

Deinde

Deinde cum sit $d. / \text{tang } \varphi = \frac{d\pi}{\text{tang}(\varphi - \pi)}$; erit

$$d. / \text{tang } \varphi = \frac{1}{2} d\pi^2 \cdot \frac{\cos(\varphi - \pi)}{\pi d\varphi} (V \sin(\varphi - \pi) + T \cos(\varphi - \pi) - \frac{R}{\text{tang } \varphi})$$

Duas ergo has aequationes loco superioris tertiae, ex qua latitudo ψ inueniri debebat, in calculum introduci conueniet; inuentis enim π et φ erit $\text{tang } \psi = \text{tang } \varphi \sin(\varphi - \pi)$.

§. 12. Hinc patet lineam nodorum nunquam esse mobilem, quin simul inclinatio φ variationi sit obnoxia. Eadem enim vis $V \sin(\varphi - \pi) + T \cos(\varphi - \pi) - \frac{R}{\text{tang } \varphi}$, quae lineae nodorum motum imprimit eius longitudinem π immutando, simul in inclinatione φ variationem generat. Nulli autem plane immutationi tam linea nodorum, quam inclinatio erunt subiectae, si vis illa euanescat, quod euenit si media directio omnium virium corpus L sollicitantium in ipsum planum ΩCL , in quo corpus semel moueri coepit, perpetuo incidat; hicque est casus, quo corpus continuo in eodem plano moueri pergit. Generatim ergo corporis a tribus viribus V, T, R sollicitati motus quatuor sequentibus aequationibus determinatur.

$$\text{I. } 2 dx d\varphi + x dd\varphi = -\frac{1}{2} T d\pi^2$$

$$\text{II. } d d\pi - \pi d\varphi^2 = -\frac{1}{2} V d\pi^2$$

$$\text{III. } d\pi = \frac{1}{2} d\pi^2 \cdot \frac{\sin(\varphi - \pi)}{\pi d\varphi} (V \sin(\varphi - \pi) + T \cos(\varphi - \pi) - \frac{R}{\text{tang } \varphi})$$

$$\text{IV. } d. / \text{tang } \varphi = \frac{d\pi}{\text{tang}(\varphi - \pi)}$$

quas ergo quouis casu oblato resolui oportet.

CAPUT

CAPUT II.

INVESTIGATIO VIRIUM LUNAM SOLLICITANTIUM.

§. 13.

Cum Lunae motus, qualis ex centro terrae spectaretur, definiri debeat, sit C terrae centrum, ad quod etiam praecipua vis, qua Luna vrgetur, directa concipitur; atque tabula exhibeat planum eclipticae, in quo nunc quidem Sol existat in S, Luna vero supra hoc planum versetur in L latitudinem habens borealem, vnde ad planum eclipticae perpendicularum demittatur LM. Hinc ductis rectis CL, CM, CS, itemque CQ initium arietis versus, vnde longitudines computari solent, fiant sequentes denominationes.

Fig. 2.

1. Longitudo Solis seu angulus ACS = θ
2. Longitudo Lunae seu angulus ACM = φ
3. Latitudo Lunae seu angulus MCL = ψ
4. Distantia Solis a Terra CS = γ
5. Distantia Lunae a Terra curtata CM = π

§. 14. Sit iam AM projectio orbitae lunaris in planum eclipticae; ac planum, in quo Luna nunc mouetur, per centrum terrae ductum, planum eclipticae intersectet secundum rectam C Ω , quae lineam nodorum pro tempore praesenti exhibebit: ac terminus Ω quidem nodum ascendentem referet, siquidem lunam secundum regionem AM promoueri ponamus. Quod si ergo porro vocemus

C

6. Lon-

6. Longitudinem nodi asc: $AC\Omega = \pi$

7. Incl. orbitae Lunae ad eclipticam $= \varphi$

hinc latitudo Lunae geocentrica ita definietur, vt sit
 $\tan \psi = \tan. \sin (\varphi - \pi)$. Vnde incrementum latitudinis $d\psi$ commodè assignabitur, cum sit, vt supra vidimus

$$d. \tan \psi = \frac{d\psi}{\cos \psi^2} = d\varphi \tan \varphi \cos (\varphi - \pi) : \text{ac}$$

praeterea ex motu nodi cognito variatio inclinationis ita

$$\text{definietur, vt sit } d. \tan \varphi = \frac{d\pi \tan \varphi}{\tan (\varphi - \pi)}, \text{ seu } d\varphi = \frac{d\pi \sin \varphi \cos \varphi}{\tan (\varphi - \pi)}.$$

§. 15. Cum nunc primum Luna ad centrum terrae C secundum directionem LC attrahatur, fit haec vis $= M$. Deinde fit vis, qua Luna ad solem S vrgetur secundum LS $= N$; atque his duabus viribus Luna proprie vrgeri censenda est. Praeterea vero cum Terra ipsa, ad quam motum Lunae referimus, in motu versetur, ut eam tanquam quiescentem considerare queamus, non solum motum Terrae, sed etiam vires, quibus Terra vrgetur, toti mundo secundum plagas oppositas imprimi concipiamus. Sit igitur vis qua Terra ad Solem vrgetur $= S$, & vis qua ad Lunam trahitur $= L$, his viribus contrario modo in lunam translatis, Luna sequentibus viribus sollicitata habebitur

1. Secundum directionem LC vi $= M + L$

2. Secundum directionem LS vi $= N$

3. Secundum directionem MR ipsi

SC parallelam vi $= S$.

§. 16. Hunc

, vt sit
 1 latitu-
 pra vi-
 $(\pi) : \text{ac}$

onis ita
 in $\varphi \cos \varphi$
 $g(\varphi - \pi)$

n terrae
 aec vis
 etur se-
 ma pro-
 rra ipsa,
 etur, ut
 non so-
 a vrge-
 imprimi
 n vrge-
 his vi-
 sequenti-

L

. Hunc

§. 16. Nunc primo has vires ad ternas directiones supra assumptas MC, MQ et LM reducamus; ac primo quidem vis $M + L$ dabit

$$\text{pro directione MC vim} = (M + L) \cos \psi$$

$$\text{pro directione LM vim} = (M + L) \sin \psi$$

Secunda vis N vero dabit

$$\text{pro directione LM vim} = \frac{LM}{LS} \cdot N$$

$$\text{pro directione MS vim} = \frac{MS}{LS} \cdot N$$

at haec viterius resoluta ducta Mr ipsi CS parallela dabit:

$$\text{pro directione MC vim} = \frac{MC}{LS} \cdot N$$

$$\text{pro directione Mr vim} = \frac{CS}{LS} \cdot N$$

Haec postrema a vi tertia S subtracta relinquet vim $S - \frac{CS}{LS} \cdot N$, qua Luna secundum directionem MR sollicitatur, quae ob angulum $CMR = SCM = \varphi - \theta$ dabit

$$\text{pro directione MC vim} = (S - \frac{CS}{LS} \cdot N) \cos (\varphi - \theta)$$

$$\text{pro directione Mq vim} = (S - \frac{CS}{LS} \cdot N) \sin (\varphi - \theta)$$

vbi directio Mq contraria est directione MQ.

§. 17. His iam viribus cum ternis initio assumptis V, T et R comparandis, inueniemus pro his viribus sequentes valores:

C 2

1. pro-

1. pro directione MC vim V =

$$(M+L) \cos \psi + \frac{MC}{LS} \cdot N + (S - \frac{CS}{LS} \cdot N) \cos(\varphi - \theta)$$

2. pro directione MQ vim T = $-(S - \frac{CS}{LS} \cdot N) \sin(\varphi - \theta)$

3. pro directione LM vim R = $(M+L) \sin \psi + \frac{LM}{LS} \cdot N$.

Cum nunc sit CM = x, CS = y, et angulus SCM = $\varphi - \theta$; erit MS = $\sqrt{(xx - 2xy \cos(\varphi - \theta) + yy)}$, et ob LM = x tang ψ erit LS = $\sqrt{(yy - 2xy \cos(\varphi - \theta) + xx \sec^2 \psi)}$, quae distantia Solis a Luna LS breuitatis gratia ponatur = u, vt sit u = $\sqrt{(yy - 2xy \cos(\varphi - \theta) + xx \sec^2 \psi)}$. His ergo valoribus introductis erunt vires nostrae:

$$1. V = (M+L) \cos \psi + \frac{Nx}{u} + S \cos(\varphi - \theta) - \frac{Ny}{u} \cos(\varphi - \theta)$$

$$2. T = -S \sin(\varphi - \theta) + \frac{Ny}{u} \sin(\varphi - \theta).$$

$$3. R = -(M+L) \sin \psi + \frac{Nx \tan \psi}{u}$$

§. 18. Quia nunc est tang $\psi = \tan \varphi \sin(\varphi - \pi)$ et sin $\psi = \tan \varphi \cos \psi$ erit:

$$\frac{R}{\tan \varphi} = (M+L) \cos \psi \sin(\varphi - \pi) + \frac{Nx \sin(\varphi - \pi)}{u}$$

tum vero habebitur

$$V \sin(\varphi - \pi) + T \cos(\varphi - \pi) = (M+L) \cos \psi \sin(\varphi - \pi) + \frac{Nx \sin(\varphi - \pi)}{u}$$

$$+ S \cos(\varphi - \theta) \sin(\varphi - \pi) - \frac{Ny \cos(\varphi - \theta) \sin(\varphi - \pi)}{u}$$

$$- S \sin(\varphi - \theta) \cos(\varphi - \pi) + \frac{Ny \sin(\varphi - \theta) \cos(\varphi - \pi)}{u}$$

quae ob $\cos(\varphi - \theta) \sin(\varphi - \pi) - \sin(\varphi - \theta) \cos(\varphi - \pi) = \sin(\varphi - \pi)$, dat
V sin

$$V \sin(\varphi - \pi) + T \cos(\varphi - \pi) - \frac{R}{\tan \varphi} = S \sin(\varphi - \pi) - \frac{Ny}{u} \sin(\varphi - \pi)$$

ex quo aequationes motum Lunae continentes erunt:

$$I. 2dx d\varphi + x dd\varphi = -\frac{1}{2} dt^2 \left(\frac{Ny}{u} - S \right) \sin(\varphi - \theta)$$

$$II. ddx - x d\varphi^2 = -\frac{1}{2} dt^2 \left((M+L) \cos \psi + \frac{Nx}{u} - \left(\frac{Ny}{u} - S \right) \cos(\varphi - \theta) \right)$$

$$III. d\pi = -\frac{1}{2} dt^2 \cdot \frac{\sin(\varphi - \pi) \sin(\varphi - \pi)}{x d\varphi} \left(\frac{Ny}{u} - S \right)$$

$$IV. d. \tan \varphi = \frac{dx}{\tan(\varphi - \pi)}$$

vbi $\theta - \pi$ exprimit angulum Ω CS seu distantiam Solis a nodo ascendente.

§. 19. Jam secundum Theoriam Newtoni, si massam Terrae ponamus = δ ac Lunae = ϵ , ob distantiam

$$CL = \frac{x}{\cos \psi}, \text{ foret vis } M = \frac{\delta \cos \psi^3}{x x}, \text{ et vis } L = \frac{\epsilon \cos \psi^3}{x x},$$

sicque vis tota $M+L = (\delta + \epsilon) \cdot \frac{\cos \psi^3}{x x}$. Quo au-

tem, si forte haec Theoria insufficiens deprehendatur, rem generalius complectarur, ponamus hanc vim:

$$M+L = (\delta + \epsilon) \cos \psi^2 \left(\frac{1}{xx} - \frac{1}{bb} \right)$$

vbi terminus $\frac{1}{bb}$ defectum huius vis a Theoria Newtoniana exhibeat; qui cum sit minimus, pro constanti haberi poterit saltem pro exigua variabilitate, quam distantia x subit. Vim autem Solis exacte Theoriae Newtonianae conformem assumere poterimus; quoniam etiam si inde recederet, differentia non solum foret
C 3 quam

cos($\varphi - \theta$)

($\varphi - \theta$)

$\frac{M}{S} \cdot N$.

= $\varphi - \theta$;

tang ψ

distantia

vt sit

ergo va-

cos($\varphi - \theta$)

- π) et

π)

($\varphi - \pi$)

($\varphi - \pi$)

($\varphi - \pi$)

r), dat

V sin

quam minima, sed quia pro Luna aequè discreparet ac pro Terra, in nostris formulis nullius plane esset momenti.

§. 20. Posita ergo Solis massa $= \odot$, erit vis, qua Terram ad se attrahit $S = \frac{\odot}{y^2}$, vis autem qua Lunam ad se trahit $N = \frac{\odot}{u^2}$. His ergo valoribus virium in calculum inductis, motus lunae ex quatuor sequentibus aequationibus determinari debet:

$$I. 2dx d\varphi + x dd\varphi = -\frac{1}{2} dt^2 \left(\frac{\odot y}{u^3} - \frac{\odot}{y^2} \right) \sin(\varphi - \theta)$$

$$II. ddx - x dd\varphi^2 = -\frac{1}{2} dt^2 (\delta + \epsilon) \cos^2 \psi^3 \left(\frac{1}{xx} - \frac{1}{bb} \right) - \frac{1}{2} dt^2 \left(\frac{\odot x}{u^3} - \frac{\odot y}{u^3} \cos(\varphi - \theta) + \frac{\odot}{yy} \cos(\varphi - \theta) \right)$$

$$III. d\pi = -\frac{1}{2} dt^2 \cdot \frac{\sin(\varphi - \pi) \sin(\theta - \pi)}{x d\varphi} \left(\frac{\odot y}{u^3} - \frac{\odot}{yy} \right)$$

$$IV. d. l \tan g \varphi = \frac{d\pi}{\tan g(\varphi - \pi)}$$

Hic iam primum curandum est, vt elementum temporis, quod est quantitas heterogenea, ex calculo eliminemus; id quod commodissime fiet, si motum medium solis vtpote tempori proportionalem, loco temporis in calculum introducemus.

§. 21. Cum igitur etiam motus Solis in his aequationibus sit ratio habenda, eum prius inuestigemus: et quoniam pro terra quiescente Sol a sola vi $\frac{\odot}{yy}$ ad ter-

ram

ram sollicitari concipiendus est, si formulas pro luna inuenras ad solem accomodemus, obtinebimus:

$$2 dy d\theta + y dd\theta = 0$$

$$ddy - y d\theta^2 = -\frac{1}{2} dt^2 \cdot \frac{\odot}{yy}$$

si iam distantiam Solis a terra mediam ponamus $= b$ ejusque anomaliam mediam $= q$; casu quo excentricitas orbitae solaris esset nulla, foret semper $y = b$ & $d\theta = dq$: vnde altera aequatio dabit $-b dq^2 = -\frac{1}{2} dt^2 \cdot \frac{\odot}{bb}$. Quare loco elementi temporis dt elementum anomaliae mediae solis ita in calculum introduci debet, vt vbique loco $\frac{1}{2} dt^2$ scribatur $\frac{b^3 dq^2}{\odot}$, id quod tam in his formulis pro Sole, quam in superioribus pro Luna fieri poterit.

§. 22. Cum iam b denotet distantiam solis a terra mediam, sit eius vera distantia $y = b\omega$, et anomalia eius vera $= s$, erit $d\theta = ds$, quandoquidem a motu apogei solis animum abstrahimus. Hinc itaque erit

$$2 d\omega ds + \omega dd s = 0$$

$$dd\omega - \omega ds^2 = -\frac{dq^2}{\omega\omega},$$

quarum prior integrata dat $\omega\omega ds = C dq$ ob dq constans, ideoque $\omega ds^2 = \frac{CC dq^2}{\omega^3}$; qui valor in altera aequatione substitutus praebet,

$$dd\omega = \frac{CC dq^2}{\omega^3} - \frac{dq^2}{\omega\omega}.$$

quae

quae per $2d\omega$ multiplicata et integrata dat:

$$\frac{d\omega^2}{dq^2} = D - \frac{CC}{\omega\omega} + \frac{2}{\omega}$$

$$\text{vnde fit } dq = \frac{\omega d\omega}{V(D\omega\omega + 2\omega - CC)}$$

$$\text{ac proinde } ds = \frac{C d\omega}{\omega V(D\omega\omega + 2\omega - CC)}$$

§. 23. Quoniam autem hinc valores finiti haud difficulter deduci possent, tamen alia vtar methodo, quae in motu Lunae maiorem praestabit utilitatem. Inuento autem $\omega\omega ds = C dq$, alteram aequationem ita transformo, ut elementi constantis dq ratio non amplius habeatur:

$$dq \cdot d \cdot \frac{d\omega}{dq} - \omega ds^2 = - \frac{dq^2}{\omega\omega}$$

Sit nunc $\omega = \frac{1}{u}$, ut habeat $ds = C u u dq$, et $d\omega$

$$= - \frac{du}{u^2}, \text{ et ob } dq = \frac{ds}{C u u} \text{ erit } \frac{d\omega}{dq} = - \frac{C du}{ds}; \text{ hinc}$$

sumto iam elemento ds constante, erit

$$\frac{-d's}{C u u} \cdot \frac{C ddu}{ds} - \frac{ds^2}{u} = - \frac{ds^2}{C C u u} \text{ seu}$$

$$ddu + u ds^2 = \frac{ds^2}{CC}$$

vnde statim elicitur $u = \frac{1 - e \cos s}{CC}$, vbi e excentricitatem orbitae solaris indicabit.

§. 24. Hinc porro habebitur $\omega = \frac{CC}{1 - e \cos s}$, et $y = \frac{CCb}{1 - e \cos s}$, anomalia vera s ab apogeo computata; vnde distantia apogei

apogei a terra posito $s = 0$ erit $= \frac{CCb}{1 - e}$, et distantia pe-

rigei posito $s = 180^\circ$ prodit $= \frac{CCb}{1 + e}$; sicque distantia me-

media fiet $= \frac{CCb}{1 - e^2}$, quae cum per hypothesin aequalis esse

debeat ipsi b , statui oportet $CC = 1 - e^2$: hincque erit

$$y = \frac{b(1 - e^2)}{1 - e \cos s} \text{ et } \omega = \frac{1 - e^2}{1 - e \cos s}$$

Porro autem aequatio $\omega\omega ds = C dq = dq V(1 - e^2)$ abit in hanc:

$$dq = \frac{(1 - e^2)^{\frac{3}{2}} ds}{(1 - e \cos s)^2} \text{ et } q = \int \frac{(1 - e^2)^{\frac{3}{2}} ds}{(1 - e \cos s)^2}$$

ex qua, uti satis constat, vel data anomalia vera s inueniri potest anomalia media q , vel vicissim. His itaque formulis motum Solis continentibus in determinatione motus Lunae utamur.

§. 25. Primo ergo loco $\frac{1}{2} ds^2$ vbique scribamus $\frac{1}{2} ds^2$ et $b\omega$ loco y , quo facto nostrae aequationes fient

$$\text{I. } 2 dx d\varphi + x dd\varphi = -b^2 dq^2 \left(\frac{b\omega}{u^2} - \frac{1}{bb\omega u} \right) \sin(\varphi - \theta)$$

$$\text{II. } ddx - x d\varphi^2 = - \frac{(b + C) b^2 dq^2}{C} \cos \psi^2 \left(\frac{1}{x^2} - \frac{1}{bb} \right) - b^2 dq^2 \left(\frac{x}{u^3} - \frac{b\omega \cos(\varphi - \theta)}{u^3} + \frac{\cos(\varphi - \theta)}{bb\omega u} \right)$$

$$\text{III. } dx = -b^2 dq^2 \frac{\sin(\varphi - \pi) \sin(\theta - \pi)}{x d\varphi} \left(\frac{b\omega}{u^3} - \frac{1}{bb\omega u} \right)$$

Ponatur porro $u = bv$, adque in calculum quoque introduca-

ducatur distantia media lunae a terra, quae sit $=a$, positoque $x=az$, prodibit:

$$I. 2dzd\phi + 2dd\phi = -\frac{bdq^2}{a} \left(\frac{\omega}{v^3} - \frac{1}{\omega\omega} \right) \sin(\phi - \theta)$$

$$II. ddz - 2d\phi^2 = -\frac{(\xi + \zeta)b^3}{a^3} dq^2 \cos\psi^3 \left(\frac{1}{zz} - \frac{aa}{bb} \right) \\ - \frac{z dq^2}{v^3} + \frac{b\omega dq^2 \cos(\phi - \theta)}{av^3} - \frac{bdq^2 \cos(\phi - \theta)}{a\omega\omega}$$

$$III. d\pi = -\frac{bdq^2}{azd\phi} \sin(\phi - \pi) \sin(\theta - \pi) \left(\frac{\omega}{v^3} - \frac{1}{\omega\omega} \right)$$

§. 26. Ponamus nunc ad abbreviandum:

$$\frac{(\xi + \zeta)b^3}{a^3} = m; \quad \frac{(\xi + \zeta)b^3}{a^3 b b} = \mu, \text{ seu } \mu = \frac{maa}{bb}$$

quarum litterarum valores m et μ per observationes defini debent; tum vero sit $\frac{a}{b} = v$, quae est quantitas valde parua a parallaxi solis pendens. Hisque valoribus introductis, aequationes nostrae sequentes induent formas:

$$I. 2dzd\phi + 2dd\phi = -\frac{1}{v} dq^2 \left(\frac{\omega}{v^3} - \frac{1}{\omega\omega} \right) (\sin \phi - \theta)$$

$$II. ddz - 2d\phi^2 = -\frac{mdq^2 \cos\psi^3}{zz} + \mu dq^2 \cos\psi^3$$

$$\left(\frac{z dq^2}{v^3} + \frac{1}{v} dq^2 \left(\frac{\omega}{v^3} - \frac{1}{\omega\omega} \right) \cos(\phi - \theta) \right)$$

$$III. d\pi = -\frac{dq^2}{vz d\phi} \sin(\phi - \pi) \sin(\theta - \pi) \left(\frac{\omega}{v^3} - \frac{1}{\omega\omega} \right)$$

$$IV. d / \text{tang } \phi = \frac{d\pi}{\text{tang}(\phi - \pi)}$$

§. 27.

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$\cos(\phi - \theta)$
 $\frac{\cos(\phi - \theta)}{a\omega\omega}$

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§. 27.

§. 27. Cum iam posuerimus:

$$x = az; \quad y = b\omega; \quad v = \frac{a}{b} \quad \text{et} \quad \theta = \frac{a^2}{b}$$

erit $u = V(b\omega\omega - 2ab\omega z \cos(\phi - \theta) + aaz \sec \psi^2)$ atque $v = V(\omega\omega - 2v\omega z \cos(\phi - \theta) + vvz \sec \psi^2)$ vbi notandum est quantitates ω , q et s ex motu solis ita inter se pendere, vt sit

$$\omega = \frac{1 - ee}{1 - e \cos r} \quad \text{et} \quad dq = \frac{(1 - ee)^{\frac{3}{2}} ds}{(1 - e \cos r)^2} = \frac{\omega \omega ds}{V(1 - ee)}$$

ita vt huic ad datum quoduis tempus, tam valor ipsius ω quam anomaliae verae s definiri possit. Modum autem has formulas ad calculum reuocandi hic non trado, quia eum alias fufius iam exposui: hoc solum hic notari conueniet, excentricitatis orbitae solaris valorem ex observationibus colligi $e = 0,01680$.

§. 28. Nunc antequam vltius progredi queamus, valorem irrationalem ipsius v tolli conueniet, quod facile per seriem praestabitur maxime conuergentem, ob v fractionem valde parua; sumta enim parallaxi solis $= 12''$, quia parallaxis lunae media est $= 3380''$, erit $\frac{a}{b} = v$

$= \frac{1}{3380} = \frac{1}{3380}$. Hinc sufficit seriei illius conuergentis, quam reperiemus, aliquot tantum terminos ab initio assumisse; quia reliqui ob paruitatem continuo magis crescentem tuto omitti poterunt. Cum autem angulus $\phi - \theta$, qui distantiam solis a luna secundum longitudinem denotat, in hac resolutione frequentissime occurreret, breuitatis gratia ponamus $\phi - \theta = \eta$

ita vt pro v sequentem habeamus valorem irrationalem

$$v = V(\omega\omega - 2v\omega z \cos \eta + vvz \sec \psi^2).$$

D 2

§. 29.

§. 29. Quoniam ergo in nostris formulis occurrit

$$\frac{1}{\omega^3} \text{ ob } \frac{1}{\omega^3} = (\omega \omega - 2\nu \omega z \cos \eta + \nu \nu z z \sec \psi^2)^{-\frac{3}{2}}, \text{ nascimur}$$

$$\frac{1}{\omega^3} = \frac{1}{\omega^3} + \frac{3\nu z \cos \eta}{\omega^4} - \frac{3\nu \nu z z \sec \psi^2}{2\omega^5} + \frac{15\nu \nu z z}{2\omega^5} \cos \eta^2$$

ubi terminos altiores ipsius ν potestates inuoluentes sine haesitatione reicere possumus; in ipsis aequationibus autem tantum in prima ipsius ν potestate subsistemus. Habebimus ergo:

$$\frac{1}{\nu} \left(\frac{\omega}{\omega^3} - \frac{1}{\omega \omega} \right) = \frac{3z \cos \eta}{\omega^3} + \frac{3\nu z z}{2\omega^4} (5 \cos \eta^2 - \sec \psi^2) \text{ seu}$$

$$\frac{1}{\nu} \left(\frac{\omega}{\omega^3} - \frac{1}{\omega \omega} \right) = \frac{3z \cos \eta}{\omega^3} + \frac{3\nu z z}{4\omega^4} (5 + 5 \cos 2\eta - 2 \sec \psi^2)$$

hincque porro:

$$\frac{1}{\nu} \left(\frac{\omega}{\omega^3} - \frac{1}{\omega \omega} \right) \sin(\phi - \theta) = \frac{3z \sin 2\eta}{2\omega^3} + \frac{3\nu z z}{8\omega^4} (5 \sin \eta + 5 \sin 3\eta - 4 \sin \eta \sec \psi^2)$$

$$\frac{1}{\nu} \left(\frac{\omega}{\omega^3} - \frac{1}{\omega \omega} \right) \cos(\phi - \theta) = \frac{3z}{2\omega^3} (1 + \cos 2\eta) + \frac{3\nu z z}{8\omega^4} (5 \cos \eta + 5 \cos 3\eta - 4 \cos \eta \sec \psi^2)$$

$$\text{atque } \frac{z}{\omega^3} = \frac{z}{\omega^3} + \frac{3\nu z z}{\omega^4} \cos \eta$$

§. 30. Substituamus hos valores in nostris aequationibus atque obtinebimus:

$$\text{I. } 2dzd\phi + zd\phi^2 = -dq^2 \left(\frac{3z \sin 2\eta}{2\omega^3} + \frac{3\nu z z}{8\omega^4} (5 \sin \eta + 5 \sin 3\eta - 4 \sin \eta \sec \psi^2) \right)$$

$$\text{II. } ddz - zd\phi^2 = -\frac{mdq^2 \cos \psi^3}{z z} + \frac{\mu dq^2 \cos \psi^3}{z} + \frac{2dq^2}{2\omega^3} + \frac{3z dq^2}{2\omega^3} \cos 2\eta + \frac{3\nu z z dq^2}{8\omega^4} (7 \cos \eta + 5 \cos 3\eta - 4 \cos \eta \sec \psi^2)$$

III. $d\pi$

s occurrit

nascimur

$$\frac{\nu z z}{\omega^5} \cos \eta^2$$

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$\frac{dq^2}{\omega^3} \cos 2\eta$

ec. ψ^2)

III. $d\pi$

$$\text{III. } d\pi = -\frac{dq^2}{2d\phi} \sin(\phi - \pi) \sin(\theta - \pi) \left(\frac{3z \cos \eta}{\omega^3} + \frac{3\nu z z}{4\omega^4} (5 + 5 \cos 2\eta - 2 \sec \psi^2) \right)$$

$$\text{IV. } d \tan \phi = \frac{d\pi}{\tan(\phi - \pi)}.$$

Hic iam observare licet, cum angulus ψ nunquam fere 5° superet, eiusque secans nonnisi in terminis iam per ν multiplicatis, ac propterea respectu reliquorum valde parvis occurrat, sine ullius erroris sensibilis metu in his terminis poni posse sec. $\psi = 1$.

§. 31. Deinde ut etiam ex maioribus terminis $\cos \psi$ eliminemus; consideremus formulam $\tan \psi = \tan \phi$

$$\sin(\phi - \pi), \text{ eritque } \sec \psi = \frac{1}{\cos \psi} = \sqrt{1 + \tan^2 \phi \sin^2(\phi - \pi)}$$

Hinc ergo habebimus:

$$\cos \psi^3 = (1 + \tan^2 \phi \sin^2(\phi - \pi))^{-\frac{3}{2}}$$

et cum $\tan \phi^2$ nunquam fere fractionem $\frac{1}{177}$ superet erit satis exacte:

$$\cos \psi^3 = 1 - \frac{3}{2} \tan^2 \phi \sin^2(\phi - \pi) \text{ vel etiam}$$

$$\cos \psi^3 = 1 - \frac{3}{2} \tan^2 \phi + \frac{3}{2} \tan^2 \phi \cos 2(\phi - \pi)$$

qui valor pro $\cos \psi^3$ in termino maiore $\frac{mdq^2 \cos \psi^3}{z z}$ sub-

stitui potest: in altero autem termino $\mu dq^2 \cos \psi^3$ quia per se est valde parvus, atque adeo secundum Theoriam Newtoni evanesceret, nihil impedit, quo minus loco $\cos \psi^3$ scribamus unitatem.

D 3

§. 31. Hoc

§. 32. Hoc ergo modo si aequationes nostras a consideratione latitudinis Lunae ψ liberemus ad sequentes perueniemus aequationes:

$$I. 2dzd\Phi + zdd\Phi = -dq^2 \left(\frac{3z\sin 2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4} (\sin \eta + 5\sin 3\eta) \right)$$

$$II. ddx - zd\Phi^2 = -\frac{mdq^2}{z^2} \left(1 - \frac{3}{2}\text{tange}^2 + \frac{3}{2}\text{tange}^2 \cos 2(\Phi - \pi) \right) + \mu dq^2 \\ + \frac{z dq^2}{2\omega^3} + \frac{3z dq^2}{2\omega^3} \cos 2\eta + \frac{3vzzdq^2}{8\omega^4} (3\cos \eta + 5\cos 3\eta)$$

$$III. d\pi = \frac{dq^2}{z d\Phi} \sin(\Phi - \pi) \sin(\theta - \pi) \left(\frac{3z \cos \eta}{\omega^3} + \frac{3vzz}{4\omega^4} (3 + 5\cos 2\eta) \right)$$

$$IV. d. l \text{ tang } \varphi = \frac{d\pi}{\text{tang}(\Phi - \pi)}.$$

Nunc igitur in hoc erit incumbendum, ut ex his quatuor aequationibus omnia motus phaenomena, quae in Luna secundum Theoriam adesse debent, sollicitè eruantur, atque tum cum observationibus conferantur.

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CAPUT III.

INTRODUCTIO ANOMALIAE VERAЕ
LUNAE IN PRAECEDENTES AEQUATIONES.

§. 33.

Quoniam nostra quaestio circa Lunam versatur, loco anomaliae mediae solis, quam pro tempore in calculum introduximus, magis conueniet motu Lunae medio vti, qui itidem tempori est proportionalis. Verum ex sequentibus patebit calculum commodiorem reddi, si loco motus medii adhibeamus anomaliā Lunae mediam, cuius incrementa itidem temporis sunt proportionalia. Sit itaque ad datum tempus anomalia media Lunae = p ; et cum eius incrementum dp ad incrementum anomaliae mediae solis eodem tempusculo acceptum datum ac per observationes cognitam teneat rationem, ponamus $dp = ndq$. Tabulae autem Astronomicae pro intervallo 365 dierum praebent: Motum anomaliae mediae Solis 11°, 29', 44", 39" = 1295079¹¹ Motum anomaliae mediae Lunae

$$13^{\text{Rev.}} 2^{\circ}, 28', 43" 13'' = 17167393^{11}$$

$$\text{vnde fit } n = \frac{17167393}{1295079} = 13, 25586$$

§. 34. Posito ergo $\frac{dp}{n}$ loco dq , aequationes nostrae erunt

$$I. 2dzd\Phi + zdd\Phi = -\frac{dp^2}{nn} \left(\frac{3z\sin 2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4} (\sin \eta + 5\sin 3\eta) \right)$$

II. ddx

$$\text{II. } ddx - zd\phi^2 = -\frac{mdp^2}{nnzz} \left(1 - \frac{1}{2} \tan^2 \varphi + \frac{1}{2} \tan^2 \varphi \cos 2(\phi - \pi)\right) + \frac{\mu dp^2}{n\pi} \\ + \frac{z dp^2}{2n^2 u^3} + \frac{3z dp^2}{2n^2 \omega^3} \cos 2\eta + \frac{3vzz dp^2}{8nn\omega^4} (3 \cos \eta + 5 \cos 3\eta)$$

$$\text{III. } d\pi = \frac{-dp^2}{nn:d\phi} \sin(\phi - \pi) \sin(\theta - \pi) \left(\frac{3z \cos \eta}{\omega^3} + \frac{3vzz}{4\omega^4} (3 + 5 \cos 2\eta) \right)$$

$$\text{IV. } d. / \tan \varphi = \frac{d\pi}{\tan(\phi - \pi)}$$

atque hic elementum dp assumptum est constans: simul autem patet terminos, qui per nn sunt diuisi, prae ceteris satis esse paruos, cum sit $nn = 175, 71795$. Quae circumstantia sequentes approximationes non mediocriter adiuuabit.

§. 35. Nunc antequam ulterius progrediamur, aequationem primam per z multiplicemus, atque integratione in priori parte instituta obtinebimus

$$zzd\phi = Cdp - \frac{dp}{nn} \int dp \left(\frac{3z^2 \sin 2\eta}{2\omega^3} + \frac{3vz^3}{8\omega^4} (\sin \eta + 5 \sin 3\eta) \right)$$

ponamus breuitatis gratia hoc membrum integrale

$$\int dp \left(\frac{3z^2}{2\omega^3} \sin 2\eta + \frac{3vz^3}{8\omega^4} (\sin \eta + 5 \sin 3\eta) \right) = S$$

quod integrale, ne introductio constantis incertitudinem pariat, ita capi assumo, ut nullum terminum mere constantem contineat, quippe qui iam in C esset comprehensus. Hoc ergo circa determinationem integrationis probe obseruato, erit $zzd\phi = dp \left(C - \frac{S}{nn} \right)$: ubi terminus S aequabilem artarum descriptionem, quam Regula Kepleri in planetis primariis infert, perturbat; est enim

$$) + \frac{\mu dp^2}{n\pi}$$

$$- 5 \cos 3\eta)$$

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Quae

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$$(\sin 3\eta)$$

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enim $\frac{1}{2} z z d\phi$ elementum areae descriptae, quod si ipsi $C dp$ esset aequale, tempori exacte esset proportionale.

§. 36. Cum igitur sit $d\phi = \frac{dp}{zz} \left(C - \frac{S}{nn} \right)$, erit $zzd\phi^2 = \frac{dp^2}{z^3} \left(CC - \frac{2}{nn} CS + \frac{1}{n^2} SS \right)$, quo valore substituto reliquae nostrae aequationes sequentes induent formas:

$$\text{II. } ddx = \frac{dp^2}{z^3} \left(CC - \frac{2}{nn} CS + \frac{1}{n^2} SS \right)$$

$$- \frac{mdp^2}{nnzz} \left(1 - \frac{1}{2} \tan^2 \varphi + \frac{1}{2} \tan^2 \varphi \cos 2(\phi - \pi) \right) + \frac{\mu dp^2}{nn} \\ + \frac{z dp^2}{2nn\omega^3} + \frac{3z dp^2}{2nn\omega^3} \cos 2\eta + \frac{3vzz dp^2}{8nn\omega^4} (3 \cos \eta + 5 \cos 3\eta)$$

$$\text{III. } d\pi = -\frac{z dp}{Cnn - S} \sin(\phi - \pi) \sin(\theta - \pi) \left(\frac{3z \cos \eta}{\omega^3} + \frac{3vzz}{4\omega^4} (3 + 5 \cos 2\eta) \right)$$

et quarta manet $d. / \tan \varphi = \frac{d\pi}{\tan(\phi - \pi)}$ vt ante.

Eo igitur pertigimus, ut inuestigari oporteat quantitates z , π et φ , quibus inuentis obtinebitur ϕ ex formula primum eruta. Cum autem sit $d\eta = d\phi - d\theta$, ob $d\theta = d\phi$ $= \frac{dqV(1-ee)}{\omega\omega} = \frac{dpV(1-ee)}{n\omega\omega}$, erit $d\eta = \frac{dp}{zz} \left(C - \frac{S}{nn} \right) - \frac{dpV(1-ee)}{n\omega\omega}$. Tum vero est vti vidimus $u = \frac{1-ee}{1-ee \cos^2 \varphi}$ vnde et huius differentiale ad dp reduci poterit.

§. 37. Si hunc calculum profsequi vellemus, tota inuestigatio tandem eo rediret, ut definiretur quantum longi-

longitudo Lunae vera ab eius longitudine media, quae ex anomalia media p haberetur, discreparet: hoc autem discrimen nonnunquam ultra 8 gradus exsurgere posset, ideoque correctiones admodum notabiles requireret. Vt igitur nobis quam minimae correctiones inuestigandae relinquantur, expediet differentiam inter locum Lunae verum, et locum corporis quod secundum regulas Kepleri in ellipsi circa Terram reuolueretur, ita tamen mobili, ut eius motus absidum cum motu apogei Lunae per obseruationes cognito conueniret. Seu quod eodem redit, quaeramus primo ex anomalia Lunae media p secundum regulas Kepleri anomaliam eius veram quae sit $= r$, vnde si longitudo apogei fuerit $= v$, quantita, $v + r$ nunquam multum ultra gradum a longitudine Lunae vera differet: vnde discrimen multo facilius inueniri poterit, si quidem debita orbitae lunaris excentricitas in calculum inducatur. Hinc loco anomaliae Lunae mediae p eius anomaliam veram, quae scilicet mediae pro excentricitate rite assumpta conueniat, in aequationes nostras inferamus.

§. 38. Tabulae quidem astronomicae excentricitatem orbitae lunaris plerumque variabilem statuunt; sed cum hic non de vera huius orbitae excentricitate quaestio sit, quam de excentricitate illius orbitae ellipticae mobilis, in qua corpus motum proxime motum Lunae referat; huius excentricitas media erit statuenda inter maximam ac minimam, quae vulgo orbitae lunari tribuuntur: vnde ista excentricitas media colligitur $= 0$, 05445. Ne autem huic conclusioni nimium fidamus genera-

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 $= 0$,
nus
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generatim hanc excentricitatem ponamus $= k$; atque anomalia vera per mediam ita determinabitur, ut sit

$$dp = \frac{(1-kk)^{\frac{3}{2}} dr}{(1-k \cos r)^2}, \text{ vel sit brevitatis gratia } \frac{1-kk}{1-k \cos r} = t,$$

ut sit $dp = \frac{tt dr}{V(1-kk)}$. Porro autem reliqua differentialia ita ad elementum dr reuocabuntur, ut sit:

$$ds = \frac{tt dr V(1-ee)}{mww V(1-kk)} = d\beta, \text{ et } d\eta = \frac{tt dr}{zz V(1-kk)} \left(C - \frac{S}{nn} \right) - \frac{tt dr V(1-ee)}{mww V(1-kk)}.$$

§. 39. Si motus Lunae cum motu huius corporis, quod imaginamur, perfecte conueniret, tum vbique foret $z = \frac{1-kk}{1-k \cos r}$ seu $z = t$: quoniam autem hi duo motus inter se non conueniunt, non erit $z = t$. Ponamus ergo esse:

$$z = tu = \frac{(1-kk)u}{1-k \cos r} \text{ seu } x = \frac{(1-kk)au}{1-k \cos r}$$

vbi primum obseruo, quantitatem u valde parum ab unitate recedere. Erit autem quantitas variabilis, quae alium terminum constantem praeter unitatem non inuolvet: nam si alium terminum constantem contineret, is in z posset comprehendi, idque indicio esset distantiam mediam z non recte esse assumptam. Habebit ergo u huiusmodi formam $1 + Z$, vbi Z ex terminis nonnisi variabilibus constabit. Praeterea autem animaduerto, hanc quantitatem Z nullum terminum huius formae $a \cos r$ complecti debere; quoniam hoc indicio esset excentricitatem k non recte esse assumptam, sed eam vel maiorem vel minorem accipi oportuisse.

E 2

§. 40. His

§. 40. His igitur notatis, quod quantitas u primo terminum constantem $= 1$ contineat, cum vero nullum terminum formae $a \cos r$ inuoluat, statuamus

$$z = tu \text{ seu } z = \frac{(1-kk)u}{1-k \cos r} \text{ posito breuitatis gratia}$$

$t = \frac{1-kk}{1-k \cos r}$. Atque cum supra elementum dp constans posuiffemus, hac conditione exuenda erit $ddz = dp \cdot d \cdot \frac{dz}{dp}$, et $\frac{ddz}{dp^2} = \frac{1}{dp} \cdot d \cdot \frac{dz}{dp}$. Divisa ergo secunda aequatione per dp^2 , erit:

$$\text{II. } \frac{1}{dp} \cdot d \cdot \frac{dz}{dp} = \frac{CC}{t^3 u^3} - \frac{2CS}{nn t^3 u^3} + \frac{SS}{n^4 t^3 u^3} - \frac{m}{nn t u} (1 - \frac{1}{2} \tan^2 \eta + \frac{1}{2} \tan^2 \eta \cos 2(\Phi - \pi)) + \frac{\mu}{nn} + \frac{t u}{2 n n \omega^3} + \frac{3 t u \cos 2 \eta}{2 n n \omega^3} + \frac{3 v t u u}{8 n n \omega^4} (3 \cos \eta + 5 \cos 3 \eta)$$

III. $d\pi = \frac{-t u dp}{C n n - S} \sin(\Phi - \pi) \sin(\theta - \pi) \left(\frac{3 t u \cos \eta}{\omega^3} + \frac{3 v t u u}{4 \omega^4} (3 + 5 \cos 2 \eta) \right)$ vbi nunc nullum differentiale assumtum est constans, sed iam pro lubitu quoduis differentiale constans assumi poterit.

§. 41. Posito autem $z = tu$ et $dp = \frac{t dr}{V(1-kk)}$ existente $t = \frac{1-kk}{1-k \cos r}$ erit primo:

$$S = \int \frac{t dr}{V(1-kk)} \left(\frac{3 t u u}{2 \omega^3} \sin 2 \eta + \frac{3 v t^3 u^3}{8 \omega^4} (\sin \eta + 5 \sin 3 \eta) \right) \text{ seu}$$

$$S = \int \frac{dr}{V(1-kk)} \left(\frac{3 t^4 u u}{2 \omega^3} \sin 2 \eta + \frac{3 v t^5 u^3}{8 \omega^4} (\sin \eta + 5 \sin 3 \eta) \right)$$

Hinc

Hinc fiet $d\Phi = \frac{dr}{u u V(1-kk)} \left(C - \frac{S}{nn} \right)$ atque

$$d\eta = \frac{dr}{u u V(1-kk)} \left(C - \frac{S}{nn} \right) - \frac{t t dr V(1-kk)}{n \omega \omega V(1-kk)}$$

Porro autem ob $dz = t du + u dt$, erit $\frac{dz}{dp} = \frac{t du + u dt}{t dr} V(1-kk)$;

at est $dz = -\frac{(1-kk)k dr \sin r}{(1-k \cos r)^2} = -\frac{k t dr \sin r}{1-kk}$; sicque fiet

$$\frac{dz}{dp} = \frac{du V(1-kk)}{t dr} - \frac{k u \sin r}{V(1-kk)}; \text{ ac posito elemento } dr \text{ constante}$$

$$\text{erit } d \cdot \frac{dz}{dp} = \frac{d du V(1-kk)}{t dr} - \frac{d u dt V(1-kk)}{t dr} - \frac{k du \sin r}{V(1-kk)} - \frac{k u dr \cos r}{V(1-kk)}$$

hincque ob $\frac{dt}{t} = -\frac{k dr \sin r}{1-kk}$ habebitur:

$$d \cdot \frac{dz}{dp} = \frac{d du V(1-kk)}{t dr} - \frac{k u dr \cos r}{V(1-kk)}$$

§. 42. Hinc iam porro obtinemus pro secunda aequatione

$$\frac{1}{dp} \cdot d \cdot \frac{dz}{dp} = \frac{(1-kk) d du}{t^3 dr^2} - \frac{k u \cos r}{t t}$$

qui valor substitutus in aequatione per $\frac{t^3}{1-kk}$ multiplicata orietur haec aequatio:

$$\text{II. } \frac{d du}{dr^2} - \frac{k u \cos r}{1-kk} = \frac{CC}{(1-kk) u^3} - \frac{2CS}{(1-kk) n n u^3} + \frac{SS}{n^4 (1-kk) u^3} - \frac{m t}{nn (1-kk) u} (1 - \frac{1}{2} \tan^2 \eta + \frac{1}{2} \tan^2 \eta \cos 2(\Phi - \pi)) + \frac{\mu t^3}{nn (1-kk)} + \frac{t^4 u}{2 n n (1-kk) \omega^3} + \frac{3 v t^5 u u}{8 n n \omega^4 (1-kk)} (3 \cos \eta + 5 \cos 3 \eta)$$

$$\text{III. } d\pi = \frac{u dt \sin(\Phi - \pi) \sin(\theta - \pi)}{(C n n - S) V(1-kk)} \left(\frac{3 t^4 \cos \eta}{\omega^3} + \frac{3 v t^5 u}{4 \omega^4} (3 + 5 \cos 2 \eta) \right)$$

Quartam

Quartam aequationem $d. / \tan \varphi = \frac{d \pi}{\tan(\Phi - \pi)}$, cum nullam mutationem subeat, superfluum foret continuo repetere.

§. 43. Conveniet autem quantitates constantes C et m , quarum valores nondum novimus, saltem vero proxime indagare, qui facilius deinceps ipsam aequationum resolutionem dirigere queamus. Perspicuum autem est, si omnes quantitates a situ solis pendentes ex calculo deleantur, tum utique fieri debere $u = 1$. Cum igitur primum S ab angulo η pendeat, terminos tam S quam η involuentes omittamus, ac pro ω quidem scribamur 1; quia tantum determinationem ad verum accedentem requirimus, quem in finem quoque inclinationem orbitae negligamus. Hinc aequatio secunda dabit:

$$\frac{k t \cos r}{1 - k k} = \frac{C C}{1 - k k} - \frac{m t}{n n (1 - k k)} + \frac{\mu t^3}{n^2 (1 - k k)} + \frac{t^4}{2 n n (1 - k k)} \text{ siue}$$

$$C C = \frac{m t}{n n} - \frac{\mu t^3}{n n} - k t \cos r - \frac{t^4}{2 n n}$$

Cum autem sit $t = \frac{1 - k k}{1 - k \cos r} = 1 + k \cos r$ proxime, ob k valde paruum habebitur.

$$C C = \frac{m}{n n} - \frac{\mu}{n n} + \frac{1}{2 n n} + \frac{m}{n n} k \cos r - \frac{3 \mu k}{n n} \cos r - k \cos r - \frac{2 k}{n n} \cos r$$

unde perspicuum esse oportere.

$$\frac{m}{n n} = 1 + \frac{2 + 3 \mu}{n n} \text{ et } C C = 1 + \frac{3 + 4 \mu}{2 n n}$$

§. 44. His

§. 44. His igitur constantium $\frac{m}{n n}$ et $C C$ valoribus proximis inuentis ponamus esse reuera:

$$\frac{m}{n n} = 1 + \frac{2 + 3 \mu + \gamma}{n n} \text{ et } C C = 1 + \frac{3 + 4 \mu + \delta}{2 n n} = \lambda \lambda$$

scribamus enim λ pro C , quia litteris maiusculis A, B, C, D etc. deinceps in operationibus sequentibus utemur: sicque fiet

$$S = \frac{1}{V(1 - k k)} \int dr \left(\frac{3 t^4 \mu}{2 \omega^3} \sin 2 \eta + \frac{3 \mu^2 \mu^3}{8 \omega^4} (\sin \eta + 5/3 \eta) \right)$$

$$d \Phi = \frac{dr}{u u V(1 - k k)} \left(\lambda - \frac{S}{n n} \right)$$

$$d \eta = \frac{dr}{u u V(1 - k k)} \left(\lambda - \frac{S}{n n} \right) - \frac{t t dr V(1 - e e)}{n \omega \omega V(1 - k k)}$$

$$\text{II. } \frac{(1 - k k) d d u}{d r^2} = k t u \cos r + \frac{\lambda \lambda}{u^3} - \frac{2 \lambda S}{n n u^3} + \frac{S S}{n^4 u^3} + \frac{\mu u^3}{n n} + \frac{t^4 u}{2 n n \omega^3}$$

$$= \frac{m t}{n n u u} (1 - \frac{3}{2} \tan \varphi^2 + \frac{3}{2} \tan \varphi^2 \cos 2(\Phi - \pi))$$

$$+ \frac{3 t^4 u \cos 2 \eta}{2 n n \omega^3} + \frac{3 \mu^2 \mu^3}{8 n n \omega^4} (3 \cos \eta + 5 \cos 3 \eta)$$

$$\text{III. } d \eta = \frac{u u dr \sin(\Phi - \pi) \sin(\theta - \pi)}{(\lambda n n - S) V(1 - k k)} \left(\frac{3 t^4}{\omega^3} \cos \eta + \frac{3 \mu^2 \mu^3}{4 \omega^4} (3 + 5 \cos 2 \eta) \right)$$

§. 45. Ponatur $\lambda = u V(1 - k k)$, ut sit $u = 1 + \frac{3 + 4 \mu + \delta}{2 n n}$ defectum enim in termino infinito δ complecti licet, existente $m = n n + 2 + 3 \mu + \gamma$; cum vero ponatur $S = (1 - k k)^{\frac{3}{2}} \int R dr$, ut sit $R = \frac{d S}{dr V(1 - k k)}$; ac si pro η et ω valores restituamus, qui erant,

$$t = \frac{1 - k k}{1 - k \cos r} \text{ et } \omega = \frac{1 - e e}{1 - e \cos s} \text{ habebimus:}$$

$$R =$$

4. His

$$R = \frac{3}{2} \frac{(1-kk)^3 (1-e \cos s)^3}{(1-ee)^3 (1-k \cos r)^4} u \sin 2\eta$$

$$+ \frac{3v(1-kk)^4 (1-e \cos s)^4}{8(1-ee)^4 (1-k \cos r)^5} u^2 (\sin \eta + 5 \sin 3\eta)$$

$$d\phi = \frac{dr}{uu} \left(u - \frac{1}{nn} R dr \right); d\theta = ds = \frac{(1-kk)^{\frac{3}{2}} (1-e \cos s)^2}{n(1-ee)^{\frac{3}{2}} (1-k \cos r)^2} dr$$

$$\frac{du}{dr} = \frac{u}{nn} \frac{fRdr}{nnu} - \frac{(1-kk)^{\frac{3}{2}} (1-e \cos s)^2}{n(1-ee)^{\frac{3}{2}} (1-k \cos r)^2}$$

§. 46. Aequatio autem secunda facta hac substitutione, si per $1-kk$ diuidatur, abibit in sequentem:

$$II. \frac{du}{dr^2} = \frac{ku \cos r}{1-k \cos r} + \frac{uu}{n^2} - \frac{2u/Rdr}{nnu^3} + \frac{(fRdr)^2}{n^4 u^3} + \frac{\mu(1-kk)^2}{nn(1-k \cos r)^3}$$

$$= \frac{m}{nn(1-k \cos r)uu} \left(1 - \frac{1}{2} \tan^2 e + \frac{1}{2} \tan^2 e \cos 2(\phi - \pi) \right)$$

$$+ \frac{(1-kk)^3 (1-e \cos s)^3}{2nn(1-ee)^3 (1-k \cos r)^4} u (1 + 3 \cos 2\eta)$$

$$+ \frac{3v(1-kk)^4 (1-e \cos s)^4}{8nn(1-ee)^4 (1-k \cos r)^5} u^2 (3 \cos \eta + 5 \cos 3\eta)$$

$$III. \frac{du}{dr} = - \frac{dr \sin(\phi - \pi) \sin(\theta - \pi)}{(nnu - R/dr)^2} \left(\frac{3(1-kk)^3 (1-e \cos s)^3}{(1-ee)^3 (1-k \cos r)^4} u \cos \eta \right)$$

$$+ \frac{3v(1-kk)^4 (1-e \cos s)^4}{4(1-ee)^4 (1-k \cos r)^5} u^2 (3 + 5 \cos 2\eta)$$

Ac si e denotet inclinationem mediam orbitae lunaris, quantitas $1 - \frac{1}{2} \tan^2 e + \frac{1}{2} \tan^2 e \cos 2(\phi - \pi)$ in has duas partes discespi poterit:

$$\left(1 - \frac{1}{2} \tan^2 e \right) + \frac{1}{2} (\tan^2 e - \tan^2 e + \tan^2 e \cos 2(\phi - \pi))$$

quarum illa est constans, haec vero proprie a nodo et inclinatione pendet.

§. 47.

§. 47. Euoluamus autem producta illa ex e et ω orta, et quoniam excentricitates k et e sunt valde parvae, sufficit ad eos vsque terminos tantum progredi, qui coefficientes habeant kk , ek et ee , eosque qui per altiores potestates sint multiplicati omittere. Hinc erit:

$$\frac{1}{1-k \cos r} = 1 + \frac{1}{2} kk + k \cos r + \frac{1}{2} k^2 \cos 2r$$

$$\frac{k \cos r}{1-k \cos r} = \frac{1}{2} kk + k \cos r + \frac{1}{2} k^2 \cos 2r$$

$$\frac{(1-kk)^2}{(1-k \cos r)^3} = 1 + 3 k \cos r, \text{ quia hic terminus per } \mu \text{ multiplicatur.}$$

$$\frac{(1-kk)^{\frac{3}{2}}}{(1-k \cos r)^2} = 1 + 2 k \cos r + \frac{1}{2} kk \cos 2r$$

$$\frac{(1-kk)^3}{(1-k \cos r)^4} = 1 + 2 kk + 4 k \cos r + 5 kk \cos 2r$$

$$\frac{(1-kk)^4}{(1-k \cos r)^5} = 1 + 5 k \cos r, \text{ quia hic terminus iam per } v \text{ est multiplicatus.}$$

§. 48. Porro vero pro terminis ex ω enatis est:

$$\frac{(1-e \cos s)^2}{(1-ee)^2} = 1 + 2 ee - 2 e \cos s + \frac{1}{2} ee \cos 2s$$

$$\frac{(1-e \cos s)^3}{(1-ee)^3} = 1 + \frac{1}{2} ee - 3 e \cos s + \frac{1}{2} ee \cos 2s$$

$$\frac{(1-e \cos s)^4}{(1-ee)^4} = 1 - 4 e \cos s, \text{ quia hic factor tantum in minimis terminis occurrit.}$$

Hinc ergo colligimus:

$$\frac{(1-kk)^{\frac{3}{2}} (1-e \cos s)^2}{(1-ee)^{\frac{3}{2}} (1-k \cos r)^2} = 1 + 2 ee + 2 k \cos r + \frac{1}{2} kk \cos 2r - 2 e \cos s$$

$$- 2 ek \cos(r+s) - 2 ek \cos(r-s) + \frac{1}{2} ee \cos 2s$$

E

(1-ee)

n 39)

tr

bstitu-

m:

-kk)

cosr)

(φ-π)

39)

cos η

maris,

duas

-π)

lo et

§. 47.

$$\frac{(1-kk)^3(1-e\cos r)^3}{(1-ee)^3(1-k\cos r)^4} = 1 + 2kk + \frac{2}{3}ee + 4k\cos r + 5kk\cos 2r$$

$$\frac{(1-kk)^4(1-e\cos r)^4}{(1-ee)^4(1-k\cos r)^5} = 1 + 5k\cos r - 4e\cos s,$$

atque hinc fiet: $d\Phi = \frac{dr}{uu} (u - \frac{1}{nn} \int R dr)$ atque

$$\frac{ds}{dr} = \frac{1+2ee}{n} + \frac{2k}{n} \cos r - \frac{2e}{n} \cos s - \frac{2ek}{n} (\cos r - s)$$

$$+ \frac{3kk}{2n} \cos 2r + \frac{ee}{2n} \cos 2s - \frac{2ek}{n} \cos(r+s)$$

$$\frac{d\eta}{dr} = \frac{n}{uu} - \frac{\int R dr}{nnuu} - \frac{1-2ee}{n} - \frac{2k}{n} \cos r + \frac{2e}{n} \cos s + \frac{2ek}{n} \cos(r-s)$$

$$- \frac{3kk}{2n} \cos 2r - \frac{ee}{2n} \cos 2s + \frac{2ek}{n} \cos(r+s)$$

§. 49. Introductis nunc his valoribus evolutis in formulas nostras, iisque, qui per sinum cosinumue alterius anguli sunt multiplicati, pariter secundum simplices angulos explicatis, obtinebimus primum valorem ipsius R, qui erit:

$$R = \frac{1}{2} u^2 \left[\begin{aligned} & (1+2kk + \frac{2}{3}ee) \sin 2\eta + 2k \sin(2\eta-r) + 2k \sin(2\eta+r) \\ & + \frac{2}{3}kk \sin(2\eta-2r) + \frac{2}{3}kk \sin(2\eta+2r) \\ & - \frac{2}{3}e \sin(2\eta-s) - \frac{2}{3}e \sin(2\eta+s) \\ & + \frac{2}{3}ee \sin(2\eta-2s) + \frac{2}{3}ee \sin(2\eta+2s) \\ & - 3ek \sin(2\eta-r+s) - 3ek \sin(2\eta+r-s) \\ & - 3ek \sin(2\eta-r-s) - 3ek \sin(2\eta+r+s) \\ & + \frac{1}{2} v u^2 \left[\begin{aligned} & \sin \eta + \frac{1}{2}k \sin(\eta-r) - 2e \sin(\eta-s) \\ & + 5 \sin 3\eta + \frac{1}{2}k \sin(\eta+r) - 2e \sin(\eta+s) \\ & + \frac{2}{3}k \sin(3\eta-r) - 10e \cos(3\eta-s) \\ & + \frac{2}{3}k \sin(3\eta+r) - 10e \cos(3\eta+s) \end{aligned} \right] \end{aligned} \right]$$

§. 50.

$\cos 2r$
 $e\cos 2s$

$\sin(r-s)$

$\cos(r+s)$

utis in
altri-
impli-
lorem

$2\eta+r$

$-2r$

)

s

$-r-s$

$-r+s$

)

)

§. 50.

§. 50. Aequatio autem secunda principalis sequentem induet formam:

$$II. \frac{ddu}{dr^2} = \frac{uu}{u^3} - \frac{2u \int R dr}{nnu^3} + \frac{(\int R dr)^2}{n^4 u^3}$$

$$+ \frac{3m \tan g^2}{4nnuu} (1 - \cos 2(\Phi - \pi)) (1 + k \cos r)$$

$$- \frac{m}{nnuu} (1 + \frac{1}{2}kk + k \cos r + \frac{1}{2}k^2 \cos 2r) + \frac{\mu}{nn} (1 + 3k \cos r)$$

$$+ u \left(\frac{1}{2}kk + k \cos r + \frac{1}{2}kk \cos 2r \right)$$

$$+ \frac{u}{2nn} \left[\begin{aligned} & (1+2kk + \frac{2}{3}ee + 4k \cos r - 3e \cos s - 6ek \cos(r-s) \\ & + 5kk \cos 2r + \frac{2}{3}ee \cos 2s - 6ek \cos(r+s) \\ & (1+2kk + \frac{2}{3}ee) \cos 2\eta + 2k \cos(2\eta-r) + 2k \cos(2\eta+r) \\ & + \frac{2}{3}kk \cos(2\eta-2r) + \frac{2}{3}kk \cos(2\eta+2r) \\ & - \frac{2}{3}e \cos 2(2\eta-s) - \frac{2}{3}e \cos(2\eta+s) \\ & + \frac{2}{3}ee \cos(2\eta-2s) + \frac{2}{3}ee \cos(2\eta+2s) \\ & - 3ek \cos(2\eta-r+s) - 3ek \cos(2\eta+r-s) \\ & - 3ek \cos(2\eta-r-s) - 3ek \cos(2\eta+r+s) \\ & + \frac{3vuu}{8nn} \left[\begin{aligned} & 3 \cos \eta + 5 \cos 3\eta + \frac{1}{2}k \cos(\eta-r) + \frac{1}{2}k \cos(\eta+r) \\ & - 6e \cos(\eta-s) - 6e \cos(\eta+s) + \frac{2}{3}k \cos(\eta-r) + \frac{2}{3}k \cos(\eta+r) \\ & - 10e \cos(3\eta-s) - 10e \cos(3\eta+s) \end{aligned} \right] \end{aligned} \right]$$

vbi terminos, qui adhuc ulteriori evolutione indigent, primo loco posui, et cum terminus tang e^2 implicans iam sit valde parvus, in eius multiplicatore secundam ipsius k potestatem omisi: sin autem alicuius momenti videantur, loco $1 + k \cos r$ scribi poterit $1 + \frac{1}{2}kk + k \cos r + \frac{1}{2}kk \cos 2r$.

§. 51. Pro longitudine vero nodi invenienda aequatio sequens prodibit resoluenda:

F 2

 $d\pi =$

$$d\pi = -\frac{3uvdr \sin(\Phi-\pi) \sin(\theta-\pi)}{unn - fRdr} \cos \eta (1 + 2kk + \frac{1}{2}ee + 4k \cos r$$

$$+ 5kk \cos 2r - 3e \cos s + \frac{1}{2}ee \cos 2s)$$

$$- \frac{3vu^3 dr \sin(\Phi-\pi) \sin(\theta-\pi)}{4(unn - fRdr)} (3 + 5 \cos 2\eta) (1 + 5k \cos r)$$

At est $\sin(\Phi-\pi) \sin(\theta-\pi) = \frac{1}{2} \cos \eta - \frac{1}{2} (\Phi + \theta - 2\pi)$; vnde
 $\sin(\Phi-\pi) \sin(\theta-\pi) \cos \eta = \frac{1}{4} + \frac{1}{4} \cos 2\eta - \frac{1}{4} \cos 2(\Phi-\pi) - \frac{1}{4} \cos 2(\theta-\pi)$
 et $\sin(\Phi-\pi) \sin(\theta-\pi) \cos 2\eta = \frac{1}{4} \cos \eta + \frac{1}{4} \cos 3\eta$
 $-\frac{1}{4} \cos(3\Phi - \theta - 2\pi) - \frac{1}{4} \cos(3\theta - \Phi - 2\pi)$

Tum vero ob $fRdr$ valde paruum prae unn , erit satis exacte

$$\frac{1}{unn - fRdr} = \frac{1}{unn} + \frac{fRdr}{unn^2} + \frac{(fRdr)^2}{unn^3}$$

vbi quidem postremus terminus tuto omitti potest.

§. 52. Praeterea vero ponatur $u = 1 + \frac{v}{nn}$, vt sit
 $\frac{d\pi}{dr} = \frac{d\pi}{du} \frac{du}{dr}$, et reiectis terminis per n^4 diuisis, qui iam

$$\frac{d\pi}{dr} = u - \frac{2uv}{nn} + \frac{3uv^2}{n^2} - \frac{fRdr}{nn} + \frac{2vfRdr}{n^2}$$

$$\frac{d\eta}{dr} = u - \frac{1-2ee}{n} - \frac{2k}{n} \cos r + \frac{2e}{n} \cos s - \frac{3kk}{2n} \cos 2r - \frac{ee}{2n} \cos 2s$$

$$+ \frac{2ek}{n} \cos(r-s) + \frac{2ek}{n} \cos(r+s)$$

$$- \frac{2uv - fRdr}{nn} + \frac{3uv^2 + 2vfRdr}{n^2} \quad \text{atque}$$

$$R = \frac{1}{2} (1 + 2kk + \frac{1}{2}ee) \sin 2\eta + 3k \sin(2\eta - r) - \frac{1}{2}e \sin(2\eta - s)$$

$$+ \frac{3v}{nn} \sin 2\eta + \frac{1}{2}k \sin(2\eta - r) + \frac{1}{2}e \sin(2\eta - s)$$

$$+ \frac{3vu}{2n^2} \sin 2\eta - \frac{1}{2}ek \sin(2\eta - r + s) - \frac{1}{2}ek \sin(2\eta + r - s)$$

$$+ \frac{1}{2}ek \sin(2\eta - r - s) - \frac{1}{2}ek \sin(2\eta + r + s)$$

$$+ \frac{6kv}{nn} \sin(2\eta - r) + \frac{6kv}{nn} \sin(2\eta + r)$$

$$- \frac{9ev}{2nn} \sin(2\eta - s) - \frac{9ev}{2nn} \sin(2\eta + s)$$

$$+ \frac{3}{2}v \sin \eta + \frac{1}{2}v k \sin(\eta - r) - \frac{1}{2}v e \sin(\eta - s)$$

$$+ \frac{1}{2}v k \sin(3\eta - r) - \frac{1}{2}v e \sin(3\eta - s)$$

$$+ \frac{1}{2}v \sin 3\eta + \frac{1}{2}v k \sin(\eta + r) - \frac{1}{2}v e \sin(\eta + s)$$

$$+ \frac{1}{2}v k \sin(3\eta + r) - \frac{1}{2}v e \sin(3\eta + s)$$

§. 53. Ipsa vero aequatio secunda per hanc substitutionem, postquam per nn fuerit multiplicata, in formam sequentem abibit.

$$\text{II. } \frac{ddv}{dr^2} = unn - 3uv + \frac{6uvv}{nn} - 2u fRdr$$

$$+ \frac{6kv}{nn} fRdr + \frac{1}{nn} (fRdr)^2$$

$$+ \frac{3mtang^2}{4} (1 - \cos 2(\Phi - \pi)) (1 + \frac{1}{2}kk + k \cos r + \frac{1}{2}kk \cos 2r)$$

$$- \frac{3mv tang^2}{2nn} (1 - \cos 2(\Phi - \pi)) (1 + k \cos r)$$

$$- \frac{3mvv}{n^2} (1 + k \cos r) - m (1 + \frac{1}{2}kk + k \cos r + \frac{1}{2}kk \cos 2r)$$

$$+ \frac{2mv}{nn} (1 + \frac{1}{2}kk + k \cos r + \frac{1}{2}kk \cos 2r) + nn (\frac{1}{2}kk + k \cos r + \frac{1}{2}kk \cos 2r)$$

$$+ v (\frac{1}{2}kk + k \cos r + \frac{1}{2}kk \cos 2r) + \mu (1 + 3k \cos r)$$

$$+ \frac{1}{2} + kk + \frac{1}{2}ee + 2k \cos r - \frac{1}{2}e \cos s - 3ek \cos(r-s)$$

$$+ \frac{1}{2}kk \cos r + \frac{1}{2}ee \cos 2s - 3ek \cos(r+s)$$

$$+ \frac{v}{2nn} (1 + 4k \cos r - 3e \cos s)$$

$$+ \frac{1}{2} (1 + 2kk + \frac{1}{2}ee) \cos 2\eta + 3k \cos(2\eta - r) - \frac{1}{2}e \cos(2\eta - s)$$

$$+ 3 \cos(2\eta + r) - \frac{1}{2}e \cos(2\eta + s)$$

$$\begin{aligned}
& + \frac{1}{4} k k \cos(2\eta - 2r) + \frac{1}{8} e e \cos(2\eta - 2r) \\
& + \frac{1}{4} k k \cos(2\eta + 2r) + \frac{1}{8} e e \cos(2\eta + 2r) \\
& - \frac{1}{2} e k \cos(2\eta - r + s) - \frac{1}{2} e k \cos(2\eta + r - s) \\
& - \frac{1}{2} e k \cos(2\eta - r - s) - \frac{1}{2} e k \cos(2\eta + r + s) \\
& + \frac{3v}{2nn} \left[\cos 2\eta + 2k \cos(2\eta - r) + 2k \cos(2\eta + r) \right. \\
& \quad \left. - \frac{1}{2} e \cos(2\eta - s) - \frac{1}{2} e \cos(2\eta + s) \right] \\
& + \frac{1}{3} v \left\{ \begin{aligned} & + 3 \cos \eta + \frac{1}{2} k \cos(\eta - r) - 6 e \cos(\eta - s) \\ & + 5 \cos 3\eta + \frac{1}{2} k \cos(\eta + r) - 6 e \cos(\eta + s) \\ & + \frac{1}{2} k \cos(3\eta - r) - 10 e \cos(3\eta - s) \\ & + \frac{1}{2} k \cos(3\eta + r) - 10 e \cos(3\eta + s) \end{aligned} \right\} \\
& + \frac{3vv}{4nn} (3 \cos \eta + 5 \cos 3\eta)
\end{aligned}$$

§. 54. Cum autem sit $m = nn + 2 + 3\mu + \gamma$ et $nn =$
 $x + \frac{3 + 4\mu + \delta}{2nn}$, si hi valores substituantur, plures
termini se mutuo destruent, aequatioque prodibit se-
quenti forma concinniori contenta: vbi quidem in ter-
minis per se minimis loco m scribi licebit nn , et i loco
 nn vel x .

II. AEQUATIO.

$$\begin{aligned}
\frac{d\pi}{dr^2} &= \frac{1}{2} \delta - \gamma + \frac{1}{2} e e - \gamma k \cos r + \frac{1}{2} k k \cos 2r \\
&- 2 \left(1 + \frac{3 + 4\mu + \delta}{4nn} \right) f R dr + \frac{1}{nn} (f R dr)^2 \\
&- v \left(1 - \frac{1}{2} k k - 3 k \cos r - \frac{1}{2} k k \cos 2r \right) \\
&+ \frac{vv}{nn} (3 - 3k \cos r - \frac{1}{2} e \cos s + \frac{1}{2} e e \cos 2s - 3ek \cos(r-s) \\
&- 3ek \cos(r+s) + \frac{1}{2} (1 + 2kk + \frac{1}{2} e e) \cos 2\eta \\
&+
\end{aligned}$$

$$\begin{aligned}
& + 3k \cos(2\eta - r) + \frac{1}{4} k k \cos(2\eta - 2r) - \frac{1}{2} e \cos(2\eta - s) \\
& + 3k \cos(2\eta + r) + \frac{1}{4} k k \cos(2\eta + 2r) - \frac{1}{2} e \cos(2\eta + s) \\
& + \frac{1}{8} e e \cos(2\eta - 2s) - \frac{1}{2} e k \cos(2\eta - r + s) - \frac{1}{2} e k \cos(2\eta - r - s) \\
& + \frac{1}{8} e e \cos(2\eta + 2s) - \frac{1}{2} e k \cos(2\eta + r - s) - \frac{1}{2} e k \cos(2\eta + r + s) \\
& + \frac{v}{nn} \left\{ \begin{aligned} & 2\gamma - \frac{1}{2} \delta + 6k \cos r + 2(3\mu + \gamma) k \cos r + 6f R dr \\ & - \frac{1}{2} e \cos s + \frac{1}{2} \cos 2\eta + 3k \cos(2\eta - r) + 3k \cos(2\eta + r) \\ & - \frac{1}{2} e \cos(2\eta - s) - \frac{1}{2} e \cos(2\eta + s) \end{aligned} \right\} \\
& + \frac{1}{3} v \left\{ \begin{aligned} & 3 \cos \eta + \frac{1}{2} k \cos(\eta - r) - 6 e \cos(\eta - s) \\ & + 5 \cos 3\eta + \frac{1}{2} k \cos(\eta + r) - 6 e \cos(\eta + s) \\ & + \frac{1}{2} k \cos(3\eta - r) - 10 e \cos(3\eta - s) \\ & + \frac{1}{2} k \cos(3\eta + r) - 10 e \cos(3\eta + s) \end{aligned} \right\} \\
& + \frac{3vv}{4nn} (3 \cos \eta + 5 \cos 3\eta) \\
& + \frac{1}{4} (nn + 2 + 3\mu + \gamma) \frac{(1-2v)}{nn} \tan g^2(1 - \cos 2(\phi - \pi)) \\
& (1 + \frac{1}{2} k k + k \cos r + \frac{1}{2} k k \cos 2r)
\end{aligned}$$

§. 55. Pro loco nodi autem inveniendō prodibit
sequens aequatio.

$$\begin{aligned}
\frac{d\pi}{dr} &= \frac{-3}{nnn} \left(1 + \frac{2vv + f R dr}{nnn} \right) (1 + 2kk + \frac{1}{2} e e) \\
& \left\{ \begin{aligned} & \frac{1}{2} + \frac{1}{2} \cos 2\eta - \frac{1}{2} \cos 2(\phi - \pi) - \frac{1}{2} \cos 2(\theta - \pi) \\ & + k \cos r + \frac{1}{2} k \cos(2\eta - r) - \frac{1}{2} k \cos(2\phi - 2\pi - r) \\ & - \frac{1}{2} e \cos s + \frac{1}{2} k \cos(2\eta + r) - \frac{1}{2} k \cos(2\phi - 2\pi + r) \\ & - \frac{1}{2} e \cos(2\eta - s) - \frac{1}{2} k \cos(2\theta - 2\pi - r) \\ & - \frac{1}{2} e \cos(2\eta + r) - \frac{1}{2} k \cos(2\theta - 2\pi + r) \end{aligned} \right\} \\
& - \frac{3v}{4nnn} \left\{ \begin{aligned} & \frac{1}{2} \cos \eta + \frac{1}{2} \cos 3\eta - \frac{1}{2} \cos(3\phi + \theta - 2\pi) \\ & - \frac{1}{2} \cos(3\phi - \theta - 2\pi) - \frac{1}{2} \cos(3\theta - \phi - 2\pi) \end{aligned} \right\}
\end{aligned}$$

At

At pro inclinatione orbitae habebitur:

$$\frac{d. / \text{tange}}{dr} = \frac{-3}{\sin} \left(1 + \frac{2\kappa v + fRdr}{\sin} \right) (1 + 2kk + \frac{e}{e})$$

$$\begin{cases} \frac{3}{4} \sin 2(\phi - \pi) + \frac{3}{4} \sin 2(\theta - \pi) - \frac{3}{4} \sin 2\eta \\ - \frac{3}{4} k \sin (2\eta - r) + \frac{3}{4} k \sin (2\phi - 2\pi - r) \\ - \frac{3}{4} k \sin (2\eta + r) + \frac{3}{4} k \sin (2\phi - 2\pi + r) \\ + \frac{3}{4} e \sin (2\eta - s) + \frac{3}{4} k \sin (2\theta - 2\pi - r) \\ + \frac{3}{4} e \sin (2\eta + s) + \frac{3}{4} k \sin (2\theta - 2\pi + r) \end{cases}$$

$$- \frac{3v}{4\sin} (-\frac{3}{4} \sin \eta - \frac{3}{4} \sin 3\eta + \frac{3}{4} \sin (\phi + \theta - 2\pi))$$

$$+ \frac{3}{4} \sin (3\phi - \theta - 2\pi) + \frac{3}{4} \sin (3\theta - \phi - 2\pi)$$

Quomodo igitur his aequationibus ad motum Lunae cognoscendum uti conveniat, in sequentibus capitibus videamus.

CAPUT IV.

INVESTIGATIO INAEQUALITATIS LUNAE ABSOLUTAE, QUAE VARIATIO DICITUR.

§. 33.

Ex his aequationibus perspicitur in determinationem motus Lunae plurimorum angulorum vel sinus vel cosinus ingredi, qui anguli formantur per variationem combinationem sequentium 4 angulorum:

1. ex distantia Solis a Luna, quem angulum posuimus $= \eta$
2. ex anomalia Lunae vera $= r$
3. ex anomalia Solis vera $= s$
4. ex distantia Lunae a nodo ascendente $= \phi - \pi$.

Ne igitur a tanta angulorum multitudine obruamur, a casibus simplicioribus ordiamur: ac primo quidem in eas tantum motus inaequalitates inquiramus, quae a solo angulo η pendeant, neque idcirco excentricitatem vel Solis vel Lunae implicent, neque ab orbitae lunaris inclinatione ad eclipticam afficiantur.

§. 57. Has igitur inaequalitates, quae a solo situ Solis respectu Lunae nascuntur, atque ab Astronomis sub nomine variationis comprehendi solent, ex praecedentibus aequationibus eliciemus, si tam excentricitatem Lunae k quam solis e pro nihilo habeamus, atque inclinationem orbitae lunaris ad eclipticam evanescentem statuamus, ita ut sit $k = 0$, $e = 0$ et $\text{tang } \varphi = 0$. Sic enim obtinebimus eas inaequalitates Lunae, quae ab his elementis non pendent, ideoque tantum per angulum

Ium η determinantur; quae cum vnica tabula comprehendendi queant, haec tabula variationem Lunae indicare dicitur. Interim tamen hic animaduerti oportet, partem quandam exiguam variationis quoque ab excentricitate orbitae Lunae k pendere, quam partem deinceps supplebimus, cum huius excentricitatis rationem sumus habituri.

§. 58. Reiectis ergo terminis k , e , et tang p continentibus, habebimus :

$$\frac{d\phi}{dr} = \kappa - \frac{2\kappa v - fR dr}{nn} + \frac{3\kappa v^2 + 2v fR dr}{n^4}$$

$$\frac{d\eta}{dr} = \kappa - \frac{1}{n} - \frac{2\kappa v - fR dr}{nn} + \frac{3\kappa v^2 + 2v fR dr}{n^4}$$

$R = \frac{1}{2} \sin 2\eta + \frac{3v}{nn} \sin 2\eta + \frac{1}{2} v \sin \eta + \frac{1}{8} v \sin 3\eta$, ac deniquē

$$\frac{ddv}{dr^2} = \frac{1}{2} \delta - \gamma - \tau \left(1 + \frac{3+4\mu+\delta}{4nn}\right) fR dr + \frac{1}{nn} (fR dr)^2 - v + \frac{3vv}{nn}$$

$$+ \frac{1}{2} \cos 2\eta + \frac{v}{nn} (2\gamma - \frac{1}{2} \delta) + \frac{3v \cos 2\eta}{2nn} + \frac{6v}{nn} fR dr$$

$$+ \frac{1}{2} \cos \eta + \frac{1}{8} v \cos 3\eta$$

Hic autem notandum est esse $\kappa = \sqrt{1 + \frac{3+4\mu+\delta}{2nn}}$;

quoniam vero valores litterarum μ et δ demum cum per consensum obseruationum, tum per indolem calculi definire instituis, hic ex obseruationibus petamus valores ipsius κ ; cum enim sit $\kappa:1 = d\phi:dr$, hoc est vt motus Lunae medius ad motum anomaliae, erit $\kappa = 1,0085272$. Fieri quidem potest, vt hic valor aliquantulum a vero differat, sed errorem si quis lateat infra detegemus, facillimeque emendabimus.

§. 59.

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ntricitate
eps sup-
1 sumus

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deniquē

$$-v + \frac{3vv}{nn}$$

$$fR dr$$

$$\frac{\mu + \delta}{2nn};$$

n cum

calculi

us va-

: est vt

: $\kappa =$

liquan-

: infra

§. 59.

§. 59. Cum igitur iam supra inuenerimus esse

$$\kappa = 1,0085272 \text{ ac proinde } nn = 175,71795$$

$$\text{erit } \frac{1}{n} = 0,075438, \text{ ideoque } \kappa - \frac{1}{n} = 0,933089$$

Hic autem numerus, qui iam quasi medium valorem rationis $\frac{d\eta}{dr}$ exprimit, in omnibus operationibus, quae sequuntur, frequentissime occurret, hincque breuitatis gratia ponamus

$$\kappa - \frac{1}{n} = \alpha, \text{ vt sit } \alpha + \frac{1}{n} = \sqrt{1 + \frac{3+4\mu+\delta}{2nn}}$$

eritque ergo $\alpha = 0,933089$, qui valor quam minime a vero discrepat, vti mox patebit. Quod autem verus ipsius α valor aliquantulum diuersus esse possit, inde primo patet, quod minurias, quae ex terminis $\frac{3vv^2 + 2v fR dr}{n^4}$

quantitati constanti accrescere potuissent, hic negleximus; tum vero fieri potest, vt ratio media differentialium $d\eta$ ad dr alia sit atque quantitarum finitarum η et r .

§. 60. Si has formulas attente contemplemur, moxprehendemus valorem integralis $\int R dr$ constare ex cosinibus angulorum 2η , η , 3η , et 4η . Quanquam enim altiora quoque multipla huius anguli ingredientur, tamen facile patet, coefficientes eorum continuo fieri minores, ita vt in quadruplo tuto subsistere possimus: similis autem erit ratio valoris ipsius v . Hinc ponamus:

$$\int R dr = A \cos 2\eta + B \cos 4\eta + a v \cos \eta + b v \cos 3\eta$$

$$v = A \cos 2\eta + B \cos 4\eta + a v \cos \eta + b v \cos 3\eta$$

atque hos valores fictitios in formulis nostris substituamus:

G 2

mus:

mus, ut inde valores istorum coefficientium assumto-
rum determinare possimus: quippe qui modus aptissi-
mus videtur ad cognitionem integralium perueniendi.
Quia autem est circiter $v = \frac{1}{282}$, pater terminos per v
multiplicatos prae reliquis tam esse exiguos, ut eos qui
multo fuerint minores, sine haesitatione praetermittere
possimus.

§. 61. Per hos ergo valores assumptos consequemur:

$$\frac{d\Phi}{dr} = \kappa - \frac{(2\kappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{(2\kappa B + \mathfrak{B})}{nn} \cos 4\eta \\ + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} \cos 2\eta + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} \cos 4\eta \\ - \frac{(2\kappa a + a)}{nn} v \cos \eta - \frac{(2\kappa b + b)}{nn} v \cos 3\eta$$

atque ob $\kappa = \frac{1}{n} = a$ erit minimis terminis omissis,

quia hi in operatione multo magis diminuerentur:

$$\frac{d\eta}{dr} = a - \frac{(2\kappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{(2\kappa B + \mathfrak{B})}{nn} \cos 4\eta \\ - \frac{(2\kappa a + a)}{nn} v \cos \eta - \frac{(2\kappa b + b)}{nn} v \cos 3\eta$$

His positis erit:

$$\frac{d \cos 2\eta}{dr} = -\sin 2\eta \cdot \frac{2d\eta}{dr} = -2a \sin 2\eta - \frac{(2\kappa B + \mathfrak{B})}{nn} \sin 2\eta + \frac{(2\kappa A + \mathfrak{A})}{nn} \sin 4\eta \\ + \frac{(2\kappa a + a)}{nn} v \sin \eta + \frac{(2\kappa b + b)}{nn} v \sin 3\eta \\ - \frac{(2\kappa b + b)}{nn} v \sin \eta$$

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$$\frac{d \cos 4\eta}{dr} = -\sin 4\eta \cdot \frac{4d\eta}{dr} = -4a \sin 4\eta + \frac{2(2\kappa A + \mathfrak{A})}{nn} \sin 2\eta$$

$$\frac{d \cos \eta}{dr} = -\sin \eta \cdot \frac{d\eta}{dr} = -a \sin \eta$$

$$\frac{d \cos 3\eta}{dr} = -\sin 3\eta \cdot \frac{3d\eta}{dr} = -3a \sin 3\eta$$

§. 62. Quod si iam secundum has formulas quan-
titas integralis $\int R dr$ differentietur, obtinebitur:

$$R = (-2a\mathfrak{A} - \frac{\mathfrak{A}(2\kappa B + \mathfrak{B})}{nn} + \frac{2\mathfrak{B}(2\kappa A + \mathfrak{A})}{nn}) \sin 2\eta \\ + (\frac{\mathfrak{A}(2\kappa A + \mathfrak{A})}{nn} - 4a\mathfrak{B}) \sin 4\eta \\ + (\frac{\mathfrak{A}(2\kappa a + a)}{nn} + \frac{\mathfrak{A}(2\kappa b + b)}{nn} - a a) v \sin \eta \\ + (\frac{\mathfrak{A}(2\kappa a + a)}{nn} - 3a b) v \sin 3\eta$$

Cum iam fit per hypothesin

$$R = \frac{1}{2} \sin 2\eta + \frac{3A}{2nn} \sin 4\eta + \frac{1}{2} v \sin \eta + \frac{1}{2} v \sin 3\eta \\ - \frac{3B}{2nn} + \frac{3a}{2nn} v + \frac{3a}{2nn} v \\ - \frac{3b}{2nn} v$$

prodibit terminis homogeneis comparandis:

$$2a\mathfrak{A} = -\frac{1}{2} - \frac{\mathfrak{A}(2\kappa B + \mathfrak{B}) + 2\mathfrak{B}(2\kappa A + \mathfrak{A})}{nn} + \frac{3B}{2nn} \\ 4a\mathfrak{B} = -\frac{3A}{2nn} + \frac{\mathfrak{A}(2\kappa A + \mathfrak{A})}{nn} \\ a a = -\frac{1}{2} - \frac{3(a-b)}{2nn} + \frac{\mathfrak{A}(2\kappa a + a) - \mathfrak{A}(2\kappa b + b)}{nn} \\ 3ab = -\frac{1}{2} - \frac{3a}{2nn} + \frac{\mathfrak{A}(2\kappa a + a)}{nn}$$

G 3

§. 63.

§. 63. Aequatio autem nostra differentio-differentialis, si pro $f/R dr$ et v valores assumti substituantur, sequentem induet formam:

$$\begin{aligned} \frac{ddv}{dr^2} = & \left(\frac{1}{2}\delta - \gamma\right) - 2\kappa A \cos 2\eta - 2\kappa B \cos 4\eta - 2\kappa a v \cos \eta - 2\kappa b v \cos 3\eta \\ & + \frac{A A}{2nn} - A - B - av - bv \\ & + \frac{3AA}{2nn} + \frac{3AA}{2nn} \\ & + \frac{3}{4} + \frac{(4\gamma - 3\delta)}{2nn} A + \frac{(4\gamma - 3\delta)}{2nn} B + \frac{3b}{4nn} v \\ & + \frac{3A}{4nn} + \frac{3B}{4nn} + \frac{3A}{4nn} + \frac{3a}{4nn} v + \frac{3a}{4nn} v \\ & + \frac{3A A}{nn} + \frac{3A A}{nn} + \frac{3}{4} v + \frac{3}{4} v \end{aligned}$$

vbi quidem perspicuum est, quinam termini respectu reliquorum tam sint parvi, vt sine errore deleri queant.

§. 64. Quaeramus ergo primum differentiale $\frac{dv}{dr}$ ac reperietur:

$$\begin{aligned} & (-2Aa - \frac{A(2\kappa B + 2\delta)}{nn} + \frac{2B(2\kappa A + 2\eta)}{nn}) \sin 2\eta \\ \frac{dv}{dr} = & (-4\kappa B + \frac{A(2\kappa A + 2\eta)}{nn}) \sin 4\eta \\ & (-\kappa a + \frac{A(2\kappa a + a)}{nn} - \frac{A(2\kappa b + b)}{nn}) v \sin \eta \\ & (-3\kappa b + \frac{A(2\kappa a + a)}{nn}) v \sin 3\eta \end{aligned}$$

pona-

ponatur autem breuitatis ergo:

$$\frac{dv}{dr} = -A' \sin 2\eta - B' \sin 4\eta - a' v \sin \eta - b' v \sin 3\eta$$

vt fit:

$$A' = 2\kappa A + \frac{A(2\kappa B + 2\delta)}{nn} - \frac{2B(2\kappa A + 2\eta)}{nn}$$

$$B' = 4\kappa B - \frac{A(2\kappa A + 2\eta)}{nn}$$

$$a' = \kappa a - \frac{A(2\kappa a + a)}{nn} + \frac{A(\kappa b + b)}{nn}$$

$$b' = 3\kappa b - \frac{A(2\kappa a + a)}{nn}$$

§. 65. Hinc cum fit:

$$\begin{aligned} \frac{d(\sin 2\eta)}{dr} &= \cos 2\eta \cdot \frac{2d\eta}{dr} = 2\kappa \cos 2\eta - \frac{(2\kappa B + 2\delta)}{nn} \cos 2\eta - \frac{(2\kappa A + 2\eta)}{nn} \cos 4\eta \\ &= \frac{(2\kappa A + 2\eta)}{nn} - \frac{(2\kappa a + a)}{nn} v \cos \eta - \frac{(2\kappa b + b)}{nn} v \cos \eta - \frac{(2\kappa a + a)}{nn} v \cos 3\eta \end{aligned}$$

$$\frac{d(\sin 4\eta)}{dr} = \cos 4\eta \cdot \frac{4d\eta}{dr} = 4\kappa \cos 4\eta - \frac{2(2\kappa A + 2\eta)}{nn} \cos 2\eta - \frac{2(2\kappa B + 2\delta)}{nn}$$

$$\frac{d(\sin \eta)}{dr} = \cos \eta \cdot \frac{d\eta}{dr} = \kappa \cos \eta; \text{ et } \frac{d(\sin 3\eta)}{dr} = \cos 3\eta \cdot \frac{3d\eta}{dr} = 3\kappa \cos 3\eta$$

prodibit

$$+ \frac{A'(2\kappa A + 2\eta)}{nn} + \frac{2B'(2\kappa B + 2\delta)}{nn}$$

$$(-2\kappa A' + \frac{A'(2\kappa B + 2\delta)}{nn} + \frac{2B'(2\kappa A + 2\eta)}{nn}) \cos 2\eta$$

$$\frac{ddv}{dr^2} = (-4\kappa B' + \frac{A'(2\kappa A + 2\eta)}{nn}) \cos 4\eta$$

$$(-\kappa a' + \frac{A'(2\kappa a + a)}{nn}) v \cos \eta$$

$$(-3\kappa b' + \frac{A'(2\kappa a + a)}{nn}) v \cos 3\eta$$

feu

seu substitutis superioribus valoribus:

$$\begin{aligned}
 & + \frac{2Aa(2\kappa A + \mathfrak{A})}{nn} + \frac{8aB(2\kappa B + \mathfrak{B})}{nn} \\
 & (-4aaA + \frac{12aB(2\kappa A + \mathfrak{A})}{nn}) \cos 2\eta \\
 \frac{ddv}{dr^2} = & (-16aaB + \frac{8aA(2\kappa A + \mathfrak{A})}{nn}) \cos 4\eta \\
 & (-aaA + \frac{3aA(2\kappa A + a)}{nn} - \frac{aA(2\kappa b + b)}{nn}) \cos \eta \\
 & (-9aaB + \frac{5aA(2\kappa A + a)}{nn}) \cos 3\eta
 \end{aligned}$$

§ 66. Hi iam termini singulatim illis, qui §. 63. sunt exhibiti, aequales statuuntur, atque sequentes prodibunt determinationes,

$$\begin{aligned}
 \frac{1}{2}\delta - \gamma + \frac{3AA + 6A\mathfrak{A} + \mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A}{4nn} - \frac{2aA(2\kappa A + \mathfrak{A})}{nn} + \frac{8aB(2\kappa B + \mathfrak{B})}{nn} \\
 - A + \frac{1}{2} - 2\kappa\mathfrak{A} + \frac{(4\gamma - 3\delta)}{2nn} A + \frac{3B}{4nn} = -4aaA + \frac{12aB(2\kappa A + \mathfrak{A})}{nn} \\
 - B - 2\kappa\mathfrak{B} + \frac{3AA + 6A\mathfrak{A} + \mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A}{4nn} + \frac{(4\gamma - 3\delta)}{2nn} B = \\
 -16aaB + \frac{8aA(2\kappa A + \mathfrak{A})}{nn}
 \end{aligned}$$

$$-A + \frac{1}{2} - 2\kappa a + \frac{3a + 3b}{4nn} = -aaA + \frac{3aA(2\kappa A + a)}{nn} - \frac{aA(2\kappa b + b)}{nn}$$

$$-b + \frac{1}{2} - 2\kappa b + \frac{3a}{4nn} = -9aaB + \frac{5aA(2\kappa A + a)}{nn}$$

vnde primum quaeri debent valores vero proximi, qui sunt:

$$\mathfrak{A} = -\frac{3}{4a}; \quad a = -\frac{3}{8a}; \quad b = -\frac{5}{8a};$$

$$A = -\frac{\frac{1}{2} + 2\kappa\mathfrak{A}}{4aa - 1}; \quad \mathfrak{A} = \frac{\frac{1}{2} - 2\kappa a}{1 - aa}; \quad b = -\frac{\frac{1}{2} + 2\kappa b}{9aa - 1}$$

§. 67.

§. 67. Calculus ergo sequenti modo instituitur:

$$a = 0,933089; \quad 1/a = 9,969923$$

$$\kappa = 1,008527; \quad 1/\kappa = 0,003687$$

$$\text{Iam est} \quad 1/2\kappa = 0,304717$$

$$\text{fubtr. a} \quad 1/8a = 0,873013$$

$$1/3 = 0,477121$$

$$1/5 = 0,698970$$

$$a = -0,402; \text{erit } 1/a = 9,604108$$

$$b = -0,635; \quad 1/b = 9,825957$$

$$\mathfrak{A} = -0,804; \quad 1/\mathfrak{A} = 9,905138$$

atque hinc conficietur:

$$A = -\frac{3,121}{4aa-1}; \quad a = +\frac{1,936}{1-aa}; \quad b = -\frac{3,156}{9aa-1}$$

quarum ergo litterarum valores proximi sunt

$$A = -1,2583; \quad a = +14,968; \quad b = -0,4613$$

§. 68. Quaeamus hinc primum valores litterarum $\mathfrak{B}$ et B .

$$\mathfrak{A} = -0,804; \quad 1/\mathfrak{A} = 9,905138$$

$$2\kappa A + \mathfrak{A} = -3,341; \text{ vnde colligitur}$$

$$4a\mathfrak{B} = +\frac{4,573}{nn} \quad 1/4,573 = 0,660201$$

hinc erit

$$1/nn = \frac{2,244816}{8,415385}$$

$$1/4a = \frac{0,571983}{7,843402}$$

$$\mathfrak{B} = +0,00697 \quad 1/\mathfrak{B} = 7,843402$$

Deinde est

$$(16aa-1)B = 2\kappa\mathfrak{B} - \frac{3A}{4nn} - \frac{3AA-6A\mathfrak{A}-\mathfrak{A}\mathfrak{A}}{2nn} + \frac{8aA(2\kappa A + \mathfrak{A})}{nn}$$

$$\text{seu } B = +\frac{0,16819}{16aa-1}; \text{ vnde reperitur}$$

$$B = +0,012792 \quad \text{et } 1/B = 8,106947$$

H

§. 69.

§. 69. His iam valoribus proxime veris inuentis quaerantur exacti, ac primo quidem

$$2\alpha\mathcal{A} = -\frac{1}{2} + \frac{\frac{1}{2}B - \mathcal{A}(2\kappa B + \mathcal{B}) + 2\mathcal{B}(2\kappa A + \mathcal{A})}{nn}$$

vnde reperitur vt ante :

$$\mathcal{A} = -0,80378 \quad . \quad . \quad . \quad l - \mathcal{A} = 9,905138$$

$$\alpha a = -\frac{1}{2} - \frac{\frac{1}{2}(a-b) + \mathcal{A}(2\kappa a + a) - \mathcal{A}(2\kappa b + b)}{nn} = -0,6536r$$

$$a = -0,70048 \quad . \quad . \quad . \quad l - a = 9,845396$$

$$3\alpha b = -\frac{1}{2} - \frac{\frac{1}{2}a + \mathcal{A}(2\kappa a + a)}{nn} = -2,13900$$

$$b = -0,76413 \quad . \quad . \quad . \quad l - b = 9,883167$$

$$(4\alpha a - 1)A = -\frac{1}{2} + 2\kappa\mathcal{A} - \frac{\frac{1}{2}B + 12\alpha B(2\kappa A + \mathcal{A})}{nn} = -3,12379$$

$$A = -1,25826 \quad . \quad . \quad . \quad l - A = 0,099771$$

$$(1 - \alpha a)a = \frac{1}{2} - 2\kappa a + \frac{\frac{1}{2}(a+b) - 3\alpha A(2\kappa a + a) + \alpha A(2\kappa b + b)}{nn} \text{ vel}$$

$$(1 - \alpha a) = \frac{3}{4nn} + \frac{(6\alpha\kappa A)}{nn} a = \frac{1}{2} - 2\kappa a$$

$$+ \frac{\frac{1}{2}b - 3\alpha Aa + \alpha A(2\kappa b + b)}{nn} = -2,53335$$

$$\text{hinc } a = +30,989 \quad \text{et} \quad l - a = 1,491207$$

Vnde patet valorem ipsius a ante inuentum non satis esse exactum, exactior ergo prodibit ex hac formula

$$(a - \frac{\mathcal{A}}{nn})a = -\frac{1}{2} - \frac{\frac{1}{2}(a-b) + \mathcal{A}(2\kappa a - 2\kappa b - b)}{nn} = -0,93709$$

$$\text{hinc } a = -0,99939 \quad \text{et} \quad l - a = 9,999735$$

vnde etiam exactius valor ipsius a reperitur, ex quo denuo

nuentis

denuo valor ipsius a corrigetur, sicque tandem satis exacte obtinebitur

$$a = -1,2537 \quad . \quad . \quad . \quad l - a = 0,098200$$

$$a = +44,48 \quad . \quad . \quad . \quad l - a = 1,648165$$

$$b = -0,95003 \quad . \quad . \quad . \quad l - b = 9,977736$$

§. 70. Hinc iam accuratius quaeramus valorem ipsius b

$$(9\alpha a - 1)b = -\frac{1}{2} + 2\kappa b - \frac{\frac{1}{2}a + 5\alpha A(2\kappa a + a)}{nn}$$

$$b = -1,0146 \quad . \quad . \quad . \quad l - b = 0,006314$$

vnde si denuo praecedentes valores corrigantur, fiet

$$a = -1,2630 \quad . \quad . \quad . \quad l - a = 0,101403$$

$$b = -0,9500 \quad . \quad . \quad . \quad l - b = 9,977736$$

$$a = +44,525 \quad . \quad . \quad . \quad l - a = 1,648604$$

$$b = -1,015 \quad . \quad . \quad . \quad l - b = 0,006400$$

$$\mathcal{A} = -0,80378 \quad . \quad . \quad . \quad l - \mathcal{A} = 9,905138$$

$$\mathcal{B} = +0,00697 \quad . \quad . \quad . \quad l - \mathcal{B} = 7,843402$$

$$A = -1,25826 \quad . \quad . \quad . \quad l - A = 0,099771$$

$$B = +0,01279 \quad . \quad . \quad . \quad l - B = 8,106947$$

His autem valoribus inuentis colligitur fore

$$\frac{1}{2}\delta - \gamma = +0,01742$$

Hic autem valor partem insuper accipit cum ab excentricitate vtriusque orbitae, tum ab inclinatione oriundam, quam deinceps determinabimus.

§. 71. Ex his ergo valoribus habebimus:

$$fRdr = -0,80378 \cos 2\eta + 0,00697 \cos 4\eta$$

$$9,905138 \quad 7,843402$$

$$-1,2630 \vee \cos \eta \quad -0,9500 \vee \cos 3\eta$$

$$0,101403 \quad 9,977736$$

$$H 2$$

$$v =$$

$$\begin{aligned}
 v &= -1,25826 \cos 2\eta + 0,01279 \cos 4\eta \\
 &\quad 0,099771 \quad 8,106947 \\
 &\quad +44,525 \nu \cos \eta - 1,015 \nu \cos 3\eta \\
 &\quad 1,648604 \quad 0,006400
 \end{aligned}$$

hincque porro

$$\begin{aligned}
 \frac{d\phi}{dr} &= \kappa + 0,019015 \cos 2\eta - 0,0000762 \cos 4\eta \\
 &\quad 8,279096 \quad 5,881955 \\
 &\quad +0,0001103
 \end{aligned}$$

$$\begin{aligned}
 &\quad -0,50381 \nu \cos \eta + 0,017068 \nu \cos 3\eta \\
 &\quad 9,702270 \quad 8,232184
 \end{aligned}$$

at est $\frac{d\eta}{dr} = \frac{d\phi}{dr} - \frac{1}{n}$, posuimusque $\kappa - \frac{1}{n} = \alpha$, existente $\kappa = V \left(1 + \frac{3+4\mu+\delta}{2mn} \right)$

§. 72. Ponatur breuitatis gratia

$$\frac{d\phi}{dr} = \Omega + \mathcal{P} \cos 2\eta - \Omega \cos 4\eta - \mathcal{R} \nu \cos \eta + \mathcal{S} \nu \cos 3\eta$$

$$\text{erit } \frac{d\eta}{dr} = \alpha + \mathcal{P} \cos 2\eta - \Omega \cos 4\eta - \mathcal{R} \nu \cos \eta + \mathcal{S} \nu \cos 3\eta$$

fitque ad integrandum:

$$\phi = \nu r + p \sin 2\eta - q \sin 4\eta - r \nu \sin \eta + s \nu \sin 3\eta$$

unde per differentiationem elicitur:

$$\frac{d\phi}{dr} = \nu + 2ap \cos 2\eta - 4aq \cos 4\eta - ar \nu \cos \eta + 3as \nu \cos 3\eta$$

$$\begin{aligned}
 \mathcal{P} p &= \Omega p \cos 2\eta + \mathcal{P} p \cos 4\eta - \mathcal{R} p \nu \cos \eta - \mathcal{R} p \nu \cos 3\eta \\
 + 2\Omega q &- 2\mathcal{P} q \cos 2\eta \quad + 2\mathcal{R} q \nu \cos 3\eta
 \end{aligned}$$

hinc ergo fit:

$$\begin{aligned}
 \nu &= \Omega - \mathcal{P} p - 2\Omega q = \kappa + 0,0001103 - \mathcal{P} p - 2\Omega q \\
 (2\alpha - \Omega) p &= \mathcal{P} + 2\mathcal{P} q; \quad 4\alpha q = \Omega + \mathcal{P} p; \\
 \alpha r &= \mathcal{R} - \mathcal{R} p; \quad 3\alpha s = \mathcal{S} + \mathcal{R} (p - 2q)
 \end{aligned}$$

Ergo

$$\begin{aligned}
 \text{Ergo } p &= 0,010191 \quad . \quad . \quad . \quad / p = 8,008208 \\
 q &= 0,000072 \quad . \quad . \quad . \quad / q = 5,859381 \\
 r &= 0,53453 \quad . \quad . \quad . \quad / r = 9,727977 \\
 s &= 0,00790 \quad . \quad . \quad . \quad / s = 7,897466 \\
 \nu &= \kappa - 0,000080
 \end{aligned}$$

§. 73. Longitudo igitur lunae ϕ quatenus pendet a sola distantia lunae a sole erit

$$\begin{aligned}
 \phi &= (\kappa - 0,000080) r + 0,010191 \sin 2\eta - 0,53453 \nu \sin \eta \\
 &\quad - 0,000072 \sin 4\eta + 0,00790 \nu \sin 3\eta
 \end{aligned}$$

Simili modo cum distantia lunae a terra posita sit $= \frac{\alpha(1-kk)\mu}{1-k\cos r}$, ob $\mu = 1 + \frac{\nu}{mn}$, quatenus valor ipsius μ a sola phasi lunae pendet, erit

$$\begin{aligned}
 \mu &= 1 - 0,00716 \cos 2\eta + 0,00287 \nu \cos \eta \\
 &\quad + 0,00007 \cos 4\eta - 0,00009 \nu \cos 3\eta
 \end{aligned}$$

Verum tamen hic valor litterae α ac praecipue ipsius μ non admodum certus videtur, cum a terminis neglectis licet minimis insignem mutationem perpeti queat. Hic enim pro α non solum $\kappa - \frac{1}{n}$ sed $\kappa - \frac{1}{n} + 0,0001103$ accipi debuiſſet; quare cum valorem ipsius α propius cognoscimus, hanc determinationem repeti conueniet.

CAPUT V.

INVESTIGATIO INAEQUALITATUM
LUNAE AB EIUS EXCENTRICITATE SIMPLICI
SOLUM PUDENTIUM.

§. 74.

Quemadmodum in praecedenti capite inaequalitas absoluta seu variatio duabus partibus constans est inuenta, quarum posterior a littera v seu a parallaxi solis pendebat, ac maiorem curam requirebat; ita etiam inaequalitates, quas hoc capite scrutamur, partes continent ab eadem parallaxi solis pendentes; quarum indagatio quoque accuratiorem cognitionem quorundam elementorum exigit. Hancobrem et praecedentis capituli et huius partes, quae litteram v inuoluunt deinceps, cum reliquis inaequalitates, a parallaxi solis non pendentes determinauerimus, seorsim inuestigabimus, atque titulo inaequalitatum parallaeticarum complectemur.

§. 75. In hoc ergo capite ac sequentibus, donec ad parallaxin solis perueniamus, terminos formularum nostrarum per v multiplicatos tantisper remouebimus; et quoniam hoc loco tantum propositum est in motus lunae inaequalitates a sola excentricitate orbitae lunaris ortas inquirere, eos terminos qui vel excentricitatem solis e vel inclinationem g continent, praetermittimus. Cum autem in formulis nostris duplicis generis termini relinquantur, quorum alteri per k , alteri per kk sunt affecti, inaequalitates ab excentricitate lunae k pendentes

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CAPUT V.

in duas partes distribui conueniet; quarum altera excentricitatem tantum simplicem k implicet, cui hoc caput destinatur, altera vero excentricitatis huius quadrato kk afficiatur, de quo in sequenti capite agemus.

§. 76. Verum tam in huius generis inaequalitates, quam in sequentes, omnes inaequalitates absolutae in praecedenti capite erutae praecipue ingrediuntur; ex quo eas quoque in calculum introduci oportebit. Retinendae ergo erunt in calculo litterae $\mathcal{A}$, $\mathcal{B}$ et A , B , quarum valores cum iam constent, calculus vehementer contrahetur: imprimis autem quia valores litterarum $\mathcal{B}$ et B per se sunt admodum parui, quatenus illi in valores sequentium terminorum influunt, effectum pro nihilo habendum praestabunt. Inuestigationem ergo nostram ita incipiemus, ut pro $\int R dr$ et v valores fictos assumamus, et quoniam $\int R dr$ nullum terminum constantem, v vero neque constantem neque terminum huius formae $a \cos r$ continere debet, ponamus:

$$\begin{aligned} \int R dr = & \mathcal{A} \cos 2\eta + \mathcal{B} \cos 4\eta + \mathcal{C} k \cos r \\ & + \mathcal{D} k \cos (2\eta - r) + \mathcal{F} k \cos (4\eta - r) \\ & + \mathcal{E} k \cos (2\eta + r) + \mathcal{G} k \cos (4\eta + r) \end{aligned}$$

$$\begin{aligned} v = & A \cos 2\eta + B \cos 4\eta \\ & + D k \cos (2\eta + r) + F k \cos (4\eta - r) \\ & + E k \cos (2\eta - r) + G k \cos (4\eta + r) \end{aligned}$$

vbi quidem facile colligere licet, coefficientes $\mathcal{F}$, $\mathcal{G}$, F et G fore minimos.

§. 77.

§. 77. Ex his autem valoribus assumtis obtinebimus ex (§. 52.) sequentes expressiones.

$$\begin{aligned} & \kappa + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} - \frac{(2\kappa A + \mathfrak{A})}{n^2} \cos 2\eta \\ & \left(- \frac{(2\kappa B + \mathfrak{B})}{n^2} + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} \right) \cos 4\eta \\ \frac{d\phi}{dr} = & \left(- \frac{\mathfrak{C}}{n^2} + \frac{3\kappa AD}{n^4} + \frac{A\mathfrak{D} + \mathfrak{A}D}{n^4} \right) k \cos r \\ & - \frac{(2\kappa D + \mathfrak{D})}{n^2} k \cos(2\eta - r) - \frac{(2\kappa E + \mathfrak{E})}{n^2} k \cos(2\eta + r) \\ & - \frac{(2\kappa F + \mathfrak{F})}{n^2} k \cos(4\eta - r) - \frac{(2\kappa G + \mathfrak{G})}{n^2} k \cos(4\eta + r) \\ & + \frac{(3\kappa AD + A\mathfrak{D} + \mathfrak{A}D)}{n^4} k \cos(4\eta - r) \end{aligned}$$

Patebit enim ex valoribus qui inueniuntur, litteras D et $\mathfrak{D}$ tantum prae reliquis fore notabiles, unde terminos ex combinatione reliquarum litterarum oriundos tuto omittere licet.

Pro valore autem ipsius $\frac{d\eta}{dr}$ etiam hi termini ex combinatione orti omitti poterunt. Posito ergo $\kappa + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4}$

$$= \frac{1}{n} \text{ seu } \kappa + 0,00003 = \frac{1}{n} = a \text{ erit:}$$

$$\begin{aligned} \frac{d\eta}{dr} = a - & \frac{(2\kappa A + \mathfrak{A})}{n^2} \cos 2\eta - \frac{2k}{n} \cos r \\ & - \frac{(2\kappa D + \mathfrak{D})}{n^2} k \cos(2\eta - r) - \frac{(2\kappa E + \mathfrak{E})}{n^2} k \cos(2\eta + r) \end{aligned}$$

Cum enim haec formula differentiationibus instituendis inferuiat, reliqui termini post primum cum aliis angulis combinantur, sicque tanto minores terminos produ-

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cunt, qui ex calculo sine errore expungi poterunt: atque ob hanc causam in expressione valoris $\frac{d\eta}{dr}$, statim terminos prae reliquis admodum paruos praetermittere visum est.

§. 78. Valorem autem ipsius R atque $\frac{ddv}{dr^2}$ accuratissime exhiberi oportet, propterea quod his expressionibus totus calculus praecipue innititur, dum valor $\frac{d\eta}{dr}$ formulam tantum subsidiariam suppeditat. Erit ergo

$$\begin{aligned} R = & \frac{3}{2} \sin 2\eta + \frac{3A}{2nn} \sin 4\eta + 3k \sin(2\eta - r) + 3k \sin(2\eta + r) \\ & + \frac{3A}{nn} k \sin(4\eta - r) + \frac{3A}{nn} k \sin(4\eta + r) \\ & + \frac{3D}{2nn} k \sin r - \frac{3E}{2nn} k \sin r \\ & + \frac{3D}{2nn} k \sin(4\eta - r) + \frac{3E}{2nn} k \sin(4\eta + r) \\ & - \frac{3A}{nn} k \sin r + \frac{3A}{nn} k \sin r \end{aligned}$$

vbi quidem terminos ab k non pendentes omittere possumus, quia illorum iam habuimus rationem, ita vt fit

$$\begin{aligned} R = & \dots + \frac{3(D-E)}{2nn} k \sin r + 3k \sin(2\eta - r) + 3k \sin(2\eta + r) \\ & + \frac{3D}{2nn} k \sin(4\eta - r) + \frac{3E}{2nn} k \sin(4\eta + r) \\ & + \frac{3A}{nn} k \sin(4\eta - r) + \frac{3A}{nn} k \sin(4\eta + r) \end{aligned}$$

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§. 79.

§. 79. Simili modo terminis a k non pendentibus omittendis habebitur:

$$\frac{ddv}{dr^2} = -\gamma k c f r + 3 k c f (2\eta - r) + 3 k c f (2\eta + r) - 2\kappa \beta k c f (4\eta - r) - 2\kappa \beta k c f (4\eta + r)$$

$$\begin{aligned} & -\frac{2\kappa \mathcal{C}}{2nn} + \frac{2\kappa \mathcal{D}}{2nn} - \frac{2\kappa \mathcal{E}}{2nn} + \frac{2\kappa \mathcal{D}}{2nn} + \frac{2\kappa \mathcal{E}}{2nn} \\ & + \frac{2\kappa \mathcal{D}}{2nn} + \frac{2\kappa \mathcal{E}}{2nn} + \frac{2\kappa \mathcal{E}}{2nn} - D - E - F - G \\ & + \frac{3AD}{nn} + \frac{3A}{nn} + \frac{3A}{nn} + \frac{3B}{nn} + \frac{3B}{nn} \\ & + \frac{3AE}{nn} + \frac{(2\gamma - \frac{3}{2}\delta)}{nn} D + \frac{(2\gamma - \frac{3}{2}\delta)}{nn} E + \frac{3AD}{nn} + \frac{3AE}{nn} \\ & - \frac{3AA}{2nn} + \frac{(3+3\mu+\gamma)}{nn} A + \frac{(3+3\mu+\gamma)}{nn} A - \frac{3AA}{4nn} - \frac{3AA}{4nn} \\ & + \frac{3AD}{nn} + \frac{3AD}{nn} + \frac{3AE}{nn} + \frac{3AE}{nn} \\ & + \frac{3AE}{nn} + \frac{3AE}{nn} + \frac{3AE}{nn} + \frac{3AE}{nn} \\ & + \frac{3AD}{nn} + \frac{3AD}{nn} + \frac{3AE}{nn} + \frac{3AE}{nn} \\ & + \frac{3AE}{nn} + \frac{3AE}{nn} + \frac{3AE}{nn} + \frac{3AE}{nn} \end{aligned}$$

Hic scilicet plures terminos, qui nullius futuri essent momenti, omisimus, ne calculus nimium implicaretur: notandum autem est esse $\kappa = \sqrt{1 + \frac{3+4\mu+\delta}{2nn}} = 1 + \frac{3+4\mu+\delta}{4nn}$

$$\text{proxime; unde } \mu = (\kappa - 1)nn - \frac{3-\delta}{4} \text{ et } \frac{3+3\mu+\gamma}{4nn} \\ = 3(\kappa - 1) + \frac{3-3\delta+4\gamma}{4nn}.$$

§. 80.

lentibus

 $k c f (4\eta + r)$

$$\frac{2\kappa \mathcal{C}}{2nn}$$

G

 $\frac{3}{2} B$

$$\frac{3AE}{nn}$$

$$\frac{3AA}{4nn}$$

$$\frac{3AE}{nn}$$

$$\frac{3AE}{nn}$$

i essent

aretur:

$$\frac{3+4\mu+\delta}{4nn}$$

$$\frac{\mu+\gamma}{nn}$$

§. 80.

§. 80. Quaeramus nunc quoque ex forma pro fR dr ficta valorem ipsius R, atque exclusis terminis ab k non pendentibus reperiemus:

$$\left(-\frac{2\kappa \mathcal{E} + \mathcal{C}}{nn} + \frac{2\kappa \mathcal{D} + \mathcal{D}}{nn} - \mathcal{C} - \frac{\mathcal{D}(2\kappa \mathcal{A} + \mathcal{A})}{nn} + \frac{\mathcal{E}(2\kappa \mathcal{A} + \mathcal{A})}{nn} \right) k \sin r$$

$$\left(+ \frac{2\mathcal{A}}{n} - (2\alpha - 1) \mathcal{D} \right) k \sin (2\eta - r)$$

$$R = \left(+ \frac{2\mathcal{A}}{n} - (2\alpha + 1) \mathcal{E} \right) k \sin (2\eta + r)$$

$$\left(+ \frac{2\kappa \mathcal{D} + \mathcal{D}}{nn} + \frac{4\mathcal{B}}{n} + \frac{\mathcal{D}(2\kappa \mathcal{A} + \mathcal{A})}{nn} - (4\alpha - 1) \mathcal{F} \right) k \sin (4\eta - r)$$

$$\left(+ \frac{2\kappa \mathcal{E} + \mathcal{C}}{nn} + \frac{4\mathcal{B}}{n} + \frac{\mathcal{E}(2\kappa \mathcal{A} + \mathcal{A})}{nn} - (4\alpha + 1) \mathcal{G} \right) k \sin (4\eta + r)$$

atque instituta comparatione inuenietur:

$$\mathcal{C} = \frac{2\kappa \mathcal{D} + \mathcal{D} - (\mathcal{D} - \mathcal{C})(2\kappa \mathcal{A} + \mathcal{A}) - \frac{1}{2}(\mathcal{D} - \mathcal{E}) - 2\kappa \mathcal{E} + \mathcal{C}}{nn}$$

$$(2\alpha - 1) \mathcal{D} = \frac{2\mathcal{A}}{n} - 3; \quad (2\alpha + 1) \mathcal{E} = \frac{2\mathcal{A}}{n} - 3;$$

$$(4\alpha - 1) \mathcal{F} = \frac{4\mathcal{B}}{n} + \frac{2\kappa \mathcal{D} + \mathcal{D} + \mathcal{D}(2\kappa \mathcal{A} + \mathcal{A}) - \frac{1}{2}(2\mathcal{A} + \mathcal{D})}{nn}$$

$$(4\alpha + 1) \mathcal{G} = \frac{4\mathcal{B}}{n} + \frac{2\kappa \mathcal{E} + \mathcal{C} - \frac{1}{2}(2\mathcal{A} + \mathcal{E}) + \mathcal{E}(2\kappa \mathcal{A} + \mathcal{A})}{nn}$$

§. 81. Pro differentiali $\frac{dv}{dr}$ inueniendo, praeter terminos supra inventos habebimus:

$$\frac{dv}{dr} = -A' \sin 2\eta - B' \sin 4\eta$$

$$\left(+ \frac{A(2\kappa \mathcal{D} + \mathcal{D})}{nn} - \frac{A(2\kappa \mathcal{E} + \mathcal{C})}{nn} - \frac{\mathcal{D}(2\kappa \mathcal{A} + \mathcal{A})}{nn} + \frac{\mathcal{E}(2\kappa \mathcal{A} + \mathcal{A})}{nn} \right) k \sin r$$

$$\left(+ \frac{2\mathcal{A}}{n} - (2\alpha - 1) \mathcal{D} \right) k \sin (2\eta - r) + \left(\frac{2\mathcal{A}}{n} - (2\alpha + 1) \mathcal{E} \right) k \sin (2\eta + r)$$

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$$\left(+ \frac{A(2\kappa D + \mathfrak{D})}{n} + \frac{4B}{n} + \frac{D(2\kappa A + \mathfrak{A})}{nn} - (4\alpha - 1)F \right) k \sin(4\eta - r)$$

$$\left(+ \frac{A(2\kappa E + \mathfrak{E})}{nn} + \frac{4B}{n} + \frac{E(2\kappa A + \mathfrak{A})}{nn} - (4\alpha + 1)G \right) k \sin(4\eta + r)$$

Ponatur autem brevitatis gratia:

$$\frac{dv}{dr} = -A' \sin 2\eta - B' \sin 4\eta - C' k \sin r - D' k \sin(2\eta - r) \\ - E' k \sin(2\eta + r) - F' k \sin(4\eta - r) - G' k \sin(4\eta + r)$$

ut sit:

$$A' = 2A\alpha + \frac{A(2\kappa B + \mathfrak{B}) - 2B(2\kappa A + \mathfrak{A})}{nn}; B' = 4\alpha B - \frac{A(2\kappa A + \mathfrak{A})}{nn}$$

$$C' = \frac{-A(2\kappa D + \mathfrak{D}) + A(2\kappa E + \mathfrak{E}) + (D - E)(2\kappa A + \mathfrak{A})}{nn}$$

$$\text{five } C' = -\frac{A(\mathfrak{D} - \mathfrak{E}) + \mathfrak{A}(D - E)}{nn}$$

$$D' = (2\alpha - 1)D - \frac{2A}{n}; E' = (2\alpha + 1)E - \frac{2A}{n}$$

$$F' = (4\alpha - 1)F - \frac{4B}{n} - \frac{A(2\kappa D + \mathfrak{D}) - D(2\kappa A + \mathfrak{A})}{nn}$$

$$G' = (4\alpha + 1)G - \frac{4B}{n} - \frac{A(2\kappa E + \mathfrak{E}) - E(2\kappa A + \mathfrak{A})}{nn}$$

§. 82. Hinc denuo differentiando obtinebitur terminis tantum per k multiplicatis scribendis:

$$\frac{ddv}{dr^2} = (-C' + \frac{A'(2\kappa D + \mathfrak{D})}{nn} + \frac{A'(2\kappa E + \mathfrak{E})}{nn} + \frac{D'(2\kappa A + \mathfrak{A})}{nn} + \frac{E'(2\kappa A + \mathfrak{A})}{nn}) k \cos r$$

$$\left(+ \frac{2A'}{n} - (2\alpha - 1)D' \right) k \cos(2\eta - r)$$

$$\left(+ \frac{2A'}{n} - (2\alpha + 1)E' \right) k \cos(2\eta + r)$$

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$$\left(+ \frac{4B'}{n} + \frac{A'(2\kappa D + \mathfrak{D})}{nn} + \frac{D'(2\kappa A + \mathfrak{A})}{nn} - (4\alpha - 1)F' \right) k \cos(4\eta - r)$$

$$\left(+ \frac{4B'}{n} + \frac{A'(2\kappa E + \mathfrak{E})}{nn} + \frac{E'(2\kappa A + \mathfrak{A})}{nn} - (4\alpha + 1)G' \right) k \cos(4\eta + r)$$

vnde comparatione instituta orietur:

$$\gamma = -2\kappa \mathfrak{E} - \frac{\frac{1}{2}AA + \frac{3}{2}A(D + E) + \frac{1}{2}A(\mathfrak{D} + \mathfrak{E}) + \frac{1}{2}\mathfrak{A}(2D + E) + \frac{1}{2}\mathfrak{A}(D + \mathfrak{E})}{nn}$$

$$- \frac{A'(2\kappa D + \mathfrak{D}) - A'(2\kappa E + \mathfrak{E}) - (D' + E')(2\kappa A + \mathfrak{A})}{nn}$$

$$\left. \begin{aligned} (2\alpha - 1)^2 D - \frac{2(2\alpha - 1)}{n} A - \frac{2A'}{n} + 3 - 2\kappa \mathfrak{D} - D \\ + \frac{1}{2} A + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{E} + (2\gamma - \frac{1}{2}\delta)D + (3 + 3\mu + \gamma)A}{nn} \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} (2\alpha + 1)^2 E - \frac{2(2\alpha + 1)}{n} A - \frac{2A'}{n} + 3 - 2\kappa \mathfrak{E} - E \\ + \frac{1}{2} A + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{E} + (2\gamma - \frac{1}{2}\delta)E + (3 + 3\mu + \gamma)A}{nn} \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} (4\alpha - 1)^2 F - \frac{4(4\alpha - 1)}{n} B - \frac{(4\alpha - 1)A(2\kappa D + \mathfrak{D}) - (4\alpha - 1)D(2\kappa A + \mathfrak{A})}{nn} \\ - \frac{4B'}{n} - \frac{A'(2\kappa D + \mathfrak{D}) - D'(2\kappa A + \mathfrak{A})}{nn} - 2\kappa \mathfrak{F} - F \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} + \frac{1}{2} B + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{D} + \frac{3}{2}AD - \frac{1}{2}AA + \frac{3}{2}A\mathfrak{D} + \frac{3}{2}\mathfrak{A}D}{nn} \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} (4\alpha + 1)^2 G - \frac{4(4\alpha + 1)}{n} B - \frac{(4\alpha + 1)A(2\kappa E + \mathfrak{E}) - (4\alpha + 1)E(2\kappa A + \mathfrak{A})}{nn} \\ - \frac{4B'}{n} - \frac{A'(2\kappa E + \mathfrak{E}) - E'(2\kappa A + \mathfrak{A})}{nn} - 2\kappa \mathfrak{G} - G \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} + \frac{1}{2} B + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{E} + \frac{3}{2}AE - \frac{1}{2}AA + \frac{3}{2}A\mathfrak{E} + \frac{3}{2}\mathfrak{A}E}{nn} \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} + \frac{1}{2} B + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{E} + \frac{3}{2}AE - \frac{1}{2}AA + \frac{3}{2}A\mathfrak{E} + \frac{3}{2}\mathfrak{A}E}{nn} \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} + \frac{1}{2} B + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{E} + \frac{3}{2}AE - \frac{1}{2}AA + \frac{3}{2}A\mathfrak{E} + \frac{3}{2}\mathfrak{A}E}{nn} \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} + \frac{1}{2} B + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{E} + \frac{3}{2}AE - \frac{1}{2}AA + \frac{3}{2}A\mathfrak{E} + \frac{3}{2}\mathfrak{A}E}{nn} \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} + \frac{1}{2} B + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{E} + \frac{3}{2}AE - \frac{1}{2}AA + \frac{3}{2}A\mathfrak{E} + \frac{3}{2}\mathfrak{A}E}{nn} \end{aligned} \right\} = 0$$

$$\left. \begin{aligned} + \frac{1}{2} B + \frac{\frac{1}{2}\mathfrak{A}\mathfrak{E} + \frac{3}{2}AE - \frac{1}{2}AA + \frac{3}{2}A\mathfrak{E} + \frac{3}{2}\mathfrak{A}E}{nn} \end{aligned} \right\} = 0$$

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§. 83.

§. 83. Incipiamus a coefficientibus $\mathfrak{D}$, $\mathfrak{E}$, et $\mathfrak{D}, \mathfrak{E}$;
et quia $\mathfrak{E}$ est quantitas admodum exigua, erit:

$$(2\alpha-1)\mathfrak{D} = -3 + \frac{2\gamma}{n}; \quad (2\alpha+1)\mathfrak{E} = -3 + \frac{2\gamma}{n}$$

$$\left((2\alpha-1)^2 - 1 + \frac{2\gamma - \frac{1}{2}\delta}{nn} \right) \mathfrak{D} = -3 - \frac{1}{2}A + 2\kappa\mathfrak{D} \\ + 2\frac{(2\alpha-1)}{n}A + \frac{2A'}{n} - (3\kappa - 3 + \frac{3-3\delta+4\gamma}{4nn})A$$

$$\left((2\alpha+1)^2 - 1 + \frac{2\gamma - \frac{1}{2}\delta}{nn} \right) \mathfrak{E} = -3 - \frac{1}{2}A + 2\kappa\mathfrak{E} \\ + 2\frac{(2\alpha+1)}{n}A + \frac{2A'}{n} - (3\kappa - 3 + \frac{3-3\delta+4\gamma}{4nn})A$$

vnde reperitur:

$$\mathfrak{D} = -3,6035 \quad ; \quad \mathfrak{D} = 0,556724$$

$$\mathfrak{E} = -1,0890 \quad ; \quad \mathfrak{E} = 0,037028$$

ac porro

$$\left(-0,24973 + \frac{(2\gamma-\frac{1}{2}\delta)}{nn} \right) \mathfrak{D} = -1,40048 - 7,4315 \left\{ \frac{(2\gamma-\frac{1}{2}\delta)}{nn} \right\} 0,629 \\ \left(+7,21497 + \frac{(2\gamma-\frac{1}{2}\delta)}{nn} \right) \mathfrak{E} = -1,40048 - 2,7403 \left\{ \frac{(2\gamma-\frac{1}{2}\delta)}{nn} \right\} 0,629$$

§. 84. Quoniam autem valorem ipsius $\frac{2\gamma-\frac{1}{2}\delta}{nn}$ non-
dum nobis, hunc terminum, cum certo sit valde par-
vus, reiciamus. Postmodum vero cum istum terminum
cognoverimus, facile erit correctionem inde oriundam,
si operae premium videbitur, invenire.

$$\mathfrak{D} = +35,366 \quad ; \quad \mathfrak{D} = 1,548664$$

$$\mathfrak{E} = -0,5739 \quad ; \quad \mathfrak{E} = 9,758988$$

Porro autem litterae $\mathfrak{F}$ et $\mathfrak{G}$ ita elicientur, ut sit:

$$(4\alpha-1)\mathfrak{F} = 0,000529 + 0,01028 + 0,02071 + 0,02638 \\ (4\alpha+1)\mathfrak{G} = 0,000529 + 0,01028 + 0,02071 + 0,02638 \\ \mathfrak{F} =$$

$\mathfrak{D}, \mathfrak{E}$;

$$- \frac{2\gamma}{n}$$

$$2\kappa\mathfrak{D}$$

$$\gamma) A$$

$$-2\kappa\mathfrak{E}$$

$$) A$$

$$\mathfrak{F} = -0,1907 \quad ; \quad \mathfrak{F} = 9,280416$$

$$\mathfrak{G} = +0,0122 \quad ; \quad \mathfrak{G} = 8,087607$$

Praeterea autem colligimus fore

$$\mathfrak{E} = -0,67465 \quad ; \quad \mathfrak{E} = 9,829072$$

vnde erit proxime $\frac{\mathfrak{E}}{2nn} = 0,00154$, ex quo accuratius

concluditur fore

$$\mathfrak{D} = +35,3724 \quad ; \quad \mathfrak{D} = 1,548664$$

$$\mathfrak{E} = -0,5741 \quad ; \quad \mathfrak{E} = 9,758988$$

§. 85. Reliquae aequationes nobis praebebunt

$$6,4655 F + 3,67820 = 0$$

$$21,3946 G - 0,29574 = 0$$

vnde obtinebitur

$$F = -0,56890 \quad ; \quad F = 9,755033$$

$$G = +0,01382 \quad ; \quad G = 8,140620$$

ac denique $\gamma = 1,40673$.

Supra autem iam inuenimus $\frac{1}{2}\delta - \gamma = 0,01742$, vnde
ambas istas quantitates γ et δ , quas initio ad veros valo-
res constantium litterarum n et κ determinandos assum-
simus, nunc cognitae habemus, erit enim:

$$\delta = 2,84830, \quad \text{et} \quad 2\gamma - \frac{1}{2}\delta = -1,45899$$

ac propterea particulae illius $\frac{2\gamma-\frac{1}{2}\delta}{nn}$ hactenus neglectae

valor erit $\frac{2\gamma-\frac{1}{2}\delta}{nn} = -0,00832$, cuius ope iam litterae

$\mathfrak{D}$ et $\mathfrak{E}$ accuratius definiri poterunt.

§. 86.

§. 86. Hinc autem potissimum valor ipsius D mutationem patitur, fiet enim re vera

$$-0,25805 D = -8,83698 \text{ seu}$$

$$D = 34,24520 \quad \text{. . .} \quad D = 1,534600$$

$$7,20665 E = -4,14578 \text{ seu}$$

$$E = -0,57527 \quad \text{. . .} \quad -E = 9,759874$$

et quoniam D parte sua tricesima diminuitur, in eadem fere ratione diminuentur valores litterarum $\mathcal{C}$ et γ , ita ut exactius sit:

$$\mathcal{C} = -0,65217 \quad \text{. . .} \quad -\mathcal{C} = 9,814361$$

$$\gamma = +1,35984 \quad \text{. . .} \quad \gamma = 0,133490$$

$$\delta = +2,75336 \quad \text{. . .} \quad \delta = 0,439863$$

$$\text{et } \frac{2\gamma - \frac{1}{2}\delta}{nn} = -0,00804$$

Deinceps autem operae erit pretium in hos valores adhuc diligentius inquirere.

§. 87. Cum igitur finxerimus sequentes valores:

$$\begin{aligned} \int R dr &= \mathcal{A} \cos 2\eta + \mathcal{B} \cos 4\eta + \mathcal{C} k \cos r \\ &\quad + \mathcal{D} k \cos(2\eta - r) + \mathcal{F} k \cos(4\eta - r) \\ &\quad + \mathcal{E} k \cos(2\eta + r) + \mathcal{G} k \cos(4\eta + r) \\ &= A \cos 2\eta + B \cos 4\eta \\ &\quad + D k \cos(2\eta - r) + F k \cos(4\eta - r) \\ &\quad + E k \cos(2\eta + r) + G k \cos(4\eta + r) \end{aligned}$$

horum coefficientium valores sunt.

$$\mathcal{A} = +0,80378 \quad \mathcal{A} = -1,25826$$

$$\mathcal{B} = +0,00697 \quad \mathcal{B} = -0,01279$$

$$\mathcal{C} = -0,65217$$

$$\mathcal{D} = -3,60350$$

$$\mathcal{E} = -1,08900$$

$$\mathcal{F} = -0,19070$$

$$\mathcal{G} = +0,01220$$

$$D = +34,24520$$

$$E = -0,57527$$

$$F = -0,56890$$

$$G = +0,01382$$

vnde

D mu-

v.

u

$\frac{d}{dr}$

ai

Φ

eadem

et γ , ita

ores ad-

lores:

$\eta - r$

$\eta + r$

$\eta - r$

$\eta + r$

F

(

vnde

vnde pro distantia lunae a terra $x = \frac{(1-kk)au}{1-k \cos r}$ fit

$$\begin{aligned} u &= 1 - 0,007161 \cos 2\eta + 0,000073 \cos 4\eta \\ &\quad + 0,194888 k \cos(2\eta - r) - 0,003274 k \cos(2\eta + r) \\ &\quad - 0,003238 k \cos(4\eta - r) + 0,000078 k \cos(4\eta + r) \end{aligned}$$

§. 88. His valoribus in §. 77. substitutis obtinebimus:

$$\begin{aligned} \frac{d\Phi}{dr} &= \kappa + 0,019015 \cos 2\eta - 0,001255 k \cos r \\ &\quad + 0,0001103 - 0,000076 \cos 4\eta \\ &\quad - 0,38410 k \cos(2\eta - r) + 0,01278 k \cos(2\eta + r) \\ &\quad + 0,002647 k \cos(4\eta - r) - 0,000229 k \cos(4\eta + r) \end{aligned}$$

ad cuius integrale inveniendum ponamus:

$$\begin{aligned} \Phi &= Or + \mathcal{A}' \sin 2\eta + \mathcal{B}' \sin 4\eta + \mathcal{C}' k \sin r \\ &\quad + \mathcal{D}' k \cos(2\eta - r) + \mathcal{E}' k \cos(2\eta + r) \\ &\quad + \mathcal{F}' k \cos(4\eta - r) + \mathcal{G}' k \cos(4\eta + r) \end{aligned}$$

eritque differentiendo et terminis iam cognitis omittendis.

$$\begin{aligned} \frac{d\Phi}{dr} &= (\mathcal{C}' - \frac{\mathcal{A}'(2\kappa D + \mathcal{D})}{nn} - \frac{\mathcal{A}'(2\kappa E + \mathcal{E})}{nn} - \frac{\mathcal{D}'(2\kappa A + \mathcal{A})}{nn} - \frac{\mathcal{E}'(2\kappa A + \mathcal{A})}{nn}) k \cos r \\ &\quad (-\frac{2\mathcal{A}'}{n} + (2\kappa - 1)\mathcal{D}') k \cos(2\eta - r) \\ &\quad (-\frac{2\mathcal{A}'}{n} + (2\kappa + 1)\mathcal{E}') k \cos(2\eta + r) \\ &\quad (-\frac{4\mathcal{B}'}{n} - \frac{\mathcal{A}'(2\kappa D + \mathcal{D})}{nn} - \frac{\mathcal{D}'(2\kappa A + \mathcal{A})}{nn} + (4\kappa - 1)\mathcal{F}') k \cos(4\eta - r) \\ &\quad (-\frac{4\mathcal{B}'}{n} - \frac{\mathcal{A}'(2\kappa E + \mathcal{E})}{nn} - \frac{\mathcal{E}'(2\kappa A + \mathcal{A})}{nn} + (4\kappa + 1)\mathcal{G}') k \cos(4\eta + r) \end{aligned}$$

Pro terminis autem iam inuentis est

$$\begin{aligned} O &= \kappa - 0,000080; \quad \mathcal{A}' = 0,010191; \quad \mathcal{B}' = -0,000072 \\ \mathcal{D}' &= 8,008208; \quad \mathcal{E}' = -5,859381 \end{aligned}$$

K

§. 89.

§. 89. Comparatione iam instituta fiet:

$$\begin{aligned} (2\alpha-1)\mathcal{D}' &= -0,38410 + \frac{2\mathcal{M}'}{n}; (2\alpha+1)\mathcal{E}' = +0,01278 + \frac{2\mathcal{M}'}{n} \\ (4\alpha-1)\mathcal{F}' &= +0,002647 + \frac{4\mathcal{B}'}{n} + \frac{\mathcal{M}'(2\kappa\mathcal{D}+\mathcal{D})+\mathcal{D}'(2\kappa\mathcal{A}+\mathcal{A})}{nn} \\ (4\alpha+1)\mathcal{G}' &= -0,000229 + \frac{4\mathcal{B}'}{n} + \frac{\mathcal{M}'(2\kappa\mathcal{E}+\mathcal{E})+\mathcal{E}'(2\kappa\mathcal{A}+\mathcal{A})}{nn} \\ \mathcal{E}' &= -0,001255 + \frac{\mathcal{M}'(2\kappa\mathcal{D}+\mathcal{D})+\mathcal{D}'(2\kappa\mathcal{A}+\mathcal{A})}{nn} \\ &+ \frac{\mathcal{M}'(2\kappa\mathcal{E}+\mathcal{E})+\mathcal{E}'(2\kappa\mathcal{A}+\mathcal{A})}{nn} \end{aligned}$$

vnde colligitur fore

$$\mathcal{E}' = +0,01083$$

$$\mathcal{D}' = -0,44167 \quad . \quad . \quad . \quad l-\mathcal{D}' = 9,645092$$

$$\mathcal{E}' = +0,00499 \quad . \quad . \quad . \quad l-\mathcal{E}' = 7,698640$$

$$\mathcal{F}' = +0,00546 \quad . \quad . \quad . \quad l-\mathcal{F}' = 7,737733$$

$$\mathcal{G}' = -0,00010 \quad . \quad . \quad . \quad l-\mathcal{G}' = 6,002537$$

ita vt fit

$$\Phi = (\kappa - 0,000080r) + 0,010191 \sin 2\eta + 0,01083 k \sin r \\ - 0,000072 \sin 4\eta$$

$$- 0,44167 k \sin (2\eta - r) + 0,00499 k \sin (2\eta + r)$$

$$+ 0,00546 k \sin (4\eta - r) - 0,00010 k \sin (4\eta + r)$$

vnde ex comparatione motus medii ad modum anomalie erit $\kappa = 1,008607$, et $\alpha = 0,933279$, qui valores iam propius ad veritatem accedunt, quam hactenus vsurpati.

CAPUT.

CAPUT VI.

INVESTIGATIO INAEQUALITATUM
LUNAE A QUADRATO EXCENTRICITATIS
IPSIVS ORBITARUM.

§. 90.

Pervenimus nunc ad alteram partem inaequalitatum in motu Lunae, quae ab eius excentricitate k pendent, eiusque quadratum inuoluunt, ita vt hic non nisi eos terminos simus contemplaturi, qui per quadratum excentricitatis lunae kk sunt multiplicati. Hic autem tam in valorem ipsius $fRdr$, quam ipsius v termini formae $kk \cos 2\eta$ et $kk \cos 4\eta$ ingredientur, qui postquam fuerint inuenti, terminis huius generis iam ante inuentis adiici debent: praeterea vero vtrinque etiam termini formae $kk \cos 2r$ accedent. Hinc ponamus:

$$\begin{aligned} fRdr &= \mathcal{A} \cos 2\eta + a \, kk \cos 2\eta + \mathcal{B} \cos 4\eta + b \, kk \cos 4\eta \\ &+ \mathcal{C} k \cos r + \mathcal{D} k \cos (2\eta - r) + \mathcal{E} k \cos (2\eta + r) \\ &+ \mathcal{F} k \cos (4\eta - r) + \mathcal{G} k \cos (4\eta + r) \\ &+ \mathcal{H} kk \cos 2r + \mathcal{I} kk \cos (2\eta - 2r) + \mathcal{K} kk \cos (2\eta + 2r) \\ &+ \mathcal{L} kk \cos (4\eta - 2r) + \mathcal{M} kk \cos (4\eta + 2r) \end{aligned}$$

$$\begin{aligned} v &= A \cos 2\eta + a \, kk \cos 2\eta + B \cos 4\eta + b \, kk \cos 4\eta \\ &+ D k \cos (2\eta - r) + E k \cos (2\eta + r) \\ &+ F k \cos (4\eta - r) + G k \cos (4\eta + r) \\ &+ H kk \cos 2r + J kk \cos (2\eta - 2r) + K kk \cos (2\eta + 2r) \\ &+ L kk \cos (4\eta - 2r) + M kk \cos (4\eta + 2r) \end{aligned}$$

K 2

§. 91.

APUT

§. 91. Nunc ad terminos, quibus ante valorem ipsius $\frac{d\phi}{dr}$ exprimi inuenimus, insuper sequentes per kk multiplicati accedent:

$$\begin{aligned} \frac{d\phi}{dr} = & \dots + \frac{D(3kD+2D)}{2n^4} k^2 + \left(-\frac{(2\kappa A+a)}{nn} + \frac{ED}{n^4} \right) k^2 \cos 2\eta \\ & \left(-\frac{(2\kappa b+b)}{nn} + \frac{D(3\kappa E+2E)}{2n^4} + \frac{E(3\kappa D+2D)}{2n^4} \right) k^2 \cos 4\eta \\ & \left[-\frac{(2\kappa H+\phi)}{nn} + \frac{D(3\kappa E+2E)}{2n^4} + \frac{E(3\kappa D+2D)}{2n^4} \right. \\ & \quad \left. + \frac{A(3\kappa J+2J)}{2n^4} + \frac{J(3\kappa A+2A)}{2n^4} \right] k^2 \cos 2r \\ & \left(-\frac{(2\kappa J+J)}{nn} + \frac{A(3\kappa H+2\phi)}{2n^4} + \frac{H(3\kappa A+2A)}{2n^4} + \frac{ED}{n^4} \right) k^2 \cos(2\eta-2r) \\ & \left(-\frac{(2\kappa K+K)}{nn} + \frac{A(3\kappa H+2\phi)}{2n^4} + \frac{H(3\kappa A+2A)}{2n^4} \right) k^2 \cos(4\eta+2r) \\ & \left(-\frac{(2\kappa L+L)}{nn} + \frac{A(3\kappa J+2J)}{2n^4} + \frac{J(3\kappa A+2A)}{2n^4} + \frac{D(3\kappa D+2D)}{2n^4} \right) k^2 \cos(4\eta-2r) \\ & - \frac{(2\kappa M+M)}{nn} k^2 \cos(4\eta+2r) \end{aligned}$$

vbi quidem terminos, quos minimos fore facile est praecuidere, omisimus.

§. 92. Terminus autem constans $\frac{D(3\kappa D+2D)}{2n^4} kk$ reperitur = 0,00175, unde posito $\kappa + 0,000285 = \frac{1}{n} = a$, quoniam valorem ipsius $\frac{d\eta}{dr}$ non opus est tam exacte nosse, sumamus:

$$\frac{d\eta}{dr}$$

valorem
per kk

$$) k^2 \cos 2\eta$$

$$k^2 \cos 4\eta$$

$$) k^2 \cos 2r$$

$$\cos(2\eta-2r)$$

$$(4\eta+2r)$$

$$(4\eta-2r)$$

cile est

$$\frac{2D}{n^4} kk$$

$$000285$$

us est

$$\frac{d\eta}{dr}$$

$$\begin{aligned} \frac{d\eta}{dr} = & a - \frac{(2\kappa A+M)}{nn} \cos 2\eta - \left(\frac{2k}{n} + \frac{Ek}{nn} \right) \cos r \\ & - \frac{(2\kappa D+D)}{nn} k \cos(2\eta-r) - \left(\frac{3kk}{2n} + \frac{(2\kappa H+\phi)}{nn} \right) \cos 2r \\ & - \frac{(2\kappa J+J)}{nn} k^2 \cos(2\eta-2r) \end{aligned}$$

Deinde vero praeter terminos iam tractatos habebitur:

$$\begin{aligned} R = & \dots 3k^2 \sin 2\eta + \frac{3D}{nn} k^2 \sin 4\eta + \frac{3(2D+J)}{2nn} k^2 \sin 2r \\ & + \left(\frac{1}{4} k^2 + \frac{3(H-L)}{2nn} \right) \sin(2\eta-2r) + \left(\frac{1}{4} k^2 + \frac{3H}{2nn} \right) \sin(2\eta+2r) \\ & + \frac{3(2D+J)}{2nn} k^2 \sin(4\eta-2r) \end{aligned}$$

atque simili modo:

$$\begin{aligned} \frac{ddv}{dr^2} = & \frac{1}{2} \delta - \gamma + \frac{2a + \frac{1}{2} D D + 3 D D + 3 D D + 3 A a + 3 M a + 3 A a}{nn} kk \\ & + \left(3 + \frac{1}{2} (A+D+E) - a - 2\kappa a \right) kk \cos 2\eta \\ & + \left(\frac{1}{2} (B+F+G) - b - 2\kappa b \right) kk \cos 4\eta \\ & + \left(\frac{1}{2} - 2\kappa \phi - H + \frac{2J+3J+3A+3A}{nn} \right) kk \cos 2r \\ & + \left(\frac{1}{4} - 2\kappa J - J + \frac{1}{2} A + \frac{1}{2} D \right) kk \cos(2\eta-2r) \\ & + \left(\frac{1}{4} - 2\kappa K - K + \frac{1}{2} A + \frac{1}{2} E \right) k^2 \cos(2\eta+2r) \\ & + (-2\kappa L - L + \frac{1}{2} B + \frac{1}{2} F) kk \cos(4\eta-2r) \\ & + (-2\kappa M - M + \frac{1}{2} B + \frac{1}{2} G) k^2 \cos(4\eta+2r) \\ & + \frac{2J+3J+3A+3A + \frac{1}{2} D D + 3 D D + 3 D D}{nn} kk \cos(4\eta-2r) \end{aligned}$$

§. 93. Eliciamus nunc quoque valorem ipsius R per differentiationem ex formula $\int R dr$, ac terminis apte dispositis habebimus

$$K_3$$

$$R =$$

R ==

$kk \sin 2\eta$	$kk \sin 4\eta$	$kk \sin 2r$	$k^2 \sin(2\eta - 2r)$
$-2\alpha a$	$+\frac{a(2\kappa A + \mathfrak{A})}{nn}$	$+\frac{\mathfrak{A}(2\kappa J + \mathfrak{J})}{nn}$	$+\frac{3\mathfrak{A}}{2n}$
$+\frac{2b(2\kappa A + \mathfrak{A})}{nn}$	$-4\alpha b$	$-2\mathfrak{J}$	$+\frac{\mathfrak{A}(2\kappa H + \mathfrak{H})}{nn}$
$+\frac{\mathfrak{D}(2n + \mathfrak{C})}{nn}$	$+\frac{\mathfrak{C}(2\kappa D + \mathfrak{D})}{nn}$	$+\frac{\mathfrak{C}(2\kappa D + \mathfrak{D})}{nn}$	$+\frac{\mathfrak{D}(2n + \mathfrak{C})}{nn}$
$+\frac{\mathfrak{C}(2n + \mathfrak{C})}{nn}$	$+\frac{2\mathfrak{J}(2n + \mathfrak{C})}{nn}$	$+\frac{\mathfrak{J}(2\kappa A + \mathfrak{A})}{nn}$	$-2(\alpha - 1)\mathfrak{J}$
$+\frac{2\mathfrak{J}(2\kappa D + \mathfrak{D})}{nn}$	$+\frac{2\mathfrak{C}(2n + \mathfrak{C})}{nn}$	$+\frac{\mathfrak{R}(2\kappa A + \mathfrak{A})}{nn}$	$+\frac{2\mathfrak{J}(2\kappa A + \mathfrak{A})}{nn}$

$kk \sin(2\eta + 2r)$	$kk \sin(4\eta - 2r)$	$kk \sin(4\eta + 2r)$
$+\frac{3\mathfrak{A}}{2n}$	$+\frac{\mathfrak{A}(2\kappa J + \mathfrak{J})}{nn}$	
$+\frac{\mathfrak{A}(2\kappa H + \mathfrak{H})}{nn}$	$+\frac{3\mathfrak{J}}{n}$	$+\frac{3\mathfrak{J}}{n}$
$+\frac{2\mathfrak{J}(2\kappa J + \mathfrak{J})}{nn}$	$+\frac{2\mathfrak{J}(2\kappa H + \mathfrak{H})}{nn}$	$+\frac{2\mathfrak{J}(2\kappa H + \mathfrak{H})}{nn}$
$+\frac{\mathfrak{C}(2n + \mathfrak{C})}{nn}$	$+\frac{\mathfrak{D}(2\kappa D + \mathfrak{D})}{nn}$	$+\frac{\mathfrak{C}(2n + \mathfrak{C})}{nn}$
$+\frac{2\mathfrak{C}(2\kappa D + \mathfrak{D})}{nn}$	$+\frac{2\mathfrak{J}(2n + \mathfrak{C})}{nn}$	$+\frac{\mathfrak{R}(2\kappa A + \mathfrak{A})}{nn}$
$-2(\alpha + 1)\mathfrak{R}$	$+\frac{\mathfrak{J}(2\kappa A + \mathfrak{A})}{nn}$	$-2(2\alpha + 1)\mathfrak{M}$
$+\frac{2\mathfrak{M}(2\kappa A + \mathfrak{A})}{nn}$	$-2(2\alpha - 1)\mathfrak{J}$	

vnde

vnde oriuntur sequentes determinaciones:

$$\begin{aligned}
 3 &= -2\alpha a + \frac{2b(2\kappa A + \mathfrak{A}) + (\mathfrak{D} + \mathfrak{C})(2n + \mathfrak{C}) + 2\mathfrak{J}(2\kappa D + \mathfrak{D})}{nn} \\
 \frac{3\mathfrak{D}}{nn} &= -4\alpha b + \frac{a(2\kappa A + \mathfrak{A}) + \mathfrak{C}(2\kappa D + \mathfrak{D}) + 2(\mathfrak{J} + \mathfrak{C})(2n + \mathfrak{C})}{nn} \\
 \frac{3(2\mathfrak{D} + \mathfrak{J})}{2nn} &= -2\mathfrak{J} + \frac{\mathfrak{A}(2\kappa J + \mathfrak{J}) + \mathfrak{C}(2\kappa D + \mathfrak{D}) - (\mathfrak{J} - \mathfrak{R})(2\kappa A + \mathfrak{A})}{nn} \\
 \frac{1}{2} + \frac{3(\mathfrak{H} - \mathfrak{L})}{2nn} &= -2(\alpha - 1)\mathfrak{J} + \frac{3\mathfrak{A}}{nn} + \frac{\mathfrak{A}(2\kappa H + \mathfrak{H})}{nn} \\
 &\quad + \frac{\mathfrak{D}(2n + \mathfrak{C})}{nn} + \frac{2\mathfrak{J}(2\kappa A + \mathfrak{A})}{nn} \\
 \frac{1}{2} + \frac{3\mathfrak{H}}{2nn} &= 2(\alpha + 1)\mathfrak{R} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H + \mathfrak{H})}{nn} \\
 &\quad + \frac{2\mathfrak{J}(2\kappa J + \mathfrak{J}) + 2\mathfrak{C}(2\kappa D + \mathfrak{D}) + \mathfrak{C}(2n + \mathfrak{C}) + 2\mathfrak{M}(2\kappa A + \mathfrak{A})}{nn} \\
 + \frac{3(2\mathfrak{D} + \mathfrak{J})}{2nn} &= -2(2\alpha - 1)\mathfrak{J} + \frac{3\mathfrak{J}}{n} + \frac{\mathfrak{A}(2\kappa J + \mathfrak{J})}{nn} \\
 &\quad + \frac{2\mathfrak{J}(2\kappa H + \mathfrak{H}) + \mathfrak{D}(2\kappa D + \mathfrak{D}) + 2\mathfrak{J}(2n + \mathfrak{C}) + \mathfrak{J}(2\kappa A + \mathfrak{A})}{nn} \\
 0 &= -2(2\alpha + 1)\mathfrak{M} + \frac{3\mathfrak{J}}{n} + \frac{2\mathfrak{J}(2\kappa H + \mathfrak{H}) + 2\mathfrak{C}(2n + \mathfrak{C})}{nn} \\
 &\quad + \frac{\mathfrak{R}(2\kappa A + \mathfrak{A})}{nn}
 \end{aligned}$$

§. 94. Deinde simili modo si ponatur:

$$\begin{aligned}
 \frac{dw}{dr} &= -A' \sin 2\eta - A' k^2 \sin 2\eta - B' \sin 4\eta - B' k^2 \sin 4\eta \\
 &\quad - C' k \sin r - D' k \sin(2\eta - r) - F' k \sin(4\eta - r) \\
 &\quad - E' k \sin(2\eta + r) - G' k \sin(2\eta + r) \\
 &\quad - H' k^2 \sin 2r - J' k^2 \sin(2\eta - 2r) - L' k^2 \sin(4\eta - 2r) \\
 &\quad - K' k^2 \sin(2\eta + 2r) - M' k^2 \sin(4\eta + 2r)
 \end{aligned}$$

erit

vnde

erit practer valores §. 81. datos:

$$a' = 2\alpha a - \frac{2b(2\kappa A + \mathfrak{A}) - (D + E)(2\pi + \mathfrak{C}) - 2F(2\kappa D + \mathfrak{D})}{\pi\pi}$$

$$b' = 4\alpha b - \frac{a(2\kappa A + \mathfrak{A}) - E(2\kappa D + \mathfrak{D})}{\pi\pi} - \frac{2(F + G)(2\pi + \mathfrak{C}) - D(2\kappa E + \mathfrak{E})}{\pi\pi}$$

$$H' = 2H - \frac{A(2\kappa J + \mathfrak{J}) - E(2\kappa D + \mathfrak{D})}{\pi\pi} + \frac{(J - K)(2\kappa A + \mathfrak{A}) + D(2\kappa E + \mathfrak{E})}{\pi\pi}$$

$$J' = 2(\alpha - 1)J - \frac{3A}{2\pi} - \frac{A(2\kappa H + \mathfrak{H}) - D(2\pi + \mathfrak{C}) - 2L(2\kappa A + \mathfrak{A})}{\pi\pi}$$

$$K' = 2(\alpha + 1)K - \frac{3A}{2\pi} - \frac{A(2\kappa H + \mathfrak{H}) - 2B(2\kappa J + \mathfrak{J}) - 2G(2\kappa D + \mathfrak{D})}{\pi\pi} - \frac{E(2\pi + \mathfrak{C}) - 2M(2\kappa A + \mathfrak{A})}{\pi\pi}$$

$$L' = 2(2\alpha - 1)L - \frac{3B}{\pi} - \frac{A(2\kappa J + \mathfrak{J}) - 2B(2\kappa H + \mathfrak{H}) - D(2\kappa D + \mathfrak{D})}{\pi\pi} - \frac{2F(2\pi + \mathfrak{C}) - J(2\kappa A + \mathfrak{A})}{\pi\pi}$$

$$M' = 2(2\alpha + 1)M - \frac{3B}{\pi} - \frac{2B(2\kappa H + \mathfrak{H}) - G(2\pi + \mathfrak{C}) - K(2\kappa A + \mathfrak{A})}{\pi\pi}$$

vbi quidem plures terminos, quos admodum paruos fore praeuidimus, omisimus.

§. 95.

§. 95. Hinc autem denuo differentiando obtinemus

valorem ipsius $\frac{ddv}{dr^2} =$

$\frac{D + \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{E + \mathfrak{E}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{E + \mathfrak{E}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{A + \mathfrak{A}}$

$\frac{+ \mathfrak{D}}{D + \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{A + \mathfrak{A}}$

$\frac{+ \mathfrak{D}}{D + \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{A + \mathfrak{A}}$

$\frac{+ \mathfrak{D}}{A + \mathfrak{A}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$\frac{+ \mathfrak{D}}{+ \mathfrak{D}}$

$k k$	$k k \text{ cof } 2 \eta$	$k k \text{ cof } 4 \eta$	$k k \text{ cof } 2 r$
$\frac{a'(2\kappa A + \mathfrak{A})}{\pi\pi}$	$\frac{2\alpha a'}{\pi\pi}$	$\frac{a'(2\kappa A + \mathfrak{A})}{\pi\pi}$	$\frac{A'(2\kappa J + \mathfrak{J})}{\pi\pi}$
$\frac{D'(2\kappa D + \mathfrak{D})}{\pi\pi}$	$\frac{2b'(2\kappa A + \mathfrak{A})}{\pi\pi}$	$\frac{4\alpha b'}{\pi\pi}$	$\frac{E'(2\kappa D + \mathfrak{D})}{\pi\pi}$
$\frac{+ \mathfrak{D}}{\pi\pi}$	$\frac{D'(2\pi + \mathfrak{C})}{\pi\pi}$	$\frac{E'(2\kappa D + \mathfrak{D})}{\pi\pi}$	$\frac{2H'}{\pi\pi}$
$\frac{+ \mathfrak{D}}{\pi\pi}$	$\frac{E'(2\pi + \mathfrak{C})}{\pi\pi}$	$\frac{2F'(2\pi + \mathfrak{C})}{\pi\pi}$	$\frac{J'(2\kappa A + \mathfrak{A})}{\pi\pi}$
$\frac{+ \mathfrak{D}}{\pi\pi}$	$\frac{2F'(2\kappa D + \mathfrak{D})}{\pi\pi}$	$\frac{2G'(2\pi + \mathfrak{C})}{\pi\pi}$	$\frac{K'(2\kappa A + \mathfrak{A})}{\pi\pi}$
$k k \text{ cof } (2\eta - 2r)$	$k k \text{ cof } (2\eta + 2r)$	$k k \text{ cof } (4\eta - 2r)$	$k k \text{ cof } (4\eta + 2r)$
$\frac{3A'}{2\pi}$	$\frac{3A'}{2\pi}$	$\frac{A'(2\kappa J + \mathfrak{J})}{\pi\pi}$	$\frac{3B'}{\pi}$
$\frac{A'(2\kappa H + \mathfrak{H})}{\pi\pi}$	$\frac{A'(2\kappa H + \mathfrak{H})}{\pi\pi}$	$\frac{3B'}{\pi}$	$\frac{3B'}{\pi}$
$\frac{D'(2\pi + \mathfrak{C})}{\pi\pi}$	$\frac{2G'(2\kappa D + \mathfrak{D})}{\pi\pi}$	$\frac{2B'(2\kappa H + \mathfrak{H})}{\pi\pi}$	$\frac{2B'(2\kappa H + \mathfrak{H})}{\pi\pi}$
$\frac{+ \mathfrak{D}}{\pi\pi}$	$\frac{+ \mathfrak{D}}{\pi\pi}$	$\frac{D'(2\kappa D + \mathfrak{D})}{\pi\pi}$	$\frac{2G'(2\pi + \mathfrak{C})}{\pi\pi}$
$\frac{- 2(\alpha - 1)J'}{2L'(2\kappa A + \mathfrak{A})}$	$\frac{- 2(\alpha + 1)K'}{2M'(2\kappa A + \mathfrak{A})}$	$\frac{2F'(2\pi + \mathfrak{C})}{\pi\pi}$	$\frac{- 2(2\alpha + 1)M'}{K'(2\kappa A + \mathfrak{A})}$
$\frac{+ \mathfrak{D}}{\pi\pi}$	$\frac{+ \mathfrak{D}}{\pi\pi}$	$\frac{- 2(2\alpha - 1)L'}{J'(2\kappa A + \mathfrak{A})}$	$\frac{+ \mathfrak{D}}{\pi\pi}$

L

vnde

vnde tandem nascimur has determinationes:

$$\frac{1}{2}\delta - \gamma + \frac{2a + \frac{1}{2}DD + 3D\mathcal{D} + \frac{1}{2}DD + 3Aa + 3\mathcal{A}a + 3Aa}{k k} \\ = \frac{a'(2\kappa A + \mathcal{A}) + D'(2\kappa D + \mathcal{D})}{k k}$$

$$3 + \frac{1}{2}(A + D + E) - a - 2\kappa a = -2a' + \frac{2b'(2\kappa A + \mathcal{A})}{k k} \\ + \frac{(D' + E')(2\kappa + \mathcal{C}) + 2F'(2\kappa D + \mathcal{D})}{k k}$$

$$\frac{1}{2}(B + F + G) - b - 2\kappa b = -4ab' + \frac{a'(2\kappa A + \mathcal{A})}{k k} \\ + \frac{E'(2\kappa D + \mathcal{D}) + 2(F' + G')(2\kappa + \mathcal{C})}{k k}$$

$$\frac{1}{2} - 2\kappa\mathcal{J} - H + \frac{2\mathcal{J} + 3\mathcal{J}J + 3A\mathcal{J} + 3AJ}{k k} = -2H' + \frac{A'(2\kappa J + \mathcal{J})}{k k} \\ + \frac{E'(2\kappa D + \mathcal{D}) + (J' + K')(2\kappa A + \mathcal{A})}{k k}$$

$$\frac{1}{2} - 2\kappa\mathcal{J} - J + \frac{1}{2}A + \frac{1}{2}D = -2(a-1)J' + \frac{3A'}{2\kappa} + \frac{A'(2\kappa H + \mathcal{H})}{k k} \\ + \frac{D'(2\kappa + \mathcal{C}) + 2L'(2\kappa A + \mathcal{A})}{k k}$$

$$\frac{1}{2} - 2\kappa\mathcal{K} - K + \frac{1}{2}A + \frac{1}{2}E = -2(a+1)K' + \frac{3A'}{2\kappa} + \frac{A'(2\kappa H + \mathcal{H})}{k k} \\ + \frac{2G'(2\kappa D + \mathcal{D}) + 2M'(2\kappa A + \mathcal{A})}{k k}$$

$$-2\kappa\mathcal{L} - L + \frac{1}{2}B + \frac{1}{2}F + \frac{2\mathcal{J} + 3\mathcal{J}J + 3A\mathcal{J} + 3AJ + \frac{1}{2}DD + 3D\mathcal{D} + \frac{1}{2}DD}{k k} = \\ -2(2a-1)L' + \frac{3B'}{2\kappa} + \frac{A'(2\kappa J + \mathcal{J}) + 2B'(2\kappa H + \mathcal{H})}{k k} \\ + \frac{D'(2\kappa D + \mathcal{D}) + 2F'(2\kappa + \mathcal{C}) + J'(2\kappa A + \mathcal{A})}{k k}$$

$$-2\kappa\mathcal{M} - M + \frac{1}{2}B + \frac{1}{2}G = -2(2a+1)M' + \frac{3B'}{2\kappa} + \frac{2B'(2\kappa H + \mathcal{H})}{k k} \\ + \frac{2G'(2\kappa + \mathcal{C}) + K'(2\kappa A + \mathcal{A})}{k k}$$

§. 96. Primum autem valoribus iam cognitis substituendis, reperitur:

$2a = -3,837 - 0,038b$ et $4ab = -1,073 - 0,019a$
hincque $a = -2,051$ et $b = -0,277$
ex quibus porro elicimus: $a' = -12,595$ et $b' = -0,086$
et $a' = -23,510$. Deinde pro reliquis litteris

$$\mathcal{J} = +32,663 \quad - \quad \mathcal{J} = 1,514059$$

$$\mathcal{K} = -1,025 \quad - \quad \mathcal{K} = 0,014776$$

$$J' = -2(1-a)J = 5,060;$$

$$K' = 2(1+a)K = 0,227;$$

$$J = -15,555 \quad - \quad J = 1,191891$$

$$K = -0,370 \quad - \quad K = 9,568589$$

$$\text{Porro } \mathcal{L} = -1,453 \quad - \quad \mathcal{L} = 0,162070$$

$$\mathcal{M} = -0,000 \quad - \quad -$$

$$L' = 2(2a-1)L = 12,786$$

$$M' = 2(2a+1)M$$

$$L = +6,252 \quad - \quad L = 0,796019$$

$$M = -0,001 \quad - \quad M = 7,000000$$

$$\text{Denique } \mathcal{H} = -0,123 \quad \text{et } H = -1,033$$

$$\text{atque } \frac{1}{2}\delta - \gamma = -7,459 \quad k k$$

§. 97. His igitur valoribus inuentis innotescet primum distantia Lunae a terra curtata, quatenus a sola
L 2 excen-

excentricitate orbitae lunaris k pendet. Cum enim haec distantia posita sit $x = \frac{(1-kk)au}{1-k\cos r}$ ob $u = 1 + \frac{v}{nn}$, erit

$$\begin{aligned} u = 1 & - 0,007161 \cos 2\eta - 0,0719 kk \cos 2\eta \\ & + 0,000073 \cos 4\eta - 0,0005 kk \cos 4\eta \\ & + 0,194888 k \cos(2\eta-r) - 0,003274 k \cos(2\eta+r) \\ & - 0,003238 k \cos(4\eta-r) + 0,000078 k \cos(4\eta+r) \\ & - 0,0059 kk \cos 2r \\ & - 0,0889 kk \cos(2\eta-2r) - 0,0021 kk \cos(2\eta+2r) \\ & + 0,0357 kk \cos(4\eta-2r) \end{aligned}$$

At pro longitudine Lunae, quatenus a sola excentricitate k pendet, prodibit $\frac{d\phi}{dr} =$

$$\begin{aligned} & + 0,000085 + 0,019015 \cos 2\eta + 0,000076 \cos 4\eta \\ & + 0,1562 kk \cos 2\eta + 0,0008 kk \cos 4\eta \\ & - 0,001255 k \cos r - 0,38410 k \cos(2\eta-r) + 0,002647 k \cos(4\eta-r) \\ & + 0,01278 k \cos(2\eta+r) - 0,000229 k \cos(4\eta+r) \\ & + 0,0118 kk \cos 2r - 0,0081 kk \cos(2\eta-2r) - 0,0076 kk \cos(4\eta-2r) \\ & + 0,0102 kk \cos(2\eta+2r) \end{aligned}$$

§. 98. Ponatur nunc:

$$\begin{aligned} \phi = 0 & + \mathcal{A}' \sin 2\eta + \mathcal{A}' kk \sin 2\eta + \mathcal{B}' \sin 4\eta + \mathcal{B}' kk \sin 4\eta \\ & + \mathcal{C}' k \sin r + \mathcal{D}' k \sin(2\eta-r) + \mathcal{F}' k \sin(4\eta-r) \\ & + \mathcal{E}' k \sin(2\eta+r) + \mathcal{G}' k \sin(4\eta+r) \\ & + \mathcal{H}' kk \sin 2r - \mathcal{I}' kk \sin(2\eta-2r) + \mathcal{L}' kk \sin(4\eta-2r) \\ & + \mathcal{K}' kk \sin(2\eta+2r) + \mathcal{M}' kk \sin(4\eta+2r) \end{aligned}$$

atque

atque differentiando orientur sequentes comparationes

$$\begin{aligned} u + 0,000285 & = 0 - \frac{\mathcal{A}'(2\kappa A + \mathcal{M}) - \mathcal{D}'(2\alpha D + \mathcal{D})}{nn} kk + 0,000190 \\ & + 0,1562 = 2\alpha \mathcal{A}' - \frac{(\mathcal{D}' + \mathcal{E}')(2n + \mathcal{C}) - 2\mathcal{F}'(2\kappa D + \mathcal{D})}{nn} \\ & + 0,0008 = 4\alpha \mathcal{B}' - \frac{\mathcal{A}'(2\kappa A + \mathcal{M}) - \mathcal{E}'(2\kappa D + \mathcal{D}) - 2(\mathcal{F}' + \mathcal{G}')(2n + \mathcal{C})}{nn} \\ & + 0,0118 = 2\mathcal{H}' - \frac{\mathcal{E}'(2\kappa D + \mathcal{D}) - \mathcal{M}'(2\kappa J + \mathcal{I}) - (\mathcal{I}' + \mathcal{K}')(2\kappa A + \mathcal{M})}{nn} \\ & - 0,0081 = 2(\alpha - 1)\mathcal{I}' - \frac{3\mathcal{M}'}{2n} - \frac{\mathcal{D}'(2n + \mathcal{C}) - \mathcal{M}'(2\kappa H + \mathcal{H})}{nn} \\ & \quad - \frac{2\mathcal{L}'(2\kappa A + \mathcal{M})}{nn} \\ & + 0,0102 = 2(\alpha + 1)\mathcal{K}' - \frac{3\mathcal{M}'}{2n} - \frac{\mathcal{M}'(2\kappa H + \mathcal{H}) - 2\mathcal{G}'(2\kappa D + \mathcal{D})}{nn} \\ & - 0,0076 = 2(2\alpha - 1)\mathcal{L}' - \frac{3\mathcal{B}'}{2n} - \frac{\mathcal{M}'(2\kappa J + \mathcal{I}) - \mathcal{I}'(2\kappa A + \mathcal{M})}{nn} \\ & \quad - \frac{\mathcal{D}'(2\kappa D + \mathcal{D}) - 2\mathcal{F}'(2n + \mathcal{C})}{nn} \\ & \bullet = 2(2\alpha + 1)\mathcal{M}' - \frac{3\mathcal{B}'}{n} - \frac{2\mathcal{G}'(2n + \mathcal{C})}{nn} \end{aligned}$$

§. 99. Ex his comparationibus elicimus:

$$\begin{aligned} \mathcal{A}' & = + 0,0509; & \mathcal{B}' & = 0,0008 \\ \mathcal{I}' & = + 0,5385; & \mathcal{K}' & = 0,0028 \\ \mathcal{L}' & = - 0,1055; & \mathcal{M}' & = 0,0000 \\ \mathcal{H}' & = + 0,0021; & \text{et } 0 & = \kappa - 0,000429 \end{aligned}$$

posito $k = 0,05445$. Hinc autem erit $\frac{1}{2}\delta - \gamma = -0,02302$
 Cum autem iam ante inuentum esset $\frac{1}{2}\delta - \gamma = -0,01742$
 erit reuera $\frac{1}{2}\delta - \gamma = -0,00560$. Tum vero inueniemus:
 $\gamma = 1,40673$, vnde erit $\frac{1}{2}\delta = 1,40113$, et $\delta = 2,80226$
 hincque $2\gamma - \frac{1}{2}\delta = -1,38993$ et $\frac{2\gamma - \frac{1}{2}\delta}{nn} = -0,00794$.

Verum ex cognita ratione motus medii ad motum ano-
 maliae est $Q = 1,0085272$, vnde $x = 1,0089562$

Verum esse debet $x = 1 + \frac{3 + 4\mu + \delta}{4nn}$; vnde foret

$0,0089562 = 0,008289 + \frac{\mu}{nn}$; ideoque $\frac{\mu}{nn} = 0,000667$

qui valor cum sit tam exiguus, merito dubitamus,
 num μ non prorsus sit $= 0$.

CAPUT VII.

CORRECTIO INAEQUALITATUM LUNAE,
ANTE INVENTARUM.

§. 100.

Quoniam nunc quidem valores litterarum γ et δ
 ita inuenimus, vt eos pro proxime veris habere
 queamus, ex iis coefficientes terminorum, quibus
 inaequalitates lunae continentur, accuratius definire po-
 terimus. Cum enim sit $\gamma = 1,40673$ et $\delta = 2,80226$,
 colligamus hic in vnum omnes formulas, quas hactenus
 pro inueniendis coefficientibus assumtis eliciuimus. Po-
 fueramus autem:

$$\begin{aligned} fRdr &= A \cos 2\eta + a k k \cos 2\eta + B \cos 4\eta + b k k \cos 4\eta \\ &+ C k \cos r + D k \cos (2\eta - r) + F k \cos (4\eta - r) \\ &+ E k \cos (2\eta + r) + G k \cos (4\eta + r) \end{aligned}$$

$$\begin{aligned} &+ H k k \cos 2r + J k^2 \cos (2\eta - 2r) + L k^2 \cos (4\eta - 2r) \\ &+ K k^2 \cos (2\eta + 2r) + M k^2 \cos (4\eta + 2r) \end{aligned}$$

$$\begin{aligned} v &= A \cos 2\eta + a k k \cos 2\eta + B \cos 4\eta + b k k \cos 4\eta \\ &+ D k \cos (2\eta - r) + F k \cos (4\eta - r) \\ &+ E k \cos (2\eta + r) + G k \cos (4\eta + r) \end{aligned}$$

$$\begin{aligned} &+ H k k \cos 2r + J k k \cos (2\eta - 2r) + L k k \cos (4\eta - 2r) \\ &+ K k k \cos (2\eta + 2r) + M k k \cos (4\eta + 2r) \end{aligned}$$

§. 101. Hinc posito $x = V(1 + \frac{3+4\mu+\delta}{2nn})$
collegimus fore

$$\begin{aligned} \frac{d\phi}{dr} = & x + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\kappa D + 2\mathfrak{D})}{2n^4} kk + \frac{A(3\kappa a + a)}{2n^4} kk \\ & + \frac{a(3\kappa A + 2\mathfrak{A})}{2n^4} kk - \frac{(2\kappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{(2\kappa B + \mathfrak{B})}{nn} \cos 4\eta \\ & + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} \cos 4\eta - \frac{\mathfrak{C}}{nn} k \cos r - \frac{(2\kappa a + a)}{nn} k^2 \cos 2\eta \\ & - \frac{(2\kappa b + \mathfrak{b})}{nn} k^2 \cos 4\eta + \frac{D(3\kappa D + 2\mathfrak{D}) + A(3\kappa D + 2\mathfrak{D})}{2n^4} k \cos r \\ & - \frac{(2\kappa D + \mathfrak{D})}{nn} k \cos(2\eta - r) - \frac{(2\kappa F + \mathfrak{F})}{nn} k \cos(4\eta - r) \\ & + \frac{D(3\kappa A + 2\mathfrak{A}) + A(3\kappa D + 2\mathfrak{D})}{2n^4} k \cos(4\eta - r) \\ & - \frac{(2\kappa E + \mathfrak{E})}{nn} k \cos(2\eta + r) - \frac{(2\kappa G + \mathfrak{G})}{nn} k \cos(4\eta + r) \\ & - \frac{(2\kappa J + \mathfrak{J})}{nn} kk \cos(2\eta - 2r) - \frac{(2\kappa H + \mathfrak{H})}{nn} kk \cos 2r \\ & + \frac{J(3\kappa A + 2\mathfrak{A}) + A(3\kappa J + 2\mathfrak{J})}{2n^4} k^2 \cos 2r \\ & - \frac{(2\kappa K + \mathfrak{K})}{nn} kk \cos(2\eta + 2r) - \frac{(2\kappa L + \mathfrak{L})}{nn} k^2 \cos(4\eta - 2r) \\ & - \frac{D(3\kappa D + 2\mathfrak{D}) + J(3\kappa A + 2\mathfrak{A}) + A(3\kappa J + 2\mathfrak{J})}{2n^4} k^2 \cos(4\eta - 2r) \\ & - \frac{(2\kappa M + \mathfrak{M})}{nn} k^2 \cos(4\eta + 2r) \end{aligned}$$

§. 102.

§. 102. Si iam ponamus

$$\begin{aligned} x + & \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\kappa D + 2\mathfrak{D})}{2n^4} kk \\ & + \frac{A(3\kappa a + 2a)}{2n^4} kk + \frac{a(3\kappa A + 2\mathfrak{A})}{2n^4} kk \\ - \frac{1}{n} = & \alpha; \text{ vt sit neglectis terminis admodum exiguis} \\ \frac{d\eta}{dr} = & \alpha - \frac{(2\kappa A + \mathfrak{A})}{nn} \cos 2\eta - \frac{(2\kappa + \mathfrak{C})}{nn} k \cos r \\ & - \frac{(2\kappa D + \mathfrak{D})}{nn} k \cos(2\eta - r) - \frac{(2\kappa E + \mathfrak{E})}{nn} k^2 \cos(2\eta + r) \end{aligned}$$

ex superioribus capitibus repetimus has determinaciones:

$$\begin{aligned} 2\alpha\mathfrak{A} = & -\frac{1}{2} \\ 4\alpha\mathfrak{B} = & -\frac{3A}{2nn} + \frac{\mathfrak{A}(3\kappa A + \mathfrak{A})}{nn} \\ \mathfrak{C} = & \frac{\mathfrak{A}(2\kappa D + \mathfrak{D})}{nn} - \frac{(\mathfrak{D} - \mathfrak{C})(2\kappa A + \mathfrak{A})}{nn} - \frac{\mathfrak{A}(2\kappa E + \mathfrak{E})}{nn} - \frac{3(\mathfrak{D} - \mathfrak{E})}{2nn} \\ (2\alpha - 1)\mathfrak{D} = & -3 + \frac{(2\kappa + \mathfrak{C})}{nn} \mathfrak{A} \\ (2\alpha + 1)\mathfrak{E} = & -3 + \frac{(2\kappa + \mathfrak{C})}{nn} \mathfrak{A} \\ (4\alpha - 1)\mathfrak{F} = & \frac{2(2\kappa + \mathfrak{C})}{nn} \mathfrak{B} + \frac{\mathfrak{A}(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{D}(2\kappa A + \mathfrak{A})}{nn} - \frac{3(2A + \mathfrak{D})}{2nn} \\ (4\alpha + 1)\mathfrak{G} = & \frac{2(2\kappa + \mathfrak{C})}{nn} \mathfrak{B} + \frac{\mathfrak{A}(2\kappa E + \mathfrak{E})}{nn} + \frac{\mathfrak{E}(2\kappa A + \mathfrak{A})}{nn} - \frac{3(2A + \mathfrak{E})}{2nn} \\ 2\alpha a = & -\frac{1}{2} + \frac{(\mathfrak{D} + \mathfrak{E})(2\kappa + \mathfrak{C})}{nn} + \frac{2\mathfrak{b}(2\kappa A + \mathfrak{A})}{nn} \\ & + \frac{2\mathfrak{F}(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{D}(2\kappa F + \mathfrak{F})}{nn} \end{aligned}$$

+ δ)

+ a) kk

$\cos 4\eta$

$\cos 2\eta$

$k \cos r$

$\eta - r$

$\eta + r$

$2r$

r

$-2r$

$+2r$

102.

M

4ab

$$\begin{aligned}
 4\alpha b &= -\frac{3D}{nn} + \frac{2(\mathfrak{F} + \mathfrak{G})(2n + \mathfrak{C})}{nn} + \frac{\mathfrak{C}(2\kappa D + \mathfrak{D})}{nn} \\
 &\quad + \frac{a(2\kappa A + \mathfrak{A})}{nn} + \frac{\mathfrak{D}(2\kappa E + \mathfrak{E})}{nn} \\
 2\mathfrak{H} &= -\frac{3(2D + \mathfrak{J})}{2nn} + \frac{\mathfrak{C}(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{A}(2\kappa \mathfrak{J} + \mathfrak{J})}{nn} - \frac{(\mathfrak{J} - \mathfrak{K})(2\kappa A + \mathfrak{A})}{nn} \\
 2(\alpha - 1)\mathfrak{J} &= -\frac{1}{2} - \frac{3(H - L)}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H + \mathfrak{H})}{nn} \\
 &\quad + \frac{\mathfrak{D}(2n + \mathfrak{C})}{nn} + \frac{2\mathfrak{L}(2\kappa A + \mathfrak{A})}{nn} \\
 2(\alpha + 1)\mathfrak{K} &= -\frac{1}{2} - \frac{3H}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H + \mathfrak{H})}{nn} \\
 &\quad + \frac{\mathfrak{C}(2n + \mathfrak{C})}{nn} + \frac{2\mathfrak{G}(2\kappa D + \mathfrak{D})}{nn} \\
 2(2\alpha - 1)\mathfrak{L} &= -\frac{3(2D + \mathfrak{J})}{2nn} + \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H + \mathfrak{H})}{nn} + \frac{2\mathfrak{G}(2n + \mathfrak{C})}{nn} \\
 &\quad + \frac{\mathfrak{A}(2\kappa \mathfrak{J} + \mathfrak{J})}{nn} + \frac{\mathfrak{D}(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{J}(2\kappa A + \mathfrak{A})}{nn} \\
 2(2\alpha + 1)\mathfrak{M} &= \dots \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H + \mathfrak{H})}{nn} + \frac{2\mathfrak{G}(2n + \mathfrak{C})}{nn} + \frac{\mathfrak{K}(2\kappa A + \mathfrak{A})}{nn}
 \end{aligned}$$

§. 103. Antequam vltcrius progrediamur, sequen-
tes notandae sunt nouae denominationes

$$\begin{aligned}
 A' &= 2\alpha A + \frac{A(2\kappa B + \mathfrak{B})}{nn} - \frac{B(2\kappa A + \mathfrak{A})}{nn} \\
 B' &= 4\alpha B - \frac{A(2\kappa A + \mathfrak{A})}{nn} \\
 C' &= \frac{\mathfrak{A}(D - E) - A(\mathfrak{D} - \mathfrak{E})}{nn} \\
 D' &= (2\alpha - 1)D - \frac{(2n + \mathfrak{C})}{nn} A
 \end{aligned}$$

$$E' =$$

$$\begin{aligned}
 F &= \frac{(2\kappa D + \mathfrak{D})}{nn} \\
 F &= \frac{2\kappa E + \mathfrak{E}}{nn} \\
 F &= \frac{\mathfrak{K}(2\kappa A + \mathfrak{A})}{nn} \\
 F &= \frac{2\kappa H + \mathfrak{H}}{nn} \\
 F &= \frac{\kappa A + \mathfrak{A}}{nn} \\
 F &= \frac{\kappa H + \mathfrak{H}}{nn} \\
 F &= \frac{2\kappa D + \mathfrak{D}}{nn} \\
 F &= \frac{2\mathfrak{G}(2n + \mathfrak{C})}{nn} \\
 F &= \frac{\mathfrak{J}(2\kappa A + \mathfrak{A})}{nn} \\
 F &= \frac{\mathfrak{K}(2\kappa A + \mathfrak{A})}{nn} \\
 &\text{; sequen-} \\
 &+ \mathfrak{A})
 \end{aligned}$$

$$E' =$$

$$\begin{aligned}
 E' &= (2\alpha + 1)E - \frac{(2n + \mathfrak{C})}{nn} A \\
 F' &= (4\alpha - 1)F - \frac{4B}{n} - \frac{A(2\kappa D + \mathfrak{D}) - D(2\kappa A + \mathfrak{A})}{nn} \\
 G' &= (4\alpha + 1)G - \frac{4B}{n} - \frac{A(2\kappa E + \mathfrak{E}) - E(2\kappa A + \mathfrak{A})}{nn} \\
 A' &= 2\alpha A - \frac{2b(2\kappa A + \mathfrak{A})}{nn} - \frac{(D + E)(2n + \mathfrak{C})}{nn} \\
 &\quad - \frac{2F(2\kappa D + \mathfrak{D})}{nn} + \frac{D(2\kappa F + \mathfrak{F})}{nn} \\
 B' &= 4\alpha B - \frac{A(2\kappa A + \mathfrak{A})}{nn} - \frac{A(2\kappa A + \mathfrak{A})}{nn} - \frac{D(2\kappa E + \mathfrak{E})}{nn} \\
 &\quad - \frac{E(2\kappa D + \mathfrak{D})}{nn} - \frac{2(F + G)(2n + \mathfrak{C})}{nn} \\
 H' &= 2H + \frac{D\mathfrak{C} - \mathfrak{D}E - A(\mathfrak{J} - \mathfrak{K}) + \mathfrak{A}(J - K)}{nn} \\
 J' &= 2(\alpha - 1)J + \frac{3A}{2n} - \frac{A(2\kappa H + \mathfrak{H})}{nn} - \frac{D(2n + \mathfrak{C})}{nn} \\
 &\quad - \frac{2\mathfrak{L}(2\kappa A + \mathfrak{A})}{nn} + \frac{A(2\kappa L + \mathfrak{L})}{nn} \\
 K' &= 2(\alpha + 1)K - \frac{3A}{n} - \frac{A(2\kappa H + \mathfrak{H})}{nn} - \frac{E(2n + \mathfrak{C})}{nn} - \frac{2G(2\kappa D + \mathfrak{D})}{nn} \\
 L' &= (2\alpha - 1)L - \frac{3B}{n} - \frac{2B(2\kappa H + \mathfrak{H})}{nn} - \frac{2F(2n + \mathfrak{C})}{nn} \\
 &\quad - \frac{D(2\kappa D + \mathfrak{D})}{nn} - \frac{A(2\kappa \mathfrak{J} + \mathfrak{J})}{nn} - \frac{J(2\kappa A + \mathfrak{A})}{nn} \\
 M' &= 2(2\alpha + 1)M - \frac{3B}{n} - \frac{2B(2\kappa H + \mathfrak{H})}{nn} - \frac{2G(2n + \mathfrak{C})}{nn} - \frac{E(2\kappa E + \mathfrak{E})}{nn}
 \end{aligned}$$

§. 104. Nunc vt terminos completos obtineamus,
faltem eos qui angulos 2η et r inuoluunt, notandum

$$M 2$$

$$\approx ft$$

est in nostris aequationibus $\sin 2\eta$ et $\cos 2\eta$ non per $\frac{3}{2}(1+2kk)$ sed per $\frac{3}{2}(1+2kk+\frac{2}{3}ee)$ esse multiplicatos. Hinc cum sit fere $\frac{2}{3}ee = \frac{2}{3}kk$, loco kk hic scribi oportebit $\frac{2}{3}kk$, unde in valore ipsius a pro 3 scripsi $3\frac{2}{3}$ seu $\frac{10}{3}$. Deinde ut in his terminis quoque rationem habeamus inclinationis orbitae, cuius medius valor sit $= \epsilon$, ponamus $\frac{2}{3}(nn + 2 + 3\mu + \gamma) \tan \epsilon^2 = f = \frac{2}{3}(\frac{2}{3}nnnn - \frac{2}{3}nn - \frac{2}{3} + \gamma - \frac{2}{3}\delta) \tan \epsilon^2$ ob $\mu = \frac{2}{3}(nn-1)nn - \frac{2}{3} - \frac{2}{3}\delta$, eritque nostra aequatio:

$$\begin{aligned} \frac{ddv}{dr^2} &= \frac{1}{2}\delta - \gamma + \frac{1}{2}kk - \gamma k \cos r + \frac{1}{2}kk \cos 2r + \frac{1}{2} \cos 2\eta + \frac{1}{2}kk \cos 2\eta \\ &+ f + \frac{1}{2}fkk + f k \cos r + \frac{1}{2}f k k \cos 2r \\ &+ 3k \cos(2\eta - r) + 3k \cos(2\eta + r) + \frac{1}{2}kk \cos(2\eta - 2r) + \frac{1}{2}kk \cos(2\eta + 2r) \\ &- 2\mu / Rdr - v \left(1 - \frac{1}{2}kk + \frac{2f}{nn} + \frac{fkk}{nn} - \frac{(2\gamma - \frac{2}{3}\delta)}{nn} \right) \\ &+ v \left(3nn + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{2}{3}\delta)}{nn} \right) k \cos r \\ &+ v \left(\frac{3}{2} - \frac{f}{nn} \right) k^2 \cos 2r \\ &+ v \left(\frac{3}{2nn} \cos 2\eta + \frac{3k}{nn} \cos(2\eta - r) + \frac{3k}{nn} \cos(2\eta + r) \right) \\ &+ \frac{1}{nn} (\sqrt{Rdr})^2 + \frac{6v}{nn} \sqrt{Rdr} + \frac{3vv}{nn} - \frac{3vv}{nn} k \cos r \end{aligned}$$

§. 105. Sit breuitatis gratia:

$$\begin{aligned} 1 + \frac{2f}{nn} - \frac{(2\gamma - \frac{2}{3}\delta)}{nn} &= g; \quad 3nn + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{2}{3}\delta)}{nn} = b \\ \text{et } 1 - \frac{1}{2}kk + \frac{2f}{nn} + \frac{fkk}{nn} - \frac{(2\gamma - \frac{2}{3}\delta)}{nn} &= c, \text{ quo} \\ &\text{termino} \end{aligned}$$

non per
plicatos.
bi opor-
3. $\frac{2}{3}$ seu
1 habea-
it $= \epsilon$,
 $f =$
 $\mu =$
0:

$k \cos 2\eta$

$(2\eta + 2r)$

$-\frac{1}{2}\delta)$

$)k \cos r$

$(\eta + r)$

$\frac{v}{1} k \cos r$

$\delta) = b$

, quo

rmimo

termino in angulis ex 2η et r compositis utemur:

$$\text{eritque } \frac{ddv}{dr^2} =$$

$$\begin{aligned} &\frac{1}{2}\delta - \gamma + \frac{1}{2}kk + f + \frac{1}{2}fkk + \frac{3A}{4nn} + \frac{3akk}{4nn} \\ &+ \frac{3Dkk}{2nn} + \frac{3Ekk}{2nn} + \frac{3Gkk}{2nn} + \frac{3Dkk}{2nn} + \frac{3Ekk}{2nn} + \frac{3Gkk}{2nn} - \frac{3ADkk}{2nn} \\ &+ \frac{3AG}{nn} + \frac{3DG}{nn} + \frac{3AA}{2nn} + \frac{3DDkk}{2nn} + \frac{3EEkk}{2nn} \\ &+ \cos 2\eta \left\{ \frac{1}{2} - 2\mu - 6A \right\} \\ &+ kk \cos 2\eta \left\{ \frac{1}{2} - 2\mu - 6A + \frac{1}{2}bD + \frac{1}{2}bE + \frac{6D}{nn} + \frac{36E}{nn} \right. \\ &+ \cos 4\eta \left\{ -2\mu - 6B + \frac{3A}{4nn} + \frac{3G}{2nn} + \frac{3AG}{nn} + \frac{3AA}{2nn} \right. \\ &+ kk \cos 4\eta \left\{ -2\mu - 6B + \frac{1}{2}bF + \frac{1}{2}bG + \frac{3E}{2nn} + \frac{3D}{2nn} \right. \\ &+ \frac{3A}{nn} + \frac{3G}{nn} + \frac{3DE}{nn} + \frac{3AE}{nn} + \frac{3AG}{nn} + \frac{3AA}{nn} \\ &+ k \cos r \left\{ -\gamma + f - 2\mu - 6C + \frac{3D}{4nn} + \frac{3E}{4nn} + \frac{3A}{nn} + \frac{3G}{nn} + \frac{3D}{nn} + \frac{3E}{nn} \right. \\ &+ \frac{3AD}{nn} + \frac{3AG}{nn} + \frac{3GD}{nn} + \frac{3GE}{nn} + \frac{3AD}{nn} + \frac{3AE}{nn} + \frac{3AA}{2nn} \\ &+ k \cos(2\eta - r) \left\{ 3 - 2\mu - 6D + \frac{1}{2}bA + \frac{3F}{4nn} + \frac{3G}{nn} + \frac{3AG}{nn} \right. \\ &+ k \cos(2\eta + r) \left\{ 3 - 2\mu - 6E + \frac{1}{2}bA + \frac{3G}{4nn} + \frac{3G}{nn} + \frac{3AG}{nn} \right. \\ &\quad \quad \quad M_3 \quad \quad \quad + \end{aligned}$$

$$\begin{aligned}
& + kk \cos 2r \left\{ \frac{1}{2} + \frac{1}{2}f - 2\kappa\delta - \epsilon H + \frac{3J}{4nn} + \frac{3K}{4nn} + \frac{3E}{2nn} + \frac{3D}{2nn} \right. \\
& \quad \left. + \frac{\mathcal{C}\mathcal{C}}{nn} + \frac{\mathcal{D}\mathcal{C}}{nn} + \frac{3\mathcal{D}\mathcal{C}}{nn} + \frac{3\mathcal{D}\mathcal{E}}{nn} + \frac{3\mathcal{D}\mathcal{E}}{nn} - \frac{3\mathcal{A}\mathcal{D}}{2nn} \right. \\
& + kk \cos(2\eta - 2r) \left\{ \frac{1}{2} - 2\kappa\delta - \epsilon J + \frac{1}{2}bD + \left(\frac{1}{2} - \frac{f}{2nn}\right)A \right. \\
& \quad \left. + \frac{3H}{4nn} + \frac{2\delta}{nn} + \frac{3A\delta}{nn} + \frac{3\mathcal{C}\mathcal{D}}{nn} \right. \\
& + kk \cos(2\eta + 2r) \left\{ \frac{1}{2} - 2\kappa\delta - \epsilon K + \frac{1}{2}bE + \left(\frac{1}{2} - \frac{f}{2nn}\right)A \right. \\
& \quad \left. + \frac{3H}{4nn} + \frac{2\delta}{nn} + \frac{3A\delta}{nn} + \frac{3\mathcal{C}\mathcal{E}}{nn} \right. \\
& + kk \cos(4\eta - 2r) \left\{ -2\kappa\mathcal{E} - \epsilon L + \left(\frac{1}{2} - \frac{f}{2nn}\right)B \right. \\
& \quad \left. + \frac{3J}{4nn} + \frac{3D}{2nn} + \frac{\mathcal{D}\mathcal{D}}{2nn} + \frac{3\mathcal{D}\mathcal{D}}{2nn} + \frac{1}{2}bF \right. \\
& + kk \cos(\eta + 4r) \left\{ -2\kappa\mathcal{M} - \epsilon M + \left(\frac{1}{2} - \frac{f}{2nn}\right)B \right. \\
& \quad \left. + \frac{3K}{4nn} + \frac{3E}{2nn} + \frac{\mathcal{C}\mathcal{C}}{2nn} + \frac{3\mathcal{E}\mathcal{E}}{2nn} + \frac{1}{2}bG \right. \\
& + k \cos(4\eta - r) \left\{ -2\kappa\mathcal{F} - \epsilon F + \frac{1}{2}bB + \frac{3D}{4nn} + \frac{3A}{2nn} \right. \\
& \quad \left. + \frac{2\mathcal{D}}{nn} + \frac{3\mathcal{A}\mathcal{D}}{nn} + \frac{3\mathcal{M}\mathcal{D}}{nn} + \frac{3\mathcal{A}\mathcal{D}}{nn} - \frac{3\mathcal{A}\mathcal{A}}{4nn} \right. \\
& + k \cos(4\eta + r) \left\{ -2\kappa\mathcal{G} - \epsilon G + \frac{1}{2}bB + \frac{3E}{4nn} + \frac{3A}{2nn} \right. \\
& \quad \left. + \frac{2\mathcal{C}}{nn} + \frac{3\mathcal{A}\mathcal{C}}{nn} + \frac{3\mathcal{M}\mathcal{E}}{nn} + \frac{3\mathcal{A}\mathcal{E}}{nn} - \frac{3\mathcal{A}\mathcal{A}}{4nn} \right.
\end{aligned}$$

§. 106.

lit

I.

$$\begin{aligned}
& - \frac{3D}{2nn} \\
& \frac{3\mathcal{A}\mathcal{D}}{2nn}
\end{aligned}$$

$$\left. \right) A$$

$$\frac{3\mathcal{C}\mathcal{D}}{nn}$$

$$\left. \right) A$$

$$\frac{3\mathcal{C}\mathcal{E}}{nn}$$

II.

$$\left. \right) B$$

III.

$$\frac{1}{2}bF$$

$$\left. \right) B$$

$$\frac{1}{2}bG$$

IV.

$$- \frac{3A}{2nn}$$

$$\frac{3\mathcal{A}\mathcal{A}}{4nn}$$

V.

$$\frac{3A}{2nn}$$

$$\frac{3\mathcal{A}\mathcal{A}}{4nn}$$

106.

§. 106. Hinc denique nascentur sequentes aequalitates.

$$\begin{aligned}
\text{I. } & \frac{1}{2}\delta - \gamma + \frac{1}{2}kk + \frac{1}{2}fkk + \frac{3A}{4nn} + \frac{2\mathcal{M}}{2nn} + \frac{3A\mathcal{M}}{nn} + \frac{3AA}{2nn} \\
& + \frac{3(D+E)}{nn}kk - \frac{3\mathcal{A}\mathcal{D}kk}{2nn} + \frac{\mathcal{C}\mathcal{C}kk}{2nn} + \frac{(\mathcal{D}\mathcal{D} + \mathcal{C}\mathcal{C})}{2nn}kk \\
& + \frac{3\mathcal{D}\mathcal{D}}{nn}kk + \frac{3(\mathcal{D}\mathcal{D} + \mathcal{E}\mathcal{E})}{2nn}kk + \frac{2\mathcal{A}}{nn}kk \\
& + \frac{3A\mathcal{A}}{nn}kk + \frac{3\mathcal{M}\mathcal{A}}{nn}kk + \frac{3A\mathcal{A}}{nn}kk = \frac{A'(2\kappa A + \mathcal{M})}{nn} + \frac{A'(2\kappa A + \mathcal{M})}{nn}kk \\
& + \frac{D'(2\kappa D + \mathcal{D})}{nn}kk + \frac{A'(2\kappa A + \mathcal{A})}{nn}kk \\
\text{II. } & \frac{1}{2} - 2\kappa\mathcal{M} - \epsilon A = -2\alpha A' + \frac{A'(2\kappa B + \mathcal{B})}{nn} + \frac{2B'(2\kappa A + \mathcal{M})}{nn} \\
\text{III. } & -2\kappa\mathcal{B} - \epsilon B + \frac{3A}{4nn} + \frac{2\mathcal{M}}{2nn} + \frac{3A\mathcal{M}}{nn} + \frac{3AA}{2nn} \\
& = -4\alpha B' + \frac{A'(2\kappa A + \mathcal{M})}{nn} \\
\text{IV. } & \frac{1}{2} - 2\kappa\mathcal{A} - \epsilon A + \frac{1}{2}b(D+E) + \frac{3\mathcal{C}(D+E)}{nn} = -2\alpha A' \\
& + \frac{(D'+E')(2\kappa + \mathcal{C})}{nn} + \frac{2b'(2\kappa A + \mathcal{M})}{nn} + \frac{2F'(2\kappa D + \mathcal{D})}{nn} \\
\text{V. } & -2\kappa\mathcal{B} - \epsilon B + \frac{1}{2}b(F+G) + \frac{3(D+E)}{nn} + \frac{3\mathcal{D}(E+J)}{nn} + \frac{3A\mathcal{A}}{nn} = -4\alpha B' \\
& + \frac{A'(2\kappa A + \mathcal{M})}{nn} + \frac{E'(2\kappa D + \mathcal{D})}{nn} + \frac{2(F'+G')(2\kappa + \mathcal{C})}{nn} + \frac{D'(2\kappa E + \mathcal{C})}{nn}
\end{aligned}$$

VI.

$$\text{VI. } -\gamma + f - 2\kappa \mathfrak{C} + \frac{3(D+E)}{4nn} + \frac{3A}{nn} - \frac{3AA}{2nn} + \frac{\mathfrak{A}(D+\mathfrak{C})}{nn} \\ + \frac{3A(D+\mathfrak{C})}{nn} + \frac{3A(D+E)}{nn} + \frac{3\mathfrak{A}(D+E)}{nn} = \\ -C + \frac{A'(2\kappa D + \mathfrak{D})}{nn} + \frac{A'(2\kappa E + \mathfrak{C})}{nn} + \frac{(D'+E')(2\kappa A + \mathfrak{A})}{nn}$$

$$\text{VII. } 3 - 2\kappa \mathfrak{D} - \mathfrak{E} D + \frac{1}{2} b A + \frac{3F}{4nn} + \frac{\mathfrak{C}(\mathfrak{A} + 3A)}{nn} = \\ - (2\kappa - 1) D' + \frac{A'(2\kappa + \mathfrak{C})}{nn}$$

$$\text{VIII. } 3 - 2\kappa \mathfrak{C} - \mathfrak{E} E + \frac{1}{2} b A + \frac{3G}{4nn} + \frac{\mathfrak{C}(\mathfrak{A} + 3A)}{nn} = \\ - (2\kappa + 1) E' + \frac{A'(2\kappa + \mathfrak{C})}{nn}$$

$$\text{IX. } -3\kappa \mathfrak{F} - \mathfrak{E} F + \frac{1}{2} b B + \frac{3A}{2nn} + \frac{3D}{4nn} + \frac{\mathfrak{A}D}{nn} + \frac{3AD}{nn} \\ + \frac{3\mathfrak{A}D}{nn} + \frac{3AD}{nn} - \frac{3AA}{4nn} = - (4\alpha - 1) F + \frac{4B'}{n} \\ + \frac{A'(2\kappa D + \mathfrak{D})}{nn} + \frac{D'(2\kappa A + \mathfrak{A})}{nn}$$

$$\text{X. } -2\kappa \mathfrak{G} - \mathfrak{E} G + \frac{1}{2} b B + \frac{3A}{2nn} + \frac{3E}{4nn} + \frac{\mathfrak{A}E}{nn} \\ + \frac{3\mathfrak{A}E}{nn} + \frac{3AE}{nn} - \frac{3AA}{4nn} = - (4\alpha + 1) G' + \frac{4B'}{n} \\ + \frac{A'(2\kappa E + \mathfrak{C})}{nn} + \frac{E'(2\kappa A + \mathfrak{A})}{nn}$$

XI.

$$\text{XI. } \frac{1}{2} + \frac{1}{2} f - 2\kappa \mathfrak{H} - \mathfrak{E} H + \frac{3(D+E)}{2nn} + \frac{3(J+K)}{4nn} \\ - \frac{3AD}{2nn} + \frac{\mathfrak{C}E}{2nn} - \frac{3AE}{2nn} + \frac{\mathfrak{D}E}{nn} + \frac{3DE}{nn} + \frac{3DE}{nn} + \frac{3DE}{nn} \\ + \frac{\mathfrak{A}J}{nn} + \frac{3\mathfrak{A}J}{nn} + \frac{3AJ}{nn} + \frac{3AJ}{nn} = -2H' + \frac{A'(2\kappa J + \mathfrak{J})}{nn} \\ + \frac{E'(2\kappa D + \mathfrak{D})}{nn} + \frac{D'(2\kappa E + \mathfrak{C})}{nn} + \frac{(J'+K')(2\kappa A + \mathfrak{A})}{nn}$$

$$\text{XII. } \frac{1}{2} - 2\kappa \mathfrak{J} - \mathfrak{E} J + \frac{1}{2} b D + \left(\frac{1}{2} - \frac{f}{2nn}\right) A + \frac{3H}{4nn} \\ + \frac{\mathfrak{H}(\mathfrak{A} + 3A)}{nn} + \frac{3\mathfrak{C}D}{nn} = -2(\alpha - 1) J' + \frac{3A'}{2n} \\ + \frac{D'(2\kappa + \mathfrak{C})}{nn} + \frac{A'(2\kappa H + \mathfrak{H})}{nn} + \frac{2L'(2\kappa A + \mathfrak{A})}{nn}$$

$$\text{XIII. } \frac{1}{2} - 2\kappa \mathfrak{K} - \mathfrak{E} K + \frac{1}{2} b E + \left(\frac{1}{2} - \frac{f}{2nn}\right) A + \frac{3H}{4nn} \\ + \frac{\mathfrak{H}(\mathfrak{A} + 3A)}{nn} + \frac{3\mathfrak{C}E}{nn} = -2(\alpha + 1) K' + \frac{3A'}{2n} \\ + \frac{E'(2\kappa + \mathfrak{C})}{nn} + \frac{A'(2\kappa H + \mathfrak{H})}{nn} + \frac{2G'(2\kappa D + \mathfrak{D})}{nn}$$

$$\text{XIV. } -2\kappa \mathfrak{L} - \mathfrak{E} L + \frac{1}{2} b F + \left(\frac{1}{2} - \frac{f}{2nn}\right) B - \frac{3AD}{2nn} \\ + \frac{3D}{2nn} + \frac{3J}{4nn} + \frac{\mathfrak{D}D}{2nn} + \frac{3DD}{nn} + \frac{3DD}{2nn} = -2(2\alpha - 1) L' + \frac{3B'}{n} \\ + \frac{2F'(2\kappa + \mathfrak{C})}{nn} + \frac{A'(2\kappa J + \mathfrak{J})}{nn} + \frac{J'(2\kappa A + \mathfrak{A})}{nn} + \frac{D'(2\kappa D + \mathfrak{D})}{nn}$$

N

XV.

$$\text{XV. } -2\kappa M - 6M + \frac{1}{2}bG + \left(\frac{1}{2} - \frac{f}{2nn}\right)B - \frac{3AE}{2nn} \\ + \frac{3E}{2nn} + \frac{3K}{4nn} + \frac{EE}{2nn} + \frac{3EE}{nn} + \frac{3EE}{2nn} = \\ -2(2\kappa+1)M' + \frac{3B'}{n} + \frac{2G'(2n+\mathcal{C})}{nn}.$$

§. 107. Nunc antequam hos valores inuenire queamus, verus valor ipsius α inuestigari debet: quod fiet ex valore integrali ipsius Φ , qui si vti §. 98. ponatur

$$\Phi = Or + \mathcal{M}' \sin 2\eta + \text{etc. obtinebitur.}$$

$$\kappa + \frac{A(3\kappa A + 2\mathcal{M})}{2n^4} + \frac{D(3\kappa D + 2\mathcal{D})}{2n^4} \frac{kk}{kk} + \frac{A(3\kappa A + a)}{2n^4} \frac{kk}{kk} + \frac{a(3\kappa A + 2\mathcal{M})}{2n^4} \frac{kk}{kk} = \\ = O - (\mathcal{M}' + a'kk) \frac{(2\kappa A + \mathcal{M})}{nn} - \frac{\mathcal{D}'(2\kappa D + \mathcal{D})}{nn} \frac{kk}{kk} = \alpha + \frac{1}{n}$$

vbi ex obseruationibus constat esse $O = 1,0085272$

Proxime autem esse supra inuenimus esse:

$$\begin{array}{lll} \mathcal{M}' = 0,01 & \mathcal{M} = -0,80 & A = -1,25 \\ a' = 0,05 & a = -2,05 & a = -12,60 \\ \mathcal{D}' = -0,44 & \mathcal{D} = -3,60 & D = 34,25 \end{array}$$

atque $\kappa = 1,0085$; $kk = 0,003$; $nn = 175,71795$

unde inuenimus $O + 0,000649 = \alpha + \frac{1}{n} = \kappa + 0,000285$

§. 108. Cum nunc sit $O = 1,0085272$, erit $\alpha + \frac{1}{n} =$

$1,009176$, et ob $\frac{1}{n} = 0,675438$, habebitur vrues valor:

$$\alpha = 0,933738 \quad \text{et} \quad l\alpha = 9,9702255$$

atque $\kappa = 1,008991$ et $l\kappa = 0,0038874$

Hinc

Hinc iam primo obinemus:

$$\mathcal{M} = -0,80313 \quad l\mathcal{M} = 9,9047898$$

Deinde cum sit satis prope $\mathcal{C} = -0,67465$, erit

$$\frac{(2n+\mathcal{C})}{nn} = 0,147037 \quad \text{et} \quad \frac{2n+\mathcal{C}}{nn} = 9,1674260$$

$$\mathcal{D} = -3,593620 \quad \dots \quad l\mathcal{D} = 0,5555310$$

$$\mathcal{C} = -1,087320 \quad \dots \quad l\mathcal{C} = 0,0363580$$

atque porro ex valore ipsius A proxime cognito erit

$$\mathcal{B} = +0,006967 \quad \dots \quad l\mathcal{B} = 7,8430540$$

et quia est satis prope $B = 0,0128$, erit $A' = 2\kappa A - 0,000720$

et $B' = 4\kappa B - 0,023926$, unde fit;

$$\frac{1}{2} + 1,62172 - \mathcal{C}A = -4\kappa A + 0,00144\kappa + 0,000374\kappa A \\ = 0,152 \kappa B + 0,00091$$

$$+ 0,01247 - \mathcal{C}B = -16\kappa B + 0,09570\kappa - 0,038032\kappa A \\ + 0,000014$$

§. 109. Nunc primum quaeri debent valores litterarum f , b et b : et cum sit $s = 5^a$, g' et $2\gamma - \frac{1}{2}\delta = -1,3899$ proxime, reperietur

$$f = 1,093757 \quad \text{et} \quad lf = 0,0389208$$

$$b = 3,0423 \quad \dots \quad lb = 0,4832020$$

$$b = 1,01591 \quad \dots \quad lb = 0,0068560$$

hincque erit

$$2,4720 A = -3,11947 - 0,142 B$$

$$12,9369 B = +0,07684 - 0,0355 A$$

unde concluditur fore:

$$A = -1,262463 \quad \dots \quad lA = 0,1912186$$

$$B = +0,009404 \quad \dots \quad lB = 7,9733114$$

N 2

Porro

Hinc

Porro verò est

$$D' = 0,867676 D + 0,185628$$

$$E' = 2,867676 E + 0,185628$$

et $A' = -2,35859 \dots I - A' = 0,3726530$

§. 110. Ex his valoribus aequationes VII et VIII induent has formas,

$$3 - 7,25185 - 6D - 1,92040 - 0,00244 + 0,01704 = -0,75286 D - 0,16106 - 0,34680$$

$$3 - 2,19420 - 6E - 1,92040 + 0,00006 + 0,01704 = -8,22357 E - 0,53232 - 0,34680$$

vnde prodibit

$$D = +33,6600 \dots I D = 1,5271130$$

$$E = -0,5785 \dots I - E = 9,7623410$$

ergo

$$D' = 29,39153$$

$$E' = -1,47347$$

Ex his nanciscemur sequentes formulas pro calculo sequenti

$$\frac{2\pi A + \pi}{\pi\pi} = -0,01884 \quad I - \frac{(2\pi A + \pi)}{\pi\pi} = 8,275051$$

$$\frac{2\pi B + \pi}{\pi\pi} = +0,000148 \quad I - \frac{2\pi B + \pi}{\pi\pi} = 6,170262$$

$$\frac{2\pi D + \pi}{\pi\pi} = +0,36611 \quad I - \frac{2\pi D + \pi}{\pi\pi} = 9,563604$$

$$\frac{2\pi E + \pi}{\pi\pi} = -0,01283 \quad I - \frac{(2\pi E + \pi)}{\pi\pi} = 8,108292$$

§. III.

§. III. Ex his iam porro inuenitur

$$C = -0,64383 \dots I - C = 9,808771$$

atque $C' = -0,13847$

Porro valores litterarum § et © determinabuntur per has aequationes

$$(4\pi - 1) § = 0,002049 - 0,294032 + 0,067699 + 0,021554 - 0,287335$$

$$(4\pi + 1) © = 0,002049 + 0,010306 + 0,020484 + 0,021554 + 0,004939$$

ex quibus reperitur

$$§ = -0,17957 \dots I - § = 9,254241$$

$$© = +0,01253 \dots I © = 8,097936$$

atque

$$F' = (4\pi - 1) F - 0,00283 + 0,46219 - 0,63411 =$$

$$(4\pi - 1) F + 1,09347$$

$$G' = (4\pi + 1) G - 0,00283 - 0,01620 - 0,01090 =$$

$$(4\pi + 1) G - 0,02993$$

vnde aequationes IX et X prodibunt.

$$+0,36238 - 1,01591 F + 0,00353 - 0,00680 - 1,00349 =$$

$$- (4\pi - 1)^2 F - 2,98415 - 0,86349 + 0,00337 - 0,55370$$

$$- 0,02528 - 1,01591 G + 0,00353 - 0,00680 + 0,04634 =$$

$$- (4\pi + 1)^2 G + 0,14473 + 0,03027 + 0,00337 + 0,02776$$

feu $6,43486 F = -3,75359$

$$21,40769 G = +0,18534$$

§. 110. Hinc prodeunt sequentes valores correcti pro F et G,

$$F = -0,58360 \dots I - F = 9,766112$$

$$G = +0,00866 \dots I G = 7,937400$$

N 3

Ex

it VIII

704 =

704 =

10 se

I

I

I

I

I

III.

Ex formula autem sexta hinc leui calculo colligitur fore:

$$\gamma - f = 1,58161 \text{ et } \gamma = 2,67537$$

Valores autem ex F et G deriuati erunt

$$F' = -0,49919 \text{ et } G' = 0,01107$$

$$\frac{2\kappa F + \mathfrak{F}}{nn} = -0,00772 \quad / \quad \frac{(2\kappa F + \mathfrak{F})}{nn} = 7,887828$$

$$\frac{2\kappa G + \mathfrak{G}}{nn} = 0,00017 \quad / \quad \frac{2\kappa G + \mathfrak{G}}{nn} = 6,232305$$

§. 113. Nunc procedamus ad valores litterarum a et b qui erunt

$$1,867676 a = -3,75000 - 0,68827 - 0,13148 - 0,03764 b \\ + 0,02776$$

$$3,735352 b = -0,57467 - 0,04913 - 0,39807 - 0,01884 a \\ + 0,04611$$

vnde reperitur:

$$a = -2,42686 \quad . \quad . \quad / -a = 0,385044$$

$$b = -0,24899 \quad . \quad . \quad / -b = 0,24899$$

huncque porro

$$a' = 2\kappa a + 0,03768 b - 4,86420 + 0,16048$$

$$b' = 4\kappa b + 0,01884 a + 0,16887 + 0,64373$$

$$+ 0,01450 a - 0,01744$$

$$\text{feu } a' = 2\kappa a + 0,03768 b - 4,70372$$

$$b' = 4\kappa b + 0,03334 a + 0,79516$$

§. 114. Aequationes IV et V hinc induent sequentes formas:

IV.

fore:

$$\text{IV. } + 3,75000 - 1,01591 a + 50,32200 - 0,36363 = -4\kappa a \\ + 4,89730$$

$$- 0,07034 b + 8,78034 + 4,10500 - 0,36551 - 0,00125 a$$

$$- 0,14067 b \quad - 0,02996$$

$$\text{V. } + 0,50246 - 1,01591 b - 0,87350 + 0,28239 - 0,95732 \\ - 0,02155 a$$

$$= -16\kappa a b - 0,12447 a - 2,96860 - 0,00142 b$$

$$- 0,03517 a + 0,17723$$

$$- 0,14354$$

$$- 0,91659$$

rum

Hinc fit

$$2,47355 a = -0,21101 b - 46,11580$$

$$12,93836 b = -0,13809 a - 2,80553$$

$$\text{et } a = -18,64200 \quad . \quad . \quad / -a = 1,270493$$

$$b = -0,01794 \quad . \quad . \quad / -b = 8,253822$$

ex quibus oriuntur:

$$a' = -39,52164 \quad . \quad . \quad b' = +0,10663$$

$$\text{et } \frac{2\kappa a + a}{nn} = -0,22790 \quad / \quad \frac{(2\kappa a + a)}{nn} = 9,357744$$

valor autem ipsius $\frac{2\kappa b + b}{nn}$ nullius plane erit momenti, vnde eum praetermittimus.

§. 115. Ex prima autem aequatione §. 106. colligitur

$$\frac{1}{2} \delta = \gamma - f + 0,02285$$

supra autem inuenimus esse $\gamma - f = 1,58161$, sicque erit $\frac{1}{2} \delta = 1,60446$ atque

$$\delta = 3,20892 \quad . \quad . \quad / \delta = 0,506358$$

Nunc cum sit proxime: $\mathfrak{H} = -0,123$; $H = -1,033$

ideoque $\frac{2\kappa H + \mathfrak{H}}{nn} = -0,0126$; ob $\mathfrak{L} = -1,453$ et

$L = +6,252$; habebimus $-0,$

$$\begin{aligned}
 -0,132324 \mathfrak{J} &= -3,75000 + 0,00882 - 0,09088 \\
 &\quad + 0,05336 - 0,01012 \\
 &\quad - 0,52839 + 0,05474 \\
 + 3,867676 \mathfrak{K} &= -3,75000 + 0,00882 - 0,09088 \\
 &\quad + 0,01012 \\
 &\quad - 0,15987 + 0,00917
 \end{aligned}$$

Hinc reperitur

$$\mathfrak{J} = 32,05945 \quad . \quad . \quad / \mathfrak{J} = 1,505956$$

$$\mathfrak{K} = -1,02714 \quad . \quad . \quad / -\mathfrak{K} = 0,011629$$

§. 116. Hinc ulterius progrediendo habebimus.

$$\begin{aligned}
 \mathfrak{J}' &= 2(\alpha-1)\mathfrak{J} + 0,28571 - 4,94924 + 0,23502 = \\
 &\quad - 0,01591 \quad - 0,08015
 \end{aligned}$$

$$2(\alpha-1)\mathfrak{J} - 4,52457$$

$$\begin{aligned}
 \mathfrak{K}' &= 2(\alpha+1)\mathfrak{K} + 0,28571 + 0,08507 - 0,00317 = \\
 &\quad - 0,01591
 \end{aligned}$$

$$2(\alpha+1)\mathfrak{K} + 0,35170$$

vnde aequationes XII et XIII fiunt

$$\begin{aligned}
 +3,75000 - 64,69550 - 1,01591\mathfrak{J} + 51,28180 \\
 - 0,94292 + 0,00321 - 0,36999 = \\
 - 0,00441)
 \end{aligned}$$

$$\begin{aligned}
 -4(\alpha-1)^2\mathfrak{J} - 0,59871 - 0,26690 + 4,32164 \\
 + 0,02972 - 0,47004
 \end{aligned}$$

$$\begin{aligned}
 +3,75000 + 2,07275 - 1,01591\mathfrak{K} - 0,88006 \\
 - 0,94292 + 0,00321 + 0,00636 = \\
 - 0,00441)
 \end{aligned}$$

$$\begin{aligned}
 -4(\alpha+1)^2\mathfrak{K} - 1,36026 - 0,26690 - 0,21665 \\
 + 0,02972 + 0,00810
 \end{aligned}$$

ex quibus colligitur fore

$$\mathfrak{J} = -14,09600 \quad . \quad . \quad / -\mathfrak{J} = 1,149096$$

$$\mathfrak{K} = -0,41676 \quad . \quad . \quad / -\mathfrak{K} = 9,619888$$

Hinc

$$\text{Hinc } \mathfrak{J}' = -2,65933 \quad . \quad . \quad \mathfrak{K}' = -1,26020$$

$$\text{atque } \frac{2\mathfrak{J} + \mathfrak{J}'}{nn} = +0,02057 \quad . \quad / \quad \frac{2\mathfrak{K} + \mathfrak{K}'}{nn} = 8,313172$$

$$\frac{2\mathfrak{K} + \mathfrak{K}'}{nn} = -0,01063 \quad . \quad / \quad \frac{2\mathfrak{K} + \mathfrak{K}'}{nn} = 8,026598$$

§. 117. Quaeramus iam valorem ipsius $\mathfrak{H}$, ex aequatione

$$2\mathfrak{H} = -0,45434 - 0,39771 - 0,01652 + 0,62330$$

$$\text{erit } \mathfrak{H} = -0,12264 \quad . \quad . \quad / -\mathfrak{H} = 9,088632$$

$$\text{hincque reperitur: } \mathfrak{H}' = 2\mathfrak{H} + 0,08011$$

vnde aequatio XI praebet:

$$\begin{aligned}
 2,04688 + 0,24748 - 1,01591\mathfrak{H} + 0,27805 - 3,06194 \\
 + 0,36275 + 0,00118 - 0,00623 \quad + 0,02224 + 0,03552 \\
 - 0,62485 - 0,33247 - 0,83753 \quad + 0,49710 = \\
 - 4\mathfrak{H} - 0,16022 - 0,04851 - 0,53944 - 0,37802 \\
 + 0,07384
 \end{aligned}$$

$$\text{feu } 2,98409\mathfrak{H} = -2,68053$$

$$\text{Ergo } \mathfrak{H} = -0,89829 \quad . \quad . \quad / -\mathfrak{H} = 9,953417$$

$$\mathfrak{H}' = -1,71647 \quad ; \quad \frac{2\mathfrak{H} + \mathfrak{H}'}{nn} = -0,01102$$

§. 118. Tandem supersunt litterae $\mathfrak{L}$ et $\mathfrak{M}$

$$\begin{aligned}
 2(2\alpha-1)\mathfrak{L} &= -0,45434 - 0,05280 - 0,01652 - 1,31570 \\
 &\quad + 0,00158 \quad - 0,60396 \\
 &\quad - 0,00015
 \end{aligned}$$

$$\begin{aligned}
 2(2\alpha+1)\mathfrak{M} &= +0,00158 + 0,00368 + 0,01935 \\
 &\quad - 0,00015
 \end{aligned}$$

$$\text{Hinc } \mathfrak{L} = +1,40715 \quad . \quad . \quad / -\mathfrak{L} = 0,148340$$

$$\mathfrak{M} = +0,00426 \quad . \quad . \quad / \mathfrak{M} = 7,629896$$

O

Deinde

Deinde vero habebitur:

$$L' = 2(2a-1) L - 0,00213 + 0,17162 - 12,31170 - 0,02596 \\ + 0,00013 \quad - 0,26555$$

$$M' = 2(2a+1) M - 0,00213 - 0,00254 - 0,00742 \\ + 0,00013$$

$$\text{feu} \quad L' = 2(2a-1) L - 12,43359 \\ M' = 2(2a+1) M - 0,01196$$

§. 119. Nunc denique aggrediamur aequationes XIV et XV

$$\text{XIV.} - 2,83960 - 1,01591 L - 0,88774 + 0,00702 + 0,36275 \\ + 0,28733 \quad - 0,06016 + 0,03675 + 7,60650 = \\ - 3,01144 L + 21,57666 + 0,00255 - 0,14680 + 10,75272$$

$$\text{XV.} + 0,00860 - 1,01591 M + 0,01317 + 0,00702 - 0,00623 \\ - 0,00494 \quad - 0,00176 + 0,00336 + 0,01360 = \\ - 32,89415 M + 0,06859 + 0,00255 + 0,00325$$

ex quibus eruitur

$$L = + 13,86720 \quad - \quad L = 1,141988$$

$$M = + 0,00131 \quad - \quad M = 7,117165$$

$$\text{hincque } L' = 11,3090 \quad \text{et } M' = - 0,00445$$

$$\frac{2\kappa L + \xi}{nn} = 0,15125 \quad - \quad \frac{2\kappa L + \xi}{nn} = 9,179684$$

$$\frac{2\kappa M + \eta}{nn} = 0,00004 \quad \frac{2\kappa M + \eta}{nn} = 5,592770$$

Ex his valoribus nouae correctiones inueniri possent, sed differentiae prodirent tam exiguae, ut operae pretium non sit eas inuestigare.

§. 120.

§. 120. His igitur valoribus inuentis, denotante iam a distantiam Lunae mediam a Terra, et eius distantia curtata $= x$, cum sit $x = \frac{(1-kk)a}{1-k \cos r}$, erit:

iones

75

50

172

23

60

34

10

tent,
pre-

120.

log. coefficient:

$x = 1 - 0,0074991 \cos 2\eta$	7,875009
$+ 0,0000532 \cos 4\eta$	5,725912
$+ 0,191557k \cos(2\eta - r)$	9,282297
$- 0,003293k \cos(2\eta + r)$	7,517525
$- 0,003321k \cos(4\eta - r)$	7,521296
$+ 0,000049k \cos(4\eta + r)$	5,692584
$- 0,00511kk \cos 2r$	7,708601
$- 0,08022kk \cos(2\eta - 2r)$	8,904280
$- 0,00237kk \cos(2\eta + 2r)$	7,375072
$+ 0,07892kk \cos(4\eta - 2r)$	8,897172
$+ 0,00001kk \cos(4\eta + 2r)$	4,872349

vbi quidem in duobus primis terminis sunt eos, qui per kk erant affecti, sumus complexi, posito $k = 0,05445$. Etiam si enim hic valor non omnino esset iustus, tamen inde in his terminis minimis nullus error nasci poterit.

O 2

§. 121.

§. 121. Porro quoque hinc ex §. 116. valorem ipsius $\frac{d\phi}{dr}$ determinabimus, quatenus a sola excentricitate orbitae lunaris pender.

$\frac{d\phi}{dr} =$	log. coeff.
1,009276	0,004010
+ 0,0195144 $\cos 2\eta$	8,290355
— 0,0000322 $\cos 4\eta$	5,507856
— 0,001231 $k \cos r$	7,090258
— 0,366103 $k \cos(2\eta - r)$	9,563604
+ 0,012832 $k \cos(2\eta + r)$	8,108292
+ 0,002829 $k \cos(4\eta - r)$	7,451633
— 0,000171 $k \cos(4\eta + r)$	6,232305
+ 0,01182 $kk \cos 2r$	8,072618
— 0,02057 $kk \cos(2\eta - 2r)$	8,313172
+ 0,01063 $kk \cos(2\eta + 2r)$	8,026598
— 0,09883 $kk \cos(4\eta - 2r)$	8,994889
— 0,00004 $kk \cos(4\eta + 2r)$	5,592770

§. 122.

ipſius
e orbi-

§. 122. Cum nunc fit $\frac{d\theta}{dr} = \frac{ds}{dr} = \frac{1+2ee}{n} + \frac{2}{n} k \cos r$
+ $\frac{3}{2n} k k \cos 2r$, erit

coeff.		log. coeff.
	$\frac{d\eta}{dr} =$ 0,933838	9,970272
10	+ 0,0195144 $\cos 2\eta$	8,290355
55	— 0,0000322 $\cos 4\eta$	5,507856
56	— 0,152101 $k \cos r$	9,182132
58	— 0,366103 $k \cos(2\eta - r)$	9,563604
24	+ 0,012829 $k \cos(2\eta + r)$	8,108292
12	+ 0,002829 $k \cos(4\eta - r)$	7,451633
13	— 0,000171 $k \cos(4\eta + r)$	6,232305
15	— 0,10133 $kk \cos 2r$	9,005738
8	— 0,02057 $kk \cos(2\eta - 2r)$	8,313172
2	+ 0,01063 $kk \cos(2\eta + 2r)$	8,026598
8	— 0,09883 $kk \cos(4\eta - 2r)$	8,994889
9	— 0,00004 $kk \cos(4\eta + 2r)$	5,592770
0		

quae formulae ad motum Lunae horarium tam absolute quam a sole adhiberi possunt, quemadmodum illa distantiam definiens diametro apparenti et parallaxi horizontali investigandae inservit.

122.

O 3

φ =

§. 23. Quaeramus nunc valorem integralem pro longitudine Lunae Φ , quatenus a sola excentricitate orbitae lunaris pendet, ac ponamus,

$$\begin{aligned}\Phi = Or + A' \sin 2\eta + a' k k \sin 2\eta + B' \sin 4\eta + b' k k \sin 4\eta \\ + C' k \sin r + D' k \sin (2\eta - r) + F' k \sin (4\eta - r) \\ + G' k \sin (2\eta + r) + H' k \sin (4\eta + r) \\ + I' k k \sin 2r + J' k k \sin (2\eta - 2r) + L' k k \sin (4\eta - 2r) \\ + K' k k \sin (2\eta + 2r) + M' k k \sin (4\eta + 2r)\end{aligned}$$

atque sequentes obtinebimus formulas:

$$\begin{aligned}+0,0188387 &= 2aA' - \frac{A'(2kA+B)}{nn} - \frac{2B'(2kA+A)}{nn} \\ -0,0000370 &= 4aB' - \frac{A'(2kA+A)}{nn} \\ -0,001231 &= C' - \frac{A'(2kD+D)}{nn} + \frac{A'(2kE+G)}{nn} \\ &\quad - \frac{(D'+G')(2kA+A)}{nn} \\ -0,366103 &= (2a-1)D' - \frac{A'(2n+G)}{nn} \\ +0,012832 &= (2a+1)G' - \frac{A'(2n+G)}{nn} \\ +0,002829 &= (4a-1)F' - \frac{4B'}{n} - \frac{A'(2kD+D)}{nn} - \frac{D'(2kA+A)}{nn} \\ -0,000171 &= (4a+1)I' - \frac{4B'}{n} - \frac{A'(2kE+G)}{nn} - \frac{G'(2kA+A)}{nn} \\ &\quad +\end{aligned}$$

em pro
itate or-

$k k \sin 4\eta$

$(\eta - r)$

$(\eta + r)$

$(1 - 2r)$

$(+ 2r)$

$A)$

$+ G)$

$+ A)$

$A+A)$

$+ A)$

$+$

$$\begin{aligned}+0,22790 &= 2aA' - \frac{(D'+G')(2n+G)}{nn} - \frac{2F'(2kD+D)}{nn} \\ &\quad - \frac{2b'(2kA+A)}{nn} \\ +0,00163 &= 4aB' - \frac{a'(2kA+A)}{nn} - \frac{G'(2kD+D)}{nn} \\ &\quad - \frac{2(F'+G')(2n+G)}{nn} \\ +0,01182 &= 2b' - \frac{G'(2kD+D)}{nn} - \frac{A'(2kJ+J)}{nn} \\ &\quad - \frac{(G'+K')(2kA+A)}{nn} - \frac{D'(2kE+G)}{nn} \\ -0,02057 &= 2(a-1)J' - \frac{3A'}{2n} - \frac{D'(2n+G)}{nn} - \frac{A'(2kH+H)}{nn} \\ &\quad - \frac{2I'(2kA+A)}{nn} \\ +0,01063 &= 2(a+1)K' - \frac{3A'}{2n} - \frac{G'(2n+G)}{nn} - \frac{A'(2kH+H)}{nn} \\ &\quad - \frac{2G'(2kD+D)}{nn} \\ -0,09883 &= 2(2a-1)L' - \frac{3B'}{n} + \frac{2F'(2n+G)}{nn} + \frac{A'(2kJ+J)}{nn} \\ &\quad - \frac{J'(2kA+A)}{nn} - \frac{D'(2kD+D)}{nn} \\ -0,00004 &= 2(2a+1)M' - \frac{3B'}{n} - \frac{2G'(2n+G)}{nn} \\ &\quad +\end{aligned}$$

§. 124. Ex his eliciuntur valores sequentes:

$A' = +0,0100887$	$- I A' = 8,003837$	$a' = 0,09140$
$B' = -0,0000409$	$- I B' = 5,611723$	$a' = 8,960934$
$C' = +0,010146$	$- I C' = 8,006295$	$b' = 0,00089$
$D' = -0,420226$	$- I D' = 9,623483$	$b' = 6,949340$
$E' = +0,004992$	$- I E' = 7,698261$	$A' + a'kk = 0,0103597$
$F' = +0,005286$	$- I F' = 7,723163$	$B' + b'kk = 0,0000382$
$G' = -0,000086$	$- I G' = 5,935307$	$(A' + a'kk) = 8,015347$
$H' = +0,00420$	$- I H' = 7,623250$	$(B' + b'kk) = 5,582063$
$I' = +0,57328$	$- I I' = 9,758367$	
$K' = +0,00318$	$- I K' = 7,502427$	
$L' = -0,15083$	$- I L' = 9,178488$	
$M' = -0,00002$	$- I M' = 5,301030$	

§. 125. Pro longitudine ergo Lunae habemus hactenus hanc formulam

	log. coeff.
$\Phi = \text{Const.} + 1,0085272 r$	0,003687
$+ 0,0103597 \sin 2\eta$	8,015347
$- 0,0000382 \sin 4\eta$	5,582063
$+ 0,010146 k \sin r$	8,006295
$- 0,420226 k \sin(2\eta - r)$	9,623483
$+ 0,004992 k \sin(2\eta + r)$	7,698261
$+ 0,005286 k \sin(4\eta - r)$	7,723163
$- 0,000086 k \sin(4\eta + r)$	5,935307
$+ 0,00420 k k \sin 2r$	7,623250
$+ 0,57328 k k \sin(2\eta - 2r)$	9,758367
$+ 0,00318 k k \sin(2\eta + 2r)$	7,402427
$- 0,15083 k k \sin(4\eta - 2r)$	9,178488
$- 0,00002 k k \sin(4\eta + 2r)$	5,301030

§. 126.

§. 126. Quodsi iam ponamus $k = 0,65445$, et hos coefficientes ad minuta secunda cum partibus decimalibus reducamus, longitudo Φ ita exprimitur ut sit:

$\Phi = \text{Const.} + 1,0085272 r$	log. coeff.
$+ 2136'',8 \sin 2\eta$	3,329772
$- 7,8 \sin 4\eta$	0,895488
$+ 113,9 \sin r$	2,056718
$- 4719,6 \sin(2\eta - r)$	3,673906
$+ 56,1 \sin(2\eta + r)$	1,748684
$+ 59,4 \sin(4\eta - r)$	1,773586
$- 1,0 \sin(4\eta + r)$	9,985720
$+ 2,5 \sin 2r$	0,409671
$+ 350,6 \sin(2\eta - 2r)$	2,544788
$+ 1,5 \sin(2\eta + 2r)$	9,188848
$- 92,2 \sin(4\eta - 2r)$	1,064909
$- 0,0 \sin(4\eta + 2r)$	8,087451

Hisque formulis praecipuae inaequalitates, quibus motus Lunae perturbatur, continentur.

CAPUT VIII.

DE MOTU APOGEI LUNAE.

§. 127.

His inventis iam arduam illam de motu apogei Lunae questionem examinare, atque adeo decidere licebit. Quamquam enim in praecedentibus calculis, ubique verum apogei motum, quem observationes ostendunt, introduxi, ita ut id ipsum, quod in controversia est, assumisse videar; tamen quoniam in hunc ipsum finem terrae vim, qua luna vrgetur, indefinitam sum contemplatus, dum rationi distantiarum reciprocae duplicatae terminum indefinitum adiunxi, vnde littera μ in calculum est ingressa, iudicium de eo apogei motu, qui Theoriae Newtonianae esset consentaneus, non erit difficile. Quodsi enim valor litterae μ nihilo aequalis reperitur, hinc concludendum erit Theoriam Newtoni cum phaenomenis perfecte consentire; sin autem pro littera μ notabilis predeat valor, Theoria ista insufficiens erit censenda.

§. 128. Motus autem apogei, quoniam huius rei in calculo nusquam mentio est facta, in ea continetur proportione, quam motus lunae medius ad motum anomaliae tenere est positus. Cum enim remotis lunae inaequalitatibus, quae regulae Keplerianae aduersantur, longitudo lunae vera obtineatur, si eius anomalia vera r ad longitudinem apogei addatur: denotet ν longitudinem apogei, eritque longitudo vera $\phi = \nu + r$, vnde

CAPUT VIII.

vnde fit $\nu = \phi - r$. Ex quo intelligitur, si $\phi - r$ quantitatem designet constantem, apogeeum in quiete relinqui, sin autem $\phi - r$ valorem variabilem obtineat, tum apogeeum quoque lunae motum esse habiturum.

§. 129. Cum autem terminos illos omnes, qui sinus angulorum implicant, ideoque inaequalitates periodicas continent, quibus apogei motus non afficitur, omitimus, per integrationem deducimur ad huiusmodi formulam $\phi = \text{Const.} + \text{Or}$, vnde propterea habetur longitudo apogei $\nu = \text{Const.} + (\text{O} - \text{I}) r$. Hinc consequimur sequentes proportionēs:

- I. Vt 1 ad $\text{O} - \text{I}$, ita motus anomaliae lunae ad motum apogei.
- II. Vt O ad 1, ita motus lunae medius ad motum anomaliae.
- III. Vt O ad $\text{O} - \text{I}$, ita motus lunae medius ad motum apogei.

§. 130. Si observationes consulamus, valor litterae O reperitur $= 1,0085272$, quem etiam in calculo ubique adhibui; propterea quod propositum erat non tam in istum valorem a priori inquirere, quam ipsam potius Theoriam ita instituere, atque si opus fuerit, emendare, ut motus inde apogei experientiae consentaneus resultaret. Vicissim autem Theoria stabilita, siue Newtoniana siue alia, quae ex determinato pro μ substituto valore oriatur, facile erit valorem ipsius O a priori eruere, quem deinceps cum valore vero 1,0085272 conferre licebit. Vel inuento valore ipsius O , apogeeum

ei Lucidare calculones introhunc nitam rocae era μ notu, i erit qualis utoni o litciens

s rellectur ano-nae utur, vera gitu-
- r, vnde

lunae intervallo mensis apogistici progreditur per spatium $(O-1) 360^\circ$, intervallo autem mensis periodici per spatium $(1-\frac{1}{O}) 360^\circ$. Secundum observationes autem apogeu promouetur

vno mense apogistico per spatium $3^\circ, 4', 11''$

vno mense periodico per spatium $3, 2, 38$

§. 131. Ex calculo autem §. 107. exposito [valor litterae O ex elementis ante assumtis ita definitur, ut sit $O + 0,000649 = x + 0,000285$ siue $O = x - 0,000364$]. Eri enim haec exigua particula 0,000364 iam ex valore ipsius x veritati consentaneae assumto est orta, tamen perspicuum est, leuem differentiam nullius hic momenti futuram fuisse. Verum littera x per Theoriam ita erat assumta, ut esset

$$x = \sqrt{1 + \frac{3+4\mu+\delta}{2nn}}$$

vbi quidem valor ipsius nn ex motu medio lunae ad motum solis relatio habetur, ita ut sit sine respectu ad motum apogei habitus $nn = 175,71795$. Ergo pro Theoria Newtoniana est

$$x = \sqrt{1 + \frac{3+\delta}{2nn}} \quad \text{et} \quad O = \sqrt{1 + \frac{3+\delta}{2nn}} - 0,000364.$$

§. 132. Illic igitur patet totam hanc inuestigationem ad inuentionem litterae δ reduci, cuius valor, ut ex superiori calculo manifestum est, a pluribus litteris et coefficientibus terminorum, quos ante eruere oportebat, pender, ita ut neglecta hac littera δ motus apogei nullo modo recte definiti queat. Initio quidem vbi hanc litteram

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teram in calculum induximus, quod factum est §. 44. haec res leuis momenti est visa; cum enim pro CC, quae erat constans per integrationem in calculum ingressa, valorem vero proximum inuenissemus $1 + \frac{3+4\mu}{2nn}$, quoniam facile erat praeuidere, reliquis adhibitis elementis ad motum lunae pertinentibus, hunc valorem aliquantum immutari posse, pro vero valore ipsius $\frac{CC}{1-\mu}$ posuimus

$1 + \frac{3+4\mu+\delta}{2nn}$. Deinde autem valor ipsius δ potissimum pendet a valore litterae γ , qua usi sumus ad verum valorem constantis $\frac{m}{nn} = 1 + \frac{2+3\mu+\gamma}{nn}$ obtinendum, cum pro-

xime verus esset inuentus $= 1 + \frac{2+3\mu}{nn}$.

§. 133. Ab his ergo litteris γ et δ , quae initio nullius fere usus esse videbatur, determinatio motus apogei potissimum pendet, quae cum ex pluribus atque adeo omnibus inaequalitatibus lunae ab excentricitate ortis determinari debeant, mirum sane non est, quod legitima motus apogei designatio, cum tantis implicata sit difficultatibus, tam dudum fuerit abscondita. Plerique enim, qui motum apogei ex sola Theoria concludere sunt annisi, ad omnes has inaequalitates non respexerunt, atque calculum perinde administraverunt, ac si hic litteras γ et δ neglexissemus. Ac si non desuere, qui sibi persuaferunt, motum apogei cum Theoria Newtoniana consentire, ii plerumque per errorem calculi seducti ad veritatem peruenisse sibi sunt visi. Quin etiam Ipse Neu-

tonus Theoriae suae in motu apogei determinando parum tribuisse videtur.

§. 134. Hinc ex neglectu harum litterarum γ et δ , seu ex alia omissione eodem recedente, factum est, ut Theoria Newtoni observationibus circa motum apogei lunae institutis plane non satisfacere sit putata; quae opinio etiam ita inualuit, ut perspicacissimus quisque hanc Theoriam insufficientem pronunciaret. Atque sagacissimus Clairautius huic opinioni vehementissime erat additus, antequam publice in contrarias partes discesserat. Eadem scilicet ratione ob neglectum minutarum illarum particularum erat deceptus, qua et ego fateri cogor, me per complures annos constanter esse opinatum, ex Theoria Newtoni pro motu apogei Lunae non ultra semissem prodire, ita ut error ultra semissem exsurgens committeretur.

§. 135. Fons itaque huius erroris, qui nisi summa circumspectio adhibeatur, vix evitatur, in eo latet, quod in calculo debita illa constantium determinatio, pro qua equidem hic litteras γ et δ adhibui, negligatur. Quemadmodum per hanc omissionem dimidius tantum apogei motus eliciatur, ostendisse iuvabit. Sit igitur $\delta = 0$, atque littera illius O secundum Theoriam Newtonianam, qua est $\mu = 0$, valor erit $O = \sqrt{1 + \frac{3}{2nn}} = 0,000364$; qui euolutus sit: $O = 1,0042592 - 0,000364$. Quare etiam si particula $0,000364$ utpote ex profundiori indagine nata praetermittatur, tamen iste valor pro $O = 1,0042592$, si cum vero per observationes cognito $O = 1,0085272$ compareretur, exacte fere dimidium motum apogei praebet;

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bet;

bet; atque adeo haec tam accurata medietas non parum digna videtur.

§. 136. Jam videamus, quam prope valorem litterae δ adhibendo ad veritatem perducamur. Inuenimus autem (115) $\delta = 3,20892$, unde prodit

$$\sqrt{1 + \frac{3 + \delta}{2nn}} = 1,0087947.$$

qui valor iam maior est quam verus $1,0085272$, sed recordandum est inde subtrahi debere $0,000364$, sicque relinquetur $O = 1,0084307$, ex quo motus progressivus apogei pro intervallo mensis apogistici prodibit $= 3^\circ 2' 9''$ et pro intervallo mensis periodici $= 3^\circ 0' 37''$, qui numeri duobus tantum minutis a vero deficiunt. Ad hunc defectum supplendum litterae μ tribui poterit valor conveniens ex formula $\mu = \frac{1}{2} (nn - 1) nn - \frac{3}{4} - \frac{1}{4} \delta$, unde reperitur $\mu = 0,03782$, qui valor tantillus est, ut nisi de motu apogei sit quaestio, semper pro nihilo haberi possit.

§. 137. Verum nullo modo affirmare possumus, valores illos pro γ et δ inventos ita esse absolutos, ut nulla amplius correctione indigeant. Quin potius, si formulas supra exhibitas attentius perpendamus, tantum abest ut eas pro completis habere possimus, ut potius manifestum sit, omnes reliquas inaequalitates motus lunae perinde ac eas quas iam definiuimus, terminos quoque in eas suppeditare. Qui etsi admodum erunt parvi, tamen omnino sufficere poterunt ad exiguum istud supplementum, quo adhuc a vero distamus, consiciendum. Cum

Cum enim sola fere inaequalitas ab angulo $2\eta - r$ pendens motum apogei a dimidio tantopere auxisset, ut valor ipsius O ab 1,0042592 vsque ad 1,0084307 increvisset, nullum fere est dubium, quin levis defectus huius numeri a vero valore 1,0085272 a reliquis inaequalitatibus proficiscatur.

§. 138. Hinc igitur concludere debemus, Theoriam Newtonianam cum motu apogei observato tam exacte convenire, ut aberratio, si quidem vlla locum habeat, tam sit exigua, ut merito pro nihilo reputari possit: neque etiam calculi ope ob summam parvitatem eam certo definire licebit. Cum itaque hoc pacto Theoria Newtoniana a fortissima obiectione sit vindicata, gloria huius insignis inventi cum industriae tum candori excellentissimi Clairauti debetur, qui primus egregium hunc Theoriae consensum cum veritate detexit et publice est professus: cui ea re eo maiores debemus gratias, quod sine eius studio summo, quod in hac investigatione consumsit, Theoria Newtoniana fortasse vix unquam ab hac suspitione insufficientiae esset liberata. Atque nunc demum pleno lumine veritas istius Theoriae, cui vni Astronomiae Theoria uniuersa innititur, fulgere est censenda, cum antea non mediocribus tenebris fuisset inuoluta.

CAPUT

— r pendisset, ut 27 incrementus huius inaequa-

Theoriam ex-cum haberi pos-suitatem: pacto indicata, cando-egregi-texit et ebemus hac in-asse vix liberata.

Theo-nititur, s tene-

PUT

CAPUT IX.

INVESTIGATIO INAEQUALITATUM LUNAE
A SOLA EXCENTRICITATE ORBITAE SOLIS
PENDENTIUM

§. 139.

Quoniam in hac investigatione excentricitas orbitae lunaris non in censum venit, inaequalitates quas scrutamur partim ab anomalia vera solis s partim ab angulo 2η pendebunt. Cum igitur sit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n} k \cos r - \frac{2}{n} e \cos s - \frac{2}{n} ek \cos(r-s) + \frac{3}{2n} kk \cos 2r + \frac{1}{2n} ee \cos 2s - \frac{2}{n} ek \cos(r+s)$$

hinc differentiale ds ad differentiale dr reducitur. Atque hoc quidem capite, quia ad excentricitatem Lunae non attendimus, erit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} - \frac{2}{n} e \cos s + \frac{1}{2n} ee \cos 2s$$

§. 140. Incipiamus ergo a formulis $\int R dr$ et v , quas omissis terminis ab angulo r pendentibus ponamus

$$\begin{aligned} \int R dr &= A \cos 2\eta + P e \cos s + Q e \cos(2\eta - s) + R e \cos(2\eta + s) \\ &\quad + S e e \cos 2s + T e e \cos(2\eta - 2s) + V e e \cos(2\eta + 2s) \\ v &= A \cos 2\eta + P e \cos s + Q e \cos(2\eta - s) + R e \cos(2\eta + s) \\ &\quad + S e e \cos 2s + T e e \cos(2\eta - 2s) + V e e \cos(2\eta + 2s) \end{aligned}$$

Q

vbi

vbi. quidem pro $\mathcal{A}$ et A valores supra inuentos completos accipi oportet, ita vt in iis termini akk et akk sint comprehensi; erit ergo

$$\mathcal{A} = -0,81033 \quad 1-\mathcal{A} = 9,908662 \\ A = -1,31773 \quad 1-A = 0,119826$$

Valores autem hinc deriuati erunt:

$$\frac{2\kappa A + \mathcal{A}}{nn} = -0,019744, \quad \frac{-(2\kappa A + \mathcal{A})}{nn} = 8,295442$$

$$A' = -2,47576 \quad 1-A' = 0,393708$$

$$\mathcal{A}' = +0,01036 \quad 1-\mathcal{A}' = 8,015347$$

Terminos autem angulum quadruplum 4η inuoluentes hic ob summam paruitatem omisi, quoniam in combinatione cum angulo s plane fierent imperceptibiles.

§. 141. Hinc iam primo colligitur:

$$\frac{d\phi}{dr} = \kappa - \frac{(2\kappa A + \mathcal{A})}{nn} \cos 2\eta - \frac{(2\kappa P + \mathcal{P})}{nn} e \cos s \\ - \frac{(2\kappa Q + \mathcal{Q})}{nn} e \cos(2\eta - s) - \frac{(2\kappa R + \mathcal{R})}{nn} e \cos(2\eta + s) \\ - \frac{(2\kappa S + \mathcal{S})}{nn} e e \cos 2s \\ - \frac{(2\kappa T + \mathcal{T})}{nn} e e \cos(2\eta - 2s) - \frac{(2\kappa V + \mathcal{V})}{nn} e e \cos(2\eta + 2s)$$

atque porro

$$\frac{d\eta}{dr} = \kappa - \frac{(2\kappa A + \mathcal{A})}{nn} \cos 2\eta - \left(\frac{2\kappa P + \mathcal{P}}{nn} - \frac{2}{n} \right) e \cos s \\ - \frac{(2\kappa Q + \mathcal{Q})}{nn} e \cos(2\eta - s) + \frac{(2\kappa R + \mathcal{R})}{nn} e \cos(2\eta + s) \\ - \left(\frac{(2\kappa S + \mathcal{S})}{nn} - \frac{1}{2n} \right) e e \cos 2s$$

Deinde

Deinde quia est proxime $kk = 9ee$, erit

$$R = \frac{1}{2} (1 + \frac{1}{2} ee) \sin 2\eta + \left(\frac{3Q}{2nn} - \frac{3R}{2nn} \right) e \sin s$$

$$- \left(\frac{2}{n} - \frac{3P}{2nn} \right) e \sin(2\eta - s) - \left(\frac{2}{n} - \frac{3P}{2nn} \right) e \sin(2\eta + s)$$

$$+ \left(\frac{3T}{2nn} - \frac{3V}{2nn} \right) e e \sin 2s$$

$$+ \left(\frac{2}{n} + \frac{3S}{2nn} \right) e e \sin(2\eta - 2s) + \left(\frac{2}{n} + \frac{3S}{2nn} \right) e e \sin(2\eta + 2s)$$

atque omisiss terminis, quibus non est opus.

$$\frac{ddv}{dr^2} = e \cos s \left\{ -\frac{2}{n} - 2\kappa \mathcal{P} - 6P - \frac{9A}{4nn} + \frac{\mathcal{A}\mathcal{Q}}{nn} + \frac{\mathcal{A}\mathcal{R}}{nn} \right. \\ \left. + \frac{3AQ}{nn} + \frac{3AR}{nn} + \frac{3\mathcal{A}(Q+R)}{nn} + \frac{3A(\mathcal{Q}+\mathcal{R})}{nn} \right.$$

$$+ e \cos(2\eta - s) \left\{ -\frac{2}{n} - 2\kappa \mathcal{Q} - 6Q - \frac{3A}{4nn} + \frac{3P}{4nn} \right. \\ \left. + \frac{\mathcal{A}\mathcal{P}}{nn} + \frac{3AP}{nn} + \frac{3\mathcal{A}P}{nn} + \frac{3A\mathcal{P}}{nn} \right.$$

$$+ e \cos(2\eta + s) \left\{ -\frac{2}{n} - 2\kappa \mathcal{R} - 6R - \frac{3A}{4nn} + \frac{3P}{4nn} \right. \\ \left. + \frac{\mathcal{A}\mathcal{P}}{nn} + \frac{3AP}{nn} + \frac{3\mathcal{A}P}{nn} + \frac{3A\mathcal{P}}{nn} \right.$$

$$+ e e \cos 2s \left\{ +\frac{2}{n} - 2\kappa \mathcal{S} - 6S - \frac{3R}{4nn} + \frac{\mathcal{P}\mathcal{P}}{2nn} \right. \\ \left. + \frac{3PP}{2nn} + \frac{3P\mathcal{P}}{nn} \right.$$

$$Q^2 +$$

$$\begin{aligned}
 + e \cos(2\eta - 2s) & \left\{ \begin{aligned} & + \frac{1}{2} - 2\kappa \mathfrak{E} - \mathfrak{E} T - \frac{9P}{8nn} + \frac{\mathfrak{A}\mathfrak{G}}{nn} \\ & + \frac{3AS}{nn} + \frac{3\mathfrak{A}S}{nn} + \frac{3A\mathfrak{G}}{nn} \end{aligned} \right. \\
 + e \cos(2\eta + 2s) & \left\{ \begin{aligned} & + \frac{1}{2} - 2\kappa \mathfrak{V} - \mathfrak{E} V - \frac{9P}{8nn} + \frac{\mathfrak{A}\mathfrak{G}}{nn} \\ & + \frac{3AS}{nn} + \frac{3\mathfrak{A}S}{nn} + \frac{3A\mathfrak{G}}{nn} \end{aligned} \right.
 \end{aligned}$$

§. 142. Quodsi iam forma pro $/R dr$ assumpta differentietur, orietur:

$$\begin{aligned}
 R = -2\alpha \mathfrak{A} \sin 2\eta \\
 + e \sin s & \left(-\frac{1}{n} \mathfrak{P} - \frac{(\mathfrak{Q}-\mathfrak{R})(2\kappa A + \mathfrak{A})}{nn} \right) \\
 + e \sin(2\eta - s) & \left(-\frac{2\mathfrak{A}}{n} + \frac{\mathfrak{A}(2\kappa P + \mathfrak{P})}{nn} - (2\alpha - \frac{1}{n}) \mathfrak{Q} \right) \\
 + e \sin(2\eta + s) & \left(-\frac{2\mathfrak{A}}{n} + \frac{\mathfrak{A}(2\kappa P + \mathfrak{P})}{nn} - (2\alpha + \frac{1}{n}) \mathfrak{R} \right) \\
 + e \sin 2s & \left(\frac{1}{n} \mathfrak{P} - \frac{2}{n} \mathfrak{G} - \frac{(\mathfrak{E}-\mathfrak{V})(2\kappa A + \mathfrak{A})}{nn} \right) \\
 + e \sin(2\eta - 2s) & \left(\frac{1}{2n} \mathfrak{A} - \frac{3}{n} \mathfrak{Q} - (2\alpha - \frac{2}{n}) \mathfrak{E} \right) \\
 + e \sin(2\eta + 2s) & \left(\frac{1}{2n} \mathfrak{A} - \frac{1}{n} \mathfrak{R} - (2\alpha + \frac{2}{n}) \mathfrak{V} \right)
 \end{aligned}$$

§. 143. Comparatione ergo instituta habebitur

$$-\frac{1}{n} \mathfrak{P} - \frac{(\mathfrak{Q}-\mathfrak{R})(2\kappa A + \mathfrak{A})}{nn} = \frac{3(\mathfrak{Q}-\mathfrak{R})}{2nn}$$

$$\frac{P}{nn} + \frac{\mathfrak{A}\mathfrak{G}}{nn}$$

$$\frac{P}{n} + \frac{\mathfrak{A}\mathfrak{G}}{nn}$$

mta dif-

$$\frac{\mathfrak{A})}{n}$$

$$-\frac{1}{n} \mathfrak{Q})$$

$$\frac{1}{n} \mathfrak{R})$$

$$\frac{A + \mathfrak{A})}{n}$$

$$\frac{2}{n} \mathfrak{E})$$

$$) \mathfrak{V})$$

tur

$$-\frac{2}{n} \mathfrak{A} + \frac{\mathfrak{A}(2\kappa P + \mathfrak{P})}{nn} - (2\alpha - \frac{1}{n}) \mathfrak{Q} = -\frac{1}{2} + \frac{3P}{2nn}$$

$$-\frac{2}{n} \mathfrak{A} + \frac{\mathfrak{A}(2\kappa P + \mathfrak{P})}{nn} - (2\alpha + \frac{1}{n}) \mathfrak{R} = -\frac{1}{2} + \frac{3P}{2nn}$$

$$\frac{1}{n} \mathfrak{P} - \frac{2}{n} \mathfrak{G} - \frac{(\mathfrak{E}-\mathfrak{V})(2\kappa A + \mathfrak{A})}{nn} = \frac{3(T-V)}{2nn}$$

$$\frac{1}{2n} \mathfrak{A} - \frac{3}{n} \mathfrak{Q} - (2\alpha - \frac{2}{n}) \mathfrak{E} = \frac{1}{2} + \frac{3S}{2nn}$$

$$\frac{1}{2n} \mathfrak{A} - \frac{1}{n} \mathfrak{R} - (2\alpha + \frac{2}{n}) \mathfrak{V} = \frac{1}{2} + \frac{3S}{2nn}$$

vnde deinceps valores litterarum germanicarum $\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, \mathfrak{G}, \mathfrak{E}, \mathfrak{V}$ sumus inuestigaturi.

§. 144. Differentietur simili modo quantitas v , ac ponatur:

$$\frac{dv}{dr} = -A' \sin 2\eta - P' e \sin s - Q' e \sin(2\eta - s) - S' e \sin 2s - T' e \sin(2\eta - 2s) - R' e \sin(2\eta + s) - V' e \sin(2\eta + 2s)$$

critque

$$A' = 2\alpha \mathfrak{A}, \text{ cuius quidem valor iam supra habetur}$$

$$P' = \frac{1}{n} P + \frac{(Q-R)(2\kappa A + \mathfrak{A})}{nn}$$

$$Q' = (2\alpha - \frac{1}{n}) Q + \frac{2}{n} A - \frac{A(2\kappa P + \mathfrak{P})}{nn}$$

$$R' = (2\alpha + \frac{1}{n}) R + \frac{2}{n} A - \frac{A(2\kappa P + \mathfrak{P})}{nn}$$

$$S' = \frac{2}{n} S - \frac{1}{n} P + \frac{(T-V)(2\kappa A + \mathfrak{A})}{nn}$$

Q 3

T'

$$T' = (2\alpha - \frac{2}{n})T + \frac{3}{n}Q - \frac{1}{2n}A$$

$$V' = (2\alpha + \frac{2}{n})V + \frac{1}{n}R - \frac{1}{2n}A$$

vnde denuo differentiando eruitur.

$$\frac{ddv}{dt^2} = e \cos s \left(-\frac{1}{n}P' + \frac{(Q'+R')(2\kappa A + \mathfrak{A})}{nn} \right)$$

$$e \cos(2\eta - s) \left(-(2\alpha - \frac{1}{n})Q' - \frac{2}{n}A' + \frac{A'(2\kappa P + \mathfrak{P})}{nn} \right)$$

$$e \cos(2\eta + s) \left(-(2\alpha + \frac{1}{n})R' - \frac{2}{n}A' + \frac{A'(2\kappa P + \mathfrak{P})}{nn} \right)$$

$$ee \cos 2s \left(-\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2\kappa A + \mathfrak{A})}{nn} \right)$$

$$ee \cos(2\eta - 2s) \left(-(2\alpha - \frac{2}{n})T' + \frac{1}{2n}A' - \frac{3}{n}Q' \right)$$

$$ee \cos(2\eta + 2s) \left(-(2\alpha + \frac{2}{n})V' + \frac{1}{2n}A' - \frac{1}{n}R' \right)$$

§. 145. Sequentes ergo aequationes resoluendae occurrent

$$-\frac{1}{4} - 2\kappa \mathfrak{P} - 6P - \frac{9A}{4nn} + \frac{(2\mathfrak{A}+3A)(Q+R)}{nn} + \frac{(3\mathfrak{A}+3A)(Q+R)}{nn} =$$

$$-\frac{1}{n}P' + \frac{(Q'+R')(2\kappa A + \mathfrak{A})}{nn}$$

$$-\frac{1}{4} - 2\kappa \mathfrak{Q} - 6Q - \frac{3A}{4nn} + \frac{3P}{4nn} + \frac{(2\mathfrak{A}+3A)\mathfrak{P}}{nn} + \frac{(3\mathfrak{A}+3A)P}{nn} =$$

$$-(2\alpha - \frac{1}{n})Q' - \frac{2}{n}A' + \frac{A'(2\kappa P + \mathfrak{P})}{nn}$$

$$-\frac{1}{4} - 2\kappa \mathfrak{R} - 6R - \frac{3A}{4nn} + \frac{3P}{4nn} + \frac{(2\mathfrak{A}+3A)\mathfrak{P}}{nn} + \frac{(3\mathfrak{A}+3A)P}{nn} =$$

$$-(2\alpha + \frac{1}{n})R' - \frac{2}{n}A' + \frac{A'(2\kappa P + \mathfrak{P})}{nn}$$

$$+\frac{1}{4} - 2\kappa \mathfrak{S} - 6S - \frac{3P}{4nn} + \frac{\mathfrak{P}\mathfrak{P} + 6P\mathfrak{P} + 3PP}{2nn} =$$

$$-\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2\kappa A + \mathfrak{A})}{nn}$$

$$+\frac{1}{4} - 2\kappa \mathfrak{T} - 6T - \frac{9P}{8nn} + \frac{(2\mathfrak{A}+3A)\mathfrak{S}}{nn} + \frac{(3\mathfrak{A}+3A)S}{nn} =$$

$$-(2\alpha - \frac{2}{n})T' + \frac{1}{2n}A' - \frac{3}{n}Q'$$

$$+\frac{1}{4} - 2\kappa \mathfrak{V} - 6V - \frac{9P}{8nn} + \frac{(2\mathfrak{A}+3A)\mathfrak{S}}{nn} + \frac{(3\mathfrak{A}+3A)S}{nn} =$$

$$-(2\alpha + \frac{2}{n})V' + \frac{1}{2n}A' - \frac{1}{n}R'$$

Neglectis primo terminis minimis, qui adhuc sunt incogniti, reperitur:

$\mathfrak{Q} = +1,3238$	$Q = +2,5714$	$Q' = 4,40924$
$\mathfrak{R} = +1,2210$	$R = +2,0571$	$R' = 3,79793$
$\mathfrak{P} = -0,0313$	$P = -1,4807$	$P' = -0,12185$

§. 146. Ex his autem accuratius ita definiuntur ut sit:

$\mathfrak{Q} = +1,33859$	$Q = 0,126649$
$\mathfrak{R} = +1,23468$	$R = 0,091545$
$Q = +2,60087$	$Q' = 0,415119$
$R = +2,00590$	$R' = 0,302308$
$Q' = 4,44801$; $R' = 3,6758\kappa$	

et

$$\begin{aligned} \text{et } \mathfrak{P} &= -0,04010 \quad . \quad . \quad . \quad \text{I-}\mathfrak{P} = 8,603133 \\ \mathfrak{P} &= -1,46488 \quad . \quad . \quad . \quad \text{I-P} = 0,165801 \\ \mathfrak{P}' &= -0,12222 \end{aligned}$$

Deinde reperitur

$$\begin{aligned} \mathfrak{C} &= +0,03911 \quad . \quad . \quad . \quad \text{I}\mathfrak{C} = 8,592230 \\ \mathfrak{S} &= +0,60882 \quad . \quad . \quad . \quad \text{I}\mathfrak{S} = 9,784486 \\ \mathfrak{E} &= -0,85267 \quad . \quad . \quad . \quad \text{I-E} = 9,930827 \\ \mathfrak{B} &= -0,62125 \quad . \quad . \quad . \quad \text{I-B} = 9,793266 \\ \mathfrak{T} &= -2,60380 \quad . \quad . \quad . \quad \text{I-T} = 0,465615 \\ \mathfrak{V} &= -1,02720 \quad . \quad . \quad . \quad \text{I-V} = 0,011672 \end{aligned}$$

Pro sequentibus vero calculis est

$$\begin{aligned} \frac{2\kappa\mathfrak{P}+\mathfrak{P}}{nn} &= -0,017051 \quad . \quad . \quad . \quad \text{I-}\frac{(2\kappa\mathfrak{P}+\mathfrak{P})}{nn} = 8,231755 \\ \frac{2\kappa\mathfrak{Q}+\mathfrak{Q}}{nn} &= +0,037487 \quad . \quad . \quad . \quad \text{I}+\frac{2\kappa\mathfrak{Q}+\mathfrak{Q}}{nn} = 8,573878 \\ \frac{2\kappa\mathfrak{R}+\mathfrak{R}}{nn} &= +0,030062 \quad . \quad . \quad . \quad \text{I}+\frac{2\kappa\mathfrak{R}+\mathfrak{R}}{nn} = 8,478023 \end{aligned}$$

§. 147. Hinc igitur pro distantia lunae a terra curvata $u = \frac{(1-kk')uu}{1-k \cos r}$, pars quantitatis u ab excentricitate orbitae solaris tantum pendens erit

	log. coeff.
$u = \text{Praeced.} - 0,008336e \cos s$	7,920985
$+ 0,014801e \cos(2\eta - s)$	8,170303
$+ 0,011415e \cos(2\eta + s)$	8,057492
$+ 0,00364ee \cos 2s$	7,539670
$- 0,01482ee \cos(2\eta - 2s)$	8,170799
$- 0,00584ee \cos(2\eta + 2s)$	7,766856
Deinde	

Deinde vero erit

	log. coeff.
$\frac{d\Phi}{ds} = \text{Praec.} + 0,017041e \cos s$	8,231755
$- 0,037487e \cos(2\eta - s)$	8,573878
$- 0,030062e \cos(2\eta + s)$	8,478023
$- 0,00722ee \cos 2s$	7,858166
$+ 0,03470ee \cos(2\eta - 2s)$	8,540319
$+ 0,015333ee \cos(2\eta + 2s)$	8,185614

§. 148. Ponatur nunc integrale

$$\begin{aligned} \Phi &= \text{Praec.} + \mathfrak{M}' \sin 2\eta + \mathfrak{P}' e \sin s \\ &+ \mathfrak{Q}' e \sin(2\eta - s) + \mathfrak{R}' e \sin(2\eta + s) \\ &+ \mathfrak{E}' ee \sin 2s + \mathfrak{T}' ee \sin(2\eta - 2s) \\ &+ \mathfrak{V}' ee \sin(2\eta + 2s) \end{aligned}$$

erit differentiatione peracta:

$$\begin{aligned} + 0,017051 &= \frac{1}{n} \mathfrak{P}' - \frac{(\mathfrak{Q}' + \mathfrak{R}') (2\kappa\mathfrak{A} + \mathfrak{M})}{nn} \\ - 0,037489 &= \left(2\alpha - \frac{1}{n}\right) \mathfrak{Q}' + \frac{2}{n} \mathfrak{M}' - \frac{\mathfrak{M}'(2\kappa\mathfrak{P} + \mathfrak{P})}{nn} \\ - 0,030062 &= \left(2\alpha + \frac{1}{n}\right) \mathfrak{R}' + \frac{2}{n} \mathfrak{M}' - \frac{\mathfrak{M}'(2\kappa\mathfrak{P} + \mathfrak{P})}{nn} \\ - 0,00722 &= \frac{2}{n} \mathfrak{E}' - \frac{1}{n} \mathfrak{P}' - \frac{(\mathfrak{T}' + \mathfrak{V}') (2\kappa\mathfrak{A} + \mathfrak{M})}{nn} \\ + 0,03470 &= \left(2\alpha - \frac{2}{n}\right) \mathfrak{T}' - \frac{1}{2n} \mathfrak{M}' + \frac{3}{n} \mathfrak{Q}' \\ + 0,01533 &= \left(2\alpha + \frac{2}{n}\right) \mathfrak{V}' - \frac{1}{2n} \mathfrak{M}' + \frac{1}{n} \mathfrak{R}' \end{aligned}$$

R

fietque

fietque his valoribus determinatis

	log. coeff.
$\phi = \text{Praec.} + 0,236034 e \sin s$	9,372974
$- 0,021889 e \sin (2\eta - s)$	8,340237
$- 0,016368 e \sin (2\eta + s)$	8,214002
$+ 0,06615 e \sin 2s$	8,820508
$+ 0,02332 e \sin (2\eta - 2s)$	8,367825
$+ 0,00840 e \sin (2\eta + 2s)$	7,924429

§. 149. Reducamus has inaequalitates etiam ad minuta secunda, ponendo excentricitatem orbitae solaris $e = 0,01680$, atque habebimus

	log. coeff.
$\phi = \text{Praec.} + 817'',9 \sin s$	2,912708
$- 75,8 \sin (2\eta - s)$	1,879971
$- 56,7 \sin (2\eta + s)$	1,753736
$+ 3,8 \sin 2s$	0,585551
$+ 1,4 \sin (2\eta - 2s)$	0,132868
$+ 0,5 \sin (2\eta + 2s)$	9,689472

Denotat hic s anomaliam veram solis; unde patet eam Lunae inaequalitatem, quae sinui huius anomaliae est proportionalis, admodum esse notabilem, dum ad $13', 38''$ exurgit. Tabulae autem Astronomicae, ubi haec inaequalitas aequatio solaris nominatur, eam multo minorem faciunt, cuius rei causam investigari adhuc conveniet.

§. 150.

§. 150. Quodsi enim litteram η' accuratius definire velimus, habebimus has formulas resoluendas:

$$-\frac{1}{n}\eta' + \frac{A(2nQ+\Omega)}{nn} - \frac{A(2nR+\mathcal{R})}{nn} - \frac{(\Omega-\mathcal{R})(2nA+\mathcal{A})}{nn} - \frac{2(Q-R)}{nn}$$

$$P' = \frac{1}{n}P - \frac{A(2nQ+\Omega)}{nn} + \frac{A(2nR+\mathcal{R})}{nn} + \frac{(Q-R)(2nA+\mathcal{A})}{nn}$$

$$-\frac{1}{2} - 2n\eta' - 6P - \frac{9A}{4nn} + \frac{(2\mathcal{A}+3A)(\Omega+\mathcal{R})}{nn} + \frac{(3\mathcal{A}+3A)(Q+R)}{nn} =$$

$$-\frac{1}{n}P' + \frac{A(2nQ+\Omega)}{nn} + \frac{A(2nR+\mathcal{R})}{nn} + \frac{(Q+R')(2nA+\mathcal{A})}{nn}$$

unde elicimus:

$$\eta' = -0,1183; P = -1,1356; \frac{P}{nn} = -0,0064$$

atque $\frac{2nP+\eta'}{nn} = -0,01376$, fierique iam oportet

$$+0,01376 = \frac{1}{n}\eta' - \frac{A(2nQ+\Omega)}{nn} - \frac{A(2nR+\mathcal{R})}{nn} - \frac{(\Omega+\mathcal{R})(2nA+\mathcal{A})}{nn}$$

unde oritur $\eta' = +0,201385$. Quare accuratius habemus

$s = \text{Praec.} - 0,006400 e \cos s$	7,806180
$\frac{d\phi}{ds} = \text{Praec.} + 0,013760 e \cos s$	8,138618
$\phi = \text{Praec.} + 0,201385 e \sin s$	9,304026
seu	

$\phi = \text{Praec.} + 701'',1 \sin s$ 2,845780
Ergo aequatio sinui Anguli e proportionalis tantum est $11', 41''$.

R 2

CAPUT

§. 150.

CAPUT X.

INVESTIGATIO INAEQUALITATVM LUNAE
AB VTRIUSQUE ORBITAE EXCENTRICITATE
SIMUL PENDENTIVM.

§. 151.

Quoniam praevidemus inaequalitates huius generis, quae altiores litterarum k et e potestates simul complectuntur, minimas esse futuras, alios terminos non scrutabimur, nisi qui producto simplici ek sint affecti. Habebimus ergo

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1}{n} + \frac{2}{n} k \cos r - \frac{2}{n} e \cos s - \frac{2}{n} ek \cos(r-s) - \frac{2}{n} ek \cos(r+s)$$

Cum igitur ad hanc investigationem opus non sit illis terminis ex praecedentibus, qui vel per k^2 vel per e^2 erant affecti, quia litterae alphabeti deficere incipiunt, litteris S, T et sequentibus denuo utemur; quare cavendum, ne istae litterae cum ante adhibitis confundantur.

§. 152. Assumtis ergo ex terminis iam ante definitis, iis qui in eos, quos iam investigamus, vim exerunt, ponamus

$$\begin{aligned} f R dr = & A \cos 2\eta + E k \cos r + D k \cos(2\eta-r) + P e \cos s + Q e \cos(2\eta-s) \\ & + E k \cos(2\eta+r) + R e \cos(2\eta+s) \\ & + E ek \cos(r-s) + B ek \cos(2\eta-r+s) + Y ek \cos(2\eta-r-s) \\ & + Z ek \cos(r+s) + X ek \cos(2\eta+r+s) + S ek \cos(2\eta+r+s) \\ v = & \end{aligned}$$

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2\eta-s)

\eta+s)

r-s)

r+s)

v =

CAPUT X.

$$\begin{aligned} v = & A \cos 2\eta + D k \cos(2\eta-r) + P e \cos s + Q e \cos(2\eta-s) \\ & + E k \cos(2\eta+r) + R e \cos(2\eta+s) \\ & + S ek \cos(r-s) + V ek \cos(2\eta-r+s) + Y ek \cos(2\eta-r-s) \\ & + T ek \cos(r+s) + X ek \cos(2\eta+r-s) + Z ek \cos(2\eta+r+s) \end{aligned}$$

erisque ex praecedentibus

$$A = -0,81033 ; \quad l-A = 9,908662$$

$$E = -0,64383 ; \quad l-E = 9,808771$$

$$D = -3,59362 ; \quad l-D = 0,555531$$

$$P = -1,08732 ; \quad l-P = 0,036358$$

$$Q = -0,11830 ; \quad l-Q = 9,072985$$

$$R = +1,33859 ; \quad l-R = 0,126649$$

$$S = +1,23468 ; \quad l-S = 0,091545$$

$$A = -1,31773 ; \quad l-A = 0,119826$$

$$D = +33,6600 ; \quad l-D = 1,527113$$

$$E = -0,5785 ; \quad l-E = 9,762341$$

$$P = -1,1356 ; \quad l-P = 0,055225$$

$$Q = +2,60087 ; \quad l-Q = 0,415119$$

$$R = +2,00590 ; \quad l-R = 0,302308$$

§. 153. Reliqui vero valores hinc derivati, quibus opus habemus, sunt:

$$A' = -2,47576 ; \quad l-A' = 0,393708$$

$$C' = -0,13847 ; \quad l-C' = 9,141356$$

$$D' = 29,39153 ; \quad l-D' = 1,468222$$

$$E' = -1,47347 ; \quad l-E' = 0,168341$$

$$P' = -0,0260 ; \quad l-P' = 8,414973$$

$$Q' = 4,40924 ; \quad l-Q' = 0,644363$$

$$R' = 3,79793 ; \quad l-R' = 0,579548$$

R 3

$$\mathfrak{A}' = +0,01036 ; \quad / \mathfrak{A}' = 8,015347$$

$$\mathfrak{C}' = +0,01015 ; \quad / \mathfrak{C}' = 8,006295$$

$$\mathfrak{D}' = -0,42023 ; \quad / \mathfrak{D}' = 9,623483$$

$$\mathfrak{E}' = +0,00499 ; \quad / \mathfrak{E}' = 7,698261$$

$$\mathfrak{Y}' = +0,20138 ; \quad / \mathfrak{Y}' = 9,304016$$

$$\mathfrak{Q}' = -0,02189 ; \quad / \mathfrak{Q}' = 8,340237$$

$$\mathfrak{R}' = -0,01637 ; \quad / \mathfrak{R}' = 8,214002$$

Sit breuitatis gratia

$$a' = \frac{2\kappa A + \mathfrak{A}}{nn} = -0,019744 ; \quad / \frac{(2\kappa A + \mathfrak{A})}{nn} = 8,295442$$

$$d' = \frac{2\kappa D + \mathfrak{D}}{nn} = +0,36611 ; \quad / \frac{2\kappa D + \mathfrak{D}}{nn} = 9,563604$$

$$e' = \frac{2\kappa E + \mathfrak{E}}{nn} = -0,01283 ; \quad / \frac{(2\kappa E + \mathfrak{E})}{nn} = 8,108292$$

$$p' = \frac{2\kappa P + \mathfrak{P}}{nn} = -0,01376 ; \quad / \frac{(2\kappa P + \mathfrak{P})}{nn} = 8,138618$$

$$q' = \frac{2\kappa Q + \mathfrak{Q}}{nn} = +0,03749 ; \quad / \frac{2\kappa Q + \mathfrak{Q}}{nn} = 8,573878$$

$$r' = \frac{2\kappa R + \mathfrak{R}}{nn} = +0,03006 ; \quad / \frac{2\kappa R + \mathfrak{R}}{nn} = 8,478023$$

§. 154. Si simili modo vterius ponatur:

$$s' = \frac{2\kappa S + \mathfrak{S}}{nn} ; \quad t' = \frac{2\kappa T + \mathfrak{T}}{nn} ; \quad v' = \frac{2\kappa V + \mathfrak{V}}{nn} ;$$

$$x' = \frac{2\kappa X + \mathfrak{X}}{nn} ; \quad y' = \frac{2\kappa Y + \mathfrak{Y}}{nn} ; \quad z' = \frac{2\kappa Z + \mathfrak{Z}}{nn}$$

habe-

habebimus :

$$\frac{d\Phi}{dr} = \text{Praec.} - a' c f 2\eta - \frac{\mathfrak{C}}{nn} k c f r - d' k c f (2\eta - r) - p' e c f s - q' e c f (2\eta - s) \\ - s' k c f (2\eta + r) - r' e c f (2\eta + s)$$

$$- s' t e k c f (r - s) - v' e k c f (2\eta - r + s) - y' e k c f (2\eta - r - s)$$

$$- s' t e k c f (r + s) - x' e k c f (2\eta + r - s) - z' e k c f (2\eta + r + s)$$

$$\text{atque posito } \frac{2}{n} + \frac{\mathfrak{C}}{nn} = c' = 0,147197 ; \quad / c' = 9,167900$$

$$\frac{d\eta}{dr} = a - a' c f 2\eta - c' k c f r - d' k c f (2\eta - r) + \left(\frac{2}{n} - p'\right) e c f s - q' e c f (2\eta - s) \\ - c' k c f (2\eta + r) - r' e c f (2\eta + s)$$

$$+ \left(\frac{2}{n} - s'\right) e k c f (r - s) - v' e k c f (2\eta - r + s) - y' e k c f (2\eta - r - s)$$

$$+ \left(\frac{2}{n} - s'\right) e k c f (r + s) - x' e k c f (2\eta + r - s) - z' e k c f (2\eta + r + s)$$

$$\text{vbi cum sit } \frac{2}{n} = 0,150876, \text{ erit } \frac{2}{n} - p' = 0,16464$$

§. 155. Nunc termini coefficiente ek affecti, qui in formis $R =$ et $\frac{ddv}{dr^2}$ insunt, colligantur: eritque

$$R = ek \sin(r-s) \left(+ \frac{3}{2nn} V - \frac{3}{2nn} X - \frac{3Q}{nn} + \frac{3R}{nn} - \frac{9D}{4nn} + \frac{9E}{4nn} \right)$$

$$ek \sin(r+s) \left(+ \frac{3}{2nn} Y - \frac{3}{2nn} Z - \frac{3R}{nn} + \frac{3Q}{nn} + \frac{9E}{4nn} - \frac{9D}{4nn} \right)$$

$$ek \sin(2\eta - r + s) \left(- \frac{3}{2nn} + \frac{3}{2nn} S + \frac{3P}{nn} \right)$$

$$ek \sin(2\eta + r - s) \left(- \frac{3}{2nn} + \frac{3}{2nn} S + \frac{3P}{nn} \right)$$

ibe-

ek

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CAPUT X.

$$ek \sin(2\eta - r - s) \left(-\frac{1}{2} + \frac{3}{2nn} T + \frac{3}{nn} P \right)$$

$$ek \sin(2\eta + r + s) \left(-\frac{1}{2} + \frac{3}{2nn} T + \frac{3}{nn} P \right)$$

$$et \frac{ddv}{dr^2} =$$

$$ek \cos(r - s) \left\{ \begin{aligned} & -3 - 2\kappa \mathfrak{C} - \mathfrak{C}S + \frac{1}{2}bP + \frac{3}{4nn} V + \frac{3}{4nn} X + \frac{3}{2nn} Q + \frac{3}{2nn} R \\ & - \frac{9D}{8nn} - \frac{9E}{8nn} + \frac{\mathfrak{A}B}{nn} + \frac{\mathfrak{A}Y}{nn} + \frac{\mathfrak{C}P}{nn} + \frac{D\Omega}{nn} + \frac{\mathfrak{C}R}{nn} \\ & + \frac{3AV}{nn} + \frac{3AX}{nn} + \frac{3DQ}{nn} + \frac{3ER}{nn} - \frac{3AQ}{2nn} - \frac{3AR}{2nn} \\ & + \frac{3\mathfrak{A}V}{nn} + \frac{3A\mathfrak{B}}{nn} + \frac{3\mathfrak{A}X}{nn} + \frac{3A\mathfrak{Y}}{nn} + \frac{3\mathfrak{C}P}{nn} \\ & + \frac{3DQ}{nn} + \frac{3D\Omega}{nn} + \frac{3\mathfrak{C}R}{nn} + \frac{3ER}{nn} \end{aligned} \right.$$

$$ek \cos(r + s) \left\{ \begin{aligned} & -3 - 2\kappa \mathfrak{C} - \mathfrak{C}T + \frac{1}{2}bP + \frac{3}{4nn} Y + \frac{3}{4nn} Z + \frac{3}{2nn} Q + \frac{3}{2nn} R \\ & - \frac{9D}{8nn} - \frac{9E}{8nn} + \frac{\mathfrak{A}Y}{nn} + \frac{\mathfrak{A}Z}{nn} + \frac{\mathfrak{C}P}{nn} + \frac{D\Omega}{nn} + \frac{\mathfrak{C}R}{nn} \\ & + \frac{3AY}{nn} + \frac{3AZ}{nn} + \frac{3DR}{nn} + \frac{3EQ}{nn} - \frac{3AQ}{2nn} - \frac{3AR}{2nn} \\ & + \frac{3\mathfrak{A}Y}{nn} + \frac{3A\mathfrak{Y}}{nn} + \frac{3\mathfrak{A}Z}{nn} + \frac{3A\mathfrak{Z}}{nn} + \frac{3\mathfrak{C}P}{nn} \\ & + \frac{3\mathfrak{C}Q}{nn} + \frac{3E\Omega}{nn} + \frac{3D\mathfrak{R}}{nn} + \frac{3D\mathfrak{R}}{nn} \end{aligned} \right.$$

ek

CAPUT X.

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$$ek \cos(2\eta - r + s) \left\{ \begin{aligned} & -\frac{1}{2} - 2\kappa \mathfrak{B} - \mathfrak{C}V + \frac{1}{2}bR + \frac{3}{4nn} S - \frac{3}{4nn} D + \frac{3}{2nn} P \\ & + \frac{\mathfrak{A}S}{nn} + \frac{\mathfrak{C}R}{nn} + \frac{D\mathfrak{P}}{nn} + \frac{3AS}{nn} + \frac{3DP}{nn} - \frac{3AP}{2nn} \\ & + \frac{3\mathfrak{A}S}{nn} + \frac{3A\mathfrak{C}}{nn} + \frac{3\mathfrak{C}R}{nn} + \frac{3D\mathfrak{P}}{nn} + \frac{3D\mathfrak{P}}{nn} \end{aligned} \right.$$

$$ek \cos(2\eta + r - s) \left\{ \begin{aligned} & -\frac{1}{2} - 2\kappa \mathfrak{X} - \mathfrak{C}X + \frac{1}{2}bQ + \frac{3}{4nn} S - \frac{3}{4nn} E + \frac{3}{2nn} P \\ & + \frac{\mathfrak{A}S}{nn} + \frac{\mathfrak{C}R}{nn} + \frac{D\mathfrak{P}}{nn} + \frac{3AS}{nn} + \frac{3EP}{nn} - \frac{3AP}{2nn} \\ & + \frac{3\mathfrak{A}S}{nn} + \frac{3A\mathfrak{C}}{nn} + \frac{3\mathfrak{C}Q}{nn} + \frac{3D\mathfrak{P}}{nn} + \frac{3E\mathfrak{P}}{nn} \end{aligned} \right.$$

$$ek \cos(2\eta - r - s) \left\{ \begin{aligned} & -\frac{1}{2} - 2\kappa \mathfrak{Y} - \mathfrak{C}Y + \frac{1}{2}bQ + \frac{3}{4nn} T - \frac{3}{4nn} D + \frac{3}{2nn} P \\ & + \frac{\mathfrak{A}S}{nn} + \frac{\mathfrak{C}R}{nn} + \frac{D\mathfrak{P}}{nn} + \frac{3AT}{nn} + \frac{3DP}{nn} - \frac{3AP}{2nn} \\ & + \frac{3\mathfrak{A}T}{nn} + \frac{3A\mathfrak{C}}{nn} + \frac{3\mathfrak{C}Q}{nn} + \frac{3D\mathfrak{P}}{nn} + \frac{3D\mathfrak{P}}{nn} \end{aligned} \right.$$

$$ek \cos(2\eta + r + s) \left\{ \begin{aligned} & -\frac{1}{2} - 2\kappa \mathfrak{Z} - \mathfrak{C}Z + \frac{1}{2}bR + \frac{3}{4nn} T - \frac{3}{4nn} E + \frac{3}{2nn} P \\ & + \frac{\mathfrak{A}S}{nn} + \frac{\mathfrak{C}R}{nn} + \frac{D\mathfrak{P}}{nn} + \frac{3AT}{nn} + \frac{3EP}{nn} - \frac{3AP}{2nn} \\ & + \frac{3\mathfrak{A}T}{nn} + \frac{3A\mathfrak{C}}{nn} + \frac{3\mathfrak{C}R}{nn} + \frac{3D\mathfrak{P}}{nn} + \frac{3E\mathfrak{P}}{nn} \end{aligned} \right.$$

S

§. 156

§. 156. Quaeramus ergo quoque ex formula assumpta $\int R dr$ differentiale, quod erit

$$R =$$

$$ek \sin(r-s) \left(+ \mathcal{A}' - \mathcal{B}' - \mathcal{D}' + \mathcal{E}' + \frac{1}{n} \mathcal{P} + \mathcal{Q}' - \mathcal{R}' - \left(1 - \frac{1}{n}\right) \mathcal{S} - \mathcal{V}' + \mathcal{X}' \right)$$

$$ek \sin(r+s) \left(+ \mathcal{A}' - \mathcal{B}' - \mathcal{D}' + \mathcal{E}' - \frac{1}{n} \mathcal{P} - \mathcal{Q}' + \mathcal{R}' - \left(1 + \frac{1}{n}\right) \mathcal{S} - \mathcal{V}' + \mathcal{Z}' \right)$$

$$ek \sin(2\eta-r+s) \left(- \mathcal{A}' \left(\frac{2}{n} - s'\right) - \mathcal{D}' \left(\frac{2}{n} - p'\right) + \mathcal{R}' - \frac{1}{n} \mathcal{R} - \left(2\alpha - 1 + \frac{1}{n}\right) \mathcal{S} \right)$$

$$ek \sin(2\eta+r-s) \left(- \mathcal{A}' \left(\frac{2}{n} - s'\right) - \mathcal{E}' \left(\frac{2}{n} - p'\right) + \mathcal{Q}' + \frac{1}{n} \mathcal{Q} - \left(2\alpha + 1 - \frac{1}{n}\right) \mathcal{X} \right)$$

$$ek \sin(2\eta-r-s) \left(- \mathcal{A}' \left(\frac{2}{n} - s'\right) - \mathcal{D}' \left(\frac{2}{n} - p'\right) + \mathcal{Q}' + \frac{1}{n} \mathcal{Q} - \left(2\alpha - 1 - \frac{1}{n}\right) \mathcal{Y} \right)$$

$$ek \sin(2\eta+r+s) \left(- \mathcal{A}' \left(\frac{2}{n} - s'\right) - \mathcal{E}' \left(\frac{2}{n} - p'\right) + \mathcal{R}' - \frac{1}{n} \mathcal{R} - \left(2\alpha + 1 + \frac{1}{n}\right) \mathcal{Z} \right)$$

§. 157. Ponatur ex differentiatione formae v ;

$$S' = \left(1 - \frac{1}{n}\right) S - A(v'-s') + Dq' - E' - \frac{1}{n} P - Qd' + R' + (V-X)al$$

$$T' = \left(1 + \frac{1}{n}\right) T - A(y'-z') + Dq' - E' + \frac{1}{n} P + Qc' - R' + (Y-Z)al$$

$$V' = \left(2\alpha - 1 + \frac{1}{n}\right) V + A\left(\frac{2}{n} - s'\right) + D\left(\frac{2}{n} - p'\right) - R' + \frac{1}{n} R$$

$$X' = \left(2\alpha + 1 - \frac{1}{n}\right) X + A\left(\frac{2}{n} - s'\right) + E\left(\frac{2}{n} - p'\right) - Q' - \frac{1}{n} Q$$

$$Y' = \left(2\alpha - 1 - \frac{1}{n}\right) Y + A\left(\frac{2}{n} - s'\right) + D\left(\frac{2}{n} - p'\right) - Q' - \frac{1}{n} Q$$

$$Z' = \left(2\alpha + 1 + \frac{1}{n}\right) Z + A\left(\frac{2}{n} - s'\right) + E\left(\frac{2}{n} - p'\right) - R' + \frac{1}{n} R$$

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vt

vt habeatur

$$\frac{dv}{dr} = -A' \sin 2\eta - C' k \sin r - D' k \sin(2\eta-r) - E' k \sin(2\eta+r)$$

$$- P' e \sin s - Q' e \sin(2\eta-s) - R' e \sin(2\eta+s)$$

$$- S' ek \sin(r-s) - V' ek \sin(2\eta-r+s) - Y' ek \sin(2\eta-r-s)$$

$$- T' ek \sin(r+s) - X' ek \sin(2\eta+r-s) - Z' ek \sin(2\eta+r+s)$$

§. 158. Haec iam forma denuo differentiata dabit

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ ek \cos(r-s) \left(+ A'(v'+s') + D'q' + E'r' - \frac{1}{n} P' + Q'd' + R'e' - \left(1 - \frac{1}{n}\right) S' + (V+X)al \right)$$

$$+ ek \cos(r+s) \left(+ A'(y'+z') + D'q' + E'r' - \frac{1}{n} P' + Q'e' + R'd' - \left(1 + \frac{1}{n}\right) T' + (Y+Z)al \right)$$

$$+ ek \cos(2\eta-r+s) \left(- A'\left(\frac{2}{n} - s'\right) - D'\left(\frac{2}{n} - p'\right) + R' - \frac{1}{n} R' - \left(2\alpha - 1 + \frac{1}{n}\right) V' \right)$$

$$+ ek \cos(2\eta+r-s) \left(- A'\left(\frac{2}{n} - s'\right) - E'\left(\frac{2}{n} - p'\right) + Q' + \frac{1}{n} Q' - \left(2\alpha + 1 - \frac{1}{n}\right) X' \right)$$

$$+ ek \cos(2\eta-r-s) \left(- A'\left(\frac{2}{n} - s'\right) - D'\left(\frac{2}{n} - p'\right) + Q' + \frac{1}{n} Q' - \left(2\alpha - 1 - \frac{1}{n}\right) Y' \right)$$

$$+ ek \cos(2\eta+r+s) \left(- A'\left(\frac{2}{n} - s'\right) - E'\left(\frac{2}{n} - p'\right) + R' - \frac{1}{n} R' - \left(2\alpha + 1 + \frac{1}{n}\right) Z' \right)$$

§. 159. Priores autem expressiones, si litterarum cognitarum valores substituantur, sequenti modo prodibunt.

$$R = ek \sin(r-s) (-0,44856 + 0,00854(V-X))$$

$$ek \sin(r+s) (-0,42826 + 0,00854(Y-Z))$$

$$ek \sin(2\eta-r+s) (-4,51938 + 9,00854 S)$$

$$ek \sin(2\eta+r-s) (-4,51938 + 0,00854 S)$$

$$ek \sin(2\eta-r-s) (-4,51939 + 0,00854 T)$$

$$ek \sin(2\eta+r+s) (-4,51938 + 0,00854 T)$$

S 2

§. 160.

§. 160. Altera vero forma pro $\frac{ddv}{dr^2}$ fit

$$\frac{ddv}{dr^2} =$$

$$\begin{aligned} & ekc(r-s)(-2,83505-2k\mathfrak{S}-\mathfrak{E}S-0,02711(\mathfrak{B}+\mathfrak{X})-0,03207(V+X)) \\ & ekc(r+s)(-3,21669-2k\mathfrak{E}-\mathfrak{E}T-2,02711(\mathfrak{Y}+\mathfrak{Z})-0,03207(Y+Z)) \\ & ekc(2\eta-r+s)(-2,26927-2k\mathfrak{B}-\mathfrak{E}V-0,02711\mathfrak{S}-0,03207S) \\ & ekc(2\eta+r-s)(-0,53441-2k\mathfrak{X}-\mathfrak{E}X-0,02711\mathfrak{S}-0,03207S) \\ & ekc(2\eta-r-s)(-1,36456-2k\mathfrak{Y}-\mathfrak{E}Y-0,02711\mathfrak{T}-0,03207T) \\ & ekc(2\eta+r+s)(-1,43912-2k\mathfrak{Z}-\mathfrak{E}Z-0,02711\mathfrak{T}-0,03207T) \end{aligned}$$

§. 161. Deinde simili modo alterae formulae per differentiationem erutae, substitutis valoribus cognitis ita fe habebunt.

$$R =$$

$$\begin{aligned} & ek \sin(r-s) (+0,59883 + \mathfrak{A}(v'-x') - (1 - \frac{1}{n})\mathfrak{S}) + 0,01974(\mathfrak{B}-\mathfrak{X}) \\ & ek \sin(r+s) (+0,54556 + \mathfrak{A}(y'-z') - (1 + \frac{1}{n})\mathfrak{E}) + 0,01974(\mathfrak{Y}-\mathfrak{Z}) \\ & ek \sin(2\eta-r+s) (+0,80251 + \mathfrak{A}s' - (2\alpha-1 + \frac{1}{n})\mathfrak{B}) \\ & ek \sin(2\eta+r-s) (+0,59936 + \mathfrak{A}s' - (2\alpha+1 - \frac{1}{n})\mathfrak{X}) \\ & ek \sin(2\eta-r-s) (+1,01199 + \mathfrak{A}s' - (2\alpha-1 - \frac{1}{n})\mathfrak{Y}) \\ & ek \sin(2\eta+r+s) (+0,38988 + \mathfrak{A}s' - (\alpha+1 + \frac{1}{n})\mathfrak{Z}) \end{aligned}$$

Porro

$$\begin{aligned} & 3207(V+X)) \\ & 3207(Y+Z)) \\ & -0,03207S) \\ & -0,03207S) \\ & 0,03207T) \\ & 0,03207T) \end{aligned}$$

mulae per
cognitis ita

$$\begin{aligned} & 974(\mathfrak{B}-\mathfrak{X}) \\ & 974(\mathfrak{Y}-\mathfrak{Z}) \\ & \mathfrak{B}) \\ & \mathfrak{X}) \\ & \mathfrak{Y}) \\ & \mathfrak{Z}) \end{aligned}$$

Porro

Porro reperiemus sequentes valores

$$\begin{aligned} S' &= (1 - \frac{1}{n})S - A(v'-x') + 0,38693 - 0,01974(V-X) \\ T' &= (1 + \frac{1}{n})T - A(y'-z') + 0,18018 - 0,01974(Y-Z) \\ V' &= (2\alpha-1 + \frac{1}{n})V - A s' + 5,19902 \\ X' &= (2\alpha+1 - \frac{1}{n})X - A s' - 0,87311 \\ Y' &= (2\alpha-1 - \frac{1}{n})Y - A s' + 4,76391 \\ Z' &= (2\alpha+1 + \frac{1}{n})Z - A s' - 0,43800 \end{aligned}$$

ac denique $\frac{ddv}{dr^2} =$ Praec.

$$\begin{aligned} & + ekc(r-s)(-(1 - \frac{1}{n})S' + A'(v'+x') + 2,62592 - 0,01974(V+X')) \\ & + ekc(r+s)(-(1 + \frac{1}{n})T' + A'(y'+z') + 2,16417 - 0,01974(Y+Z')) \\ & + ekc \cos(2\eta-r+s)(-(2\alpha-1 + \frac{1}{n})V' + A's' - 4,19296) \\ & + ekc \cos(2\eta+r-s)(-(2\alpha+1 - \frac{1}{n})X' + A's' + 1,59777) \\ & + ekc \cos(2\eta-r-s)(-(2\alpha-1 - \frac{1}{n})Y' + A's' - 3,48384) \\ & + ekc \cos(2\eta+r+s)(-(2\alpha+1 + \frac{1}{n})Z' + A's' + 0,88865) \end{aligned}$$

S 3

§. 162.

§. 162. Hinc ergo pro determinandis coefficientibus sequentes obtinemus aequationes

$$(1 - \frac{1}{n}) \mathcal{C} = 1,04739 - 0,00854(V-X) + A'(v'-x') + 0,01974(\mathcal{B}-\mathcal{X})$$

$$(1 + \frac{1}{n}) \mathcal{E} = 0,97382 - 0,00858(Y-Z) + A'(y'-z') + 0,01974(\mathcal{Y}-\mathcal{Z})$$

$$(2\alpha - 1 + \frac{1}{n}) \mathcal{B} = 5,32189 - 0,00854 S + A' s'$$

$$(2\alpha + 1 - \frac{1}{n}) \mathcal{X} = 5,11874 - 0,00854 S + A' s'$$

$$(2\alpha - 1 - \frac{1}{n}) \mathcal{Y} = 5,53137 - 0,00854 T + A' t'$$

$$(2\alpha + 1 + \frac{1}{n}) \mathcal{Z} = 4,90926 - 0,00854 T + A' t'$$

Deinde

$$+5,46097 = (1 - \frac{1}{n}) S' - 2\alpha \mathcal{C} - \mathcal{E} S - 0,02711(\mathcal{B} + \mathcal{X}) - 0,03207(V + X) - A'(v' + x') + 0,01974(V' + X')$$

$$+5,38086 = (1 + \frac{1}{n}) T' - 2\alpha \mathcal{E} - \mathcal{B} T - 0,02711(\mathcal{Y} + \mathcal{Z}) - 0,03207(Y + Z) - A'(y' + z') + 0,01974(Y' + Z')$$

$$-1,92371 = (2\alpha - 1 + \frac{1}{n}) V' - 2\alpha \mathcal{B} - \mathcal{E} V - 0,02711 \mathcal{C} - 0,03207 S - A' s'$$

$$+2,13218 = (2\alpha + 1 - \frac{1}{n}) X' - 2\alpha \mathcal{X} - \mathcal{E} X - 0,02711 \mathcal{C} - 0,03207 S - A' s'$$

$$-2,11928 = (2\alpha - 1 - \frac{1}{n}) Y' - 2\alpha \mathcal{Y} - \mathcal{E} Y - 0,02711 \mathcal{E} - 0,03207 T - A' t'$$

$$+2,32777 = (2\alpha + 1 + \frac{1}{n}) Z' - 2\alpha \mathcal{Z} - \mathcal{E} Z - 0,02711 \mathcal{E} - 0,03207 T - A' t'$$

§. 163.

§. 163. Pro ulteriori calculo est

$$1 - \frac{1}{n} = 0,924562 \dots \dots \dots I(1 - \frac{1}{n}) = 9,965935$$

$$1 + \frac{1}{n} = 1,075438 \dots \dots \dots I(1 + \frac{1}{n}) = 0,031570$$

$$2\alpha - 1 + \frac{1}{n} = 0,942914 \dots \dots \dots I(2\alpha - 1 + \frac{1}{n}) = 9,965935$$

$$2\alpha + 1 - \frac{1}{n} = 2,792038 \dots \dots \dots I(2\alpha + 1 - \frac{1}{n}) = 0,445915$$

$$2\alpha - 1 - \frac{1}{n} = 0,792038 \dots \dots \dots I(2\alpha - 1 - \frac{1}{n}) = 9,898747$$

$$2\alpha + 1 + \frac{1}{n} = 2,942914 \dots \dots \dots I(2\alpha + 1 + \frac{1}{n}) = 0,468775$$

Hinc in aequationibus posterioribus valores litterarum S' , T' , V' substituantur, et ob $\mathcal{E} = 1,01591$, erit

$$0,16110 S = -5,10323 - 2\alpha \mathcal{C} - A'(v' + x') - 0,02711(\mathcal{B} + \mathcal{X}) + 0,01974(V' + X') + 1,21835(v' - x') - 0,03207(V + X) - 0,01825(V - X)$$

$$0,14059 T = +5,18709 + 2\alpha \mathcal{E} + A'(y' + z') + 0,02711(\mathcal{Y} + \mathcal{Z}) - 0,01974(Y' + Z') - 1,41710(y' - z') + 0,03207(Y + Z) + 0,02124(Y - Z)$$

$$0,12683 V = +6,82591 - 2\alpha \mathcal{B} + 3,718 s' - 0,02711 \mathcal{C} - 0,03207 S$$

$$6,77934 X = +4,56988 + 2\alpha \mathcal{X} - 6,1548 s' + 0,02711 \mathcal{C} + 0,03207 S$$

$$0,38858 Y = +5,89253 - 2\alpha \mathcal{Y} + 3,5194 s' - 0,02711 \mathcal{E} - 0,03207 T$$

$$7,64474 Z = +3,61676 + 2\alpha \mathcal{Z} - 6,3536 s' + 0,02711 \mathcal{E} + 0,03207 T$$

§. 164.

§. 164. Commodissime hi coefficientes inueniri videntur, si primo $\mathfrak{B}$, $\mathfrak{X}$, $\mathfrak{Y}$, $\mathfrak{Z}$ et V , X , Y , Z proxime quaerantur, quod fiet terminos minimos negligendo:

$$\begin{aligned}\mathfrak{B} &= + 5,7560 & \dots & \mathfrak{B} = 0,760125 \\ \mathfrak{X} &= + 1,8334 & \dots & \mathfrak{X} = 0,263245 \\ \mathfrak{Y} &= + 6,9837 & \dots & \mathfrak{Y} = 0,844088 \\ \mathfrak{Z} &= + 1,6643 & \dots & \mathfrak{Z} = 0,221240 \\ V &= - 37,7650 & \dots & V = 1,577086 \\ X &= + 1,2198 & \dots & X = 0,086293 \\ Y &= - 21,1040 & \dots & Y = 1,324360 \\ Z &= + 0,9125 & \dots & Z = 9,960209 \\ V' &= - 29,7170 & \dots & V' = - 0,398 \\ X' &= + 2,5326 & \dots & X' = + 0,024 \\ Y' &= - 11,9510 & \dots & Y' = - 0,201 \\ Z' &= + 2,2472 & \dots & Z' = + 0,020\end{aligned}$$

§. 165. Hic autem valores pro V' et Y' tam sunt magni, ut vicissim post inuentas litteras S , T minimi valores modo erutos afficiant, unde necesse erit resolutionem harum aequationum ordinario modo instruere. Reperitur ergo

$$\begin{aligned}\mathfrak{B} &= 5,7560 - 0,0193 S - 0,0050 T \\ \mathfrak{X} &= 1,8334 - 0,0064 S - 0,0016 T \\ \mathfrak{Y} &= 6,9837 - 0,0225 T - 0,0058 T \\ \mathfrak{Z} &= 1,6643 - 0,0061 T - 0,0016 T \\ 2\mathfrak{B} &= 11,6155 - 0,0390 S - 0,0100 T \\ 2\mathfrak{X} &= 3,6966 - 0,0129 S - 0,0033 T \\ 2\mathfrak{Y} &= 14,0930 - 0,0455 T - 0,0118 T \\ 2\mathfrak{Z} &= 3,3586 - 0,0122 T - 0,0032 T\end{aligned}$$

qui

si vi-
time
:

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qui valores substituti dant:

$$\begin{aligned}V &= - 37,7647 + 0,3912 S + 0,0031 T \\ X &= + 1,2198 - 0,0076 S - 0,0016 T \\ Y &= - 21,1038 + 0,1385 T + 0,0121 T \\ Z &= + 0,9125 - 0,0069 T - 0,0016 T \\ 2\mathfrak{V} &= - 76,2085 + 0,7893 S + 0,0064 T \\ 2\mathfrak{X} &= + 2,4616 - 0,0153 S - 0,0033 T \\ 2\mathfrak{Y} &= - 42,5870 + 0,2794 T + 0,0244 T \\ 2\mathfrak{Z} &= + 1,8413 - 0,0140 T - 0,0033 T\end{aligned}$$

§. 166. Hinc porro valores deriuati erunt

$$\begin{aligned}V' &= - 29,7167 + 0,3767 S + 0,0094 T \\ X' &= + 2,5326 - 0,0061 S + 0,0029 T \\ Y' &= - 11,9511 + 0,1248 T + 0,0161 T \\ Z' &= + 2,2472 - 0,0054 T + 0,0028 T \\ V' &= - 0,4009 + 0,0044 S + 0,0001 T \\ X' &= + 0,0244 \\ Y' &= - 0,2026 + 0,0015 T + 0,0001 T \\ Z' &= + 0,0200\end{aligned}$$

ac porro

$$\begin{aligned}\mathfrak{B} - \mathfrak{X} &= 3,9227 - 0,0129 S - 0,0034 T \\ \mathfrak{Y} - \mathfrak{Z} &= 5,3194 - 0,0164 T - 0,0042 T \\ V - X &= - 38,9845 + 0,3988 S + 0,0047 T \\ Y - Z &= - 22,0163 + 0,1454 T + 0,0137 T \\ \mathfrak{B} + \mathfrak{X} &= 7,5893 - 0,0257 S - 0,0066 T \\ \mathfrak{Y} + \mathfrak{Z} &= 8,6480 - 0,0286 T - 0,0074 T \\ V + X &= - 36,5449 + 0,3836 S + 0,0015 T \\ Y + Z &= - 20,1913 + 0,1316 T + 0,0105 T\end{aligned}$$

T

V'+X'

$$V' + X' = -27,1841 + 0,3706 S + 0,0123 \mathcal{S}$$

$$Y' + Z' = -9,7039 + 0,1194 T + 0,0189 \mathcal{E}$$

$$v' - z' = -0,4253 + 0,0044 S + 0,0001 \mathcal{S}$$

$$y' - z' = -0,2226 + 0,0015 T + 0,0001 \mathcal{E}$$

$$v' + z' = -0,3765 + 0,0044 S + 0,0001 \mathcal{S}$$

$$y' + z' = -0,1826 + 0,0015 T + 0,0001 \mathcal{E}$$

§. 167. His valoribus substitutis reperitur

$$\left(1 - \frac{1}{n}\right) \mathcal{S} = +1,8024 - 0,0070 S$$

$$\left(1 + \frac{1}{n}\right) \mathcal{E} = +1,4453 - 0,0027 T$$

unde concluditur

$$\mathcal{S} = 1,9495 - 0,0075 S$$

$$\mathcal{E} = 1,3440 - 0,0025 T$$

$$0,1401 S = -9,2444$$

$$2\mathcal{S} = 3,9340 - 0,0160 S$$

$$2\mathcal{E} = 2,7121 - 0,0053 T$$

$$0,1479 T = +7,9767$$

§. 168. Nunc igitur habebimus

$$S = -66,6980 \quad \text{. . .} \quad / -S = 1,824113$$

$$T = +53,9330 \quad \text{. . .} \quad / T = 1,731855$$

$$\mathcal{S} = +2,4497 \quad \text{. . .} \quad / \mathcal{S} = 0,389113$$

$$\mathcal{E} = +1,2092 \quad \text{. . .} \quad / \mathcal{E} = 0,082498$$

$$s' = -0,75204 \quad \text{. . .} \quad / -s' = 9,876241$$

$$z' = +0,62626 \quad \text{. . .} \quad / z' = 9,796755$$

$$v' = -0,6942 \quad \text{. . .} \quad / -v' = 9,841484$$

$$x' = +0,0244 \quad \text{. . .} \quad / x' = 8,387390$$

$$y' = -0,1216 \quad \text{. . .} \quad / -y' = 9,084933$$

$$z' = +0,0200 \quad \text{. . .} \quad / z' = 9,301030$$

$$\mathfrak{B} =$$

160 S

353 T

1767

$$\mathfrak{B} = +7,0311 \quad \text{. . .} \quad / \mathfrak{B} = 0,847029$$

$$\mathfrak{E} = +2,2561 \quad \text{. . .} \quad / \mathfrak{E} = 0,353358$$

$$\mathfrak{Y} = +5,7659 \quad \text{. . .} \quad / \mathfrak{Y} = 0,760867$$

$$\mathfrak{Z} = +1,3339 \quad \text{. . .} \quad / \mathfrak{Z} = 0,125123$$

$$V = -63,8498 \quad \text{, , ,} \quad / -V = 1,805160$$

$$X = +1,7451 \quad \text{. . .} \quad / X = 0,241820$$

$$Y = -13,6222 \quad \text{. . .} \quad / -Y = 1,134241$$

$$Z = +0,5356 \quad \text{. . .} \quad / Z = 9,728840$$

§. 169. His iam valoribus inuentis pro distantia lunae $x = \frac{(1-kk) au}{1-k \cos r}$ erit valoris ipsius n portio ab his terminis pendens:

	Log. coeff.
$n = \text{Praec.} - 0,3796 \text{ ek } \cos(r-s)$	9,579297
$+ 0,3069 \text{ ek } \cos(r+s)$	9,487093
$- 0,3634 \text{ ek } \cos(2\eta-r+s)$	9,560344
$+ 0,0099 \text{ ek } \cos(2\eta+r-s)$	7,997004
$- 0,0775 \text{ ek } \cos(2\eta-r-s)$	8,889425
$+ 0,0030 \text{ ek } \cos(2\eta+r+s)$	7,484024

et pro longitudine lunae

$$\frac{d\Phi}{dr} = \text{Praec.} + 0,7520 \text{ ek } \cos(r-s)$$

$$- 0,6263 \text{ ek } \cos(r+s)$$

$$+ 0,6942 \text{ ek } \cos(2\eta-r+s)$$

$$- 0,0244 \text{ ek } \cos(2\eta+r-s)$$

$$+ 0,1216 \text{ ek } \cos(2\eta-r-s)$$

$$- 0,0200 \text{ ek } \cos(2\eta+r+s)$$

T 2

cuius

cuius integrale si ponatur:

$$\Phi = \text{Præc.} + \mathcal{E}'ek \sin(r-s) + \mathcal{B}'ek \sin(2\eta-r+s) + \mathcal{Y}'ek \sin(2\eta-r-s) \\ + \mathcal{Z}'ek \sin(r+s) + \mathcal{X}'ek \sin(2\eta+r-s) + \mathcal{S}'ek \sin(2\eta+r+s)$$

erit

$$+0,7520 = (1 - \frac{1}{n}) \mathcal{E}' - \mathcal{Y}'(v' + x') - \mathcal{D}'q' - \mathcal{E}'r' + \frac{1}{n} \mathcal{Y}' - \mathcal{Q}'d' - \mathcal{R}'e' - (\mathcal{B}' + \mathcal{X}')a'$$

$$-0,6263 = (1 + \frac{1}{n}) \mathcal{Z}' - \mathcal{X}'(y' + z') - \mathcal{D}'r' - \mathcal{E}'q' + \frac{1}{n} \mathcal{Y}' - \mathcal{Q}'d' - \mathcal{R}'d' - (\mathcal{Y}' + \mathcal{S}')a'$$

$$+0,6942 = (2n - 1 + \frac{1}{n}) \mathcal{B}' + \mathcal{Y}'(\frac{2}{n} - s') + \mathcal{D}'(\frac{2}{n} - p') - \mathcal{R}'(c' - \frac{1}{n})$$

$$-0,0244 = (2n + 1 - \frac{1}{n}) \mathcal{X}' + \mathcal{Y}'(\frac{2}{n} - s') + \mathcal{E}'(\frac{2}{n} - p') - \mathcal{Q}'(c' + \frac{1}{n})$$

$$+0,1216 = (2n - 1 - \frac{1}{n}) \mathcal{Y}' + \mathcal{X}'(\frac{2}{n} - s') + \mathcal{D}'(\frac{2}{n} - p') - \mathcal{Q}'(c' + \frac{1}{n})$$

$$-0,0200 = (2n + 1 + \frac{1}{n}) \mathcal{S}' + \mathcal{Y}'(\frac{2}{n} - s') + \mathcal{E}'(\frac{2}{n} - p') - \mathcal{R}'(c' - \frac{1}{n})$$

§. 170. Hinc autem reperitur

$$\mathcal{E}' = +0,7467 \quad . \quad . \quad / \quad \mathcal{E}' = 9,873165$$

$$\mathcal{Z}' = -0,6185 \quad . \quad . \quad / \quad \mathcal{Z}' = 9,791317$$

$$\mathcal{B}' = +0,8143 \quad . \quad . \quad / \quad \mathcal{B}' = 9,910800$$

$$\mathcal{X}' = -0,0142 \quad . \quad . \quad / \quad \mathcal{X}' = 8,150690$$

$$\mathcal{Y}' = +0,2396 \quad . \quad . \quad / \quad \mathcal{Y}' = 9,379550$$

$$\mathcal{S}' = -0,0061 \quad . \quad . \quad / \quad \mathcal{S}' = 7,788910$$

§. 171.

§. 171. Quatenus ergo longitudo Lunae ab excentricitate orbitae solis pendet, erit

$-r-s$

$-r+s$

$\mathcal{E}'a'$

$\mathcal{S}'a'$

log. coeff.

$\Phi = \text{Præc.} + 0,201385e \sin s$	9,304026
$- 0,021889e \sin(2\eta-s)$	8,340237
$- 0,016368e \sin(2\eta+s)$	8,214002
$+ 0,06615ee \sin 2s$	8,820508
$+ 0,02332ee \sin(2\eta-2s)$	8,367825
$+ 0,00840ee \sin(2\eta+2s)$	7,924429
$+ 0,7476ek \sin(r-s)$	9,873165
$- 0,6185ek \sin(r+s)$	9,791317
$+ 0,8143ek \sin(2\eta-r+s)$	9,910800
$- 0,0142ek \sin(2\eta-r-s)$	8,150690
$+ 0,2396ek \sin(2\eta-r-s)$	9,379550
$- 0,0061ek \sin(2\eta+r+s)$	7,788910

T 3

§. 172.

§. 172. Hae autem singulae inaequalitates ad numerum minorum secundorum reductae dabunt:

	log. coeff.
$\phi = \text{Praec. } +701'', 1 \sin s$	2,845780
$- 75, 8 \sin (2\eta - s)$	1,879971
$- 56, 7 \sin (2\eta + s)$	1,753736
$+ 3, 8 \sin 2s$	0,585551
$+ 1, 4 \sin (2\eta - 2s)$	0,132862
$+ 0, 5 \sin (2\eta + 2s)$	9,689472
$+ 140, 9 \sin (r - s)$	2,148800
$- 116, 7 \sin (r + s)$	2,067000
$+ 153, 7 \sin (2\eta - r + s)$	2,186530
$- 2, 7 \sin (2\eta + r - s)$	0,426300
$+ 45, 2 \sin (2\eta - r - s)$	1,655200
$+ 1, 2 \sin (2\eta + r + s)$	0,064600

Hic scilicet et inaequalitates, quas in capite praecedente inuenimus, et istas in hoc capite eruras simul sum complexus, ut coniunctim conspectui exponerentur,

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CAPUT XI.

INVESTIGATIO INAEQUALITATUM LUNAE
A PARALLAXI SOLIS PENDENTIUM.

§. 173.

Jam in formulis nostris primariis ad eos quoque terminos progrediamur, qui littera v sunt affecti, et quoniam est $1:v$ ut distantia Solis media ad distantiam Lunae mediam a Terra, erit $1:v$ ut parallaxis Lunae media ad parallaxin solis: ex quo inaequalitates Lunae, quae hinc oriuntur, a parallaxi solis pendere dicuntur. Quoniam vero valor ipsius v est valde parvus, quippe $\frac{1}{280}$ propemodum, alios terminos non contemplabimur, nisi qui per v ac per vk et ve sunt multiplicati, propterea quod magis compositi fiant minimi.

§. 174. Ex terminis ergo iam inuentis hic retineamus eos, qui sunt alicuius momenti, et cum iis novos determinandos coniungamus; sit ergo:

$$\begin{aligned}
 fRdr = & A \cos 2\eta + E k \cos r + D k \cos (2\eta - r) + P e \cos s + Q e \cos (2\eta - s) \\
 & + E k \cos (2\eta + r) + R e \cos (2\eta + s) \\
 & + F v \cos v + G vk \cos (\eta - r) + H ve \cos (\eta - s) \\
 & + I v \cos 3\eta + J vk \cos (\eta + r) + L ve \cos (\eta + s) \\
 v = & A \cos 2\eta + D k \cos (2\eta - r) + P e \cos s + Q e \cos (2\eta - s) \\
 & + E k \cos (2\eta + r) + R e \cos (2\eta + s) \\
 & + F v \cos \eta + H vk \cos (\eta - r) + K ve \cos (\eta - s) \\
 & + G v \cos 3\eta + J vk \cos (\eta + r) + L ve \cos (\eta + s)
 \end{aligned}$$

Non

Non difficulter enim praevidere licet, terminos, qui angulos r et s cum angulo 3η habeant coniunctos, fore tam exiguos, ut sine errore praetermitti queant.

§. 175. Quodsi iam retentis litterarum §. 153. inductarum valoribus, praeterea ponamus:

$$f' = \frac{2\kappa F + \mathfrak{F}}{nn}; \quad g' = \frac{2\kappa G + \mathfrak{G}}{nn}; \quad b' = \frac{2\kappa H + \mathfrak{H}}{nn}$$

$$i' = \frac{2\kappa J + \mathfrak{J}}{nn}; \quad k' = \frac{2\kappa K + \mathfrak{K}}{nn}; \quad l' = \frac{2\kappa L + \mathfrak{L}}{nn}$$

habebimus:

$$\frac{d\Phi}{dr} = \text{Praec.} - a'cf2\eta - \frac{\mathfrak{C}}{nn} kcf - d'kcf(2\eta - r) - p'ecf s - q'ecf(2\eta - s)$$

$$\quad \quad \quad - e'kcf(2\eta + r) \quad \quad - r'ecf(2\eta + s)$$

$$- f'v \cos \eta - b'vk \cos(\eta - r) - k've \cos(\eta - s)$$

$$- g'v \cos 3\eta - i'vk \cos(\eta + r) - l've \cos(\eta + s)$$

atque

$$\frac{d\eta}{dr} = a - a'cf2\eta - e'kcf - d'kcf(2\eta - r) + \left(\frac{2}{n} - p'\right)ecf s - q'ecf(2\eta - s)$$

$$\quad \quad \quad - e'kcf(2\eta + r) \quad \quad - r'ecf(2\eta + s)$$

$$- f'v \cos \eta - b'vk \cos(\eta - r) - k've \cos(\eta - s)$$

$$- g'v \cos \eta - i'vk \cos(\eta + r) - l've \cos(\eta + s)$$

§. 176. Jam vero pro his terminis ab v pendentibus sequentes colligemus aequationes.

$$R = v \sin \eta \left(\frac{1}{3} + \frac{3F - 3G}{2nn} \right)$$

$$v \sin 3\eta \left(\frac{1}{3} + \frac{3F}{2nn} \right)$$

vk

$$p'k \sin(\eta - r) \left(\frac{1}{3} + \frac{3J}{2nn} + \frac{3F}{nn} - \frac{3G}{nn} \right)$$

$$q'k \sin(\eta + r) \left(\frac{1}{3} + \frac{3H}{2nn} - \frac{3G}{nn} + \frac{3F}{nn} \right)$$

$$r'ec \sin(\eta - s) \left(-\frac{1}{3} + \frac{3L}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn} \right)$$

$$s'ec \sin(\eta + s) \left(-\frac{1}{3} + \frac{3K}{2nn} + \frac{9G}{4nn} - \frac{9F}{4nn} \right)$$

$$\frac{ddv}{dr^2} =$$

$$v \cos \eta \left\{ \frac{1}{3} - 6F + \frac{3F}{4nn} + \frac{3G}{4nn} - 2\kappa \mathfrak{F} + \frac{\mathfrak{H}\mathfrak{F}}{nn} + \frac{\mathfrak{H}\mathfrak{G}}{nn} \right\}$$

$$v \cos 3\eta \left\{ \frac{1}{3} - 6G + \frac{3F}{4nn} - 2\kappa \mathfrak{G} + \frac{\mathfrak{H}\mathfrak{F}}{nn} \right\}$$

$$vk \cos(\eta - r) \left\{ \frac{1}{3} - 6H + \frac{1}{2}bF + \frac{3J}{4nn} + \frac{3F}{2nn} + \frac{3G}{2nn} - 2\kappa \mathfrak{H} \right.$$

$$\left. + \frac{\mathfrak{H}\mathfrak{J}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{\mathfrak{D}\mathfrak{F}}{nn} + \frac{\mathfrak{C}\mathfrak{G}}{nn} \right\}$$

$$vk \cos(\eta + r) \left\{ \frac{1}{3} - 6J + \frac{1}{2}bF + \frac{3H}{4nn} + \frac{3G}{2nn} + \frac{3F}{2nn} - 2\kappa \mathfrak{J} \right.$$

$$\left. + \frac{\mathfrak{H}\mathfrak{J}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{\mathfrak{D}\mathfrak{G}}{nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} \right\}$$

V

$v \cos$

$$v e \cos (\eta-s) \left\{ \begin{array}{l} -\frac{3}{2} - 6K + \frac{3L}{4nn} - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2 \kappa R \\ + \frac{2E}{nn} + \frac{3P}{nn} + \frac{3Q}{nn} + \frac{3R}{nn} \end{array} \right.$$

$$v e \cos (\eta+s) \left\{ \begin{array}{l} -\frac{3}{2} - 6L + \frac{3K}{4nn} - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2 \kappa R \\ + \frac{2E}{nn} + \frac{3P}{nn} + \frac{3Q}{nn} + \frac{3R}{nn} \end{array} \right.$$

sequentes partes seorsim exponamus:

$$v \cos \eta \left\{ \begin{array}{l} + \frac{32F}{nn} + \frac{32G}{nn} + \frac{32H}{nn} + \frac{32I}{nn} + \frac{32J}{nn} + \frac{32K}{nn} \\ + \frac{9A}{8nn} + \frac{15A}{8nn} \end{array} \right.$$

$$v \cos 3\eta \left\{ \begin{array}{l} + \frac{32F}{nn} + \frac{32G}{nn} + \frac{32H}{nn} + \frac{9A}{8nn} \end{array} \right.$$

$$v k \cos (\eta-r) \left\{ \begin{array}{l} \frac{32J}{nn} + \frac{32F}{nn} + \frac{32G}{nn} + \frac{32H}{nn} + \frac{32I}{nn} + \frac{32K}{nn} \\ + \frac{32L}{nn} + \frac{32M}{nn} + \frac{32N}{nn} + \frac{32O}{nn} + \frac{32P}{nn} + \frac{32Q}{nn} \\ + \frac{9D}{8nn} + \frac{15E}{8nn} \end{array} \right.$$

$$v k \cos (\eta+r) \left\{ \begin{array}{l} \frac{32H}{nn} + \frac{32F}{nn} + \frac{32G}{nn} + \frac{32I}{nn} + \frac{32J}{nn} + \frac{32K}{nn} \\ + \frac{32L}{nn} + \frac{32M}{nn} + \frac{32N}{nn} + \frac{32O}{nn} + \frac{32P}{nn} + \frac{32Q}{nn} \\ - \frac{32R}{2nn} - \frac{32S}{2nn} + \frac{9E}{8nn} + \frac{15D}{8nn} \end{array} \right. v e \cos$$

$$v e \cos (\eta-s) \left\{ \begin{array}{l} \frac{32L}{nn} + \frac{32F}{nn} + \frac{32G}{nn} + \frac{32H}{nn} + \frac{32I}{nn} \\ + \frac{32J}{nn} + \frac{32K}{nn} + \frac{32L}{nn} + \frac{32M}{nn} + \frac{32N}{nn} + \frac{32O}{nn} \\ + \frac{32P}{nn} + \frac{9P}{8nn} + \frac{9Q}{8nn} + \frac{15R}{8nn} \end{array} \right.$$

$$v e \cos (\eta+s) \left\{ \begin{array}{l} \frac{32K}{nn} + \frac{32F}{nn} + \frac{32G}{nn} + \frac{32H}{nn} + \frac{32I}{nn} + \frac{32J}{nn} \\ + \frac{32L}{nn} + \frac{32M}{nn} + \frac{32N}{nn} + \frac{32O}{nn} + \frac{32P}{nn} + \frac{32Q}{nn} \\ + \frac{9P}{8nn} + \frac{9R}{8nn} + \frac{15Q}{8nn} \end{array} \right.$$

§. 177. Verum differentiando nanciscemur

R

$$v \sin \eta \left[2f' - 2g' - \alpha f - \frac{1}{2} f' + \frac{1}{2} g' \right]$$

$$v \sin 3\eta \left[2f' + \frac{1}{2} f' - 3\alpha f \right]$$

$$v k \sin (\eta-r) \left[2f' + 2g' - 2g' + \frac{1}{2} f' - \frac{1}{2} f' + \frac{1}{2} g' - (\alpha-1) f - \frac{1}{2} f' \right]$$

$$v k \sin (\eta+r) \left\{ \begin{array}{l} 2f' - 2g' + 2f' + \frac{1}{2} f' - \frac{1}{2} f' + \frac{1}{2} g' - (\alpha+1) f \\ - (\alpha+1) f \end{array} \right.$$

$$v e \sin (\eta-s) \left\{ \begin{array}{l} + 2f' + 2g' - 2g' - \frac{1}{2} (\frac{2}{n} - p) f - \frac{1}{2} f' + \frac{1}{2} g' \\ - (\alpha - \frac{1}{n}) f - \frac{1}{2} f' \end{array} \right.$$

$$v e \sin (\eta+s) \left\{ \begin{array}{l} + 2f' - 2g' + 2f' - \frac{1}{2} (\frac{2}{n} - p) f - \frac{1}{2} f' + \frac{1}{2} g' \\ - (\alpha + \frac{1}{n}) f - \frac{1}{2} f' \end{array} \right.$$

V 2

Deinde

Deinde posito

$$F' = \alpha F - A(f' - g') + \frac{1}{2} F' d' - \frac{1}{2} G' d'$$

$$G' = 3\alpha G - A f' - \frac{1}{2} F' d'$$

$$H' = (\alpha - 1)H - A d' - D f' + E g' - \frac{1}{2} F' d' + \frac{1}{2} F' d' - \frac{1}{2} G' d' + \frac{1}{2} J' d'$$

$$J' = (\alpha + 1)J - A b' + D g' - E f' - \frac{1}{2} F' d' + \frac{1}{2} F' d' - \frac{1}{2} G' d' + \frac{1}{2} H' d'$$

$$K' = (\alpha - 1)K - A' - Q f' + R g' + \frac{1}{2} F'(\frac{2}{n} - p') + \frac{1}{2} F' g' - \frac{1}{2} G' d' + \frac{1}{2} L' d'$$

$$L' = (\alpha + 1)L - A k' + Q g' - R f' + \frac{1}{2} F'(\frac{2}{n} - p') + \frac{1}{2} F' d' - \frac{1}{2} G' d' + \frac{1}{2} K' d'$$

erit

$$\frac{du}{dr} = -A' \sin 2\eta - D' k \sin(2\eta - r) - P' e \sin s - Q' e \sin(2\eta - s)$$

$$= E' k \sin(2\eta + r) = R' e \sin(2\eta + s)$$

$$= F' v \sin \eta = H' v k \sin(\eta - r) = K' v e \sin(\eta - s)$$

$$= G' v \sin 3\eta = J' v k \sin(\eta + r) = L' v e \sin(\eta + s)$$

§. 178. Hinc iam denuo differentiando consequemur:

$$\frac{d^2 u}{dr^2} = \text{Præc.}$$

$$+ v \cos \eta \left\{ + A' f' + A' g' - \alpha F' + \frac{1}{2} F' d' + \frac{1}{2} G' d' \right.$$

$$+ v \cos 3\eta \left\{ + A' f' + \frac{1}{2} F' d' - 3\alpha G' \right.$$

$$+ v k \cos(\eta - r) \left\{ + A' f' + D' f' + E' g' + \frac{1}{2} F' d' + \frac{1}{2} F' d' + \frac{1}{2} G' d' \right. \\ \left. - (\alpha - 1) H' + \frac{1}{2} J' d' \right.$$

$$+ v k \cos(\eta + r) \left\{ + A' b' + E' f' + D' g' + \frac{1}{2} F' d' + \frac{1}{2} F' d' + \frac{1}{2} G' d' \right. \\ \left. + \frac{1}{2} H' d' - (\alpha + 1) J' \right.$$

$$+ v e \cos(\eta - s) \left\{ + A' p' + Q' f' + R' g' - \frac{1}{2}(\frac{2}{n} - p') F' + \frac{1}{2} F' g' + \frac{1}{2} G' d' \right. \\ \left. - (\alpha - \frac{1}{n}) K' + \frac{1}{2} L' d' \right.$$

+

+ $\frac{1}{2} J' d'$

+ $\frac{1}{2} H' d'$

+ $\frac{1}{2} L' d'$

+ $\frac{1}{2} K' d'$

(2\eta - s)

(2\eta + s)

(\eta - s)

(\eta + s)

æque-

d'

G' d'

J' d'

G' d'

1) J'

G' d'

L' d'

+

$$+ v e \cos(\eta + s) \left\{ + A' k' + R' f' + Q' g' - \frac{1}{2}(\frac{2}{n} - p') F' + \frac{1}{2} F' d' + \frac{1}{2} G' d' \right. \\ \left. + \frac{1}{2} K' d' - (\alpha + \frac{1}{n}) L' \right.$$

qui valores cum antecedentibus comparari debent, ut inde valores coefficientium eliciantur.

§. 179. Sumamus primo duos valores ab initio positos, quoniam hi a sequentibus non pendent, atque habebimus,

$$e = \alpha \mathfrak{F} - \mathfrak{A} f' + \mathfrak{A} g' + \frac{1}{2} \mathfrak{F} d' - \frac{1}{2} \mathfrak{G} d' + \frac{1}{2} + \frac{3\mathfrak{F} - 3\mathfrak{G}}{2nn}$$

$$e = 3\alpha \mathfrak{G} - \mathfrak{A} f' - \frac{1}{2} \mathfrak{F} d' + \frac{1}{2} + \frac{3\mathfrak{F}}{2nn}$$

$$e = \alpha F' - A' f' - A' g' - \frac{1}{2} F' d' - \frac{1}{2} G' d' + \frac{1}{2} - 6F - 2\pi \mathfrak{F} \\ + \frac{3(F+G)}{4nn} + \frac{(2+3A)}{nn} (\mathfrak{F} + \mathfrak{G}) + \frac{(3A+3A)}{nn} (F+G) + \frac{3A}{nn}$$

$$e = 3\alpha G' - A' f' - \frac{1}{2} F' d' + \frac{1}{2} - 6G - 2\pi \mathfrak{G} \\ + \frac{3F}{4nn} + \frac{(2+3A)}{nn} \mathfrak{F} + \frac{(32+3A)}{nn} F + \frac{9A}{8nn}$$

$$\text{et } F' = \alpha F - A(f' - g') + \frac{1}{2} F' d' - \frac{1}{2} G' d' \\ G' = 3\alpha G - A f' - \frac{1}{2} F' d'$$

$$\text{at est } f' = \frac{2\pi F + \mathfrak{F}}{nn} \text{ et } g' = \frac{2\pi G + \mathfrak{G}}{nn}$$

Hic igitur primum litterarum, quae sunt cognitae, valores in numeris substituantur, eritque

V 3

F' =

$$\begin{aligned} F' &= 0,93905 F + 0,00753 \mathfrak{F} + 0,01443 G - 0,00753 \mathfrak{G} \\ G' &= 2,80121 G + 0,02505 F + 0,00753 \mathfrak{F} \\ 0 &= 0,92847 \mathfrak{F} + 0,01776 F - 0,01776 G - 0,02501 \mathfrak{G} \\ &\quad + 0,37500 \end{aligned}$$

$$0 = 2,80121 \mathfrak{G} + 0,01447 \mathfrak{F} + 0,01776 F + 1,87500$$

hincque

$$\mathfrak{F} = -0,01930 F + 0,01913 G - 0,42145$$

$$\mathfrak{G} = -0,00623 F - 0,00010 G - 0,66717$$

§. 180. Inde porro colligemus

$$F' = 0,93895 F + 0,01457 G + 0,00185$$

$$G' = 0,02491 F + 2,80135 G - 0,00317$$

$$f' = 0,01138 F + 0,00011 G - 0,00240$$

$$g' = -0,00004 F + 0,01148 G - 0,00380$$

quibus valoribus substitutis pervenimus ad has aequationes:

$$0,09337 F = +0,05421 G + 1,96867$$

$$6,83135 G = -0,08830 F - 3,20944$$

unde fit:

$$F = 20,65700 \quad \dots \quad / \quad F = 1,315067$$

$$G = -0,73681 \quad \dots \quad / \quad G = 9,867356$$

$$\mathfrak{F} = -0,83418 \quad \dots \quad / \quad \mathfrak{F} = 9,921260$$

$$\mathfrak{G} = -0,79580 \quad \dots \quad / \quad \mathfrak{G} = 9,900804$$

$$F' = +19,38712 \quad \dots \quad / \quad F' = 1,287512$$

$$G' = -1,55266 \quad \dots \quad / \quad G' = 0,191075$$

$$f' = +0,23259 \quad \dots \quad / \quad f' = 9,366501$$

$$g' = -0,01309 \quad \dots \quad / \quad g' = 8,116940$$

§. 181.

§. 181. His valoribus, qui ad inaequalitates absolutas pertinent, expeditis, progrediamur ad eos, qui ab excentricitate orbitae lunaris pendent, ac his aequationibus continentur:

$$(\alpha-1) \mathfrak{H} - \mathfrak{A}' + \frac{1}{2} \mathfrak{J}' - \mathfrak{D}' + \mathfrak{E}' - \frac{1}{2} \mathfrak{F} (c' - d') - \frac{1}{2} \mathfrak{G} e'$$

$$+ \frac{1}{2} \mathfrak{H} + \frac{3\mathfrak{J}}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$(\alpha+1) \mathfrak{J} - \mathfrak{A}b' + \frac{1}{2} \mathfrak{H}a' + \mathfrak{D}g' - \mathfrak{E}f' - \frac{1}{2} \mathfrak{F} (c' - e') - \frac{1}{2} \mathfrak{G} d'$$

$$+ \frac{1}{2} \mathfrak{H} + \frac{3\mathfrak{H}}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$H' = (\alpha-1) H - A'a' + \frac{1}{2} J'a' - D'f' + E'g' - \frac{1}{2} F (c' - d') - \frac{1}{2} G e'$$

$$J' = (\alpha+1) J - Ab' + \frac{1}{2} Ha' + D'g' - E'f' - \frac{1}{2} F (c' - e') - \frac{1}{2} G d'$$

$$(\alpha-1) H' - A'a' - \frac{1}{2} J'a' - D'f' - E'g' - \frac{1}{2} F (c' + d') - \frac{1}{2} G e'$$

$$+ \frac{1}{2} \mathfrak{H} - 5H - 2\mathfrak{H} + \frac{1}{2} bF + \frac{3\mathfrak{J}}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{G}\mathfrak{F}}{nn} + \frac{3\mathfrak{E}F}{nn}$$

$$+ \frac{(\mathfrak{A}+3A)}{nn} \mathfrak{J} + \frac{(3\mathfrak{A}+3A)}{nn} J + \frac{(\mathfrak{D}+3D)}{nn} \mathfrak{F} + \frac{(3\mathfrak{D}+3D)}{nn} F$$

$$+ \frac{(\mathfrak{E}+3E)}{nn} \mathfrak{G} + \frac{(3\mathfrak{E}+3E)}{nn} G - \frac{3A(F+G)}{2nn} + \frac{9D+15E}{8nn} = 0$$

$$(\alpha+1) J' - A'b' - \frac{1}{2} H'a' - E'f' - D'g' - \frac{1}{2} F (c' + e') - \frac{1}{2} G d'$$

$$+ \frac{1}{2} \mathfrak{H} - 5J - 2\mathfrak{J} + \frac{1}{2} bF + \frac{3\mathfrak{H}}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{G}\mathfrak{F}}{nn} + \frac{3\mathfrak{E}F}{nn}$$

$$+ \frac{(\mathfrak{A}+3A)}{nn} \mathfrak{H} + \frac{(3\mathfrak{A}+3A)}{nn} H + \frac{(\mathfrak{D}+3D)}{nn} \mathfrak{G} + \frac{(3\mathfrak{D}+3D)}{nn} G$$

$$+ \frac{(\mathfrak{E}+3E)}{nn} \mathfrak{F} + \frac{(3\mathfrak{E}+3E)}{nn} F - \frac{3A(F+G)}{2nn} + \frac{9E+15D}{8nn} = 0$$

181.

§. 182.

§. 182. In his aequationibus substituantur valores iam cogniti, atque obtinebimus,

$$-0,06626 \mathfrak{H} + 0,81033 \mathfrak{H}' - 0,00987 \mathfrak{J} + 0,00854 \mathfrak{J}' + 2,04620 = 0$$

$$1,83374 \mathfrak{J} + 0,81033 \mathfrak{H}' - 0,00987 \mathfrak{H} + 0,00854 \mathfrak{H}' + 2,10646 = 0$$

$$\mathfrak{H}' = -0,06626 \mathfrak{H} + 1,31773 \mathfrak{H}' - 0,00937 \mathfrak{J} - 5,57458$$

$$\mathfrak{J}' = 1,83374 \mathfrak{J} + 1,31773 \mathfrak{H}' - 0,00987 \mathfrak{H} - 1,55427$$

$$-0,06626 \mathfrak{H}' + 2,47576 \mathfrak{J}' + 0,00987 \mathfrak{J}' - 11,86106$$

$$-1,01591 \mathfrak{H} + 0,00427 \mathfrak{J} - 0,02711 \mathfrak{J}' + 2,81250$$

$$-2,01798 \mathfrak{H} - 0,03634 \mathfrak{J} + 31,31044$$

$$+ 10,60834$$

$$1,83374 \mathfrak{J}' + 2,47576 \mathfrak{H}' + 0,00987 \mathfrak{H}' + 0,27762$$

$$-1,01591 \mathfrak{J} + 0,00427 \mathfrak{H} - 0,02711 \mathfrak{H} + 2,81250$$

$$-2,01798 \mathfrak{J} - 0,03634 \mathfrak{H} + 31,81380$$

$$- 0,81380$$

§. 183. Substituamus primo loco $\mathfrak{H}'$ et $\mathfrak{J}'$ valores, atque nostrae aequationes reducuntur ad formas sequentes,

$$-0,06626 \mathfrak{H} + 0,01785 \mathfrak{J} - 0,00526 \mathfrak{J}' + 2,04620 = 0$$

$$+ 1,83374 \mathfrak{J} + 0,01785 \mathfrak{H} - 0,00526 \mathfrak{H}' + 2,10646 = 0$$

$$\mathfrak{H}' = -0,06626 \mathfrak{H} + 0,00526 \mathfrak{J} + 0,00750 \mathfrak{J}' - 5,57458$$

$$\mathfrak{J}' = + 1,83374 \mathfrak{J} + 0,00526 \mathfrak{H} + 0,00750 \mathfrak{H}' - 1,55427$$

qui valores in sequentibus substituti dant

$$-1,01147 \mathfrak{H} - 2,01791 \mathfrak{H}' + 0,01417 \mathfrak{J} - 0,01352 \mathfrak{J}' + 33,22429 = 0$$

$$+ 2,34674 \mathfrak{J} - 2,01791 \mathfrak{J}' + 0,00535 \mathfrak{H}' + 0,00073 \mathfrak{H} + 30,68146 = 0$$

unde elicimus:

$$\mathfrak{H} = \pm 0,00077 \mathfrak{H} + 0,26935 \mathfrak{J} + 30,96780$$

$$\mathfrak{J} = -0,00974 \mathfrak{H} + 0,00077 \mathfrak{J} - 1,05989$$

§. 184.

§. 184. Hi autem valores in posterioribus aequationibus substituti producent

$$1,01290 \mathfrak{H} + 0,52937 \mathfrak{J} + 29,25137 = 0$$

$$2,34539 \mathfrak{J} + 0,02500 \mathfrak{H} + 32,84282 = 0$$

hincque tandem concluditur:

$$\mathfrak{H} = -21,68110 \quad \mathfrak{H}' = 1,336081$$

$$\mathfrak{J} = -13,77206 \quad \mathfrak{J}' = 1,138999$$

$$\mathfrak{H} = +27,24155 \quad \mathfrak{H}' = 1,435232$$

$$\mathfrak{J} = -1,25933 \quad \mathfrak{J}' = 0,100140$$

$$\mathfrak{H}' = -0,09396 \quad \mathfrak{H} = 8,972943$$

$$\mathfrak{J}' = -0,16533 \quad \mathfrak{J} = 9,218352$$

§. 185. Nunc pro excentricitate orbitae solaris hae restant aequationes,

$$\left(\alpha - \frac{1}{n}\right) \mathfrak{K} - \mathfrak{A}' + \frac{1}{2} \mathfrak{L}' - \mathfrak{Q}' + \mathfrak{R}' + \frac{1}{2} \left(\frac{2}{n} - p'\right) \mathfrak{F} + \frac{1}{2} \mathfrak{F}' - \frac{1}{2} \mathfrak{G}'$$

$$- \frac{1}{2} + \frac{3\mathfrak{L}}{2nn} - \frac{9\mathfrak{F}}{4nn} + \frac{9\mathfrak{G}}{4nn} = 0$$

$$\left(\alpha + \frac{1}{n}\right) \mathfrak{L} - \mathfrak{A}'' + \frac{1}{2} \mathfrak{K}' + \mathfrak{Q}' - \mathfrak{R}' + \frac{1}{2} \left(\frac{2}{n} - p'\right) \mathfrak{F} + \frac{1}{2} \mathfrak{F}' - \frac{1}{2} \mathfrak{G}'$$

$$- \frac{1}{2} + \frac{3\mathfrak{K}}{2nn} - \frac{9\mathfrak{F}}{4nn} + \frac{9\mathfrak{G}}{4nn} = 0$$

$$\mathfrak{K}' = \left(\alpha - \frac{1}{n}\right) \mathfrak{K} - \mathfrak{A}' + \frac{1}{2} \mathfrak{L}' - \mathfrak{Q}' + \mathfrak{R}' + \frac{1}{2} \left(\frac{2}{n} - p'\right) \mathfrak{F} + \frac{1}{2} \mathfrak{F}' - \frac{1}{2} \mathfrak{G}'$$

$$\mathfrak{L}' = \left(\alpha + \frac{1}{n}\right) \mathfrak{L} - \mathfrak{A}'' + \frac{1}{2} \mathfrak{K}' + \mathfrak{Q}' - \mathfrak{R}' + \frac{1}{2} \left(\frac{2}{n} - p'\right) \mathfrak{F} + \frac{1}{2} \mathfrak{F}' - \frac{1}{2} \mathfrak{G}'$$

$$\left(\alpha - \frac{1}{n}\right) \mathfrak{K}' - \mathfrak{A}' + \frac{1}{2} \mathfrak{L}' - \mathfrak{Q}' + \mathfrak{R}' + \frac{1}{2} \left(\frac{2}{n} - p'\right) \mathfrak{F} + \frac{1}{2} \mathfrak{F}' - \frac{1}{2} \mathfrak{G}'$$

$$- \frac{1}{2} - 6 \mathfrak{K} - 2 \mathfrak{L} - \frac{15\mathfrak{F}}{8nn} - \frac{9\mathfrak{G}}{8nn} + \frac{3\mathfrak{L}}{4nn} + \frac{(\mathfrak{A} + 3\mathfrak{A})}{nn} \mathfrak{L} + \frac{(3\mathfrak{A} + 3\mathfrak{A})}{nn} \mathfrak{L}$$

X

+

c valores

$$14620 = 0$$

$$0646 = 0$$

$$57458$$

$$55427$$

$$106$$

$$250$$

$$344$$

$$334$$

$$762$$

$$150$$

$$180$$

$$180$$

$$i'$$

$$valo-$$

$$formas$$

$$20 = 0$$

$$46 = 0$$

$$57458$$

$$55427$$

$$1429 = 0$$

$$1146 = 0$$

§. 184.

$$+\frac{(3\mathfrak{F}+3\mathfrak{F})(\mathfrak{P}+\mathfrak{Q})}{nn}+\frac{(3\mathfrak{F}+3\mathfrak{F})(\mathfrak{P}+\mathfrak{Q})}{nn}+\frac{(\mathfrak{G}+3\mathfrak{G})\mathfrak{R}}{nn}+\frac{(3\mathfrak{G}+3\mathfrak{G})\mathfrak{R}}{nn}$$

$$+\frac{9\mathfrak{P}}{8nn}+\frac{9\mathfrak{Q}}{8nn}+\frac{15\mathfrak{R}}{8nn}=0$$

$$(\alpha+\frac{1}{n})\mathfrak{L}-\mathfrak{A}'\mathfrak{K}'-\frac{1}{2}\mathfrak{K}'\mathfrak{A}'-\mathfrak{R}'\mathfrak{F}'-\mathfrak{Q}'\mathfrak{G}'+\frac{1}{2}(\frac{2}{n}-\mathfrak{P}')\mathfrak{F}'-\frac{1}{2}\mathfrak{F}'\mathfrak{P}'-\frac{1}{2}\mathfrak{G}'\mathfrak{Q}'$$

$$-\frac{1}{2}\mathfrak{G}'\mathfrak{Q}'-\frac{1}{2}\mathfrak{L}-\frac{1}{2}\mathfrak{K}'\mathfrak{A}'+\frac{3\mathfrak{K}}{4nn}-\frac{15\mathfrak{F}}{8nn}-\frac{9\mathfrak{G}}{8nn}+\frac{(\mathfrak{A}+3\mathfrak{A})\mathfrak{R}}{nn}+\frac{(3\mathfrak{A}+3\mathfrak{A})\mathfrak{K}}{nn}$$

$$+\frac{(3\mathfrak{F}+3\mathfrak{F})(\mathfrak{P}+\mathfrak{R})}{nn}+\frac{(3\mathfrak{F}+3\mathfrak{F})(\mathfrak{P}+\mathfrak{R})}{nn}+\frac{(\mathfrak{G}+3\mathfrak{G})\mathfrak{Q}}{nn}+\frac{(3\mathfrak{G}+3\mathfrak{G})\mathfrak{Q}}{nn}$$

$$+\frac{9\mathfrak{P}}{8nn}+\frac{9\mathfrak{R}}{8nn}+\frac{15\mathfrak{Q}}{8nn}=0$$

§. 186. Hic autem obseruo, hanc determinationem maxime esse lubricam, cum coëfficiens litterae L, quem postremo est habitura, admodum fiat parvus; unde is a terminis, quos omisimus, non mediocrem mutationem perpeti possit. Hanc ob causam consultum iudico, in calculum quoque terminos $3\eta-s$ et $3\eta+s$ introducere, quia praeuideo ab iis coëfficiens terminorum, quos quaerimus, non leuiter affici. Sequenti ergo modo calculum redintegro.

§. 187. In hunc finem quoque rationem habemus angulorum $3\eta-s$ et $3\eta+s$, sitque

$$\mathfrak{R}dr = \mathfrak{A} \cos 2\eta + \mathfrak{P} e \cos s + \mathfrak{Q} e \cos (2\eta-s)$$

$$+ \mathfrak{R} e \cos (2\eta+s)$$

$$+ \mathfrak{F} v \cos \eta + \mathfrak{K} v e \cos (\eta-s) + \mathfrak{M} v e \cos (3\eta-s)$$

$$+ \mathfrak{G} v \cos \eta + \mathfrak{L} v e \cos (\eta+s) + \mathfrak{N} v e \cos (3\eta+s)$$

$$=$$

$$\frac{\mathfrak{G}+3\mathfrak{G}}{nn}\mathfrak{R}$$

$$-\frac{1}{2}\mathfrak{G}'\mathfrak{Q}'$$

$$\frac{\mathfrak{A}+3\mathfrak{A}}{nn}\mathfrak{K}$$

$$\frac{\mathfrak{G}+3\mathfrak{G}}{nn}\mathfrak{Q}$$

em ma-
quem
de is a
tionem
iudico,
oduce-
norum,
i ergo

habea-

$$(2\eta-s)$$

$$2\eta+s)$$

$$\eta-s)$$

$$\eta+s)$$

$$=$$

$$\varphi = \mathfrak{A} \cos 2\eta + \mathfrak{P} e \cos s + \mathfrak{Q} e \cos (2\eta-s)$$

$$+ \mathfrak{R} e \cos (2\eta+s)$$

$$+ \mathfrak{F} v \cos \eta + \mathfrak{K} v e \cos (\eta-s) + \mathfrak{M} v e \cos (3\eta-s)$$

$$+ \mathfrak{G} v \cos \eta + \mathfrak{L} v e \cos (\eta+s) + \mathfrak{N} v e \cos (3\eta+s)$$

§. 188. Quodsi iam ponamus:

$$\frac{2\mathfrak{K}+\mathfrak{R}}{nn} = \mathfrak{K}' ; \frac{2\mathfrak{L}+\mathfrak{L}}{nn} = \mathfrak{L}'$$

$$\frac{2\mathfrak{M}+\mathfrak{M}}{nn} = \mathfrak{M}' ; \frac{2\mathfrak{N}+\mathfrak{N}}{nn} = \mathfrak{N}'$$

erit

$$\frac{d\varphi}{dr} = \text{Praec.} - \mathfrak{A}' \cos 2\eta - \mathfrak{P}' e \cos s - \mathfrak{Q}' e \cos (2\eta-s)$$

$$- \mathfrak{R}' e \cos (2\eta+s)$$

$$- \mathfrak{F}' v \cos \eta - \mathfrak{K}' v e \cos (\eta-s) - \mathfrak{M}' v e \cos (3\eta-s)$$

$$- \mathfrak{G}' v \cos \eta - \mathfrak{L}' v e \cos (\eta+s) - \mathfrak{N}' v e \cos (3\eta+s)$$

$$\text{atque ob } \frac{ds}{dr} = \frac{1}{n} - \frac{2}{n} e \cos s \text{ erit}$$

$$\frac{d\eta}{dr} = \alpha - \mathfrak{A}' \cos 2\eta + (\frac{2}{n} - \mathfrak{P}') e \cos s - \mathfrak{Q}' e \cos (2\eta-s)$$

$$- \mathfrak{R}' e \cos (2\eta+s)$$

$$- \mathfrak{F}' v \cos \eta - \mathfrak{K}' v e \cos (\eta-s) - \mathfrak{M}' v e \cos (3\eta-s)$$

$$- \mathfrak{G}' v \cos \eta - \mathfrak{L}' v e \cos (\eta+s) - \mathfrak{N}' v e \cos (3\eta+s)$$

§. 189. Formulas nunc assumtas differentiemus, solosque terminos, quibus opus habemus, in calculo exprimamus ac reperiemus:

$$\mathfrak{R} = \text{Praec.}$$

$$+ v \sin (\eta-s) (\mathfrak{A}' - \mathfrak{A} m' + \mathfrak{Q}' \mathfrak{P}' - \mathfrak{M}' \mathfrak{G}' - \frac{1}{2} \mathfrak{G}' (\frac{2}{n} - \mathfrak{P}') - \frac{1}{2} \mathfrak{G}' \mathfrak{Q}' + \frac{1}{2} \mathfrak{G}' \mathfrak{R} (\alpha - \frac{1}{n}))$$

$$+ \frac{1}{2} \mathfrak{L}' \mathfrak{A}' + \frac{1}{2} \mathfrak{M}' \mathfrak{A}'$$

$$X 2$$

$$+ve\sin(\eta+s)\left\{A'k' - A'n' + R'f' - Q'g' - \frac{1}{2}F'\left(\frac{2}{n} - p'\right) + \frac{1}{2}F'q' - \frac{1}{2}R'a' + \frac{1}{2}M'a'\right. \\ \left. - \frac{1}{2}K'a' + \frac{1}{2}N'a'\right\}$$

$$+ve\sin(3\eta-s)\left\{A'k' + Q'f' + \frac{1}{2}F'q' - \frac{1}{2}G'\left(\frac{2}{n} - p'\right) + \frac{1}{2}K'a' - M'\left(3a - \frac{1}{n}\right)\right\}$$

$$+ve\sin(3\eta-s)\left\{A'n' + R'f' + \frac{1}{2}F'q' - \frac{1}{2}G'\left(\frac{2}{n} - p'\right) + \frac{1}{2}K'a' - M'\left(3a + \frac{1}{n}\right)\right\}$$

Ac si breuitatis gratia ponamus:

$$K' = \left(a - \frac{1}{n}\right)K - A(k' - n') - Q'f' + R'g' + \frac{1}{2}F'\left(\frac{2}{n} - p'\right) + \frac{1}{2}F'q' - \frac{1}{2}G'a' \\ + \frac{1}{2}L'a' - \frac{1}{2}M'a'$$

$$L' = \left(a + \frac{1}{n}\right)L - A(k' + n') - R'f' + Q'g' + \frac{1}{2}F'\left(\frac{2}{n} - p'\right) + \frac{1}{2}F'q' - \frac{1}{2}G'a' \\ + \frac{1}{2}K'a' - \frac{1}{2}N'a'$$

$$M' = \left(3a - \frac{1}{n}\right)M - A k' - Q'f' - \frac{1}{2}F'q' + \frac{1}{2}G'\left(\frac{2}{n} - p'\right) - \frac{1}{2}K'a'$$

$$N' = \left(3a + \frac{1}{n}\right)N - A n' - R'f' - \frac{1}{2}F'q' + \frac{1}{2}G'\left(\frac{2}{n} - p'\right) - \frac{1}{2}L'a'$$

erit

$$\frac{dv}{dr} = -A' \sin 2\eta - P'e \sin s - Q'e \sin(2\eta - s)$$

$$- F'e \sin \eta - K'v e \sin(\eta - s) - M'v e \sin(3\eta - s)$$

$$- G'e \sin 3\eta - L'v e \sin(\eta + s) - N'v e \sin(3\eta + s)$$

§. 190. Hinc iam denuo differentiando nanciscemur

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve\cos(\eta-s)\left\{A'k' + A'n' + Q'f' + R'g' - \frac{1}{2}F'\left(\frac{2}{n} - p'\right) + \frac{1}{2}F'q' \right. \\ \left. + \frac{1}{2}G'q' + \frac{1}{2}L'a' + \frac{1}{2}M'a' - \left(a - \frac{1}{n}\right)K' \right. \\ \left. + \right.$$

+

$$+ve\cos(\eta+s)\left\{A'k' + A'n' + R'f' + Q'g' - \frac{1}{2}F'\left(\frac{2}{n} - p'\right) + \frac{1}{2}F'q' \right. \\ \left. + \frac{1}{2}G'q' + \frac{1}{2}K'a' + \frac{1}{2}N'a' - \left(a + \frac{1}{n}\right)L' \right.$$

$$+ve\cos(3\eta-s)\left\{A'k' + Q'f' + \frac{1}{2}F'q' - \frac{1}{2}G'\left(\frac{2}{n} - p'\right) + \frac{1}{2}K'a' \right. \\ \left. - \left(3a - \frac{1}{n}\right)M' \right.$$

$$+ve\cos(3\eta+s)\left\{A'k' + R'f' + \frac{1}{2}F'q' - \frac{1}{2}G'\left(\frac{2}{n} - p'\right) + \frac{1}{2}L'a' \right. \\ \left. - \left(3a + \frac{1}{n}\right)N' \right.$$

§. 191. Quodsi autem valores iam inuenti substituantur, habebitur

R = Praec.

$$+ve\sin(\eta-s)\left\{-0,85830R + 0,00526L - 0,02500M + 0,37592\right\} \\ - 0,00931L + 0,00931M$$

$$+ve\sin(\eta+s)\left\{-1,00918L + 0,00526R - 0,02500M + 0,34115\right\} \\ - 0,00931K + 0,00931N$$

$$+ve\sin(3\eta-s)\left\{-2,72578M - 0,01448R + 0,49223\right\} \\ - 0,00931K$$

$$+ve\sin(3\eta+s)\left\{-2,87666M - 0,01448L + 0,47116\right\} \\ - 0,00931L$$

X 3

K' =

$$\begin{aligned}
 K' &= 0,85830K + 0,00527L + 0,01447M + 1,48970 \\
 &\quad + 0,00750\mathfrak{L} - 0,00750\mathfrak{M} \\
 L' &= 1,00916L + 0,00527K + 0,01447N + 1,55183 \\
 &\quad + 0,00750\mathfrak{K} - 0,00750\mathfrak{N} \\
 M' &= 2,72528M + 0,02501K - 1,17408 \\
 &\quad + 0,00750\mathfrak{K} \\
 N' &= 2,87666N + 0,02501L - 0,95901 \\
 &\quad + 0,00750\mathfrak{L}
 \end{aligned}$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+vecl(\eta-s) \left\{ \begin{aligned} &-0,85830K' - 0,02843L - 0,02843M - 0,32674 \\ &-0,00987L' - 0,01409\mathfrak{L} - 0,01409\mathfrak{M} \\ &-0,02961M' \end{aligned} \right\}$$

$$+vecl(\eta+s) \left\{ \begin{aligned} &-1,00918L' - 0,02843K - 0,02843N - 0,56620 \\ &-0,00987K' - 0,01409\mathfrak{K} - 0,01409\mathfrak{N} \\ &-0,02981N' \end{aligned} \right\}$$

$$+vecl(3\eta-s) \left\{ \begin{aligned} &-2,72578M' - 0,02843K + 0,74683 \\ &-0,00987K' - 0,01409\mathfrak{K} \end{aligned} \right\}$$

$$+vecl(3\eta+s) \left\{ \begin{aligned} &-2,87661N' - 0,02853L + 0,67486 \\ &-0,00987L' - 0,01409\mathfrak{L} \end{aligned} \right\}$$

§. 192. Valores autem litterarum commate notarum hic substitutae dabunt

$$\frac{ddv}{dr^2} =$$

$$vecl(\eta-s) \left\{ \begin{aligned} &-0,73747K - 0,04264L - 0,12156M - 0,00014N \\ &-0,00029\mathfrak{K} - 0,02053\mathfrak{L} - 0,00765\mathfrak{M} + 0,00008\mathfrak{N} \\ &-1,58590 \end{aligned} \right\}$$

ve

18970

5183

574

520

1024

1N

1N

ve

$$vecl(\eta+s) \left\{ \begin{aligned} &-1,01923L - 0,04222K - 0,12821N - 0,00014M \\ &-0,00029\mathfrak{L} - 0,02166\mathfrak{K} - 0,00642\mathfrak{M} + 0,00008\mathfrak{N} \\ &-2,11858 \end{aligned} \right\}$$

$$vecl(3\eta-s) \left\{ \begin{aligned} &-7,43058M - 0,09507K - 0,00005L + 3,93257 \\ &+0,00008\mathfrak{M} - 0,03453\mathfrak{K} - 0,00007\mathfrak{L} \end{aligned} \right\}$$

$$vecl(3\eta+s) \left\{ \begin{aligned} &-8,27529N - 0,11338L - 0,00005K + 3,41830 \\ &+0,00008\mathfrak{N} - 0,03566\mathfrak{L} - 0,00007\mathfrak{K} \end{aligned} \right\}$$

§. 193. His expressionibus ita euolutis atque ad calculum numericum praeparatis, quaeramus easdem expressiones ex formulis supra traditis pro R et $\frac{ddv}{dr^2}$, quae continentur in §. 52 et 54. Inde autem omitendis terminis, quos iam tractauimus, consequemur.

$$R = \text{Pr.} + vefin(\eta-s) \left(-\frac{1}{2} + \frac{3L}{2nn} - \frac{3M}{2nn} - \frac{9F}{4nn} + \frac{9G}{4nn} \right)$$

$$+vefin(\eta+s) \left(-\frac{1}{2} + \frac{3K}{2nn} - \frac{3N}{2nn} + \frac{9G}{4nn} - \frac{9F}{4nn} \right)$$

$$+vefin(3\eta-s) \left(-\frac{1}{2} + \frac{3K}{2nn} - \frac{9F}{4nn} \right)$$

$$+vefin(3\eta+s) \left(-\frac{1}{2} + \frac{3L}{2nn} - \frac{9F}{4nn} \right)$$

ddv

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve \cos(\eta-s) \left[-\frac{1}{2} + \frac{9P}{8nn} + \frac{9Q}{8nn} + \frac{15R}{8nn} - 6K + \frac{3L}{4nn} + \frac{3M}{4nn} \right. \\ \left. - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2K + \frac{(X+3A)}{nn} (R+M) \right. \\ \left. + \frac{(3X+3A)}{nn} (L+M) + \frac{(P+3P)}{nn} P + \frac{(3P+3P)}{nn} F \right. \\ \left. + \frac{(Q+3Q)}{nn} P + \frac{(3Q+3Q)}{nn} F + \frac{(R+3R)}{nn} G \right. \\ \left. + \frac{(3R+3R)}{nn} G \right]$$

$$+ve \cos(\eta+s) \left[-\frac{1}{2} + \frac{9P}{8nn} + \frac{9R}{8nn} + \frac{15Q}{8nn} - 6L + \frac{3K}{4nn} + \frac{3N}{4nn} \right. \\ \left. - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2K + \frac{(X+3A)}{nn} (R+N) \right. \\ \left. + \frac{(3X+3A)}{nn} (K+N) + \frac{(P+3P)}{nn} F + \frac{(3P+3P)}{nn} F \right. \\ \left. + \frac{(Q+3Q)}{nn} G + \frac{(3Q+3Q)}{nn} G + \frac{(R+3R)}{nn} F \right. \\ \left. + \frac{(3R+3R)}{nn} F \right]$$

$$+ve \cos(3\eta-s) \left[-\frac{1}{2} + \frac{15P}{8nn} + \frac{9Q}{8nn} - 6M + \frac{3K}{4nn} - \frac{3G}{4nn} \right. \\ \left. - \frac{9F}{8nn} - 2K + \frac{(X+3A)}{nn} R + \frac{(3X+3A)}{nn} K + \frac{(P+3P)}{nn} G \right. \\ \left. + \frac{(3P+3P)}{nn} G + \frac{(Q+3Q)}{nn} F + \frac{(3Q+3Q)}{nn} F \right]$$

+ve

$$+ve \cos(3\eta+s) \left[-\frac{1}{2} + \frac{15P}{8nn} + \frac{9R}{8nn} - 6N + \frac{3L}{4nn} + \frac{3G}{4nn} + \frac{9F}{8nn} - 2K \right. \\ \left. + \frac{(X+3A)}{nn} R + \frac{(3X+3A)}{nn} L + \frac{(P+3P)}{nn} G + \frac{(3P+3P)}{nn} G \right. \\ \left. + \frac{(R+3R)}{nn} F + \frac{(3R+3R)}{nn} F \right]$$

§. 194. Introducantur hic quoque valores iam
cogniti, ac prodibit

$$R = \text{Pr.} +ve \sin(\eta-s) [-1,02394 + 0,00854L - 0,00854M] \\ +ve \sin(\eta+s) [-1,02394 + 0,00854K - 0,00854N] \\ +ve \sin(3\eta-s) [-4,01450 + 0,00854K] \\ +ve \sin(3\eta+s) [-4,01450 + 0,00854L]$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve \cos(\eta-s) \left[-1,01591K - 0,03207L - 0,03207M - 1,168671 \right] \\ \left[-2,01798R - 0,02711L - 0,02711M \right]$$

$$+ve \cos(\eta+s) \left[-1,01591L - 0,03207K - 0,03207N - 1,93904 \right] \\ \left[-2,01798R - 0,02711R - 0,02711R \right]$$

$$+ve \cos(3\eta-s) \left[-1,01591M - 0,03207K - 2,49545 \right] \\ \left[-2,01798M - 0,02711R \right]$$

$$+ve \cos(3\eta+s) \left[-1,01591N - 0,03207L - 2,78450 \right] \\ \left[-2,01798R - 0,02711R \right]$$

Y

§. 194.

§. 195. Hinc ergo octo sequentes aequationes resultabunt

$$I. 0,85830 \mathfrak{R} = + 0,00526 \mathfrak{L} - 0,02500 \mathfrak{M} + 1,39986 \\ - 0,01785 \mathfrak{L} + 0,01785 \mathfrak{M}$$

$$II. 1,00918 \mathfrak{L} = + 0,00526 \mathfrak{R} - 0,02500 \mathfrak{N} + 1,36509 \\ - 0,01785 \mathfrak{K} + 0,01785 \mathfrak{N}$$

$$III. 2,72578 \mathfrak{M} = - 0,01448 \mathfrak{R} + 4,50673 \\ - 0,01785 \mathfrak{K}$$

$$IV. 2,87666 \mathfrak{N} = - 0,01448 \mathfrak{L} + 4,48566 \\ - 0,01785 \mathfrak{L}$$

$$V. +0,27844 \mathfrak{K} - 0,01057 \mathfrak{L} - 0,08949 \mathfrak{M} - 0,00014 \mathfrak{N} + 0,10081 = \\ + 2,01769 \mathfrak{R} + 0,00658 \mathfrak{L} - 0,01946 \mathfrak{M} + 0,00008 \mathfrak{N}$$

$$VI. -0,00332 \mathfrak{L} - 0,01015 \mathfrak{K} - 0,09614 \mathfrak{N} - 0,00014 \mathfrak{M} - 0,17954 = \\ + 2,01769 \mathfrak{L} + 0,00545 \mathfrak{R} + 0,02069 \mathfrak{M} + 0,00008 \mathfrak{N}$$

$$VII. -6,41487 \mathfrak{M} - 0,06300 \mathfrak{K} - 0,00005 \mathfrak{L} + 6,42802 = \\ + 2,01806 \mathfrak{M} - 0,00742 \mathfrak{R} - 0,00007 \mathfrak{L}$$

$$VIII. -7,25938 \mathfrak{N} - 0,08131 \mathfrak{L} - 0,00005 \mathfrak{K} + 6,20280 = \\ + 2,01806 \mathfrak{N} - 0,00855 \mathfrak{L} - 0,00007 \mathfrak{R}$$

§. 196. Ex aequationibus III et IV statim efficiuntur hi valores

$$\mathfrak{M} = - 0,00531 \mathfrak{R} - 0,00655 \mathfrak{K} + 1,65336$$

$$\mathfrak{N} = - 0,00503 \mathfrak{L} - 0,00621 \mathfrak{L} + 1,55933$$

qui

equationes

1,39986

1,36509

0081=

7954=

102=

80=

efficiuntur

qui

qui in I et II substituti praebent :

$$0,85817 \mathfrak{R} = 1,35853 + 0,00526 \mathfrak{L} - 0,01785 (\mathfrak{L} - \mathfrak{M}) + 0,00016 \mathfrak{K}$$

$$1,00905 \mathfrak{L} = 1,32611 + 0,00526 \mathfrak{R} - 0,01785 (\mathfrak{K} - \mathfrak{N}) + 0,00015 \mathfrak{L}$$

unde obtinetur :

$$\mathfrak{R} = + 1,59116 - 0,02080 (\mathfrak{L} - \mathfrak{M}) + 0,00008 \mathfrak{K} + 0,00010 \mathfrak{N}$$

$$\mathfrak{L} = + 1,32251 - 0,01769 (\mathfrak{K} - \mathfrak{N}) + 0,00005 \mathfrak{L} + 0,00011 \mathfrak{M}$$

$$\mathfrak{M} = + 1,64491 - 0,00655 \mathfrak{K} + 0,00010 (\mathfrak{L} - \mathfrak{M})$$

$$\mathfrak{N} = + 1,55268 - 0,00621 \mathfrak{L} + 0,00009 (\mathfrak{K} - \mathfrak{N})$$

§. 197. His valoribus substitutis caeterae aequationes abibunt in formas sequentes :

$$0,27862 \mathfrak{K} - 0,05252 \mathfrak{L} - 0,04754 \mathfrak{M} + 0,00017 \mathfrak{N} + 3,35313 =$$

$$- 0,00346 \mathfrak{L} - 0,04584 \mathfrak{K} - 0,06045 \mathfrak{N} + 0,00020 \mathfrak{M} + 2,53002 =$$

$$- 6,41502 \mathfrak{M} - 0,07622 \mathfrak{K} + 0,00030 \mathfrak{L} + 9,73587 =$$

$$- 7,25971 \mathfrak{N} - 0,09384 \mathfrak{L} + 0,00028 \mathfrak{K} + 9,32499 =$$

ex quarum binis postremis statim obtinetur :

$$\mathfrak{M} = - 0,01188 \mathfrak{K} + 0,00005 \mathfrak{L} + 1,51765$$

$$\mathfrak{N} = - 0,01293 \mathfrak{L} + 0,00004 \mathfrak{K} + 1,28448$$

unde colligitur :

$$+ 0,27918 \mathfrak{K} - 0,05252 \mathfrak{L} + 3,28120 =$$

$$- 0,00268 \mathfrak{L} - 0,04584 \mathfrak{K} + 2,45267 =$$

ac denique

$$\mathfrak{K} = + 49,44710 \dots / \mathfrak{K} = 1,606887$$

$$\mathfrak{L} = + 368,40200 \dots / \mathfrak{L} = 2,566322$$

$$\mathfrak{M} = + 1,05555 \dots / \mathfrak{M} = 0,023478$$

$$\mathfrak{N} = - 3,47730 \dots / \mathfrak{N} = 0,541242$$

Y 2

§. 197.

§. 198. Litterarum germanicarum valores hinc erunt:

$$\begin{aligned} R &= -5,04677 & \dots & \dots & I-R &= 0,703013 \\ R &= +0,56302 & \dots & \dots & I-R &= 9,750524 \\ M &= +1,41672 & \dots & \dots & I-M &= 0,151283 \\ N &= -0,73119 & \dots & \dots & I-N &= 9,864030 \end{aligned}$$

ac litterarum hinc derivatarum:

$$\begin{aligned} K &= +0,43578 & \dots & \dots & I-K &= 9,639267 \\ L &= +4,23401 & \dots & \dots & I-L &= 0,626752 \\ M &= +0,02018 & \dots & \dots & I-M &= 8,304921 \\ N &= -0,04409 & \dots & \dots & I-N &= 8,644340 \end{aligned}$$

§. 199. Nunc igitur intelligimus inaequalitates ab angulis $3\eta - s$ et $3\eta + s$ pendentes tam esse paruas, vt sine vilo errore reici queant, etiamsi valores K et L aliquantum immutauerint. Distantia ergo lunae curvata a terra $x = \frac{(1-kk) au}{1-k \cos r}$ ita ab his inaequalitatibus parallacticis pendebit, vt sit

	Log. coeff.
$x = \text{Praec.} + 0,11756 \vee \cos \eta$	9,070249
$- 0,00419 \vee \cos 3\eta$	7,622540
$- 0,1234 \vee k \cos (\eta - r)$	9,091265
$- 0,0784 \vee k \cos (\eta + r)$	8,894183
$+ 0,2302 \vee \cos (\eta - s)$	9,362071
$+ 2,0965 \vee \cos (r + s)$	0,321506

Motus

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Motus autem momentaneus ita hinc afficietur, vt sit

$\frac{d\phi}{dt} = \text{Praec.} - 0,23259 \vee \cos \eta$	9,366591
$+ 0,01309 \vee \cos 3\eta$	8,116940
$+ 0,0939 \vee k \cos (\eta - r)$	8,972943
$+ 0,1653 \vee k \cos (\eta + r)$	9,218352
$- 0,4358 \vee \cos (\eta - s)$	9,639267
$- 4,2340 \vee \cos (r + s)$	0,626752

§. 200. Quodsi iam ipsam longitudinem lunae, quatenus ab his inaequalitatibus parallacticis pendet, ponamus:

$$\begin{aligned} \phi = \text{Praec.} &+ \mathcal{F}' \vee \sin \eta + \mathcal{H}' \vee k \sin (\eta - r) + \mathcal{K}' \vee \sin (\eta - s) \\ &+ \mathcal{G}' \vee \sin 3\eta + \mathcal{J}' \vee k \sin (\eta + r) + \mathcal{L}' \vee \sin (\eta + s) \end{aligned}$$

sequentes obtinebimus aequationes pro horum coefficientium determinatione:

$$\begin{aligned} -0,23259 &= a \mathcal{F}' - \mathcal{H}' \mathcal{F}' - \mathcal{K}' \mathcal{G}' - \frac{1}{2} \mathcal{J}' \mathcal{L}' - \frac{1}{2} \mathcal{G}' \mathcal{L}' \\ + 0,01303 &= 3a \mathcal{G}' - \mathcal{H}' \mathcal{F}' - \frac{1}{2} \mathcal{F}' \mathcal{L}' \\ + 0,0939 &= (a-1) \mathcal{H}' - \mathcal{H}' \mathcal{L}' - \frac{1}{2} \mathcal{J}' \mathcal{L}' - \mathcal{D}' \mathcal{F}' - \mathcal{G}' \mathcal{G}' \\ &\quad - \frac{1}{2} \mathcal{F}' \mathcal{L}' - \frac{1}{2} \mathcal{F}' \mathcal{L}' - \frac{1}{2} \mathcal{G}' \mathcal{L}' \\ + 0,1653 &= (a+1) \mathcal{J}' - \mathcal{H}' \mathcal{L}' - \frac{1}{2} \mathcal{H}' \mathcal{L}' - \mathcal{G}' \mathcal{F}' - \mathcal{D}' \mathcal{G}' \\ &\quad - \frac{1}{2} \mathcal{F}' \mathcal{L}' - \frac{1}{2} \mathcal{F}' \mathcal{L}' - \frac{1}{2} \mathcal{G}' \mathcal{L}' \end{aligned}$$

Y 3

$$\begin{aligned}
 -0,4353 &= (4 - \frac{1}{p}) R' - 2' H' - \frac{1}{2} 2' A' - \Omega' P' - R' G' \\
 &\quad + \frac{1}{2} 3' (\frac{2}{p} - p') - \frac{1}{2} 3' q' - \frac{1}{2} 3' r' \\
 -4,2340 &= (4 + \frac{1}{p}) 2' - 2' H' - \frac{1}{2} R' A' - R' P' - \Omega' G' \\
 &\quad + \frac{1}{2} 3' (\frac{2}{p} - p') - \frac{1}{2} 3' r' - \frac{1}{2} 3' q'
 \end{aligned}$$

§. 201. Valoribus autem iam cognitis hic substitutis, aequationes istae in sequentes abibunt formas;

$$\begin{aligned}
 -0,23031 &= +0,94361 3' + 0,02961 3' \\
 +0,01550 &= +2,80122 3' + 0,00987 3' \\
 -0,0056 &= -0,06626 3' + 0,00987 3' - 0,25665 3' + 0,01926 3' \\
 +0,1710 &= +1,93374 3' + 0,00987 3' - 0,06717 3' - 0,54918 3' \\
 -0,3968 &= +0,85830 R' + 0,00987 2' + 0,06357 3' - 0,04509 3' \\
 -4,2330 &= +1,00918 2' + 0,00987 R' + 0,06729 3' - 0,05625 3'
 \end{aligned}$$

unde colligitur fore

$$\begin{aligned}
 3' &= -0,24427 \quad . \quad . \quad . \quad 1-3' = 9,387868 \\
 3' &= +0,00639 \quad . \quad . \quad . \quad 1-3' = 7,805991 \\
 3' &= +1,1959 \quad . \quad . \quad . \quad 1-3' = 0,077694 \\
 3' &= +0,0757 \quad . \quad . \quad . \quad 1-3' = 8,879096 \\
 R' &= -0,3959 \quad . \quad . \quad . \quad 1-R' = 9,597508 \\
 2' &= -4,1738 \quad . \quad . \quad . \quad 1-2' = 0,620530
 \end{aligned}$$

§. 202.

§. 202. Hinc ergo habebimus sequentes partes pro longitudine Lunae, quas simul ope valorum proxime cognitorum pro v, k, e ad minuta secunda reducamus:

	Log. coeff.	Val. coeff. in min. sec.
$\Phi = \text{Præc.} - 0,24427 \ v \sin \eta$	9,387868	175"
$+ 0,00639 \ v \sin 3\eta$	7,805991	4"
$+ 1,1959 \ vk \sin (\eta - r)$	0,077694	59"
$+ 0,0757 \ vk \sin (\eta + r)$	8,879096	4"
$- 0,3959 \ ve \sin (\eta - s)$	9,597508	5"
$- 4,1738 \ ve \sin (\eta + s)$	0,620530	49"

Sicque omnes iam adepti sumus motus lunae inaequalitates, quae quidem ab inclinatione eius orbitae ad eclipticam non pendent. Interim tamen non diffiteor, dari aliquas insuper inaequalitates, quae alicuius forte sint momenti, quas in hac investigatione praeterimus, cuiusmodi sunt eae, quae ab angulis $2\eta - 3r$ et $2\eta - 2r + s$ pendent, quae ad plura minuta secunda assurgere posse videntur. Verum earum determinatio tam est caediosa, ut malim eam observationibus relinquere.

§. 203. Quae ergo hactenus inuenimus, in unum colligamus ac primo pro distantia lunae a terra curvata

$$s = \frac{(1-kk') \Delta u}{1-k \cos r} \text{ erit}$$

	Log. coeff.	coeff. integri
$\pi = 1$	0,0074991	0,007499
$+ 0,0000532 \cos 4\eta$	5,725912	+ 0,000053
$+ 0,191557k \cos(2\eta - r)$	9,282297	+ 0,010430
$- 0,003293k \cos(2\eta + r)$	7,517525	- 0,000179
$- 0,003321k \cos(4\eta - r)$	7,521296	- 0,000181
$+ 0,000049k \cos(4\eta + r)$	5,692584	+ 0,000005
$- 0,00511kk \cos 2r$	7,708601	- 0,000015
$- 0,08022kk \cos(2\eta - 2r)$	8,904280	- 0,000238
$- 0,00237kk \cos(2\eta + 2r)$	7,373072	- 0,000007
$+ 0,07892kk \cos(4\eta - 2r)$	8,897172	+ 0,000234
$+ 0,00001kk \cos(4\eta + 2r)$	4,872349	+ 0,000000
$- 0,006400e \cos s$	7,806180	- 0,000206
$+ 0,014801e \cos(2\eta - s)$	8,170303	+ 0,000249
$+ 0,011415e \cos(2\eta + s)$	8,057492	+ 0,000192
$+ 0,00364ee \cos 2s$	7,539670	+ 0,000001
$- 0,01482ee \cos(2\eta - 2s)$	8,170799	- 0,000004
$- 0,00584ee \cos(2\eta + 2s)$	7,766856	- 0,000001
$- 0,37957ek \cos(r - s)$	9,579297	- 0,000347
$+ 0,30693ek \cos(r + s)$	9,487039	+ 0,000281
$- 0,36337ek \cos(2\eta - r + s)$	9,560344	- 0,000332
$+ 0,00993ek \cos(2\eta + r - s)$	7,997004	+ 0,000009
$- 0,07752ek \cos(2\eta - r - s)$	8,889425	- 0,000071
$+ 0,00305ek \cos(2\eta + r + s)$	7,484024	+ 0,000003
$+ 0,11756v \cos \eta$	9,070249	+ 0,000408
$- 0,00419v \cos 3\eta$	7,622540	- 0,000015
$- 0,1234vk \cos(\eta - r)$	9,091265	- 0,000024
$- 0,0784vk \cos(\eta + r)$	8,894183	- 0,000015
$+ 0,2302ve \cos(\eta - s)$	9,362071	+ 0,000013
$+ 2,0965ve \cos(\eta + s)$	0,321506	+ 0,000122

Hic ad latus adiunxi valores coefficientium integrorum in numeris absolutis expressos, ponendo $k = 0,05445$, $e = 0,01680$ et $v = \frac{1}{11}$; quos proinde, si hi valores alter per observationes determinentur, facile erit emendare.

§. 204.

§. 204. Pro motu autem lunae momentaneo, ex quo eius motus horarius definiri poterit, habebimus:

	Log. coeff.	Coeff. integri
$\frac{d\phi}{dr} = 1,009176$	0,003967	+ 1,009176
$+ 0,0195144 \cos 2\eta$	8,290355	+ 0,019514
$- 0,0000322 \cos 4\eta$	5,507856	- 0,000032
$- 0,001231k \cos r$	7,090258	- 0,000067
$- 0,366103k \cos(2\eta - r)$	9,563604	- 0,019934
$+ 0,012832k \cos(2\eta + r)$	8,108292	+ 0,000699
$+ 0,002829k \cos(4\eta - r)$	7,451633	+ 0,000154
$- 0,000171k \cos(4\eta + r)$	6,232305	- 0,000009
$+ 0,01182kk \cos 2r$	8,072618	+ 0,000035
$- 0,02057kk \cos(2\eta - 2r)$	8,313172	- 0,000061
$+ 0,01063kk \cos(2\eta + 2r)$	8,026598	+ 0,000032
$- 0,09883kk \cos(4\eta - 2r)$	8,994889	- 0,000293
$- 0,00004kk \cos(4\eta + 2r)$	5,592770	- 0,000000
$+ 0,013760e \cos s$	8,138618	+ 0,000231
$- 0,037487e \cos(2\eta - s)$	8,573878	- 0,000630
$- 0,030062e \cos(2\eta + s)$	8,478023	- 0,000505
$- 0,00722ee \cos 2s$	7,858166	- 0,000002
$+ 0,03470ee \cos(2\eta - 2s)$	8,540319	+ 0,000010
$+ 0,01533ee \cos(2\eta + 2s)$	8,185614	+ 0,000005
$+ 0,75204ek \cos(r - s)$	9,876241	+ 0,000688
$- 0,62626ek \cos(r + s)$	9,796755	- 0,000573
$+ 0,69420ek \cos(2\eta - r + s)$	9,841484	+ 0,000635
$- 0,02440ek \cos(2\eta + r - s)$	8,387390	- 0,000022
$+ 0,12160ek \cos(2\eta - r - s)$	9,084933	+ 0,000111
$- 0,02000ek \cos(2\eta + r + s)$	9,301030	- 0,000018
$- 0,23259v \cos \eta$	9,366591	- 0,000808
$+ 0,01309v \cos 3\eta$	8,116940	+ 0,000045
$+ 0,0939vk \cos(\eta - r)$	8,972943	+ 0,000018
$+ 0,1653vk \cos(\eta + r)$	9,218352	+ 0,000031
$- 0,4358ve \cos(\eta - s)$	9,639267	- 0,000025
$- 4,2340ve \cos(\eta + s)$	0,626753	- 0,000247

Z

§. 105.

§. 205. Si iam longitudo Lunae per solam excentricitatem secundum regulas Keplerianas determinata ponatur $=\zeta$, ita ut posita eius anomalia vera $=r$, futurum sit $\zeta = C + 1,0085272 r$, erit longitudo vera per haec inuentas inaequalitates.

	Log. coeff.	Val. coeff. in min. sec.
$\Phi = \zeta + 0,0103597 \sin 2 \eta$	8,015347	+2137 ¹¹
$- 0,0000382 \sin 4 \eta$	5,582063	— 8
$+ 0,010146k \sin r$	8,006295	+114
$- 0,420226k \sin (2\eta - r)$	9,623483	-4720
$+ 0,004992k \sin (2\eta + r)$	7,698261	+56
$+ 0,005286k \sin (4\eta - r)$	7,723163	+59
$- 0,000086k \sin (4\eta + r)$	5,935307	— 1
$+ 0,00420kk \sin 2r$	7,623250	+2 $\frac{1}{2}$
$+ 0,57328kk \sin (2\eta - 2r)$	9,758367	+351
$+ 0,00318kk \sin (2\eta + 2r)$	7,402427	+1 $\frac{1}{2}$
$- 0,15083kk \sin (4\eta - 2r)$	9,178488	— 92
$- 0,00002kk \sin (4\eta + 2r)$	5,301030	— 0
$+ 0,20138s \sin s$	9,304026	+701
$- 0,021889s \sin (2\eta - s)$	8,340237	— 76
$- 0,016368s \sin (2\eta + s)$	8,214002	— 57
$+ 0,00615se \sin 2s$	8,820508	+4
$+ 0,02332se \sin (2\eta - 2s)$	8,367825	+1
$+ 0,00840se \sin (2\eta + 2s)$	7,924429	+ $\frac{1}{2}$
$+ 0,74760ek \sin (r - s)$	9,873165	+141
$- 0,61850ek \sin (r + s)$	9,791317	—118
$+ 0,81430ek \sin (2\eta - r + s)$	9,910800	+154
$- 0,01420ek \sin (2\eta + r - s)$	8,150690	— 3
$+ 0,23960ek \sin (2\eta - r - s)$	9,379550	+45
$- 0,00610ek \sin (2\eta + r + s)$	7,788910	— 1
$- 0,24427v \sin \eta$	9,387868	—175
$+ 0,00639v \sin 3\eta$	7,805991	+4
$+ 1,1959vk \sin (\eta - r)$	0,077694	+59
$+ 0,0757vk \sin (\eta + r)$	8,879096	+4
$- 0,3939ve \sin (\eta - s)$	9,597508	— 5
$- 4,1738ve \sin (\eta + s)$	0,620530	— 49

CAPUT

II

tricitatur
 $\zeta = C$
 uentur
 l. coeff.
 min. sec.
 -2137¹¹
 8
 114
 4720
 56
 59
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 351
 1 $\frac{1}{2}$
 92
 0
 701
 76
 57
 4
 1
 $\frac{1}{2}$
 141
 118
 154
 3
 45
 1
 175
 4
 59
 4
 5
 49

CAPUT

CAPUT XII.

INVESTIGATIO INAEQUALITATUM MOTUM
LINEAE NODORUM AFFICIENTIUM.

§. 206.

Antequam reliquas motus Lunae inaequalitates, quae ab inclinatione eius orbitae ad eclipticam pendent, definire licet, cum variationes, quae in motu lineae nodorum Lunae, tum eas, quae in ipsa inclinatione eius orbitae ad eclipticam deprehenduntur, investigari oportet. Residua enim pars aequationis nostrae principalis, qua omnes motus Lunae inaequalitates continentur, litteras π et ϱ implicat, quarum illa longitudinem nodi ascendentis, haec vero ϱ inclinationem ad eclipticam designat. Nisi igitur utriusque huius quantitatis incrementa vel decrementa ad differentiale dr reduxerimus, residuas motus Lunae inaequalitates determinare non poterimus.

207. Aequatio autem supra (§5) pro motu lineae nodorum tradita, cum sit $\frac{2\pi v + f R dr}{nn} = u + \frac{1+2ee}{nn} - \frac{d\Phi}{dr}$ ideoque $\frac{2\pi v + f R dr}{nn} = a' \cos 2\eta - (\frac{2}{n} - c') k \cos r + d' k \cos (2\eta - r) + e' k \cos (2\eta + r) + p' e \cos e + q' e \cos (2\eta - s) + r' e \cos (2\eta + s)$

Z 2

Si

si ponamus breuitatis gratia $\frac{3(1+2kk+\frac{2}{3}cc)}{nnnn} = i$, ut sit $i =$
0,0168918, induet formam sequentem:

$$\frac{d\pi}{dr} = -i \left(n + a' \cos 2\eta - \left(\frac{2}{n} - c' \right) k \cos r + d' k \cos(2\eta - r) + \text{etc.} \right) \\ + e' k \cos(2\eta + r) \\ \left(1 + 4k \cos r + 5kk \cos 2r - 6ek \cos(r-s) \right) \left(\frac{1}{4} + \frac{1}{4} \cos 2\eta - \frac{1}{4} \cos(2\Phi - 2\pi) \right) \\ - 3r \cos r + \frac{1}{4} cc \cos 2s - 6ek \cos(r+s) \left(\frac{1}{4} + \frac{1}{4} \cos 2\eta - \frac{1}{4} \cos(2\theta - 2\pi) \right) \\ - \frac{xy}{4} \left(\frac{1}{4} \cos \eta + \frac{1}{4} \cos \eta - \frac{1}{4} \cos(\Phi + \theta - 2\pi) - \frac{1}{4} \cos(3\Phi - \theta - 2\pi) \right) \\ - \frac{1}{4} \cos(3\theta - \Phi - 2\pi) \right)$$

vbi quidem plurimi termini tam sunt parui, vt facile negligi queant.

§. 208. Productum autem ex duobus prioribus factoribus, quoniam id in formula pro inclinatione recurrit, seorsum exhibeamus: fiet id autem reiectis terminis, qui prae reliquis admodum sunt parui vt sequitur:

$$\begin{aligned} -xi + 2i \left(\frac{2}{n} - c' \right) kk - \frac{1}{2} ip' ee &= -0,017043 \\ \cos 2\eta \quad (-id' - 2id' kk) &+ 0,000161 \cos 2\eta \\ k \cos r \quad \left(i \left(\frac{2}{n} - c' \right) - 4ki \right) &- 0,068110 k \cos r \\ k \cos(2\eta - r) \quad (-id' - 2id' &- 0,005791 k \cos(2\eta - r) \\ k \cos(2\eta + r) \quad (-id' - 2id' &+ 0,000828 k \cos(2\eta + r) \\ kk \cos 2r \quad (-5ki + 2i \left(\frac{2}{n} - c' \right) &- 0,085091 kk \cos 2r \\ e \cos s \quad (-ip' + 3xi) &+ 0,051362 e \cos s \\ e \cos(2\eta - s) \quad (ig' + \frac{1}{2} id' &- 0,000929 e \cos(2\eta - s) \\ e \cos(2\eta + s) \quad (-id' + \frac{1}{2} id' &- 0,000804 e \cos(2\eta + s) \\ ee \cos 2s \quad (-\frac{1}{2} xi + \frac{1}{2} ip' &- 0,025914 ee \cos 2s \end{aligned}$$

§. 209.

§. 209. His valoribus substitutis prodibit

$$\begin{aligned} \frac{d\pi}{dr} &= -0,004261 + 0,000020 \\ &+ \cos 2\eta \quad (+0,000040 - 0,004261 \\ &+ k \cos r \quad (-0,017043 - 0,000620 \\ &+ k \cos(2\eta - r) \quad (-0,001448 - 0,008514) - 0,010636 k \cos(2\eta - 2r) \\ &+ k \cos(2\eta + r) \quad (+0,000207 - 0,008514) - 0,010636 k \cos(2\eta + 2r) \\ &+ kk \cos 2r \quad (-0,021273 \\ &+ e \cos s \quad (+0,012840 - 0,000217 \\ &+ e \cos(2\eta - s) \quad (-0,000232 + 0,006420) - 0,003239 e \cos(2\eta - 2s) \\ &+ e \cos(2\eta + s) \quad (-0,000201 + 0,006420) - 0,003239 e \cos(2\eta + 2s) \\ &+ ee \cos 2s \quad (-0,006479) \\ &+ \cos 2(\Phi - \pi) \quad (+0,003261 - 0,000020) - 0,000041 \cos \eta \\ &+ 0,000019 \cos(3\Phi - \theta - 2\pi) \\ &+ \cos 2(\theta - \pi) \quad (+0,004261 - 0,000020) - 0,000019 \cos 3\eta \\ &+ 0,000019 \cos(3\theta - \Phi - 2\pi) \\ &+ k \cos(2\Phi - 2\pi - r) \quad (+0,008514 + 0,000724) \\ &+ 0,000022 \cos(\Phi + \theta - 2\pi) \\ &+ k \cos(2\Phi - 2\pi + r) \quad (+0,008514 - 0,000103) \\ &+ 0,010636 k \cos 2(\Phi - \eta - \pi) \\ &+ k \cos(2\theta - 2\pi - r) \quad (+0,008514 - 0,000103) \\ &+ k \cos(2\theta - 2\pi + r) \quad (+0,008514 + 0,000724) \\ &+ e \cos(2\Phi - 2\pi - s) \quad (-0,006420 + 0,000116) \\ &+ e \cos(2\Phi - 2\pi + s) \quad (-0,006420 + 0,000100) \\ &+ e \cos(2\theta - 2\pi - s) \quad (-0,006420 + 0,000100) \\ &+ 0,003239 ee \cos 2(\theta - s - \pi) \\ &+ e \cos(2 - \theta - 2\pi + s) \quad (-0,006420 + 0,000116) \end{aligned}$$

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§. 110.

fit $i =$

+ etc.)

$\Phi - 2\pi$)

$\theta - 2\pi$)

$\cdot 2\pi$)

$\cdot 2\pi$)

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27 + r)

r

27 - s)

27 + s)

s

§. 209.

§. 210. Habebimus ergo

$$\begin{aligned}
\frac{d\pi}{dr} = & -0,004241 & -0,000041 \cos \eta \\
& -0,004221 \cos 2\eta & -0,000019 \cos 3\eta \\
& -0,017663 k \cos r & +0,000020 \cos 4\eta \\
& -0,009962 k \cos (2\eta - r) & +0,004241 \cos (2\Phi - 2\pi) \\
& -0,008307 k \cos (2\eta + r) & +0,004241 \cos (2\theta - 2\pi) \\
& -0,010636 kk \cos (2\eta - 2r) & +0,000022 \cos (\Phi + \theta - 2\pi) \\
& -0,010636 kk \cos (2\eta + 2r) & +0,000019 \cos (3\Phi - \theta - 2\pi) \\
& -0,021273 kk \cos r & +0,000019 \cos (3\theta - \Phi - 2\pi) \\
& +0,012623 e \cos s & \\
& +0,006188 e \cos (2\eta - s) & \\
& +0,006219 e \cos (2\eta + s) & \\
& -0,006479 ee \cos 2s & \\
& -0,003239 ee \cos (2\eta - 2s) & \\
& -0,003239 ee \cos (2\eta + 2s) & \\
& +0,009238 k \cos (2\Phi - 2\pi - r) & -0,006304 e \cos (2\Phi - 2\pi - s) \\
& +0,008411 k \cos (2\Phi - 2\pi + r) & -0,006320 e \cos (2\Phi - 2\pi + s) \\
& +0,008411 k \cos (2\theta - 2\pi - r) & -0,006320 e \cos (2\theta - 2\pi - s) \\
& +0,009238 k \cos (2\theta - 2\pi + r) & -0,006304 e \cos (2\theta - 2\pi + s) \\
& +0,010636 kk \cos (2\Phi - 2\pi - 2r) & +0,003239 ee \cos (2\theta - 2\pi - 2s)
\end{aligned}$$

§. 211. Quanquam plurimi horum terminorum tam sunt parui, ut in se spectati tuto reici possent, tamen quidam per integrationem ad magnitudinem satis notabilem excrecere possunt. Huius autem indolis sunt illi termini, qui eiusmodi complectuntur angulos, quorum

rum differentialia ad dr admodum parvam tenent rationem, cuiusmodi sunt anguli s , $2s$, $2\theta - 2\pi$, $2\theta - 2\pi - s$, $2\theta - 2\pi + s$, $2\Phi - 2\pi - 2r$ et $2\theta - 2\pi - 2s$; quorum natura differentialium ex sequentibus formulis colligi potest:

$$\begin{aligned}
\frac{d\eta}{dr} = & a - a' \cos 2\eta - c'k \cos r - d'k \cos (2\eta - r) - e'k \cos (2\eta + r) \\
& + \left(\frac{2}{n} - p'\right) e \cos s - q'e \cos (2\eta - s) - r'e \cos (2\eta + s)
\end{aligned}$$

$$\begin{aligned}
\frac{d\Phi}{dr} = & a + \frac{1+2ee}{n} - a' \cos 2\eta + \left(\frac{2}{n} - c'\right) k \cos r \\
& - d'k \cos (2\eta - r) - p'e \cos s - q'e \cos (2\eta - s) \\
& - e'k \cos (2\eta + r) - r'e \cos (2\eta + r)
\end{aligned}$$

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n} k \cos r - \frac{2}{n} e \cos s$$

§. 212. Quaeramus primo inaequalitates motus nodorum, quae neque ab excentricitate orbitae lunaris neque solaris pendent, sitque:

$$\pi = \text{Const.} - Or + \mathfrak{A} \sin 2\eta + \mathfrak{B} \sin (2\Phi - 2\pi) + \mathfrak{C} \sin (2\theta - 2\pi)$$

reiectis reliquis terminis, quos praevidemus fore minimos, ac differentiando obtinebimus:

$$\begin{aligned}
\frac{d\pi}{dr} = & -O - \mathfrak{A} a' - 0,004241 \mathfrak{B} - 0,004241 \mathfrak{C} \\
& + \cos 2\eta (2a\mathfrak{A} - 0,004241 \mathfrak{B} - 0,004241 \mathfrak{C}) \\
& + \cos (2\Phi - 2\pi) \left(2\left(a + \frac{1}{n}\right) \mathfrak{B} + 0,008482 \mathfrak{B} + 0,004221 \mathfrak{C}\right) \\
& + \cos (2\theta - 2\pi) \left(-\mathfrak{B} a' + 0,004221 \mathfrak{B} + \frac{2\mathfrak{C}}{n} + 0,008482 \mathfrak{C}\right)
\end{aligned}$$

unde

vnde oritur :

$$\begin{aligned} O - 0,019744 \mathcal{A} + 0,004241 \mathcal{B} + 0,004241 \mathcal{C} &= 0,004241 \\ 1,867476 \mathcal{A} - 0,004241 \mathcal{B} - 0,004241 \mathcal{C} &= -0,004221 \\ 2,026834 \mathcal{B} + 0,004221 \mathcal{C} &= 0,004241 \\ 0,023965 \mathcal{B} + 0,159358 \mathcal{C} &= 0,004241 \end{aligned}$$

§. 213. Valores hinc igitur prodibunt sequentes :

$$\begin{aligned} O &= + 0,004078 \quad . \quad . \quad . \quad / \quad O = 7,610447 \\ \mathcal{A} &= - 0,002196 \quad . \quad . \quad . \quad / \quad \mathcal{A} = 7,341634 \\ \mathcal{B} &= + 0,002037 \quad . \quad . \quad . \quad / \quad \mathcal{B} = 7,308991 \\ \mathcal{C} &= + 0,026307 \quad . \quad . \quad . \quad / \quad \mathcal{C} = 8,420081 \end{aligned}$$

Vbi primum obseruo valorem ipsius O iam proxime accedere ad motum medium lunae nodorum, vti per observationes constat; inde enim esse deberet $O = 0,004053$ facile autem intelligitur, hunc exiguum defectum per reliquas inaequalitates suppleri posse. Quocirca hinc erit

$\pi = \text{Const.} - 0,004078 r$	Valores in min. sec.
$- 0,002196 \sin 2\eta$	
$+ 0,002037 \sin (2\phi - 2\pi)$	
$+ 0,026307 \sin (2\theta - 2\pi)$	
	$- 453''$
	$+ 420$
	$+ 5426$

quae inaequalitates mirifice conveniunt cum observationibus. His addi potest terminus :

$$+ 0,000336 \sin (4\theta - 4\pi)$$

cuius in minutis secundis valor est $+ 69''$, qui terminus cum postremo illo facile coniungi potest.

§. 214. Quæramus iam seorsim inaequalitates, quae ab excentricitate orbitae lunaris pendent, sitque

$$\pi =$$

$$\begin{aligned} \pi &= \text{Const.} - Or + \mathcal{A} \sin 2\eta + \mathcal{B} \sin (2\phi - 2\pi) + \mathcal{C} \sin (2\theta - 2\pi) \\ &\quad + \mathcal{D} k \sin (2\eta - r) + \mathcal{E} k \sin (2\eta + r) + \mathcal{F} k \sin r \\ &\quad + \mathcal{G} k k \sin (2\eta - 2r) + \mathcal{H} k k \sin 2r + \mathcal{I} k \sin (2\phi - 2\pi - r) \\ &\quad + \mathcal{K} k \sin (2\phi - 2\pi + r) + \mathcal{L} k \sin (2\theta - 2\pi - r) + \mathcal{M} k \sin (2\theta - 2\pi + r) \\ &\quad + \mathcal{N} k k \sin (2\phi - 2\pi - 2r) \end{aligned}$$

eritque differentiando

$$\begin{aligned} \frac{d\pi}{dr} &= \text{Pr.} + k \cos (2\eta - r) (-\mathcal{A}' - 0,009238 \mathcal{B} - 0,009238 \mathcal{C} + (2\alpha - 1) \mathcal{D} \\ &\quad + k \cos (2\eta + r) (-\mathcal{A}' - 0,008411 \mathcal{B} - 0,008411 \mathcal{C} + (2\alpha + 1) \mathcal{E} \\ &\quad + k \cos r (-\mathcal{A}' - \mathcal{H}' - 0,017649 \mathcal{B} - 0,017649 \mathcal{C} + \mathcal{F}' - \mathcal{D}' - \mathcal{G}' \\ &\quad + k k \cos 2r \quad (-0,010636 \mathcal{B} + 2 \mathcal{H}' - \mathcal{D}' - \mathcal{G}' - \mathcal{I}') \\ &\quad + k k \cos (2\eta - 2r) (-0,010636 \mathcal{C} + 2 (\alpha - 1) \mathcal{G}' - \mathcal{D}') \\ &\quad + k \cos (2\theta - 2\pi - r) \left\{ \begin{aligned} &-\mathcal{B}' + 0,008307 \mathcal{B} + \frac{2}{n} \mathcal{C} + 0,017663 \mathcal{C} \\ &\quad + \left(\frac{2}{n} - 1 \right) \mathcal{L} + 0,008482 \mathcal{L} \end{aligned} \right. \\ &\quad + k \cos (2\theta - 2\pi + r) \left\{ \begin{aligned} &-\mathcal{B}' + 0,009962 \mathcal{B} + \frac{2}{n} \mathcal{C} + 0,017663 \mathcal{C} \\ &\quad + \left(\frac{2}{n} + 1 \right) \mathcal{M} + 0,008482 \mathcal{M} \end{aligned} \right. \\ &\quad + k \cos (2\phi - 2\pi - r) \left\{ \begin{aligned} &+ 0,017663 \mathcal{B} + 0,009962 \mathcal{C} + 2 \left(\alpha + \frac{1}{n} \right) \mathcal{I} \\ &\quad - \mathcal{J} + 0,008482 \mathcal{J} \end{aligned} \right. \\ &\quad + k \cos (2\phi - 2\pi + r) \left\{ \begin{aligned} &+ 0,017663 \mathcal{B} + 0,008307 \mathcal{C} + 2 \left(\alpha + \frac{1}{n} \right) \mathcal{K} \\ &\quad + \mathcal{R} + 0,008482 \mathcal{R} \end{aligned} \right. \\ &\quad + k k \cos (2\phi - 2\pi - 2r) \left\{ \begin{aligned} &+ 0,021273 \mathcal{B} + 0,010636 \mathcal{C} + 2 \left(\alpha + \frac{1}{n} \right) \mathcal{N} \\ &\quad - 2 \mathcal{M} + 0,008482 \mathcal{N} \end{aligned} \right. \end{aligned}$$

A a

§. 215.

§. 215. Superfluum foret maiorem curam in his differentialibus adhibere, quia vero proxime tantum rem determinare sufficit; erit ergo:

$$0,867476 \mathcal{D} = \text{---} 0,009962$$

$$2,867476 \mathcal{E} = \text{---} 0,008307$$

Hincque et in min sec.

$$\mathcal{D} = \text{---} 0,011480 \quad | \quad \mathcal{D} k = \text{---} 129''$$

$$\mathcal{E} = \text{---} 0,002900 \quad | \quad \mathcal{E} k = \text{---} 33''$$

quae inaequalitates in loco nodi vix alicuius sunt momenti, vnde eas exactius determinare non est opus.

§. 216. Calculo autem euoluto erit

$$\begin{array}{lcl} \pi = \text{Pr.} - 0,011480k \sin(2\eta - r) & | & 8,059940 - 129'' \\ - 0,002900k \sin(2\eta + r) & | & 7,462400 - 33 \\ - 0,017663k \sin r & | & 8,247064 - 198 \\ + 0,090497kk \sin(2\eta - 2r) & | & 8,956634 + 55 \\ - 0,011978kk \sin 2r & | & 8,078384 - 7 \\ + 0,008707k \sin(2\Phi - 2\pi - r) & | & 7,939851 + 98 \\ + 0,002701k \sin(2\Phi - 2\pi + r) & | & 7,431516 + 30 \\ - 0,004680k \sin(2\theta - 2\pi - r) & | & 7,670224 - 53 \\ + 0,004685k \sin(2\theta - 2\pi + r) & | & 7,670680 + 53 \\ + 0,384848kk \sin(2\Phi - 2\pi - 2r) & | & 9,585229 + 235 \end{array}$$

§. 217. Simili modo inuestigemus inaequalitates motus nodorum, quae pendent ab excentricitate orbitae solaris sitque;

$$\pi =$$

$$\begin{aligned} \pi = \text{Const.} - Or + \mathcal{A} \sin 2\eta + \mathcal{B} \sin(2\Phi - 2\pi) + \mathcal{C} \sin(2\theta - 2\pi) \\ + \mathcal{D} e \sin s + \mathcal{E} e \sin(2\eta - s) + \mathcal{F} e \sin(2\eta + s) \\ + \mathcal{G} e e \sin 2s + \mathcal{H} e \sin(2\Phi - 2\pi - s) + \mathcal{K} e \sin(2\theta - 2\pi - s) \\ + \mathcal{L} e \sin(2\Phi - 2\pi + s) + \mathcal{M} e \sin(2\theta - 2\pi + s) \\ + \mathcal{N} e e \sin(2\theta - 2\pi - 2s) \end{aligned}$$

vnde differentiando pro terminis quaesitis erit:

$$\frac{d\pi}{dr} = \text{Praec.}$$

$$\begin{aligned} + e \cos s \left\{ -\mathcal{A}' - \mathcal{A}r' + 0,006304\mathcal{B} + 0,006320\mathcal{E} + \frac{1}{n}\mathcal{D} - \mathcal{E}' - \mathcal{F}' \right. \\ \left. + 0,006320\mathcal{B} + 0,006304\mathcal{C} \right\} \\ + e \cos(2\eta - s) \left\{ \mathcal{A} \left(\frac{2}{n} - p' \right) + 0,006304\mathcal{B} + 0,006304\mathcal{E} + \left(2\alpha - \frac{1}{n} \right) \mathcal{C} \right. \\ \left. + e \cos(2\eta + s) \left\{ \mathcal{A} \left(\frac{2}{n} - p' \right) + 0,006320\mathcal{B} + 0,006320\mathcal{E} + \left(2\alpha + \frac{1}{n} \right) \mathcal{F} \right\} \right. \\ \left. + e e \cos 2s \left\{ -0,003239\mathcal{C} - \frac{1}{n}\mathcal{D} - \mathcal{E}' - \mathcal{F}' + \frac{2}{n}\mathcal{G} \right\} \right. \\ \left. + e \cos(2\Phi - 2\pi - s) \left\{ -\mathcal{B}p' - 0,012623\mathcal{B} - 0,006188\mathcal{C} \right. \right. \\ \left. \left. + \left(2\alpha \pm \frac{1}{n} + 0,008482 \right) \mathcal{H} \right\} \right. \\ \left. + e \cos(2\Phi - 2\pi + s) \left\{ -\mathcal{B}p' - 0,012623\mathcal{B} - 0,006219\mathcal{C} \right. \right. \\ \left. \left. + \left(2\alpha + \frac{3}{n} + 0,008482 \right) \mathcal{J} \right\} \right. \end{aligned}$$

$$+ e \cos(2\theta - 2\pi - s) \left\{ \begin{array}{l} - \mathfrak{B}' - 0,006219\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0,012623\mathfrak{C} \\ + \left(\frac{1}{n} + 0,008482\right)\mathfrak{R} \end{array} \right.$$

$$+ e \cos(2\theta - 2\pi + s) \left\{ \begin{array}{l} - \mathfrak{B}'' - 0,006188\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0,012623\mathfrak{C} \\ + \left(\frac{3}{n} + 0,008482\right)\mathfrak{R} \end{array} \right.$$

$$+ e e \cos(2\theta - 2\pi - 2s) \left\{ \begin{array}{l} + 0,003239\mathfrak{B} + \frac{1}{2n}\mathfrak{C} + 0,006479\mathfrak{C} \\ + 0,008482\mathfrak{M} - \frac{1}{n}\mathfrak{R} - 0,012623\mathfrak{R} \end{array} \right.$$

§. 218. Hinc reperiuntur sequentes valores

$$\begin{array}{lll} \mathfrak{D} = 0,159070 & . & \mathfrak{D} = 9,201585 ; \mathfrak{D} e = 551'' \\ \mathfrak{C} = 0,003562 & . & \mathfrak{C} = 7,551680 ; \mathfrak{C} e = 12\frac{1}{2}'' \\ \mathfrak{B} = 0,003301 & . & \mathfrak{B} = 7,518677 ; \mathfrak{B} e = 11\frac{1}{2}'' \\ \mathfrak{G} = 0,031650 & . & \mathfrak{G} = 8,587191 ; \mathfrak{G} e = 2'' \\ \mathfrak{H} = -0,003153 & . & \mathfrak{H} = 7,498692 ; \mathfrak{H} e = -11'' \\ \mathfrak{Z} = -0,002932 & . & \mathfrak{Z} = 7,467118 ; \mathfrak{Z} e = -10'' \\ \mathfrak{R} = -0,025750 & . & \mathfrak{R} = 8,410784 ; \mathfrak{R} e = -90'' \\ \mathfrak{L} = -0,009076 & . & \mathfrak{L} = 7,957885 ; \mathfrak{L} e = -32'' \end{array}$$

At valor ipsius $\mathfrak{M}$ tam fit parvus, ut merito pro nihilo haberi possit.

§. 219.

§. 219. Colligamus ergo has inaequalitates in vnam summam, atque obtinebimus longitudinem veram nodi ascendentis

$x = \text{Const.}$	r	Valor, in minut, sec.
—	0,004053	—
—	0,002196 sin 2 η	— 453 ^{ll}
+	0,002037 sin (2 Φ - 2 π)	+ 420
+	0,026307 sin (2 θ - 2 π)	+ 5426
+	0,000370 sin (4 θ - 4 π)	+ 75
—	0,017666 sin r	— 198
—	0,011486 sin (2 η - r)	— 129
—	0,002906 sin (2 η + r)	— 33
+	0,090566 sin (2 η - 2 r)	+ 55
—	0,012066 sin 2 r	— 7
+	0,008716 sin (2 Φ - 2 π - r)	+ 98
+	0,002706 sin (2 Φ - 2 π + r)	+ 30
—	0,004686 sin (2 θ - 2 π - r)	— 53
+	0,004686 sin (2 θ - 2 π + r)	+ 53
+	0,384866 sin (2 Φ - 2 π - 2 r)	+ 235
+	0,159076 sin s	+ 551
—	0,025756 sin (2 θ - 2 π - s)	— 90
—	0,009076 sin (2 θ - 2 π + s)	— 32

omissis scilicet iis inaequalitatibus, quae non supra 30^o exsurgunt.

Aa 3

CAPUT

CAPUT XIII.

INVESTIGATIO INCLINATIONIS ORBITAE
LUNARIS AD ECLIPTICAM.

§ 220.

Pro inclinatione orbitae lunaris ad eclipticam inuenienda, forma §. 208. euoluta multiplicari debet per $-\frac{1}{4} \sin 2\eta + \frac{1}{4} \sin 2(\Phi - \pi) + \frac{1}{4} \sin 2(\theta - \pi)$, ac productum erit $= \frac{d \cdot \text{tang} e}{dr}$: Hinc ergo habebitur:

$$\begin{aligned} \frac{d \cdot \text{tang} e}{dr} = & +0,004261 \sin 2\eta - 0,000020 \sin 4\eta \\ & +0,008514k \sin(2\eta - r) \\ & +0,008514k \sin(2\eta + r) \\ & +0,000827k \sin r \\ & -0,004261 \sin(2\Phi - 2\pi) \text{ adice} \\ & -0,004261 \sin(2\theta - 2\pi) \text{ adcoeff.} \\ & -0,008514k \sin(2\Phi - 2\pi - r) - 0,000724 \\ & -0,008514k \sin(2\Phi - 2\pi + r) + 0,000103 \\ & -0,008514k \sin(2\theta - 2\pi - r) + 0,000103 \\ & -0,008514k \sin(2\theta - 2\pi + r) - 0,000724 \\ & -0,010636kk \sin(2\Phi - 2\pi - 2r) \\ & +0,006420e \sin(2\theta - 2\pi - r) - 0,000100 \\ & +0,006420e \sin(2\theta - 2\pi + r) - 0,000116 \end{aligned}$$

§. 221.

BITAE

m inue-
ri debet
:($\theta - \pi$),
ebitur:

220 sin 4 η

E

24

03

03

24

00

16

§. 221.

CAPUT XIII.

Ept

§. 221. Quaeramus primo terminos, qui a neutra excentricitate pendent, sitque

$$\frac{\text{tang} e}{\text{tang} e} =$$

$\mathcal{A} \cos 2\eta + a \cos 4\eta + \mathcal{B} \cos(2\Phi - 2\pi) + \mathcal{C} \cos(2\theta - 2\pi) + e \cos(4\theta - 4\pi)$
eritque differentiendo:

$$\frac{d \cdot \text{tang} e}{dr} = \sin 2\eta [-2a\mathcal{A} + 0,004241\mathcal{B} - 0,004241\mathcal{C}]$$

$$\sin 4\eta [-4aa + \mathcal{A}a']$$

$$\sin(2\Phi - 2\pi) \left\{ -2(a + \frac{1}{n})\mathcal{B} - 0,008482\mathcal{B} - 0,004227\mathcal{C} \right\}$$

$$\sin(2\theta - 2\pi) \left\{ -\frac{2}{n}\mathcal{C} + \mathcal{B}a' - 0,004221\mathcal{B} - 0,008482\mathcal{C} \right\}$$

$$\sin(4\theta - 4\pi) \left\{ +0,004241\mathcal{C} - \frac{4}{n}e \right\}$$

§. 222. Ex his iam reperitur:

$$\mathcal{A} = -0,002630 \quad \text{. . .} \quad \mathcal{A} = 7,419914$$

$$\mathcal{B} = +0,002037 \quad \text{. . .} \quad \mathcal{B} = 7,308991$$

$$\mathcal{C} = +0,026307 \quad \text{. . .} \quad \mathcal{C} = 8,420081$$

$$a = +0,000019 \quad \text{. . .} \quad a = 5,278753$$

$$e = +0,000370 \quad \text{. . .} \quad e = 6,567931$$

ita vt hinc fit:

$$\frac{\text{tang} e}{\text{tang} e} = -0,002630 \cos 2\eta$$

$$+0,000019 \cos 4\eta$$

$$+0,002037 \cos(2\Phi - 2\pi)$$

$$+0,026307 \cos(2\theta - 2\pi)$$

$$+0,000370 \cos(4\theta - 4\pi)$$

§. 223.

§. 223. Quaeramus iam seorsim terminos ab excentricitate Lunae pendentes: sitque

$$\frac{1}{\text{tang} \varphi} = \mathcal{A} \cos 2\eta + \mathcal{B} \cos(2\Phi - 2\pi) + \mathcal{C} \cos(2\theta - 2\pi) \\ + \mathcal{D} k \cos(2\eta - r) + \mathcal{E} k \cos(2\eta + r) + \mathcal{F} k \cos r \\ + \mathcal{G} k \cos(2\Phi - 2\pi - r) + \mathcal{H} k \cos(2\theta - 2\pi - r) \\ + \mathcal{I} k \cos(2\Phi - 2\pi + r) + \mathcal{K} k \cos(2\theta - 2\pi + r) \\ + \mathcal{L} k^2 \cos(2\Phi - 2\pi - 2r)$$

unde differentialibus sumendis habebitur: $\frac{d. 1/\text{tang} \varphi}{dr} =$

$$k \sin(2\eta - r) \left\{ + \mathcal{A}' + 0,09238 \mathcal{B} - 0,009238 \mathcal{C} + 0,004241 \mathcal{G} \right. \\ \left. + (2\pi - 1) \mathcal{D} - 0,004241 \mathcal{K} \right.$$

$$k \sin(2\eta + r) \left\{ + \mathcal{A}' + 0,008411 \mathcal{B} - 0,008411 \mathcal{C} + (2\pi + 1) \mathcal{E} \right. \\ \left. + 0,004241 \mathcal{H} - 0,004241 \mathcal{I} \right.$$

$$k \sin r \left\{ + \mathcal{A}' - \mathcal{A}' + 0,000827 \mathcal{B} - 0,000827 \mathcal{C} - \mathcal{F} - 0,004241 \mathcal{G} \right. \\ \left. + 0,004241 \mathcal{H} - \mathcal{D}' + \mathcal{E}' - 0,004241 \mathcal{I} + 0,004241 \mathcal{K} \right.$$

$$k \sin(2\Phi - 2\pi - r) \left\{ - 0,017663 \mathcal{B} - 0,009962 \mathcal{C} - 2\left(\pi + \frac{1}{\pi}\right) \mathcal{G} + \mathcal{G} \right. \\ \left. - 0,008482 \mathcal{H} - 0,004221 \mathcal{I} \right.$$

$$k \sin(2\Phi - 2\pi + r) \left\{ - 0,017663 \mathcal{B} - 0,008307 \mathcal{C} - 2\left(\pi + \frac{1}{\pi}\right) \mathcal{H} - \mathcal{H} \right. \\ \left. - 0,008482 \mathcal{I} - 0,004221 \mathcal{K} \right.$$

$$k \sin(2\theta - 2\pi - r) \left\{ + \mathcal{B}' - 0,008307 \mathcal{B} - \frac{2}{\pi} \mathcal{C} - 0,017663 \mathcal{C} \right. \\ \left. + \mathcal{G}' - 0,004221 \mathcal{G} - \frac{2}{\pi} \mathcal{I} + \mathcal{I} - 0,008482 \mathcal{I} \right.$$

$k \sin$

ab ex-

$2\theta - 2\pi$

$k \cos r$

$-2\pi - r$

$-2\pi + r$

$2\pi - 2r$

$\frac{1}{\text{tang} \varphi}$

$241 \mathcal{G}$

$+ 1) \mathcal{E}$

$241 \mathcal{G}$

$241 \mathcal{K}$

$\mathcal{H} + \mathcal{G}$

$\mathcal{H} - \mathcal{H}$

$63 \mathcal{C}$

$82 \mathcal{I}$

$k \sin$

$$k \sin(2\theta - 2\pi + r) \left\{ + \mathcal{B}' - 0,009962 \mathcal{B} - \frac{2}{\pi} \mathcal{C} - 0,017663 \mathcal{C} \right. \\ \left. + \mathcal{H}' - 0,004221 \mathcal{H} - \frac{2}{\pi} \mathcal{K} - \mathcal{K} - 0,008482 \mathcal{K} \right.$$

$$k \sin(2\Phi - 2\pi - 2r) \left\{ - 0,021273 \mathcal{B} - 0,010636 \mathcal{C} - 2\left(\pi + \frac{1}{\pi}\right) \mathcal{I} + 2 \mathcal{I} \right. \\ \left. - 0,008482 \mathcal{I} \right.$$

hincque reperitur:

$$\mathcal{D} = 0,010487 \quad \dots \quad \mathcal{D} = 8,020538$$

$$\mathcal{E} = 0,003166 \quad \dots \quad \mathcal{E} = 7,500439$$

$$\mathcal{F} = -0,001600 \quad \dots \quad \mathcal{F} = 7,204120$$

$$\mathcal{G} = +0,008719 \quad \dots \quad \mathcal{G} = 7,940484$$

$$\mathcal{H} = +0,002699 \quad \dots \quad \mathcal{H} = 7,431136$$

$$\mathcal{I} = -0,004460 \quad \dots \quad \mathcal{I} = 7,649305$$

$$\mathcal{K} = +0,004717 \quad \dots \quad \mathcal{K} = 7,623628$$

$$\mathcal{L} = +0,384890 \quad \dots \quad \mathcal{L} = 9,585335$$

§. 224. Nunc denique pro inaequalitatibus ab excentricitate orbitae solaris pendentibus ponatur.

$$\frac{1}{\text{tang} \varphi} = \mathcal{A} \cos 2\eta + \mathcal{B} \cos(2\Phi - 2\pi) + \mathcal{M} \cos(2\theta - 2\pi - r) \\ + \mathcal{C} \cos(2\theta - 2\pi) + \mathcal{N} \cos(2\theta - 2\pi + r)$$

ac differentiendo prodibit: $\frac{d. 1/\text{tang} \varphi}{dr} =$

$$e \sin(2\theta - 2\pi - r) \left\{ + \mathcal{B}' + 0,006219 \mathcal{B} + \frac{2}{\pi} \mathcal{C} + 0,012623 \mathcal{C} \right. \\ \left. - \frac{1}{\pi} \mathcal{M} - 0,008482 \mathcal{M} \right.$$

Bb

e sin

$$\sin(2\theta - 2\pi + s) \left\{ \begin{aligned} &+ 3q' + 0,06188 B + \frac{2}{n} C + 0,012623 C \\ &- \frac{3}{n} M - 0,008482 M \end{aligned} \right.$$

unde reperitur

$$M = -0,024034 \quad . \quad . \quad I-M = 8,380835$$

$$M = -0,008519 \quad . \quad . \quad I-M = 7,930332$$

§. 225. Si ergo ϵ denotet inclinationem mediam orbitae lunaris ad eclipticam, et φ inclinationem veram, erit

$\frac{\tan \varphi}{\tan \epsilon}$		log. coeff.
$-0,002630$	$\cos 2\eta$	7,419915
$+0,000019$	$\cos 4\eta$	5,278753
$+0,002037$	$\cos(2\Phi - 2\pi)$	7,308991
$+0,026307$	$\cos(2\theta - 2\pi)$	8,420081
$+0,000370$	$\cos(4\theta - 4\pi)$	6,567931
$+0,01049k$	$\cos(2\eta - r)$	8,020638
$+0,00317k$	$\cos(2\eta + r)$	7,500439
$-0,00160k$	$\cos r$	7,204120
$+0,00872k$	$\cos(2\Phi - 2\pi - r)$	7,940484
$+0,00270k$	$\cos(2\Phi - 2\pi + r)$	7,431136
$-0,00446k$	$\cos(2\theta - 2\pi - r)$	7,649305
$+0,00472k$	$\cos(2\theta - 2\pi + r)$	7,673628
$+0,3849kk$	$\cos(2\Phi - 2\pi - 2r)$	9,585335
$-0,02403\epsilon$	$\cos(2\theta - 2\pi - s)$	8,380835
$-0,00852\epsilon$	$\cos(2\theta - 2\pi + s)$	7,930332

§. 226.

§. 226. Quodsi iam ponatur $\frac{\tan \varphi}{\tan \epsilon} = S$, erit ad numeros ipsos procedendo $\frac{\tan \varphi}{\tan \epsilon} = 1 + S + \frac{1}{2} S S$
Hinc igitur negligendo terminos minimos, consequemur:

$$\begin{aligned} \frac{\tan \varphi}{\tan \epsilon} = & 1 - 0,002604 \cos 2\eta \\ & + 0,000020 \cos 4\eta \\ & + 0,002003 \cos(2\Phi - 2\pi) \\ & + 0,026307 \cos(2\theta - 2\pi) \\ & + 0,000490 \cos(4\theta - 4\pi) \\ & - 0,00160k \cos r \\ & + 0,01049k \cos(2\eta - r) \\ & + 0,00317k \cos(2\eta + r) \\ & + 0,00885k \cos(2\Phi - 2\pi - r) \\ & + 0,00270k \cos(2\Phi - 2\pi + r) \\ & - 0,00448k \cos(2\theta - 2\pi - r) \\ & + 0,00472k \cos(2\theta - 2\pi + r) \\ & + 0,3849kk \cos(2\Phi - 2\pi - 2r) \\ & - 0,02403\epsilon \cos(2\theta - 2\pi - s) \\ & - 0,00852\epsilon \cos(2\theta - 2\pi + s) \end{aligned}$$

Bb 2

§. 227.

§. 227. Cum in aequatione nostra principali, quae motum Lunae continet, infit terminus $\frac{\tan \varphi^2}{\tan \varepsilon^2}$, huiusquoque valorem euolui conueniet: erit ergo

$$\frac{\tan \varphi^2}{\tan \varepsilon^2} = 1$$

$$\begin{aligned} & - 0,005156 \cos 2\eta : + 0,000940 \cos 4\eta \\ & + 0,003938 \cos (2\Phi - 2\pi) \\ & + 0,052614 \cos (2\theta - 2\pi) + 0,001320 \cos (4\theta - 4\pi) \\ & - 0,002906 \cos r \\ & + 0,020986 \cos (2\eta - r) \\ & + 0,006346 \cos (2\eta + r) \\ & + 0,017966 \cos (2\Phi - 2\pi - r) \\ & + 0,005566 \cos (2\Phi - 2\pi + r) \\ & - 0,008966 \cos (2\theta - 2\pi - r) \\ & + 0,009406 \cos (2\theta - 2\pi + r) \\ & + 0,769866 \cos (2\Phi - 2\pi - 2r) \\ & + 0,048066 \cos (2\theta - 2\pi - r) \\ & + 0,017046 \cos (2\theta - 2\pi + r) \end{aligned}$$

Hicque ergo valor in superiori illa aequatione substitui poterit.

§. 228.

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-4π)

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8.

§. 228. Celeb. autem Clairaut conclusit inclinationem mediam ε ex observationibus exquisitissimis $5^\circ 8' 9''$, ex qua igitur ad quoduis tempus inclinationem veram elicere licebit. Sit enim $\varphi = \varepsilon + \omega$, erit $\tan \varphi = \frac{\tan \varepsilon + \omega}{1 - \omega \tan \varepsilon} = \tan \varepsilon + \frac{\omega}{\cos \varepsilon^2} = V \tan \varepsilon$, ponendo V pro expressione ipsius $\frac{\tan \varphi}{\tan \varepsilon}$. Hinc erit $\omega = (V-1) \sin \varepsilon \cos \varepsilon = \frac{1}{2} (V-1) \sin 2\varepsilon = 0,08915 (V-1)$: vnde reperitur in minutis secundis

$$\begin{aligned} \varphi = \varepsilon & - 48'' \cos 2\eta \\ & + 36 \cos (2\Phi - 2\pi) \\ & + 484 \cos (2\theta - 2\pi) \\ & + 9 \cos (4\theta - 4\pi) \\ & - 2 \cos r \\ & + 11 \cos (2\eta - r) \\ & + 3 \cos (2\eta + r) \\ & + 9 \cos (2\Phi - 2\pi - r) \\ & + 3 \cos (2\Phi - 2\pi + r) \\ & - 5 \cos (2\theta - 2\pi - r) \\ & + 5 \cos (2\theta - 2\pi + r) \\ & + 23 \cos (2\Phi - 2\pi - 2r) \\ & - 7 \cos (2\theta - 2\pi - r) \\ & - 3 \cos (2\theta - 2\pi + r) \end{aligned}$$

Bb 3

§. 229.

§. 229. Hic notandum est, etiamsi valor inclinationis mediae & aliquantillum immutetur, aequationes has tamen inde vix alterari, ita ut eae semper eadem sint mansurae. Perspicuum quoque est in calculo astronomico sufficere tres inaequalitates primores, et reliquas omnes sine errore sensibili praetermitti posse; nisi forte aequatio 23 $\cos(2\phi - 2\pi - 2r)$ retinenda censeatur, quae inter reliquas est maxima. Exprimit autem angulus $2\phi - 2\pi - 2r$ duplam distantiam apogei Lunae ab eius nodo, a quo angulo quoque locum nodi non mediocriter affici vidimus, cum correctio hinc oriunda pro loco nodi usque ad 235'' assurgere possit.

CAPUT

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la pro

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(o)

CAPUT XIV.

INVESTIGATIO INAEQUALITATUM MOTUS LUNAE AB EIUS INCLINATIONE AD ECLIP- TICAM ORIUNDARUM.

§. 230.

Ponamus more adhuc usitato:

$$\begin{aligned} fRdr = & A\cos 2\eta + E\cos r + Dk\cos(2\eta - r) + P\cos s + Q\cos(2\eta - s) \\ & + E\cos(2\eta + r) + R\cos(2\eta + s) \\ & + F\cos 2\eta + G\cos(2\phi - 2\pi) + Jfk\cos r + Kfk\cos(2\eta - r) \\ & + Hf\cos(2\theta - 2\pi) + Lfk\cos(2\eta + r) \\ & + Mfk\cos(2\phi - 2\pi - r) + Sfk\cos(2\theta - 2\pi - r) \\ & + Nfk\cos(2\phi - 2\pi + r) + Tfk\cos(2\theta - 2\pi + r) \\ & + Ofkk\cos(2\phi - 2\pi - 2r) + Ufk\cos(2\theta - 2\pi - s) \\ & + Vfk\cos(2\theta - 2\pi + s) \end{aligned}$$

$$\begin{aligned} \text{et } v = & A\cos 2\eta + Dk\cos(2\eta - r) + P\cos s + Q\cos(2\eta - s) \\ & + E\cos(2\eta + r) + R\cos(2\eta + s) \\ & + F\cos 2\eta + G\cos(2\phi - 2\pi) + Jfk\cos r + Kfk\cos(2\eta - r) \\ & + Hf\cos(2\theta - 2\pi) + Lfk\cos(2\eta + r) \\ & + Mfk\cos(2\phi - 2\pi - r) + Sfk\cos(2\theta - 2\pi - r) \\ & + Nfk\cos(2\phi - 2\pi + r) + Tfk\cos(2\theta - 2\pi + r) \\ & + Ofkk\cos(2\phi - 2\pi - 2r) + Ufk\cos(2\theta - 2\pi - s) \\ & + Vfk\cos(2\theta - 2\pi + s) \end{aligned}$$

§. 231.

ciat correctiones inde resultantibus proxime saltem determinasse; praesertim cum haec praetermissio vix ad aliquot minuta secunda sit ascensura.

§. 234. Ob eandem rationem licebit in valoribus differentialium $\frac{d\phi}{dr}$ et $\frac{d\eta}{dr}$ particulas ab inclinatione pendentes negligere, unde erit: $\frac{dv}{dr} =$

$$\begin{aligned} f \sin 2\eta & [-2aF = -F'] \\ f \sin(2\phi - 2\pi) & [-2(a + \frac{1}{n})G - 0,008482G = -G'] \\ f \sin(2\theta - 2\pi) & [+G'd' - \frac{2}{n}H - 0,008482H = -H'] \\ fk \sin r & [Fd' - Fe' - J = -J'] \\ fk \sin(2\eta - r) & [+Fe' - (2a-1)K = -K'] \\ fk \sin(2\eta + r) & [+Fe' - (2a+1)L = -L'] \\ fk \sin(2\phi - 2\pi - r) & [-2(a + \frac{1}{n})M + M' - 0,008482M = -M'] \\ fk \sin(2\phi - 2\pi + r) & [-2(a + \frac{1}{n})N - N' - 0,008482N = -N'] \\ fkl \sin(2\phi - 2\pi - 2r) & [-2(a + \frac{1}{n})O + 2O' - 0,008482O = -O'] \\ fk \sin(2\theta - 2\pi - r) & [+G'd' - \frac{2}{n}H - \frac{2}{n}S + S' - 0,008482S = -S'] \\ fk \sin(2\theta - 2\pi + r) & [+G'd' - \frac{2}{n}H - \frac{2}{n}T - 0,008482T = -T'] \\ fe \sin(2\theta - 2\pi - r) & [+G'd' + \frac{2}{n}H - \frac{1}{n}U - 0,008482U = -U'] \\ fe \sin(2\theta - 2\pi + r) & [+G'd' + \frac{2}{n}H - \frac{3}{n}V - 0,008482V = -V'] \end{aligned}$$

§. 235.

deter-
d ali-ribus
pen-

§. 235. Si nunc simili modo denuo differendemus, prodibit: $\frac{ddv}{dr^2} =$

$$\begin{aligned} f \cos 2\eta & [-2aF'] \\ f \cos(2\phi - 2\pi) & [-2(a + \frac{1}{n})G' - 0,008482G'] \\ f \cos(2\theta - 2\pi) & [+G'd' - \frac{2}{n}H' - 0,008482H'] \\ fk \cos r & [F'd' + Fe' - J'] \\ fk \cos(2\eta - r) & [F'e' - (2a-1)K'] \\ fk \cos(2\eta + r) & [F'e' - (2a+1)L'] \\ fk \cos(2\phi - 2\pi - r) & [-2(a + \frac{1}{n})M' + M' - 0,008482M'] \\ fk \cos(2\phi - 2\pi + r) & [-2(a + \frac{1}{n})N' - N' - 0,008482N'] \\ fkl \cos(2\phi - 2\pi - 2r) & [-2(a + \frac{1}{n})O' + 2O' - 0,008482O'] \\ fk \cos(2\theta - 2\pi - r) & [G'd' - \frac{2}{n}H' - \frac{2}{n}S' + S' - 0,008482S'] \\ fk \cos(2\theta - 2\pi + r) & [G'd' - \frac{2}{n}H' - \frac{2}{n}T' - 0,008482T'] \\ fe \cos(2\theta - 2\pi - r) & [G'd' + \frac{2}{n}H' - \frac{1}{n}U' - 0,008482U'] \\ fe \cos(2\theta - 2\pi + r) & [G'd' + \frac{2}{n}H' - \frac{3}{n}V' - 0,008482V'] \end{aligned}$$

§. 236. Hinc autem sequentes eliciuntur valores

$$\begin{aligned} F &= 0,01273 & \dots & / F = 8,104833 \\ G &= 0,32213 & \dots & / G = 9,508032 \\ H &= 0,06976 & \dots & / H = 8,843599 \\ J &= -1,87800 & \dots & / J = 0,273719 \end{aligned}$$

Cc 2

K =

$K = +0,05615$	$I-K = 8,749352$
$L = -0,00077$	$I-L = 6,888904$
$M = -0,29638$	$I-M = 9,471854$
$N = +0,00012$	$I-N = 6,089109$
$O = +0,32287$	$I-O = 9,509034$
$S = +0,39091$	$I-S = 9,592073$
$T = +0,69579$	$I-T = 9,842475$
$U = -0,07922$	$I-U = 8,898830$
$V = -0,05141$	$I-V = 8,711093$

§. 237. Pro distantia ergo lunae a sole curvata $x = \frac{(1-kk) \sin u}{1-k \cos r}$ erit

$u = \text{Praec.}$

	Log. coeff.	Valores coeff. integri.
$+ 0,000072f \cos 2\eta$	5,860017	$+ 0,000079$
$+ 0,001833f \cos(2\Phi-2\pi)$	7,263216	$+ 0,002005$
$+ 0,000397f \cos(2\theta-2\pi)$	6,598774	$+ 0,000434$
$- 0,01069fk \cos r$	8,028894	$- 0,000634$
$+ 0,00032fk \cos(2\eta-r)$	6,504536	$+ 0,000019$
$- 0,00000fk \cos(2\eta+r)$	4,644088	$- 0,000080$
$- 0,00169fk \cos(2\Phi-2\pi-r)$	7,227038	$- 0,000100$
$+ 0,00000fk \cos(2\Phi-2\pi+r)$	3,834293	$+ 0,000000$
$+ 0,00184fk^2 \cos(2\Phi-2\pi-2r)$	7,264218	$+ 0,000006$
$+ 0,00223fk \cos(2\theta-2\pi-r)$	7,347257	$+ 0,000132$
$+ 0,00396fk \cos(2\theta-2\pi+r)$	7,597659	$+ 0,000235$
$- 0,00045fk \cos(2\theta-2\pi-s)$	6,654014	$- 0,000000$
$- 0,00029fk \cos(2\theta-2\pi+s)$	6,466277	$- 0,000000$

vbi notandum est esse $f = 1,093756$, et $I/f = 0,038921$.

§. 238.

§. 238. Deinde pro motu momentaneo habebitur

$$\frac{d\Phi}{dr} = \text{Praec.}$$

	Log. coeff.	Valores coeff. in numeris.
$- 0,000146f \cos 2\eta$	6,164934	$- 0,000160$
$- 0,003700f \cos(2\Phi-2\pi)$	7,568133	$- 0,004746$
$- 0,000801f \cos(2\theta-2\pi)$	6,903691	$- 0,000876$
$+ 0,02157fk \cos r$	8,333811	$+ 0,001286$
$- 0,00065fk \cos(2\eta-r)$	6,809453	$- 0,000038$
$+ 0,00000fk \cos(2\eta+r)$	4,949005	$+ 0,000001$
$+ 0,00340fk \cos(2\Phi-2\pi-r)$	7,531955	$+ 0,000203$
$- 0,00000fk \cos(2\Phi-2\pi+r)$	4,139210	$- 0,000000$
$- 0,00371fk^2 \cos(2\Phi-2\pi-2r)$	7,569135	$- 0,000012$
$- 0,00449fk \cos(2\theta-2\pi-r)$	7,652174	$- 0,000267$
$- 0,00799fk \cos(2\theta-2\pi+r)$	7,902576	$- 0,000476$
$+ 0,00091fk \cos(2\theta-2\pi-s)$	6,958931	$+ 0,000000$
$+ 0,00059fk \cos(2\theta-2\pi+s)$	6,771194	$+ 0,000000$

§. 239. Pro correctione longitudinis verae hinc oriunda ponatur,

$\Phi = \text{Praec.}$

$$\begin{aligned}
 & + X' f \sin 2\eta + Y' f k \sin r + Z' f k \sin(2\Phi-2\pi-r) \\
 & + G' f \sin(2\Phi-2\pi) + R' f k \sin(2\eta-r) + M' f k \sin(2\Phi-2\pi+r) \\
 & + P' f \sin(2\theta-2\pi) + V' f k \sin(2\eta+r) + Q' f k k \sin(2\Phi-2\pi-2r) \\
 & + S' f k \sin(2\theta-2\pi-r) + W' f e \sin(2\theta-2\pi-s) \\
 & + Z' f k \sin(2\theta-2\pi+r) + B' f e \sin(2\theta-2\pi+s)
 \end{aligned}$$

Cc 3

erit.

eritque :

$$2a \mathcal{G}' = -0,000146$$

$$0,026834 \mathcal{G}' = -0,003700$$

$$0,159358 \mathcal{H}' - \mathcal{G}'d' = -0,000801$$

$$\mathcal{G}' - \mathcal{G}'d' - \mathcal{G}'e' = +0,02157$$

$$0,867476 \mathcal{K}' - \mathcal{G}'d' = -0,00065$$

$$2,867476 \mathcal{L}' - \mathcal{G}'d' = -0,00000$$

$$1,026834 \mathcal{M}' = +0,00340$$

$$3,026834 \mathcal{N}' = -0,00000$$

$$0,026834 \mathcal{O}' = -0,00371$$

$$-0,840642 \mathcal{G}' - \mathcal{G}'d' + \frac{2}{n} \mathcal{H}' = -0,00449$$

$$+1,159358 \mathcal{H}' - \mathcal{G}'d' + \frac{2}{n} \mathcal{H}' = -0,00799$$

$$+0,083920 \mathcal{U}' - \mathcal{G}'r' - \frac{2}{n} \mathcal{H}' = +0,00091$$

$$+0,234796 \mathcal{V}' - \mathcal{G}'q' - \frac{2}{n} \mathcal{H}' = +0,00059$$

§. 240. Expeditis igitur his formulis orietur :

	Log.coeff.	coeff. tot. in sec.
$\Phi = \text{Pr.} - 0,000078f \sin 2\eta$	5,893680	— 18''
$-0,001825f \sin(2\Phi - 2\pi)$	7,261316	— 422
$-0,004800f \sin(2\theta - 2\pi)$	7,681286	— 1083
$+0,02154fk \sin r$	8,333246	+ 264
$-0,00074fk \sin(2\eta - r)$	6,867925	— 9
$+0,00332fk \sin(2\Phi - 2\pi - r)$	7,520465	+ 41
$-0,13818fk^2 \sin(2\Phi - 2\pi - 2r)$	9,140450	— 92
$+0,00446fk \sin(2\theta - 2\pi - r)$	7,649421	+ 55
$-0,00685fk \sin(2\theta - 2\pi + r)$	7,835624	— 84
$+0,00310fe \sin(2\theta - 2\pi - s)$	7,491107	+ 11
$-0,00030fe \sin(2\theta - 2\pi + s)$	6,474418	— 1

§. 242.

§. 241. Haec omnia satis conveniunt cum notis inaequalitatibus motus lunae, nisi quod inaequalitas ab angulo $2\theta - 2\pi$ pendens plane aduersari videatur, cum nullum eius vestigium in tabulis astronomicis occurrat; quod quidem eo magis est mirandum, cum correctio inde oriunda ad $18'$, $3''$ exsurgat. Lubens equidem agnosco, in hoc calculo non omnem curam esse adhibitam, ut hanc aequationem tanquam omnibus numeris absolutam spectare liceat, quoniam ad plurimos terminos, quos formulae nostrae suppeditant, non respexi. Interim tamen calculum repetenti mox patebit, non admodum enormiter esse aberratum, praesertim cum aequatio ab angulo $2\Phi - 2\pi$ pendens, quae pari passu procedit, veritati perquam consentanea prodierit, cum ea reductio lunae ad eclipticam contineatur. Ac si quidem haec inaequalitas ad semissem vsque diminuatur, tamen tanta remanet, ut merito dubitare debeamus, eius effectum ab Astronomis non esse animaduersum; cum eius omissio vix per aliam aequationem compensari queat. Hancobrem, siue omissio terminorum neglectorum sit in causa, siue etiam in calculo numerico error fuerit admissus, quod facile euenire potuit, istam inuestigationem in capite sequenti accuratius suscipiamus.

r :
coeff. tot.
in sec.

— 18''
— 422
— 1083
— 264
— 9
— 41
— 92
— 55
— 84
— 11
— 1

§. 242.

CAPUT XV.

ACCURATIO INVESTITIO INAEQUALI-
TATUM LUNAE AB INCLINATIONE EJUS
ORBITAE PENDENTIUM.

§. 242.

Quoniam praecipuum dubium circa inaequalitatem
ab angulo $2\theta - 2\pi$ pendentem versatur, nostram
investigationem ab iis inaequalitatibus, quae simul
ab alterutra excentricitate pendent, abstrahamus. Po-
namus ergo:

$$fRdr = A\cos 2\eta + Ff\cos 2\eta + Gf\cos(2\Phi - 2\pi) \\ + Hf\cos(2\theta - 2\pi) + Jf\cos(4\theta - 4\pi) \\ \text{et } v = A\cos 2\eta + Ff\cos 2\eta + Gf\cos(2\Phi - 2\pi) \\ + Hf\cos(2\theta - 2\pi) + Jf\cos(4\theta - 4\pi)$$

$$\text{Positoque } \frac{2\pi F + G}{nn} = f'; \quad \frac{2\pi G + G}{nn} = g'; \quad \frac{2\pi H + H}{nn} = b';$$

$$\frac{2\pi J + J}{nn} = i' \text{ erit:}$$

$$\frac{d\Phi}{dr} = a + \frac{1}{\pi} - a'\cos 2\eta - f'f\cos 2\eta - g'f\cos(2\Phi - 2\pi) \\ - b'f\cos(2\theta - 2\pi) - i'f\cos(4\theta - 4\pi)$$

$$\frac{d\eta}{dr} = a - a'\cos 2\eta - f'f\cos 2\eta - g'f\cos(2\Phi - 2\pi) \\ - b'f\cos(2\theta - 2\pi) - i'f\cos(4\theta - 4\pi)$$

$$\frac{d\theta}{dr} = \frac{1}{\pi} \text{ et } \frac{d\pi}{dr} = -0,004241 - 0,004221 \cos 2\eta \\ + 0,004241 \cos(2\Phi - 2\pi) \\ + 0,004241 \cos(2\theta - 2\pi)$$

§. 243.

UALI-
JS

litatem
ostram
simul
Po-

$(\theta - 4\pi)$

$(\theta - 4\pi)$

$= b';$

$(\theta - 4\pi)$

$(\theta - 4\pi)$

$\cos 2\eta$

243.

CAPUT XV.

§. 243. His valoribus in formulis principalibus
substitutis habebimus has aequationes:

$$R = -\frac{3G}{2nn} f \sin(2\theta - 2\pi) + \frac{3H}{2nn} f \sin(2\Phi - 2\pi)$$

$$\frac{ddv}{dr^2} = f \cos 2\eta \left\{ -6F - 2\pi G \right.$$

$$f \cos(2\Phi - 2\pi) \left\{ -6G + \frac{3H}{4nn} - 2\pi G + \frac{2H}{nn} + \frac{3AH}{nn} \right. \\ \left. + \frac{3AH}{nn} + \frac{3AH}{nn} \right.$$

$$f \cos(2\theta - 2\pi) \left\{ -6H + \frac{3G}{4nn} - 2\pi H + \frac{2G}{nn} + \frac{3AG}{nn} \right. \\ \left. + \frac{3AG}{nn} + \frac{3AG}{nn} \right.$$

$$f \cos(4\theta - 4\pi) [-6J - 2\pi J]$$

$$f \cos 2\eta \left(-0,005156 - 0,026307 + 0,001969 \frac{A}{nn} \right)$$

$$f \cos(2\Phi - 2\pi) \left(+0,003938 - 1 - 0,052614 \frac{A}{nn} - 0,002578 \frac{A}{nn} \right)$$

$$f \cos(2\theta - 2\pi) \left(+0,052614 + 0,002578 - 0,001969 \frac{2A}{nn} + \frac{A}{nn} \right)$$

$$f \cos(4\theta - 4\pi) \left(+0,001320 + 0,026307 \frac{A}{nn} \right)$$

§. 244. Vel in numeris erit

$$R = 0,008540 H f \sin(2\Phi - 2\pi) - 0,008540 G f \sin(2\theta - 2\pi)$$

Dd

ddv =

$$\frac{ddv}{dr^2} =$$

$$f \cos 2\eta [-1,01591 F - 2\kappa \mathfrak{F} - 0,031478 \\ fcl(2\Phi-2\pi) \{-1,01591 G - 2\kappa \mathfrak{G} + 0,000636 H - 0,845648 \\ - 0,027110 \mathfrak{H} \\ fcl(2\theta-2\pi) \{-1,01591 H - 2\kappa \mathfrak{H} + 0,000636 G + 0,047722 \\ - 0,027110 \mathfrak{G} \\ fcl(4\theta-4\pi) \{-1,01591 J - 2\kappa \mathfrak{J} + 0,001123$$

Nunc autem ex formulis assumtis erit

$$R = f \sin 2\eta [-2\kappa \mathfrak{F} + 0,004241 \mathfrak{G} - 0,004241 \mathfrak{H} \\ f \sin(2\Phi-2\pi) [+ \mathfrak{A} b' - 2,026834 \mathfrak{G} + 0,004221 \mathfrak{H} \\ f \sin(2\theta-2\pi) [- \mathfrak{A} g' - 0,023965 \mathfrak{G} - 0,159358 \mathfrak{H} \\ f \sin(4\theta-4\pi) [+ 0,004241 \mathfrak{H} - 0,318716 \mathfrak{J}]$$

at est

$$\mathfrak{A} b' = -0,00931 H - 0,00461 \mathfrak{H}; \text{ et } \mathfrak{A} g' = -0,00931 G - 0,00461 \mathfrak{G} \\ \text{vnde fit: } 1,867476 \mathfrak{F} = 0,004241 (\mathfrak{G} - \mathfrak{H}) \\ 2,026874 \mathfrak{G} = -0,01785 H - 0,00039 \mathfrak{H} \\ 0,159358 \mathfrak{H} = +0,01785 G - 0,01935 \mathfrak{G} \\ 0,318716 \mathfrak{J} = +0,004241 \mathfrak{H}$$

§. 245. Tum simili modo differentiando valorem ipsius v ponatur

$$F' = 1,867476 F - 0,004241 G + 0,004241 H \\ G' = 2,026834 G + 0,01091 H + 0,00750 \mathfrak{H} \\ H' = 0,159358 H + 0,03909 G - 0,00750 \mathfrak{G} \\ J' = 0,318716 F - 0,004241 H$$

vi

$$\text{vt fit } \frac{dv}{dr} =$$

$$A' \sin 2\eta - F' f \sin 2\eta - G' f \sin(2\Phi-2\pi) \\ - H' f \sin(2\theta-2\pi) - J' f \sin(4\theta-4\pi)$$

eritque

$$\frac{ddv}{dr^2} = f \cos 2\eta [-2\kappa F' + 0,004241 G' + 0,004241 H' \\ f \cos(2\Phi-2\pi) [+ A' b' - 2,026834 G' - 0,004221 H' \\ f \cos(2\theta-2\pi) [+ A' g' - 0,159358 H' - 0,023965 G' \\ f \cos(4\theta-4\pi) [+ 0,004241 H' - 0,318716 J']$$

$$\text{feu } \frac{ddv}{dr^2} =$$

$$f \cos 2\eta \{-3,48745 F + 0,01668 G - 0,00721 H - 0,00003 \mathfrak{G} \\ + 0,00007 \mathfrak{H} \\ fcl(2\Phi-2\pi) \{-4,10820 G - 0,05121 H - 0,02929 \mathfrak{H} + 0,00003 \mathfrak{G} \\ fcl(2\theta-2\pi) \{-0,02565 H - 0,08323 G - 0,01290 \mathfrak{G} - 0,00018 \mathfrak{H} \\ fcl(4\theta-4\pi) \{-0,10158 J + 0,00016 G + 0,00202 H - 0,00003 \mathfrak{G}$$

§. 246. Hinc pro $\mathfrak{F}$ et $\mathfrak{J}$ substitutis valoribus

$$\mathfrak{F} = 0,00227 (\mathfrak{G} - \mathfrak{H}) \text{ et } \mathfrak{J} = 0,01331 \mathfrak{H}$$

habebimus has aequationes

$$+ 2,47154 F - 0,01668 G + 0,00721 H = 0,031478 \\ - 0,00456 \mathfrak{G} + 0,00456 \mathfrak{H} \\ + 3,09229 G + 0,05185 H + 0,00218 \mathfrak{H} = 0,995648 \\ - 2,01799 \mathfrak{G} \\ - 0,99026 H + 0,08387 G - 0,01421 \mathfrak{G} = -0,047722 \\ - 2,01780 \mathfrak{H} \\ - 0,91433 J - 0,00016 G - 0,00202 H = -0,001123 \\ + 0,00003 \mathfrak{G} - 0,02685 \mathfrak{H}$$

D d 2

§. 247.

45648

47722

41. $\mathfrak{H}$ 21. $\mathfrak{H}$ 58. $\mathfrak{H}$ 461. $\mathfrak{G}$

)

3

lorem

H

 $\mathfrak{H}$ $\mathfrak{G}$

vi

§. 247. Deinde reperitur

$$\Theta = -0,00881 H - 0,00002 G$$

$$\Phi = +0,11201 G + 0,00107 H$$

qui valores substituti praebeant:

$$+2,47154 F - 0,01618 G + 0,00725 H = 0,031478$$

$$+3,09256 G + 0,06964 H = 0,995648$$

$$+0,99229 H + 0,14239 G = 0,047722$$

$$-0,91430 J - 0,00297 G - 0,00205 H = -0,001123$$

vnde tandem reperitur

$$F = +0,014830 \quad / \quad F = 8,171166$$

$$G = +0,321910 \quad / \quad G = 9,507734$$

$$H = +0,001901 \quad / \quad H = 7,278927$$

$$J = +0,000180 \quad / \quad J = 6,256381$$

atque

$$\mathfrak{F} = -0,000080 \quad / \quad \mathfrak{F} = 5,903090$$

$$\Theta = -0,000023 \quad / \quad \Theta = 5,361728$$

$$\Phi = +0,036056 \quad / \quad \Phi = 8,556977$$

$$\mathfrak{J} = +0,000480 \quad / \quad \mathfrak{J} = 6,681241$$

§. 248. Hinc ergo pro distantia $x = \frac{(1-kk)an}{1-k\cos r}$ reperitur

	log.coeff.	val.coeff.
$x = \text{Pr.} + 0,000084f \cos 2\eta$	5,926350	+0,000092
$+ 0,001832f \cos(2\Phi - 2\pi)$	7,262918	+0,002004
$+ 0,000011f \cos(2\theta - 2\pi)$	5,034111	+0,000012
$+ 0,000001f \cos(4\theta - 4\pi)$	4,011565	+0,000001

At

At pro motu momentaneo erit

	log.coeff.	val.coeff.
$\frac{d\Phi}{dt} = \text{Pr.} - 0,000169f \cos 2\eta$	6,227887	-0,000185
$- 0,003697f \cos(2\Phi - 2\pi)$	7,567849	-0,004043
$- 0,000227f \cos(2\theta - 2\pi)$	6,356026	-0,000248
$- 0,000005f \cos(4\theta - 4\pi)$	4,698970	-0,000005

vnde quidem iam patet inaequalitatem ab angulo $2\theta - 2\pi$ pendentem multo fore minorem, quam supra inueneramus, in quo non paruum veritatis criterium cernitur.

§. 249. Pro ipsa iam longitudine lunae ponamus:

$$\Phi = \text{Pr.} + \mathfrak{M}' \sin 2\eta + \mathfrak{F}' \sin 2\eta + \Theta' \sin(2\Phi - 2\pi) + \mathfrak{H}' \sin(2\theta - 2\pi) + \mathfrak{J}' \sin(4\theta - 4\pi)$$

atque obtinebimus has aequationes:

$$\begin{aligned} 2 \mathfrak{F}' - 0,004241 (\Theta' + \mathfrak{H}') &= -0,000169 \\ 2,026834 \Theta' + 0,004221 \mathfrak{H}' - 0,000227 \mathfrak{M}' &= -0,003697 \\ 0,159358 \mathfrak{H}' + 0,023965 \Theta' - 0,003697 \mathfrak{M}' &= -0,000227 \\ 0,318716 \mathfrak{J}' - 0,004241 \mathfrak{H}' &= -0,000005 \end{aligned}$$

vnde erit

	log. coeff.	val. tot. coeff. in min. sec.
$\Phi = \text{Pr.} - 0,000096f \sin 2\eta$	5,984018	-22"
$- 0,001823f \sin(2\Phi - 2\pi)$	7,260310	-41"
$- 0,000910f \sin(2\theta - 2\pi)$	6,959131	-20"
$- 0,000028f \sin(4\theta - 4\pi)$	5,450835	-6"

Patet ergo reuera aequationem ab angulo $2\theta - 2\pi$ ortam multo esse minorem, quam capite praecedente inueneramus; atque nunc quidem non ultra 205" seu 3', 25" ascendere. Nullum igitur est dubium, quin haec aequatio tabulas lunares ad multo maiorem perfectionem sit effectura.

Dd 3

§. 250.

§. 250. Cum igitur neglectus terminorum, minimorum tantum errorem pepererit in aequatione ab angulo $2\theta - 2\pi$ pendente, operae erit pretium, etiam aequationes insuper ab excentricitate orbitae lunaris pendentes curatius inuestigare, quae quidem aliquibus videntur esse momenti. In hunc finem ponamus.

$$\begin{aligned} fRdr = & A \cos 2\eta + E \cos r + D \cos(2\eta - r) + E \cos(2\eta + r) + 32kk \cos(2\eta - 2r) \\ & + G \cos(2\theta - 2\pi) + H \cos(2\theta - 2\pi) \\ & + 3fk \cos r + Mfk \cos(2\Phi - 2\pi - r) + Gfk \cos(2\theta - 2\pi - r) \\ & + Ofk \cos(2\Phi - 2\pi - 2r) + Tfk \cos(2\theta + 2\pi + r) \\ \text{et } v = & A \cos 2\eta + D \cos(2\eta - r) + E \cos(2\eta + r) - 14kk \cos(2\eta - 2r) \\ & + G \cos(2\Phi - 2\pi) + H \cos(2\theta - 2\pi) \\ & + Jfk \cos r + Mfk \cos(2\Phi - 2\pi - r) + Sfk \cos(2\theta - 2\pi - r) \\ & + Ofk \cos(2\Phi - 2\pi - 2r) + Tfk \cos(2\theta - 2\pi + r) \end{aligned}$$

§. 251. His valoribus in aequationibus nostris principalibus substitutis habebimus:

$$R = fk \sin r \quad (o)$$

$$fk \sin(2\Phi - 2\pi - r) \quad (0,00854S + 0,01708H)$$

$$fk \sin(2\Phi - 2\pi - 2r) \quad (0,01708S)$$

$$fk \sin(2\theta - 2\pi - r) \quad (-0,00854M - 0,01708G)$$

$$fk \sin(2\theta - 2\pi + r) \quad (-0,01708G)$$

et:

$$\frac{ddv}{dr^2} =$$

$$\begin{aligned} f k \cos r \quad & \left[\begin{aligned} & -6J - 2K - 0,00290 - 0,00893 - 0,00278 \\ & -0,00098 - 0,00098 - 0,00448 \frac{A}{nn} + 0,00470 \frac{A}{nn} \\ & + 0,01315 \frac{A}{nn} + 0,01315 \frac{A}{nn} + 0,0263 \frac{D}{nn} \\ & + 0,0263 \frac{E}{nn} \end{aligned} \right] \end{aligned}$$

fk

$$\begin{aligned} & \left[\begin{aligned} & -6M - 2N + \frac{1}{2}G + 0,00427S - 0,00854H \\ & -0,02711G - 0,03634S + \frac{G}{nn} + \frac{3G}{nn} \\ & + 0,55422H + 0,51332H - \frac{3AH}{2nn} + 0,01796 \\ & + 0,00145 + 0,001969 - \frac{1}{2} + 0,00896 \frac{A}{nn} \\ & + 0,01049 \frac{A}{nn} - 0,02631 \frac{A}{nn} - 0,00197 \frac{A}{nn} \\ & - 0,00129 \frac{A}{nn} - 0,05261 \frac{D}{nn} - 0,00258 \frac{E}{nn} \end{aligned} \right] \\ & f k \cos(2\Phi - 2\pi - r) \quad \left[\begin{aligned} & -6O - 2P + \frac{1}{2}M + \frac{1}{2}G - \frac{10}{nn}H + \frac{54}{nn}H \\ & - \frac{3DH}{2nn} + 0,7698 + 0,00898 + 0,00098 \\ & + 0,00072 - \frac{1}{2} + 0,00448 \frac{A}{nn} + 0,0052 \frac{A}{nn} \\ & - 0,00064 \frac{A}{nn} + 0,00896 \frac{D}{nn} - 0,01315 \frac{A}{nn} \\ & - 0,02630 \frac{D}{nn} + 0,01049 \frac{E}{nn} - 0,00129 \frac{E}{nn} \\ & - 0,00129 \frac{E}{nn} - 0,00042 \end{aligned} \right] \\ & f k \cos(2\Phi - 2\pi - 2r) \quad \left[\begin{aligned} & -6S - 2T + \frac{1}{2}H + 0,00427M - 0,00854G \\ & -0,02711M - 0,03634M + \frac{G}{nn} + \frac{3G}{nn} \\ & - 0,01606G - 0,02844G - \frac{3AG}{2nn} - 0,00896 \\ & - 0,00327 + 0,026307 + 0,00129 - 0,00179 \frac{A}{nn} \\ & - 0,00145 \frac{A}{nn} + \frac{A}{2nn} - 0,00394 \frac{E}{nn} + \frac{E}{nn} \end{aligned} \right] \\ & f k \end{aligned}$$

$$\begin{aligned}
 & \left[-\mathfrak{E}T - 2\kappa\mathfrak{E} + \frac{1}{2}bH + 0,00854G + \frac{\mathfrak{C}}{nn}\mathfrak{H} + \frac{3\mathfrak{C}}{nn}H \right. \\
 & \left. + 0,55422\mathfrak{G} + 0,51332G - \frac{3AG}{2nn} + 0,00940 \right. \\
 & \left. fk\cos(2\theta - 2\pi + r) - 0,01049 + 0,026307 + 0,00129 - 0,00556\frac{A}{nn} \right. \\
 & \left. - 0,00145\frac{A}{nn} - 0,003938\frac{A}{nn} + \frac{A}{2nn} \right. \\
 & \left. - 0,0394\frac{D}{nn} + \frac{D}{nn} \right]
 \end{aligned}$$

§. 252. Hae autem formulae intricatae reducuntur ad sequentes

$$\begin{aligned}
 \frac{ddv}{dr} &= Pr. + fk\cos r (-\mathfrak{E}J - 2\kappa\mathfrak{J} - 0,01182) \\
 & fk\cos(2\phi - 2\pi - r) (-\mathfrak{E}M - 2\kappa\mathfrak{M} - 0,03207S + 0,53619) \\
 & \quad - 0,02711\mathfrak{S} \\
 & fkk\cos(2\phi - 2\pi - r) (-\mathfrak{E}O - 2\kappa\mathfrak{O} + 1,52115M + 0,76615) \\
 & fk\cos(2\theta - 2\pi - r) (-\mathfrak{E}S - 2\kappa\mathfrak{S} - 0,03207M + 0,00286) \\
 & \quad - 0,02711\mathfrak{M} \\
 & fk\cos(2\theta - 2\pi + r) (-\mathfrak{E}T - 2\kappa\mathfrak{T} + 0,38147)
 \end{aligned}$$

§. 253. Nunc eosdem valores ex formulis assumtis eruamus, ac posito more adhuc recepto $\frac{2\kappa J + \mathfrak{J}}{nn} = v'$, $\frac{2\kappa M + \mathfrak{M}}{nn} = m'$ etc. erit

$$\begin{aligned}
 \frac{d\phi}{dr} &= a + \frac{1}{n} - a'\cos 2\eta - d'k\cos(2\eta - r) - g'f\cos(2\phi - 2\pi) \\
 & \quad - e'k\cos(2\eta + r) - b'f\cos(2\theta - 2\pi) \\
 & - i'fk\cos r - m'fk\cos(2\phi - 2\pi - r) - s'fk\cos(2\theta - 2\pi - r) \\
 & - n'fkk\cos(2\phi - 2\pi - 2r) - t'fk\cos(2\theta - 2\pi + r)
 \end{aligned}$$

$$\frac{d\eta}{dr} = + \frac{3\mathfrak{C}}{nn}H$$

$$00940$$

$$0556\frac{A}{nn}$$

$$\frac{d\theta}{dr} = + \frac{A}{2nn}$$

$$-0,0$$

$$-0,0$$

$$-0,0$$

$$+ 0$$

$$+ 0$$

$$615)$$

$$86)$$

$$tie$$

$$qu$$

$$um-$$

$$= v'$$

$$- 2\pi)$$

$$- 2\pi)$$

$$- r)$$

$$+ r)$$

$$d\eta$$

$$\begin{aligned}
 \frac{d\eta}{dr} &= a - a'\cos 2\eta - e'k\cos r - d'k\cos(2\eta - r) - g'k\cos(2\phi - 2\pi) \\
 & \quad - e'k\cos(2\eta + r) - b'k\cos(2\theta - 2\pi) \\
 & \quad - i'fk\cos r - m'fk\cos(2\phi - 2\pi - r) - s'fk\cos(2\theta - 2\pi - r) \\
 & \quad - n'fkk\cos(2\phi - 2\pi - 2r) - t'fk\cos(2\theta - 2\pi + r)
 \end{aligned}$$

$$\frac{d\theta}{dr} = \frac{1}{v} + \frac{2}{n}k\cos r + \frac{3}{2n}kk\cos 2r \text{ atque}$$

$$\frac{d\pi}{dr} =$$

$$-0,004241 + 0,004241\cos(2\phi - 2\pi)$$

$$-0,004221\cos 2\eta + 0,004241\cos(2\theta - 2\pi)$$

$$-0,01766k\cos r - 0,00996k\cos(2\eta - r) - 0,010636kk\cos(2\eta - 2r)$$

$$- 0,00831k\cos(2\eta + r) - 0,021273kk\cos 2r$$

$$+ 0,00924k\cos(2\phi - 2\pi - r) + 0,00841k\cos(2\theta - 2\pi - r)$$

$$+ 0,00841k\cos(2\phi - 2\pi + r) + 0,00924k\cos(2\theta - 2\pi + r)$$

$$+ 0,01064kk\cos(2\phi - 2\pi - 2r)$$

§. 254. His iam valoribus introducendis differentiemus formulas nostras assumtas pro $\int R dr$ et v ; atque obtinebimus primo:

$R =$ Praec.

$$\begin{aligned}
 & + f k \sin r \left\{ + 0,00924\mathfrak{G} + 0,00841\mathfrak{H} - 0,004241\mathfrak{M} \right. \\
 & \quad \left. - 0,00841\mathfrak{G} - 0,00924\mathfrak{H} - 0,004241\mathfrak{S} \right. \\
 & \quad \left. + 0,004241\mathfrak{E} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + f k \sin(2\phi - 2\pi - r) \left\{ + \mathfrak{M}' - 0,01766\mathfrak{G} - 0,00996\mathfrak{H} \right. \\
 & \quad \left. - 0,026834\mathfrak{M} - 0,004221\mathfrak{S} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + f k \sin(2\phi - 2\pi - 2r) \left\{ + \mathfrak{D}' + 32\mathfrak{H}' - 0,02127\mathfrak{G} - 0,01064\mathfrak{H} \right. \\
 & \quad \left. - 0,026834\mathfrak{O} - 0,01766\mathfrak{M} - 0,00996\mathfrak{S} \right\}
 \end{aligned}$$

Et

+

$$+fk\sin(2\theta-2\pi-r)\left\{\begin{array}{l} -M' + D'b' + G'e' - \frac{2}{n}h + 0,840642S + M' \\ -G'e' - 0,00831G - 0,01766h - 0,004221M \end{array}\right\}$$

$$+fk\sin(2\theta-2\pi+r)\left\{\begin{array}{l} -D'g' + E'b' + G'd' - \frac{2}{n}h - 1,159358E \\ + 0,00996G - 0,01766h \end{array}\right\}$$

§. 255. Hinc igitur consequimur istas aequationes

$$0,00083 (G-H) - J' - 0,004241 (M+S-T) = 0$$

$$M' - 0,01766G - 0,00996h - 0,004221S - 1,026834M = 0,01708H + 0,00854S$$

$$D's' + 32b' - 0,002127G - 0,01064h - 0,026834D - 0,01766M - 0,00996S = 0,01708S$$

$$-M' + D'b' - G'e' + G'e' - 0,00831G - \frac{2}{n}h - 0,01766h + M' - 0,004221M + 0,840642S = -0,01708G - 0,00854M$$

$$-D'g' + E'b' + G'd' - 0,00996G - \frac{2}{n}h - 0,01766h - 1,159358E = -0,01708G$$

§. 256. Ponatur nunc ulterius:

$$J' = J - 0,00083 (G-H) + 0,004241 (M+S-T)$$

$$M' = 1,026834M - A's' + 0,01766G + 0,00996H + 0,004221S$$

$$O' = 0,026834O - D's' + 14b' + 0,02127G + 0,01064H + 0,01766M + 0,00996S$$

$$S' = -0,840642S + A'm' - D'b' + E'g' - G'e' + 0,00831G + \frac{2}{n}H + 0,01766H - M' + 0,004221M$$

$$T' = 1,159358T + D'g' - E'b' - G'd' + 0,00996G + \frac{2}{n}H + 0,01766H$$

eritque

M'
 $1M$

$18E$

ones

$n =$

$6M$

n'

$1M$

$=$

$1S$

H

S

H

I

I

I

eritque $\frac{d\delta v}{dr} = \text{Præc.}$

$$+fk\cos r [+0,01766 (G'+H') - J' + 0,004241 (M'+S'+T')]$$

$$+fk\cos (2\varphi-2\pi-r) \left\{ \begin{array}{l} A's' - 0,01766 G' - 0,00996 H' \\ - 0,004221 S' - 1,026834 M' \end{array} \right\}$$

$$+fk\cos (2\varphi-2\pi-2r) \left\{ \begin{array}{l} D's' - 2,6b' - 0,02127 G' - 0,01064 H' \\ - 0,01766 M' - 0,00996 S' \\ - 0,026834 O' \end{array} \right\}$$

$$+fk\cos (2\theta-2\pi-r) \left\{ \begin{array}{l} A'm' + D'b' + E'g' + G'e' + 0,840642 S' \\ - 0,00831 G' - \frac{2}{n} H' - 0,01766 H' \\ + M'a' - 0,004221 M' \end{array} \right\}$$

$$+fk\cos (2\theta-2\pi+r) \left\{ \begin{array}{l} D'g' + E'b' + G'd' - 0,00996 G - \frac{2}{n} H' \\ - 0,01766 H' - 1,159358 T' \end{array} \right\}$$

§. 257. Prioris ordinis aequationes huc reducuntur:

$$J = -0,004241 (M+S+T)$$

$$1,026834 M = -0,01785 S - 0,00883 G - 0,00039$$

$$0,026834 O = -0,05835 S - 0,03041 G - 0,01766 M + 0,00690$$

$$-0,840642 S = +0,01785 M - 0,01935 M + 0,00263$$

$$1,159358 E = +0,01247$$

Hinc fit

$$J = +0,00007 S + 0,00008 M + 0,00006 \text{ seu } J = 0$$

$$M = -0,01738 S + 0,00018 M - 0,00035$$

$$O = -2,16266 S + 0,02394 M + 0,26092$$

$$G = -0,00040 S - 0,02123 M - 0,00313$$

$$E = -0,01076$$

E e 2 §. 258.

§. 258. Porro reperietur

$$J' = J + 0,004241 (M + S - T) - 0,00026$$

$$M' = 1,026834 M + 0,01935 S - 0,00016 M + 0,00568$$

$$O' = 0,026834 O - 0,37652 S + 0,01805 M + 0,01063$$

$$S' = 0,840642 S + 0,00013 S + 0,00883 M - 0,00167$$

$$T' = 1,159358 T + 0,01023$$

ac succinctius habebitur $\frac{ddv}{dr^2} = \text{Præc.}$

$$+fk \cos r [-J' + 0,004241 (M' + S' + T') + 0,01175]$$

$$+fk \cos(2\phi - 2\pi - r) \left\{ -1,026834 M' - 0,00422 S' - 0,02842 S' \right. \\ \left. + 0,00030 M - 0,01160 \right\}$$

$$+fk \cos(2\phi - 2\pi - 2r) \left\{ -0,026834 O' - 0,01766 M' - 0,00354 M' \right. \\ \left. - 0,01511 - 0,00996 S' + 0,33746 S \right\}$$

$$+fk \cos(2\theta - 2\pi - r) \left\{ +0,840642 S' - 0,02396 M' - 0,02843 M' \right. \\ \left. - 0,01472 + 0,00025 S \right\}$$

$$+fk \cos(2\theta - 2\pi + r) [-1,159358 T' + 0,33856]$$

§. 259. Hinc tandem valores quæfuit eliciuntur

$$J = -0,81144 \quad \text{. . .} \quad I - J = 9,909256$$

$$M = -1,25325 \quad \text{. . .} \quad I - M = 0,098046$$

$$O = -2,12630 \quad \text{. . .} \quad I - O = 0,327624$$

$$S = -0,13490 \quad \text{. . .} \quad I - S = 9,130012$$

$$T = -0,10080 \quad \text{. . .} \quad I - T = 9,003441$$

ac

$$+ 0,00568$$

$$+ 0,01063$$

$$- 0,00167$$

$$0,01175]$$

$$0,02842 S]$$

$$0,01160]$$

$$10354 M]$$

$$33746 S]$$

$$2843 M]$$

$$0025 S]$$

tur

ac

$$\text{ac } \mathfrak{J} = -0,00005 \quad \text{. . .} \quad I - \mathfrak{J} = 5,698970$$

$$\mathfrak{M} = +0,00177 \quad \text{. . .} \quad I - \mathfrak{M} = 7,247973$$

$$\mathfrak{O} = +0,52266 \quad \text{. . .} \quad I - \mathfrak{O} = 9,718219$$

$$\mathfrak{S} = +0,02355 \quad \text{. . .} \quad I - \mathfrak{S} = 8,371991$$

$$\mathfrak{T} = +0,01076 \quad \text{. . .} \quad I - \mathfrak{T} = 8,031812$$

§. 260. Ex his iam pro distantia Lunae a terra erit
 $u = \text{Præc.}$

	Log. coeff.	Valor. coeff. totius
— 0,00462 $fk \cos 2r$	7,664440	— 0,000275
— 0,00773 $fk \cos(2\phi - 2\pi - r)$	7,853230	— 0,000424
— 0,01210 $fk \cos(2\phi - 2\pi - 2r)$	8,082808	— 0,000039
— 0,00077 $fk \cos(2\theta - 2\pi - r)$	6,885196	— 0,000046
— 0,00057 $fk \cos(2\theta - 2\pi + r)$	6,758625	— 0,000034

et pro motu momentaneo

$$\frac{d\phi}{dr} = \text{Præc.}$$

	Log. coeff.	Val. coeff.
+ 0,00932 $fk \cos r$	7,969416	+ 0,000555
+ 0,01558 $fk \cos(2\phi - 2\pi - r)$	8,192568	+ 0,000928
+ 0,02142 $fk \cos(2\phi - 2\pi - 2r)$	8,330819	+ 0,000069
+ 0,00142 $fk \cos(2\theta - 2\pi - r)$	7,152288	+ 0,000085
+ 0,00109 $fk \cos(2\theta - 2\pi + r)$	7,037426	+ 0,000064

§. 261. Pro longitudine autem lunae sequentes
resolui debent aequationes:

$$+0,00932 = \mathfrak{J}' - 0,01766 (\mathfrak{O}' + \mathfrak{S}') - 0,004241 (\mathfrak{M}' + \mathfrak{S}' + \mathfrak{T}') \\ + 0,01558 = 1,026834 \mathfrak{M}' - \mathfrak{M}' S' + 0,01766 \mathfrak{O}' + 0,00996 \mathfrak{S}' \\ + 0,004221 \mathfrak{S}'$$

Ee 3

+

$$\begin{aligned}
 +0,02142 &= 0,026834 \mathcal{Q}' - \mathcal{D}' + 2,6b' + 0,02127 \mathcal{U}' \\
 &+ 0,01064 \mathcal{H}' + 0,01766 \mathcal{M}' + 0,00996 \mathcal{E}' \\
 +0,00142 &= -0,840642 \mathcal{E}' - \mathcal{M}' - \mathcal{D}'b' - \mathcal{E}'g' + 0,02114 \mathcal{U}' \\
 &+ 0,16853 \mathcal{H}' + 0,02396 \mathcal{M}' \\
 +0,00109 &= 1,159358 \mathcal{E}' - \mathcal{D}'g' - \mathcal{E}'b' - 0,35615 \mathcal{U}' \\
 &+ 0,16850 \mathcal{H}'
 \end{aligned}$$

hincque prodit pro longitudine vera:

	Log. coeff.	Val. coeff. in min. sec.
$\Phi = \text{Pr.} + 0,00932fk \sin r$	7,969416	+115
$+ 0,01521fk \sin(2\Phi - 2\pi - r)$	8,182130	+187
$+ 0,79079fkk \sin(2\Phi - 2\pi - 2r)$	9,898060	+529
$- 0,00121fk \sin(2\theta - 2\pi - r)$	7,083939	-15
$- 0,00082fk \sin(2\theta - 2\pi + r)$	6,913527	-10

§. 262. Ob inclinationem ergo orbitae lunaris ad eclipticam omnes correctiones huc redeunt, vt sit

I. Pro distantia lunae a terra:

$\# = \text{Pracc.}$

	Log. coeff.	Val. coeff.
$+ 0,000084f \cos 2\eta$	5,926350	+0,000092
$+ 0,001832f \cos(2\Phi - 2\pi)$	7,262918	+0,002004
$+ 0,000011f \cos(2\theta - 2\pi)$	5,034111	+0,000012
$+ 0,000001f \cos(4\theta - 4\pi)$	4,011565	+0,000001
$- 0,00462fk \cos r$	7,664440	-0,000275
$- 0,00773fk \cos(2\Phi - 2\pi - r)$	7,853230	-0,000424
$- 0,01210fkk \cos(2\Phi - 2\pi - 2r)$	8,082808	-0,000039
$- 0,00077fk \cos(2\theta - 2\pi - r)$	6,885196	-0,000046
$- 0,00057fk \cos(2\theta - 2\pi + r)$	6,758625	-0,000034

H. Pro

II. Pro motu momentaneo:

$$\frac{d\Phi}{dr} = \text{Pracc.}$$

	Log. coeff.	Val. coeff.
$- 0,000169f \cos 2\eta$	6,227887	-0,000185
$- 0,003697f \cos(2\Phi - 2\pi)$	7,567849	-0,004043
$- 0,000227f \cos(2\theta - 2\pi)$	6,356026	-0,000248
$- 0,000005f \cos(4\theta - 4\pi)$	4,698970	-0,000005
$+ 0,00932fk \cos r$	7,969416	+0,000555
$+ 0,01558fk \cos(2\Phi - 2\pi - r)$	8,192568	+0,000928
$+ 0,02142fkk \cos(2\Phi - 2\pi - 2r)$	8,330819	+0,000069
$+ 0,00142fk \cos(2\theta - 2\pi - r)$	7,152288	+0,000085
$+ 0,00109fk \cos(2\theta - 2\pi + r)$	7,037426	+0,000064

III. Pro longitudine lunae vera

	Log. coeff.	Val. coeff. in min. sec.
$\Phi = \text{Pr.} - 0,000096f \sin 2\eta$	5,984018	-22 ⁰⁰
$- 0,001823f \sin(2\Phi - 2\pi)$	7,260310	-411
$- 0,000910f \sin(2\theta - 2\pi)$	6,959131	-205
$- 0,000028f \sin(4\theta - 4\pi - r)$	5,450835	-6
$+ 0,00932fk \sin r$	7,969416	+115
$+ 0,01521fk \sin(2\Phi - 2\pi - r)$	8,182130	+187
$+ 0,79079fkk \sin(2\Phi - 2\pi - 2r)$	9,898060	+529
$- 0,00121fk \sin(2\theta - 2\pi - r)$	7,083939	-15
$- 0,00082fk \sin(2\theta - 2\pi + r)$	6,913527	-10

CAPUT XVI.

EXPOSITIO INAEQUALITATUM LUNAE
HACTENUS INVENTARUM.

§. 263.

Quas igitur inuenimus hactenus lunae inaequalitates eae primum, si originem earum spectemus, ad sex classes reducuntur. Quatenus enim luna in motu suo a regulis Keplerianis, in quibus quidem motum apogei compeſtimur, recedit, eius errores vel primo a solo lunae aspectu, seu eius distantia a sole pendent, seu quod eodem redit, per angulum η tantum definiuntur, quibus variatio lunae continetur. Ad secundam classem refero eas lunae inaequalitates, quae insuper ab excentricitate eius orbitae pendent. Tertiae classis eas complectitur inaequalitates, quae ab excentricitate orbitae solis ortum trahunt. Quarta vero eas, quae per utramque excentricitatem coniunctim determinantur. Quintae porro classi annumeramus eas inaequalitates, quae parallaxin solis inuoluunt, atque errores quatuor ante memoratorum generum implicant. Sexta denique classis suppeditat eas inaequalitates, quae praeterea ab inclinatione orbitae lunaris ad eclipticam pendent.

§. 264. Quodsi vero ad vsum harum inaequalitatum attendamus, prouti eae, ad lunam accommodari debent, tum eae in quinque classes commodissime distribuuntur. Primo enim perpendendae sunt eae inaequali-

qualitates, quarum ope vera distantia lunae a terra determinatur, vt inde porro tam lunae diameter apparens, quam eius parallaxis horizontalis assignari possit. Secundo loco formulae erunt collocandae illae, quae motui momentaneo definiendo inseruiunt, ex quibus deinceps motus lunae horarius accurate exhiberi poterit. Tertium locum occupabunt eae inaequalitates, quae veram longitudinem lunae ad eclipticam relatum praebent. Quarto vero positio lineae nodorum lunae, seu longitudo nodi ascendentis; ac quinto vera inclinatio orbitae lunaris ad eclipticam inueniri debet; vt deinde vera lunae latitudo concludi possit. Manifestum autem est, has inaequalitates plurimum inter se permisceri, ita vt vix vllum habeatur genus, cuius inaequalitates non a reliquis generibus pendeant; cui tamen incommodo facile medela adhibetur.

§. 265. Quamquam numerus inaequalitatum, quas sumus consecuti, tantopere increuit, vt calculus sine maxima molestia expediri nequeat, tamen iam monui, non omnes inaequalitates, quibus motus Lunae perturbatur, esse definitas, sed potius earum numerum omnino esse infinitum. Facile quidem intelligitur, plerasque has praetermissas inaequalitates nullius fere esse momenti, atque sine notabili errore iis supersederi posse: verum tamen sunt inter eas nonnullae, quae ad plura minuta secunda affurgere videntur, quarum argumenta supra iam inui; ex quo omnino operae esset pretium in eas omni cura inquirere. Sed earum inuestigatio tam est lubrica et incerta, vt leuissima omisso in calculo facta eas

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maxime afficiat. Cum igitur in calculo plurimos terminos reicere cogamur, istam inuestigationem frustra plane suscipere, quamdiu scilicet rem sine appropinquatione exequi non licet. Cuius defectus eximium habemus exemplum in inaequalitatibus postremo loco inuentis, quae statim atque in negligendo minus largi fueramus, mirum quantum prodierunt immutatae; ac nulum plane est dubium, si calculum adhuc accuratius prosequi liceret, quin valores inuenti notabilem insuper mutationem sint subiturae. Imprimis autem aequatio ab angulo $2\phi - 2\pi - 2r$ seu a dupla distantia apogei a nodo pendens, est suspecta, ac minime pro certa haberi potest, cum leuissima circumstantia eam magnopere perturbare valeat.

§. 266. Si enim in causam inquiramus, cur analysis posterior tam diuersos valores pro his inaequalitatibus suppeditauerit, primo quidem statim patet, neglectum litterarum germanicarum $\mathfrak{J}$, $\mathfrak{M}$, $\mathfrak{O}$, etc. in calculo priori potissimum hoc discrimen produxisse: ingens enim valor litterae $\mathfrak{O}$ imprimis aequationem ab angulo $2\phi - 2\pi - 2r$ pendente tantopere auxit. Praeterea vero etiam non parum augmenti haec aequatio inde est nata, quod in calculo posteriori rationem quoque habuimus termini $\cos(2\eta - 2r)$, qui tam in valore $\frac{d\phi}{dr}$ quam $\frac{d\eta}{dr}$ ineffe est deprehensus; vnde tuto colligere licet, si alios quoque terminos similes veluti $2\theta - 2\pi - 2r$, etsi per se sunt minimi, in calculum introduxissemus, co-

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efficientes terminorum $2\phi - 2\pi - 2r$ non mediocrem inde mutationem subituros fuisse. Quamobrem plus hinc colligere non possumus, nisi inaequalitatem Lunae ab angulo hoc $2\phi - 2\pi - 2r$ pendente minime esse contemnendam, etiam si fortasse tanta non sit, quam inuenimus. Vera autem eius quantitas certius ex observationibus quam ex Theoria colligi posse videtur.

§. 267. Quoniam vero hae inaequalitates omnes ad anomaliam Lunae veram referuntur, antequam eas ad usum adhibere liceat, modum tradi conveniet ad quodvis tempus propositum anomaliam Lunae veram determinandi. Cognita autem excentricitate orbitae lunaris k et motu anomaliae mediae, inde ad quodvis tempus facile anomalia media p colligitur. Verum ex anomalia media p et excentricitate k anomalia vera r

definiri debet ope huius aequationis $dp = \frac{(1-kk)^{\frac{3}{2}} dr}{(1-k\cos r)^2}$;

vnde quidem non difficulter, si nota esset anomalia vera r , vicissim inveniri posset anomalia media p . Calcule enim peracto, si breuitatis gratia ponatur

$$\delta = \frac{1 - \sqrt{1-kk}}{k} = \frac{1}{2}k + \frac{1.1}{2.4}k^3 + \frac{1.1.3}{2.4.6}k^5 + \text{etc.}$$

reperitur:

$$p = r + 2k \sin r + 2\delta(k - \frac{1}{2}\delta) \sin 2r + 2\delta\delta(k - \frac{1}{2}\delta) \sin 3r + 2\delta^2(k - \frac{1}{2}\delta) \sin 4r \text{ etc.}$$

cuius seriei progressio primo intuitu patet. Cum autem k ac proinde δ sit valde paruum, erit satis exacte:

Ff 2

p =

$$p = r + 2k \sin r + \left(\frac{1}{2}kk + \frac{1}{8}k^4 + \frac{3}{8}k^6\right) \sin 2r + \left(\frac{1}{8}k^3 + \frac{1}{8}k^5\right) \sin 3r \\ + \left(\frac{1}{8}k^4 + \frac{3}{8}k^6\right) \sin 4r + \frac{1}{40}k^5 \sin 5r + \frac{1}{128}k^6 \sin 6r$$

§. 268. Data ergo anomalia media Lunae p , eius anomalia vera r elici debebit ex hac aequatione

$$r = p - 2k \sin r - \left(\frac{1}{2}kk + \frac{1}{8}k^4 + \frac{3}{8}k^6\right) \sin 2r - \left(\frac{1}{8}k^3 + \frac{1}{8}k^5\right) \sin 3r \\ - \left(\frac{1}{8}k^4 + \frac{3}{8}k^6\right) \sin 4r - \frac{1}{40}k^5 \sin 5r - \frac{1}{128}k^6 \sin 6r$$

eius quidem ope, si cognita fuerit excentricitas k , calculus non difficulter expeditur. Quoniam enim termini sinus inuoluentes sunt admodum parui, in iis statim poni poterit $r = p$, unde valor verior pro r eruetur, qui deinde iterum in his terminis adhibitus, iustiore valore pro r suppeditabit. Atque hoc modo post aliquot operationes verus tandem valor pro anomalia vera r obtinebitur. Interim tamen, quo iste calculus facilius perfici queat, aequatio haec ita potest transformari, ut loco sinuum anomaliae verae r , sinus anomaliae mediae p introducantur; id quod sequenti modo praestabitur.

§. 269 Ponatur breuitatis gratia:

$$\frac{1}{2} + \frac{1}{8}kk + \frac{3}{8}k^4 = a; \quad \frac{1}{8} + \frac{1}{8}kk = b; \quad \frac{1}{8} + \frac{3}{8}kk = \gamma \\ \frac{1}{40} = \delta \quad \text{et} \quad \frac{1}{128} = \epsilon$$

ut sit:

$$r = p - 2k \sin r - ak \sin 2r - bk^3 \sin 3r - \gamma k^4 \sin 4r - \delta k^5 \sin 5r - \epsilon k^6 \sin 6r$$

ac ponatur denuo

$$2k \sin r + ak \sin 2r + bk^3 \sin 3r + \gamma k^4 \sin 4r + \delta k^5 \sin 5r + \epsilon k^6 \sin 6r = R$$

et

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$k^5 \sin 3r$

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$k = \gamma$

$\sin 6r$

$r = R$
et

et cum sit $r = p - R$ habebitur:

$$\sin r = \left(1 - \frac{1}{2}RR + \frac{1}{24}R^4\right) \sin p - \left(R - \frac{1}{8}R^3 + \frac{1}{128}R^5\right) \cos p$$

$$\sin 2r = \left(1 - 2RR + \frac{1}{3}R^4\right) \sin 2p - \left(2R - \frac{1}{3}R^3\right) \cos 2p$$

$$\sin 3r = \left(1 - \frac{3}{2}RR\right) \sin 3p - \left(3R - \frac{1}{2}R^3\right) \cos 3p$$

$$\sin 4r = \left(1 - 8RR\right) \sin 4p - 4R \cos 4p$$

$$\sin 5r = \sin 5p - 5R \cos 5p$$

$$\sin 6r = \sin 6p$$

negligendo scilicet terminos, qui ipsius k potestates sexta altiores continent.

§. 270. Euolutio autem huius calculi fit maxime prolixa, si quidem ad sextam potestatem ipsius k ascendere velimus. Facile autem expeditur, si ad quartam substituamus, tum autem reperietur:

$$r = p - \left(2k - \frac{1}{4}k^3\right) \sin p + \left(\frac{1}{4}kk - \frac{1}{4}k^4\right) \sin 2p - \frac{1}{4}k^3 \sin 3p - \frac{1}{4}k^4 \sin 4p$$

quae expressio satis accurate pro quavis anomalia media p conuenientem anomaliam veram indicabit. Calculus autem, si excentricitas k constet, facili negotio absoluetur. Formula haec quoque ita repraesentari poterit, ut sit

$$r = p - 2k \left(1 - \frac{1}{8}kk\right) \sin p + \frac{1}{4}kk \left(1 - \frac{1}{8}kk\right) \sin 2p - \frac{1}{4}k^3 \sin 3p - \frac{1}{4}k^4 \sin 4p$$

Hinc igitur tabulam construere conueniet, unde pro quavis anomalia media proposita ipsi respondens anomalia vera excerpi queat.

§. 271. Cum autem inuenta fuerit anomalia vera r , longitudo Lunae regula Kepleriana inuenienda, quam
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supra (206) per ξ indicauimus, ita exprimetur, vt sit

$$\xi = C + 1,0085272p$$

$$-1,0085272 \left(2k \sin r + \frac{1}{2} k^2 (1 + \frac{1}{2} k k + \frac{1}{8} k^4) \sin 2r + \frac{1}{3} k^3 (1 + \frac{1}{2} k k) \sin 3r \right. \\ \left. + \frac{1}{4} k^4 (1 + \frac{1}{2} k k) \sin 4r + \frac{1}{5} k^5 \sin 5r + \frac{1}{6} k^6 \sin 6r \right)$$

vbi $C + 1,0085272p$ exhibet longitudinem Lunae mediam; quae si vocetur $= \xi$, atque in coefficientium partibus minimis pro k scribatur valor proximus 0,0545, erit

	log. coeff.	val. in min. sec.
$\xi = \xi - 2,0170544k \sin r$	0,304718	22675'' = 6°, 17', 55''
$- 0,756770k^2 \sin 2r$	9,878964	464 = 7, 44
$- 0,336551k^3 \sin 3r$	9,527051	11½
$- 0,15786k^4 \sin 4r$	9,198282	½

vnde patet superfluum futurum fuisse, si superiores expressiones ultra quartam potestatem ipsius k extendere voluissimus.

vt sit

$$\frac{1}{2} k k \sin 3r$$

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CAPUT XVII.

INVESTIGATIO ELEMENTORUM
MOTUS LUNAE

§. 272.

Inuentis iam per Theoriam hisce inaequalitatibus, quibus motus Lunae perturbatur, antequam eas ad computum astronomicum accommodare liceat, elementa, quae in eas ingrediuntur, per obseruationes determinari oportet. Primo scilicet ad datam epocham cum longitudo Lunae media, tum eius anomalia media, ac locus nodi medius constitui debet, vt eadem res inde ad quoduis aliud tempus assignari queant. Deinde quoque ex obseruationibus verus valor excentricitatis lunaris colligi debet, a quo potissimum quantitas praecipuarum inaequalitatum pender. Excentricitas autem orbitae solaris pro satis certa haberi poterit, cum sit $e = 0,0168$. Lunae vero excentricitas tam prope iam constat, vt inde sine errore ad quamlibet anomalias mediam vera satis exacte assignari possit. Est enim in anomalia vera error aliquot minutorum primorum committitur, inaequalitates Lunae inde non ultra aliquot minuta secunda afficiuntur.

§. 273. Quodsi autem statim quasuis Lunae obseruationes ad hunc finem adhibere velimus, ob tam ingentem inaequalitatum numerum, investigatio elementorum maxime molesta redderetur. Quocirca ex obser-

observationibus eas eligi conveniet, pro quibus numerus inaequalitatum multo fiat minor; dum scilicet distantia Lunae a sole seu angulus η datum obtinet valorem. Commodissimae ergo erunt eae observationes, quae in ipsis momentis coniunctionis vel oppositionis sunt institutae. Accuratas itaque observationes eclipsium lunarium ad hoc negotium adhibebo, quoniam praeter haec tempora, vera vel coniunctionis vel oppositionis momenta non satis certo ex observationibus colligi licet.

§. 274. Momento autem oppositionis verae Lunae et Solis, longitudo Lunae sex signis distat a longitudine solis, ita ut sit $\theta = \Phi \pm 180^\circ$, ideoque angulus $\eta = 180^\circ$. Posito autem pro η hoc valore longitudo Lunae vera Φ ex media ξ per sequentes formulas definiatur, in quas formulae hactenus inuentae abeunt:

$$\Phi = \xi - 2,0170544k \sin r - 0,756770kk \sin 2r - 0,33655k^3 \sin 3r$$

$$+ 0,0101460 + 0,004200$$

$$+ 0,4202260 - 0,573280$$

$$+ 0,0049920 + 0,003180$$

$$- 0,0052860 + 0,150830$$

$$- 0,000860 - 0,000002$$

$$+ 1,1959v$$

$$- 0,0757v$$

$$+ 0,00932f$$

$$+ (0,201385 + 0,021889 - 0,016368 - 0,39597 + 4,1738v) \sin r$$

$$+ (0,06645 - 0,02332 + 0,00840) \sin 2r$$

$$+ (0,74760 - 0,81430 - 0,01420) k \sin(r - r)$$

$$+ (-0,61850 - 0,23960 - 0,00610) k \sin(r + s)$$

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$$- (0,002823 + 0,000910) f \sin(2\Phi - 2\pi)$$

$$- 0,000028 f \sin(4\Phi - 4\pi)$$

$$+ (0,01521 - 0,00121) f k \sin(2\Phi - 2\pi - r)$$

$$+ 0,79079 f k k \sin(2\Phi - 2\pi - 2r)$$

§. 275. Cum igitur sit $f = 1,09375$, et $v = \frac{1}{27v}$, erit has formulas colligendo:

$$\Phi = \xi - 1,572993k \sin r - 1,17186kk \sin 2r - 0,3365k^3 \sin 3r$$

$$+ 0,21998e \sin s + 0,05123ee \sin 2s - 0,0809ek \sin(r - s)$$

$$- 0,8642ek \sin(r + s)$$

$$- 0,002989 \sin(2\Phi - 2\pi) + 0,01531k \sin(2\Phi - 2\pi - r)$$

$$- 0,000031 \sin(4\Phi - 4\pi)$$

$$+ 0,86493kk \sin(2\Phi - 2\pi - 2r)$$

et cum sit $e = 0,0168$, erit hoc valore substituto:

$$\Phi = \xi - 1,572993k \sin r - 1,17186kk \sin 2r - 0,3365k^3 \sin 3r$$

$$+ 0,003697 \sin s + 0,000014 \sin 2s - 0,001359k \sin(r - s)$$

$$- 0,014523k \sin(r + s)$$

$$- 0,002989 \sin(2\Phi - 2\pi) + 0,01531k \sin(2\Phi - 2\pi - r)$$

$$- 0,000031 \sin(4\Phi - 4\pi)$$

$$+ 0,86493kk \sin(2\Phi - 2\pi - 2r)$$

§. 276. Assumta iam hypothefi quapiam non nimis a vero aberrante, vnde ad datum quoduis tem-

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pus definiri possit tam longitudo lunae, quam eius anomalia media, ex qua praeterea ope excentricitatis proxime cognitae anomalia vera assignari queat: haec elementa correctione indigebunt, quam ex observationibus elici oporteat. Ponamus ergo longitudinem mediam ex tabulis desumptam augeri debere n minutis secundis. Tum vero excentricitas supposita, quae sit $= 0,0545$, augeri debeat $\frac{n}{10000}$, ut sit $k = 0,0545 + \frac{n}{10000}$: ipsa vero anomalia vera tabularis, quae sit $= v$, augmentum requirat μ minutorum secundorum, ut sit $r = v + \mu''$: eritque $\sin r = \sin v + \mu'' \cos v$; $\sin 2r = \sin 2v + 2\mu'' \cos 2v$; et $\sin 3r = \sin 3v$ in terminis enim minimis haec correctio praetermitti poterit.

§. 277. Quod si haec omnia in minuta secunda conuertantur, prodibit longitudo lunae vera

$$\Phi = \text{Long. med.} + n''$$

$$\begin{aligned} & -17682'' \sin v - 32,445 n'' \sin v - 0,085728 \mu'' \cos v \\ & - 718'' \sin 2v - 2,635 n'' \sin 2v - 0,006962 \mu'' \cos 2v \\ & - 11'' \sin 3v + 762'' \sin s + 3'' \sin 2s \\ & - 15'' \sin (r-s) - 163'' \sin (r+s) \\ & - 616'' \sin (2\Phi - 2\pi) + 172'' \sin (2\Phi - 2\pi - r) \\ & - 6'' \sin (4\Phi - 4\pi) + 530 \sin (2\Phi - 2\pi - 2r) \end{aligned}$$

Cum

Cum autem postremus terminus sit suspectus, loco eius coefficientis 530 malumus ponere coefficientem indefinitum 1000, atque ex observationibus valorem ipsius y indagare. Deinde sit error anomaliae verae i minutorum primorum, ut calculus commodior reddatur, atque ob $\mu = 60i$, neglectis terminis minimis erit:

$$\Phi = \text{Long. med.} + n''$$

$$\begin{aligned} & -17682'' \sin v - 32,445 n'' \sin v - 5,143 i'' \cos v \\ & - 718'' \sin 2v - 2,635 n'' \sin 2v - 0,417 i \cos 2v \\ & + 762 \sin s - 15 \sin (r-s) - 163 \sin (r+s) \\ & - 616 \sin (2\Phi - 2\pi) + 172'' \sin (2\Phi - 2\pi - r) \\ & + 1000 \sin (2\Phi - 2\pi - 2r) \end{aligned}$$

§. 278. Oblata autem observatione eclipsis lunae, quaeratur primum momentum medium huius eclipsis, pro quo colligatur longitudo solis, itemque longitudo nodi ascendentis. Punctum autem soli oppositum nondum erit longitudo lunae vera in ecliptica; verumtamen longitudo lunae pro hoc momento eclipsis medio inueniri poterit ope sequentis tabellae.

Gg 2

Sub-

Subtrahatur longitudo nodi a longitudine solis, et aequatio tabulae secundum titulos adscriptos applicetur puncto soli opposito in ecliptica.

gr.	{ O Sign. VI. Sign. subtrahere }		
	O'	O''	
0	0,	0''	30
1	0,	32	29
2	1,	6	28
3	1,	39	27
4	2,	12	26
5	2,	45	25
6	3,	17	24
7	3,	49	23
8	4,	21	22
9	4,	53	21
10	5,	24	20
11	5,	56	19
12	6,	26	18
	{ V Sign. XI. Sign. }		gr.

§. 279. Quanquam autem hoc momento, ad quod lunae longitudinem hinc colligimus; non vera lunae oppositio existit, sed luna secundum longitudinem a puncto soli opposito distat particula, quam haec tabula monstrat; tamen tuto pro hoc momento ex formula nostra longitudinem lunae inuestigare poterimus, visuri, quam

quam exacte ea conveniat cum longitudine eius ad hoc tempus ex observatione conclusa. Cum enim luna hoc tempore nunquam ultra 5' a vero oppositionis loco distet, si formula nostra generali uti vellemus, foret angulus 7 minor 5 minutis primis; vnde facile perspicitur, discrimen in loco lunae inde oriundum vix unquam 12'' esse iuperaturum. Quoniam itaque medium cuiusque eclipsis momentum ipsum tam accurate definiri nequit, ut non error dimidii minuti primi sit pertimescendus, superfluum sane foret in calculo ad istiusmodi minutias attendere.

§. 280. Hanc ob causam quoque ex calculo, quem inibo, non summam praecisionem expectari conveniet; quia ipsae observationes, quibus utar, non plenae accurationis sunt capaces. Plus igitur me non effecturum confido, quam ut satis prope tam excentricitatem orbitae lunaris, quam longitudinem et anomaliam lunae media ad datam epocham definiam. Quod cum fuerit factum maiori confidentia theoriam ad quasvis alias observationes transferre licebit; quae si nullis erroribus fuerint inquinatae, non admodum erit difficile reliquas elementorum correctiones, quibus formulae nostrae sunt innixae, inde concludere. Imprimis autem hic calculus veram excentricitatem orbitae lunaris satis exacte manifestabit, ut deinceps accuratius pro quavis anomalia media convenientem anomaliam veram definire valeamus. Hunc igitur in finem nonnullas eclipses lunares Parisiis institutas calculo subiiciam.

§. 281. Primae igitur eclipsis medium contigisse reperio Parisiis A. 1712. Jan. 23^d, 7^h, 55ⁱ, 16ⁱⁱ temp. medio. Pro quo momento colligitur:

Longitudo solis θ . . . 10°, 3°, 0', 54"

Anomalia vera solis s . . . 6, 24, 25, 13

Deinde ex tabulis meis-

Longitudo lunae media . . . 4, 7, 18, 55

Anomalia lunae media . . . 2, 0, 18, 20

Anomalia lunae vera $v =$ 1, 25, 6, 27

Longitudo nodi vera $\pi =$ 9, 24, 34, 32

Dist. nodi a sole $\theta - \pi =$ 0, 8, 26, 22

Hinc aequatio loci lunae . . . 4, 33

Ergo longitudo lunae vera $\phi =$ 4, 2, 56, 21

§. 282. Hinc calculus sequenti modo instituetur:

$v =$ 1, 25, 6, 27 ; $\sin v = + \sin 55^\circ, 6', 27''$
 $\cos v = +$

$2v =$ 3, 20, 12, 54 ; $\sin 2v = + \sin 69, 47, 6$
 $\cos 2v = -$

$s =$ 6, 24, 25, 13 ; $\sin s = - \sin 24, 25, 13$

$v - s =$ 7, 0, 41, 14 ; $\sin = - \sin 30, 41, 14$

$v + s =$ 8, 19, 31, 40 ; $\sin = - \sin 79, 31, 40$

$\phi - \pi =$ 6, 8, 21, 49

$2\phi - 2\pi =$ 0, 16, 43, 38 ; $\sin = + \sin 16, 43, 38$

$r =$ 1, 25, 6, 27

$2\phi - 2\pi + r =$ 10, 21, 37, 11 ; $\sin = - \sin 38, 22, 49$

$2\phi - 2\pi + 2r =$ 8, 26, 30, 44 ; $\sin = - \sin 86, 30, 44$

+

igisse
temp.

$$\begin{array}{r} + 9,91393 + 9,9139 + 9,7575 \\ - 4,24753 - 1,5111 - 0,7104 \\ - 4,16146 - 1,42508 - 0,46792 \end{array}$$

$$\begin{array}{r} + 9,9724 + 9,9724 - 9,5385 \\ - 2,8561 - 0,4208 - 9,6201 \\ - 2,8285 - 0,39328 + 9,15862 \end{array}$$

$$\begin{array}{r} - 9,6163 - 9,7078 - 9,9927 \\ + 2,8819 - 1,1761 - 2,2122 \\ - 2,4982 + 0,8839 + 2,2049 \end{array}$$

$$\begin{array}{r} + 9,4588 - 9,7930 \\ - 2,7896 + 2,2355 - 99,87 \\ - 2,2484 - 2,0285 \end{array}$$

aeq. +	aeq. -	- 26, 68
+ 8	- 14493	- 2, 58
+ 160	- 674	- 2, 92
+ 168	- 315	+ 0, 12
- 15766	- 177	- 99, 87
- 15598	- 107	
- 259', 58"	- 15766	
- 4°, 19, 58 . .	aequatio	

Long. med. 4, 7, 18, 55 + m

aeq. - 4, 19, 58

$$4, 2, 58, 57 - 29, 18 - 2, 82 - 99, 87 = 4, 2, 56, 21$$

Ergo $\phi = 2, 36 - 29, 18 - 2, 82 - 99, 87 + m$

§. 283. Secundae eclipsi medium contigit:

Parisiis A. 1713. Dec. 1^a, 15^b, 26^c, 34^d temp. med.

Pro quo momento colligitur

Longitudo Solis θ	$\equiv$	8, 9, 53, 40
Anomalia vera Solis s	$\equiv$	5, 1, 46, 43
Longitudo Lunae media	$\equiv$	2, 5, 2, 26
Anomalia Lunae media	$\equiv$	9, 12, 27, 42
Anomalia Lunae vera v	$\equiv$	9, 18, 24, 49
Longitudo nodi π	$\equiv$	8, 17, 46, 10
Distantia Solis a nodo	$\equiv$	11, 22, 7, 30
Aequatio loci Lunae	$\equiv$	+ 4, 17
Longitudo Lunae vera ϕ	$\equiv$	2, 9, 57, 57

§. 284. Hinc calculus sequens instituitur:

v	$\equiv$	9, 18, 24, 49	; sin v	$\equiv$	- sin 71, 35, 11
			cos v	$\equiv$	+
$2v$	$\equiv$	7, 6, 49	; sin $2v$	$\equiv$	- sin 36, 49
			cos $2v$	$\equiv$	-
s	$\equiv$	5, 1, 46	; sin s	$\equiv$	+ sin 28, 14
$v-s$	$\equiv$	4, 16, 39	; sin	$\equiv$	+ sin 43, 21
$v+s$	$\equiv$	2, 20, 11	; sin	$\equiv$	+ sin 80, 11
$\phi-\pi$	$\equiv$	11, 22, 12			
$2\phi-2\pi$	$\equiv$	11, 14, 24	; sin	$\equiv$	- sin 15, 36
r	$\equiv$	9, 18, 25			
$2\phi-2\pi-r$	$\equiv$	1, 25, 59	; sin	$\equiv$	+ sin 55, 59
$2\phi-2\pi-2r$	$\equiv$	4, 7, 34	; sin	$\equiv$	+ sin 52, 26

- 9,97717	-	9,9772	+ 9,4996
- 4,24753	-	1,5111	- 0,7104
+ 4,22470	+	1,4883 ⁿ	- 0,2100 ⁱ
- 9,7776	-	9,7776	- 9,9034
- 2,8561	-	0,4208	- 9,6201
+ 2,6337	+	0,1984 ⁿ	+ 9,5235 ⁱ
+ 9,6749	+	9,8366	+ 9,9936
+ 2,8819	-	1,1761	- 2,2122
+ 2,5568	-	1,0127	- 2,2058
- 9,4296	+	9,9185	+ 79, 2 ^j
- 2,7866	+	2,2355	
+ 2,2192	+	2,1530	

aeq. aff.	aequat.	
+ 16776	- 10	+ 30, 8 ⁿ
+ 430	- 161	+ 1, 6 ⁿ
+ 360	- 171	- 1, 6 ⁱ
+ 166		+ 0, 3 ⁱ
+ 143		
+ 17875		
- 171		
+ 17704		
+ 2961 ^d , 4 ^h	aequat.	

Long. media	$\equiv$	2, 5, 2, 26	+ m
aeq.	$\equiv$	+ 4, 55, 4	
Long. vera	$\equiv$	2, 9, 57, 30	+ m
obf.	$\equiv$	2, 9, 57, 57	

Ergo ϕ $\equiv$ - 0', 27^h + m + 32, 4ⁿ - 1, 3ⁱ + 79, 2^y

Hh

§. 285.

§. 285. Tertiae eclipsis medium contigit
Parisiis A. 1717 Mart. 26^d, 15^b, 21', 20'' temp. med.
Pro quo tempore colligitur

Longitudo solis vera $\theta = 0^{\circ}, 6', 19'', 56'''$,

Anomalia solis vera $s = 8, 28, 0, 17$

Longitudo media lunae $\dots 6, 1, 37, 2$

Anomalia media lunae $\dots 8, 24, 7, 21$

Anomalia lunae vera $v = 9, 0, 19, 10$

Longitudo nodi vera $\pi = 6, 13, 30, 22$

Distantia nodi a sole $\dots 5, 22, 49, 29$

aeq. pro loco lunae $\dots + 3, 57$

Ergo longitudo lunae vera $\phi = 6, 6, 23, 53$

§. 286. Calculus igitur ita se habebit

$v = 9, 0, 19, 10$; $\sin v = -\sin 89^{\circ}, 40', 50''$
cos. $= +$

$2v = 6, 0, 38,$; $\sin 2v = -\sin 0^{\circ}, 38'$
cos $= -$

$s = 8, 28, 0$; $\sin s = -\sin 88^{\circ}, 0'$

$v - s = 0, 2, 19$; $\sin = +\sin 2^{\circ}, 19'$

$v + s = 5, 28, 19$; $\sin = +\sin 1, 41$

$\phi - \pi = 5, 22, 53$

$2\phi - 2\pi = 11, 15, 46$; $\sin = -\sin 14, 14$

$r = 9, 0, 19$

$2\phi - 2\pi - r = 2, 15, 27$; $\sin = +\sin 75, 27$

$2\phi - 2\pi - 2r = 5, 15, 8$; $\sin = +\sin 14, 52$

$- 9,99999 - 10,0000 + 7,7425$
 $- 4,24753 - 1,5111 - 0,7104$
 $+ 4,24752 + 1,5111 = - 8,4526$

$- 8,0435 - 8,0435 - 9,9999$
 $- 2,8561 - 0,4208 - 9,6201$
 $+ 0,8996 + 8,4643 = + 9,6200$

$- 9,9997 + 8,6066 + 8,4680$
 $+ 2,8819 - 1,1761 - 2,2122$
 $- 2,8816 - 9,7827 - 0,0802$

$- 9,3907 + 9,9858 + 25,657$
 $- 2,7893 + 2,2355$
 $+ 2,1893 + 2,2213$

aeq. aff.	aeq. neq.	
+ 17682	- 762	+ 32, 48
+ 8	- 1	+ 0, 08
+ 152	- 4	- 0, 01
+ 166	- 767	+ 0, 21
+ 18008	+ 18008	
	+ 17241	
	+ 287, 21	
	+ 4, 47, 21	aequatio

Long. media $\zeta = 6, 1, 37, 2$

aeq. $+ 4, 47, 21$

Long. δ vera $= 6, 6, 24, 23$

obl. $6, 6, 23, 53$

Ergo $\theta = + 30 + m + 32, 48 + 0, 21 + 25, 657$

CAPUT XVII.

§. 289. Quintae eclipsis medium erat:

Parisiis A. 1719 Aug. 29^d, 8^h, 33', 19" temp. med.

Pro quo tempore colligitur:

Longitudo solis vera $\theta = 5, 5, 47, 14$
 Anomalia solis vera $s = 1, 27, 25, 24$
 Longitudo lunae media . 11, 2, 9, 40
 Anomalia lunae media . . 10, 15, 59, 25
 Anomalia lunae vera $v = 10, 20, 5, 19$
 Longitudo nodi vera $\pi = 4, 27, 44, 39$
 Distantia nodi a sole $= 0, 8, 2, 35$
 Aequ. pro loco lunae . . $- - 4, 22$
 Long. lunae observata . . 11, 5, 42, 52

§. 290. Calculus ergo ita se habebit:

$$v = 10, 20, 5, 19 ; \sin = - \sin 39, 54, 41 \\ \cos = +$$

$$2v = 9, 10, 11 ; \sin 2v = - \sin 79, 49 \\ \cos = +$$

$$v = 10, 20, 5$$

$$s = 1, 27, 25 ; \sin s = + \sin 57, 25$$

$$v-s = 8, 22, 40 ; \sin = - \sin 82, 40$$

$$v+s = 0, 17, 30 ; \sin = + \sin 17, 30$$

$$\phi - \pi = 0, 7, 58$$

$$2\phi - 2\pi = 0, 15, 16 ; \sin = + \sin 15, 56$$

$$r = 10, 20, 5$$

$$2\phi - 2\pi - r = 1, 25, 15 ; \sin = + \sin 55, 51$$

$$2\phi - 2\pi - 2r = 3, 5, 46 ; \sin = + \sin 84, 14$$

CAPUT XVII.

$$\begin{array}{r r r r} - & 9, 80726 & - & 9, 8073 & + & 9, 8849 \\ - & 4, 24753 & - & 1, 5111 & - & 0, 7104 \\ + & 4, 05479 & + & 1, 31848 & - & 0, 5953 \\ \hline - & 9, 9931 & - & 9, 9931 & + & 9, 2475 \\ - & 2, 8561 & - & 0, 4208 & - & 9, 6201 \\ + & 2, 8492 & + & 0, 41398 & - & 8, 8676 \\ \hline + & 9, 9256 & - & 9, 9964 & + & 9, 4781 \\ + & 2, 8819 & - & 1, 1761 & - & 2, 2122 \\ + & 2, 8075 & + & 1, 1725 & - & 1, 6903 \\ \hline + & 9, 4386 & + & 9, 9178 & + & 99, 57 \\ - & 2, 7896 & + & 2, 2355 & & \\ - & 2, 2282 & + & 2, 1533 & & \end{array}$$

aeq. aff.	aeq. neg.	+ 20, 88
+ 11345	- 49	+ 2, 68
+ 707	- 170	
+ 642	- 219	- 3, 98
+ 15	+ 12851	- 0, 18
+ 142	+ 12632	
+ 12851	210, 32	
	+ 3, 30, 32	aequatio

Long. lunae med. 11, 2, 9, 40

$$\text{aeq.} \quad + 3, 30, 32$$

Long. lunae vera 11, 5, 40, 12

$$\text{obl.} \quad 11, 5, 42, 52$$

$$= -2, 40 + m + 23, 48 = 4, 08 + 99, 57$$

§. 291.

§. 291. Sextae eclipsis medium erat
 Parisiis A. 1722. Jun. 28^d, 13^h, 58ⁱ, 41^u temp. med.
 Pro quo tempore habetur:

Longitudo solis vera $\theta = 3, 6, 51, 7$
 Anomalia solis vera $s = 11, 28, 26, 56$
 Longitudo lunae media . $9, 9, 31, 50$
 Anomalia lunae media . $4, 28, 8, 18$
 Anomalia lunae vera $v = 4, 24, 39, 53$
 Longitudo nodi vera $\pi = 3, 2, 36, 2$
 Distantia nodi a sole . . $0, 4, 15, 5$
 Aequatio loci lunae . . $— 2, 20$
 Longitudo lunae observata $9, 6, 48, 47$

§. 292. Calculus ergo ita se habebit

$v = 4, 24, 39, 53$; $\sin v = + \sin 35^{\circ}, 20', 7''$
 $\cos v = -$
 $2v = 9, 19, 20$; $\sin 2v = - \sin 70, 40$
 $\cos 2v = +$
 $r = 4, 24, 40$
 $s = 11, 28, 27$; $\sin s = - \sin 1, 33$
 $r-s = 4, 26, 13$; $\sin = + \sin 33, 47$
 $r+s = 4, 23, 7$; $\sin = + \sin 36, 53$
 $\phi - \pi = 0, 4, 13$
 $2\phi - 2\pi = 0, 8, 26$; $\sin = + \sin 8, 26$
 $r = 4, 24, 40$
 $2\phi - 2\pi - r = 7, 13, 46$; $\sin = - \sin 43, 46$
 $2\phi - 2\pi - 2r = 2, 19, 6$; $\sin = + \sin 79, 6$

+

$+ 9, 76220 + 9, 7622 - 9, 9116$
 $- 4, 24753 - 1, 5111 - 0, 7104$
 $- 4, 00973 - 1, 2733 + 0, 6220$
 $- 9, 9748 - 9, 9748 + 9, 5199$
 $- 2, 8561 - 0, 4208 - 9, 6201$
 $+ 2, 8309 + 0, 3956 - 9, 1400$
 $- 8, 4321 + 9, 7451 + 9, 7783$
 $+ 2, 8819 - 1, 1761 - 2, 2122$
 $- 1, 3140 - 0, 9212 - 1, 9905$
 $+ 9, 1663 - 9, 8399 + 98, 29$
 $- 2, 7896 + 2, 2355$
 $- 1, 9559 - 2, 0754$

aeq. aff.	aeq. neg.
$+ 678$	$- 10227 - 18, 8''$
$- 10563$	$- 21 + 2, 5''$
$- 9885$	$- 8$
$164, 45$	$- 98 + 4, 2''$
aeq. = $- 2, 44, 45$	$- 90 - 0, 1''$
	$- 119$
	$- 101563$

Long. lun. med. $9, 9, 31, 50$
 $- 2, 44, 45$

Long. lun. $9, 6, 47, 5$
 obs. $9, 6, 48, 47$

Ergo $\phi = - 1, 42 + m - 16, 3'' + 4, 1'' + 98, 99$

li

§. 293.

CAPUT XVII.

§. 293. Septimae eclipsis medium observatum est
Parisiis A. 1724 Oct. 31^d, 15^h, 34ⁱ, 17^u temp. med.
Pro quo tempore colligitur

Longitudo solis vera θ	=	7, 8, 56, 1
Anomalia solis vera s	=	4, 0, 29, 44
Longitudo lunae media		1, 9, 23, 59
Anomalia lunae media		5, 22, 38, 2
Anomalia lunae vera v	=	5, 21, 46, 51
Longitudo nodi vera		1, 16, 36, 22
Distantia nodi a sole		5, 22, 19, 39
aequatio loci lunae	+	4, 10
Long. lunae observata		1, 9, 0, 11

§. 294. Calculus ergo ita ineatur:

$v = 5, 21, 46, 51$	; $\sin v = + \sin 8^\circ, 13', 9''$
	$\cos = -$
$2v = 11, 13, 34,$	; $\sin 2v = - \sin 16^\circ, 26'$
$r = 5, 21, 47$	$\cos = +$
$s = 4, 0, 30$	; $\sin s = + \sin 59^\circ, 30'$
$r - s = 1, 21, 17$	; $\sin = + \sin 51, 17$
$r + s = 9, 22, 17$	; $\sin = - \sin 67, 43$
$\phi - \pi = 5, 22, 24$	
$2\phi - 2\pi = 11, 14, 48$	; $\sin = - \sin 15, 12$
$r = 5, 21, 47$	
$2\phi - 2\pi - r = 5, 23, 1$	; $\sin = + \sin 6, 59$
$2\phi - 2\pi - 2r = 0, 1, 14$	; $\sin = + \sin 1, 14$

+

n est
ed.

3', 9"

26'

30'

17

13

12

9

4

+

CAPUT XVII.

251

+	9, 15520	+	9, 1552	-	9, 9955
-	4, 24753	-	1, 5111	-	0, 7104
-	3, 40273	-	0, 6663 ⁿ	+	0, 7059 ⁱ
-	9, 4516	-	9, 4516	+	9, 9819
-	2, 8561	-	0, 4208	-	9, 6201
+	2, 3077	+	9, 8724 ⁿ	-	9, 6020 ⁱ
+	9, 9353	+	9, 8922	-	9, 9663
+	2, 8819	-	1, 1761	-	2, 2122
-	2, 8172	-	1, 0683	+	2, 1785
-	9, 4186	+	9, 0849	+	2, 157
-	2, 7896	+	2, 2355		
+	2, 2082	+	1, 3024		

aeq. aff.	aeq. neg.	
+	203	- 2528 - 4, 6 ⁿ
+	657	- 11 - 7, 5 ⁿ
+	151	- 2539
+	161	+ 1193 + 5, 1 ⁱ
+	21	- 1346 - 0, 4 ⁱ
+	1193	- 22, 26 aequatio

Long. D med. 1, 9, 23, 59

— 22, 26

Long. calc. 1, 9, 1, 33

Long. obs. 1, 9, 0, 11

$$e = + 1, 22 + m - 3, 9^n + 4, 7^i + 2, 15^y$$

li 2

§. 295.

§. 295. Octavae eclipsis medium observatum est
Parisiis A. 1729. Febr. 13^d, 9^h, 6^m, 56^u temp. med.
Pro quo tempore colligitur:

Longitudo solis vera θ	=	10 ^o , 25 ^o , 13 ['] , 23 ^u
Anomalia solis vera s	=	7, 16, 43, 34
Longitudo lunae media	.	5, 0, 5, 27
Anomalia lunae media	.	3, 18, 53, 24
Anomalia lunae vera v	=	3, 12, 54, 9
Longitudo nodi vera π	=	10, 24, 4, 30
Distantia nodi a sole	=	0, 1, 8, 53
aequatio pro long. lunae	.	- 0, 37
Longitudo lunae observata		4, 25, 12, 46

§. 296. Calculus ergo ita se habebit:

$v = 3, 12, 54, 9$	; $\sin v = + \sin 77^{\circ}, 5', 51''$
	$\cos v = -$
$2v = 6, 25, 48$	; $\sin 2v = - \sin 25, 48$
$r = 3, 12, 54$	$\cos 2v = -$
$s = 7, 16, 44$	; $\sin s = - \sin 46, 44$
$r - s = 7, 26, 10$	; $\sin = - \sin 56, 10$
$r + s = 10, 29, 38$	; $\sin = - \sin 30, 22$
$\phi - \pi = 0, 1, 8,$	
$2\phi - 2\pi = 0, 2, 16,$	; $\sin = + \sin 2, 16$
$r = 3, 12, 54$	
$2\phi - 2\pi - r = 8, 19, 22$	; $\sin = - \sin 79, 22$
$2\phi - 2\pi - 2r = 5, 6, 28$	; $\sin = + \sin 23, 32$

+

+ 9,98889	+ 9,9889	- 9,3488
- 4,24757	- 1,5111	- 0,7104
- 4,23642	- 1,50000	+ 0,0502
- 9,6387	- 9,6387	- 9,9544
- 2,8561	- 0,4208	- 9,6201
+ 2,4948	+ 0,0595	+ 9,5745
- 9,8622	- 9,9194	- 9,7037
+ 2,8819	- 1,1761	- 2,2122
- 2,7441	+ 1,0955	+ 1,9159
+ 8,5971	- 9,9925	+ 39, 9y
- 2,7896	+ 2,2355	
- 1,3867	- 2,2280	

aeq. aff.	aeq. neg.	- 31, 7 ⁿ
+ 312	- 17235	+ 1, 1 ⁿ
+ 12	- 555	
+ 82	- 24	+ 1, 1 ⁿ
+ 406	- 169	+ 0, 3 ⁿ
	- 17983	
	+ 406	
	- 17577	
	- 292, 57	
	- 4,52,57	aequatio

Long. lunae media	=	5, 0, 5, 27
aeq.		- 4, 52, 57
Long. lunae calc.		4, 25, 12, 30
Long. lunae obs.		4, 25, 12, 46
	=	- 16 + m - 30, 6 ⁿ + 1, 4 ⁿ + 39, 9y

li 3

§. 297.

§. 297. Nonae eclipsis medium obseruatum est
Parisiis A. 1729. Aug. 8^d, 13^b, 14ⁱ, 14^u temp. med.
Pro quo tempore reperitur.

Longitudo solis vera $\theta = 4, 16, 17, 29$

Anomalia solis vera $s = 1, 7, 47, 12$

Longitudo lunae media $10, 11, 23, 57$

Anomalia lunae media $8, 10, 36, 19$

Anomalia lunae vera $v = 8, 16, 34, 40$

Longitudo nodi vera $\pi = 10, 14, 58, 21$

Distantia nodi a sole $6, 1, 19, 8$

Aequatio pro loco lunae $- 43$

Long. lunae obseruata $10, 16, 16, 46$

§. 298. Calculus ergo ita instituetur:

$v = 8, 16, 34, 40$; $\sin v = - \sin 76, 34, 40$

$\cos v = -$

$2v = 5, 3, 9$; $\sin 2v = + \sin 26, 51$

$\cos 2v = -$

$r = 8, 16, 35$

$s = 1, 7, 47$; $\sin s = + \sin 37, 47$

$r - s = 7, 8, 48$; $\sin = - \sin 38, 48$

$r + s = 9, 24, 22$; $\sin = - \sin 65, 38$

$\phi - \pi = 6, 1, 19$

$2\phi - 2\pi = 0, 2, 38$; $\sin = + \sin 2, 38$

$r = 8, 16, 35$

$2\phi - 2\pi - r = 3, 16, 3$; $\sin = + \sin 73, 57$

$2\phi - 2\pi - 2r = 6, 29, 28$; $\sin = - \sin 29, 28$

$- 9,93797 - 9,9880 - 9,3655$
 $- 4,24755 - 1,5111 - 0,7104$
 $+ 4,23550 + 1,49918 + 0,07591$

$+ 9,6548 + 9,6548 - 9,9505$
 $- 2,8561 - 0,4208 - 9,6201$
 $- 2,5109 - 0,07568 + 9,57061$

$+ 9,7872 - 9,7970 - 9,9595$
 $+ 2,8819 - 1,1761 - 2,2122$
 $+ 2,6691 + 0,9731 + 2,1717$

$+ 8,6622 + 9,9827 - 49, 2y.$
 $- 2,7896 + 2,2355$
 $- 1,4518 + 2,2182$

aeq. aff.	aeq. neg.	
$+ 17199$	$- 324$	$+ 31, 68$
$+ 467$	$- 28$	$- 1, 28$
$+ 9$	$- 352$	$+ 1, 28$
$+ 148$	$+ 17988$	$+ 0, 48$
$+ 165$	$+ 17636$	
$+ 17988$	$+ 293, 56$	
	$+ 4, 53, 56$	aequatio

Long. $\mathcal{D}$ med. $= 10, 11, 23, 57$
aeq. $+ 4, 53, 56$

Long. $\mathcal{D}$ calc. $= 10, 16, 17, 53$

Long. $\mathcal{D}$ obl. $10, 16, 16, 46$

$e = + 1, 7 + m + 30, 48 + 1, 61 = 49, 2y$

§. 299. Decimae eclipsis medium obseruatum est
Parisiis A. 1731. Jun. 19^d, 13^h, 55ⁱ, 13ⁱⁱ. temp. med.
Pro quo tempore colligitur

Longitudo solis vera $\theta = 2^{\circ}, 28^{\circ}, 5', 41''$

Anomalia solis vera $s = 11, 19, 48, 47$

Longitudo lunae media . $9, 1, 45, 1$

Anomalia lunae media . $4, 15, 9, 43$

Anomalia lunae vera $v = 4, 10, 34, 21$

Longitudo nodi vera $\pi = 9, 8, 6, 38$

Distantia nodi a sole . $5, 19, 59, 3$

Aequatio pro loco lunae . $+ 5, 24$

Longitudo lunae obseruata $8, 28, 11, 5$

§. 300. Calculus ergo ita instituetur

$v = 4, 10, 34, 21$; $\sin v = + \sin 49, 25, 39$

$\cos v = -$

$2v = 8, 21, 9$; $\sin 2v = - \sin 81, 9$

$\cos 2v = -$

$r = 4, 10, 34$

$s = 11, 19, 49$

$r-s = 4, 20, 45$

$r+s = 4, 0, 23$

$\phi - \pi = 5, 20, 4$

$2\phi - 2\pi = 11, 10, 8$

$\dot{s} = 4, 10, 34$

$2\phi - 2\pi - r = 6, 29, 34$

$2\phi - 2\pi - 2r = 2, 19, 0$

; $\sin s = - \sin 10, 11$

; $\sin = + \sin 39, 15$

; $\sin = + \sin 59, 37$

; $\sin = - \sin 29, 52$

; $\sin = - \sin 29, 34$

; $\sin = + \sin 79, 0$

+

est

$+ 9,88057 + 9,8806 - 9,8131$

$- 4,24753 - 1,5111 - 0,7104$

$- 4,12810 - 1,39178 + 0,21002$

$- 9,9948 - 9,9948 - 9,1871$

$- 2,8561 - 0,4208 - 9,6201$

$+ 2,8509 + 0,41568 + 8,80722$

$- 9,2475 + 9,8012 + 9,9358$

$+ 2,8819 - 1,1761 - 2,2122$

$- 2,1294 - 0,9773 - 2,1480$

$- 9,5313 - 9,6932 + 98, 1y$

$- 2,7896 + 2,2355$

$+ 2,3209 - 1,9287$

aeq. aff. aeq. neg.

$+ 709 - 13431 - 24, 8m$

$+ 209 - 135 + 2, 6m$

$+ 918 - 9 + 3, 3s$

$- 13801 - 141 + 0, 1s$

$- 12883 - 85$

$- 214, 43 - 13801$

aeq. $- 3, 34, 43$

Long. lunae media $9, 1, 45, 1$

aeq. $- 3, 34, 43$

Long. lunae calc. $8, 28, 10, 18,$

Long. lunae obs. $8, 28, 11, 5$

Ergo $e = - 47'' + m - 22, 2m + 3, 4i + 98, 1y$

Kk

§. 301.

§. 301. Eclipsis undecimae medium observatum est
 Parisiis A. 1732 Dec. 1^a, 9^h, 48', 23" temp. med.
 Pro quo tempore colligitur:

Longitudo solis vera θ	=	8, 10, 3, 6
Anomalia solis vera s	=	5, 1, 29, 50
Longitudo lunae media	.	2, 6, 8, 19
Anomalia lunae media	.	7, 19, 24, 12
Anomalia lunae vera v	=	7, 24, 19, 39
Longitudo nodi vera π	=	8, 10, 41, 14
Distantia nodi a sole	=	11, 29, 21, 52
Aequ. pro loco lunae	.	+ 21
Long. lunae observata	.	2, 10, 3, 27

§. 302. Calculus ergo ita se habebit:

$v = 7, 24, 19, 39$	; sin $v = -$ sin 54, 19, 39
	cos $v = -$
$2v = 3, 18, 39$	; sin $2v = +$ sin 71, 21
	cos $= -$
$r = 7, 24, 20$	
$s = 5, 1, 30$	; sin $s = +$ sin 28, 30
$r - s = 2, 22, 50$	; sin $= +$ sin 82, 50
$r + s = 0, 25, 50$	; sin $= +$ sin 25, 50
$\phi - \pi = 11, 29, 22$	
$2\phi - 2\pi = 11, 28, 44$	; sin $= +$ sin 1, 16
$r = 7, 24, 40$	
$2\phi - 2\pi - r = 4, 4, 4$	; sin $= +$ sin 55, 56
$2\phi - 2\pi - 2r = 8, 9, 24$	; sin $= +$ sin 69, 24

um est

), 39

— 9,90975	—	9,9097	—	9,7657
— 4,24753	—	1,5111	—	0,7104
+ 4,15728	+	1,4208 ⁿ	+	0,4761 ⁱ
+ 9,9766	+	9,9766	—	9,5048
— 2,8561	—	0,4208	—	9,6201
— 2,8327	—	0,3974 ⁿ	+	9,1249 ⁱ
+ 9,6787	+	9,9969	+	9,6444
+ 2,8819	—	1,1761	—	2,2122
+ 2,5606	—	1,1730	—	1,8566
— 8,3445	+	9,9182	—	93, 6 ^y
— 2,7896	+	2,2355		
+ 1,1341	+	2,1537		

aeq. aff.	aeq. neq.	+ 26, 4 ^m
+ 14364	— 680	— 2, 5 ⁿ
+ 364	— 15	+ 3, 0 ⁱ
+ 14	— 72	+ 0, 1 ⁱ
+ 142	— 767	
+ 14884	+ 14884	
	+ 14117	
	+ 235, 17	
	+ 3, 55, 17	aequatio

Long. lunae media	2, 6, 8, 19
aeq.	+ 3, 55, 17
Long. lunae calc.	2, 10, 3, 36
obl.	2, 10, 3, 27
	= + 9 + ^m + 23, 9 ⁿ + 3, 1 ⁱ = 93, 6 ^y

Kk 2

§. 303

§. 303. Eclipsis duodecimae medium obseruatum est
Parisiis A. 1736 Mart. 26^d, 12^b, 14ⁱ, 36^u temp. med.
Pro quo tempore colligitur

Longitudo solis vera $\theta = 0^{\circ}, 6^{\circ}, 35', 42''$
Anomalia solis vera $s = 8, 27, 58, 24$
Longitudo lunae media $6, 4, 5, 0$
Anomalia lunae media $7, 3, 25, 43$
Anomalia lunae vera $v = 7, 7, 2, 56$
Longitudo nodi vera $\pi = 6, 6, 24, 31$
Distantia nodi a sole $6, 0, 11, 11$
aeq. pro long. lunae 6
Longitudo lunae obl. $6, 6, 35, 36$

§. 304. Calculus ergo ita instituat.

$v = 7, 7, 2, 56$; $\sin v = - \sin 57, 2, 56$
 $\cos = -$
 $2v = 2, 14, 6$; $\sin 2v = + \sin 74, 6$
 $\cos = +$
 $r = 7, 7, 3$
 $s = 8, 27, 58$; $\sin s = - \sin 87, 58$
 $r - s = 10, 9, 5$; $\sin = - \sin 50, 55$
 $r + s = 4, 5, 1$; $\sin = + \sin 54, 59$
 $\phi - \pi = 6, 0, 11$
 $2\phi - 2\pi = 0, 0, 22$; $\sin = + \sin 0, 22$
 $r = 7, 7, 3$
 $2\phi - 2\pi - r = 4, 23, 19$; $\sin = + \sin 36, 41$
 $2\phi - 2\pi - 2r = 9, 16, 16$; $\sin = - \sin 73, 44$

um est
med.

56

$- 9,77995 - 9,7799 - 9,9021$
 $- 4,24753 - 1,5111 - 0,7104$
 $+ 4,02748 + 1,29108 + 0,61251$
 $+ 9,9831 + 9,9831 + 9,4377$
 $- 2,8561 - 0,4208 - 9,6201$
 $- 2,8392 - 0,40398 - 9,05781$
 $- 9,9997 - 9,8900 + 9,9133$
 $+ 2,8819 - 1,1761 - 2,2122$
 $- 2,8816 + 1,0661 - 2,1255$
 $+ 7,8061 + 9,7763 - 96, 07$
 $- 2,7896 + 2,2355$
 $- 0,5957 + 2,0118$

aeq. aff.	aeq. neg.	+ 19, 68
+ 10653	- 691	- 2, 58
+ 12	- 761	
+ 103	- 133	+ 4, 11
+ 10768	- 4	- 0, 11
- 1589	- 1589	
+ 9179		
+ 152, 59		
aeq. = + 2, 32, 59		

Long. C med. $6, 4, 5, 0$

aeq. $+ 2, 32, 59$

Long. D calc. $6, 6, 37, 59$

obl. $6, 6, 35, 36$

$= + 2', 23'' + m + 17, 17 + 4, 01 = 96, 07$

K k 3

§. 305.

§. 305. Elipsis decima tertiae medium observatum est
Parisiis A. 1736 Sept. 19^d, 14^b, 59ⁱ, 36^u temp. med.
Pro quo tempore colligitur

$$\text{Longitudo solis vera } \theta = 5^{\circ}, 27^{\circ}, 21', 39''$$

$$\text{Anomalia solis vera } s = 2^{\circ}, 18, 43, 51$$

$$\text{Longitudo lunae media } \dots 11, 27, 48, 53$$

$$\text{Anomalia lunae media } \dots 0, 7, 25, 42$$

$$\text{Anomalia lunae vera } v = 0, 6, 40, 44$$

$$\text{Longitudo nodi vera } 5, 27, 15, 4$$

$$\text{Distantia nodi a sole } \dots 0, 0, 6, 35$$

$$\text{aeq. pro long. lunae } \dots 4$$

$$\text{Longitudo lunae observata } 11, 27, 21, 35$$

§. 306. Calculus ergo ita instituetur

$$v = 0, 6, 40, 44; \sin v = + \sin 6, 40, 50$$

$$\cos v = +$$

$$2v = 0, 13, 21; \sin 2v = + \sin 13, 21$$

$$\cos 2v = +$$

$$r = 0, 6, 41$$

$$s = 2, 18, 44; \sin s = + \sin 88, 44$$

$$r - s = 9, 17, 57; \sin = - \sin 72, 3$$

$$r + s = 2, 25, 25; \sin = + \sin 85, 25$$

$$\phi - \pi = 0, 0, 6$$

$$2\phi - 2\pi = 0, 0, 12; \sin = + \sin 0, 12$$

$$r = 0, 6, 41$$

$$2\phi - 2\pi - r = 11, 23, 31; \sin = - \sin 6, 29$$

$$2\phi - 2\pi - 2r = 11, 16, 50; \sin = - \sin 13, 10$$

atum est
med.

2^u,

1

1

.

1

.

—

0, 50

21

44

3

25

12

19

10

+

+

$$\begin{array}{rcl} + & 9, 06561 & + 9, 0656 + 9, 9970 \\ - & 4, 24753 & - 1, 5111 - 0, 7104 \\ - & 3, 31314 & - 0, 5767^m - 0, 7074^d \end{array}$$

$$\begin{array}{rcl} + & 9, 3634 & + 9, 3634 + 9, 9881 \\ - & 2, 8561 & - 0, 4208 - 9, 6201 \\ - & 2, 2195 & - 9, 7842^m - 9, 6082^d \end{array}$$

$$\begin{array}{rcl} + & 9, 9915 & - 9, 9783 + 9, 9986 \\ + & 2, 8819 & - 1, 1751 - 2, 2122 \\ + & 2, 8734 & + 1, 1534 - 2, 2108 \end{array}$$

$$\begin{array}{rcl} + & 7, 5429 & - 9, 0527 - 22, 6y \\ - & 2, 7896 & + 2, 2355 \\ - & 0, 3325 & - 1, 2882 \end{array}$$

aeq. aff.	aeq. neg.	— 3, 8 ^m
+ 747	— 2057	— 0, 6 ^m
+ 14	— 166	
+ 761	— 2	— 5, 1 ^d
— 2406	— 19	— 0, 4 ^d
— 1645	— 2406	
— 27, 25 ^u	aequatio	

$$\text{Long. } \mathcal{D} \text{ med. } 11, 27, 48, 53$$

$$\text{aeq. } \dots 27, 25$$

$$\text{Long. } \mathcal{D} \text{ calc. } 11, 27, 21, 28$$

$$\text{Long. } \mathcal{D} \text{ obl. } 11, 27, 21, 35$$

$$e = -0, 7^{u} + m - 4, 4^m - 5, 5^d - 22, 6y$$

§. 307. Ex his ergo tredecim eclipsibus nacti sumus aequationes, ex quibus cum tabularum, quibus sum usus, correctiones, tum verus valor aequationis ab angulo $2\phi - 2\pi - 2r$ pendens definiri debet:

Aequationes autem inde ortae sunt sequentes.

- I. $\phi = + 156'' + m - 29,1n - 2,8i - 99,8y$
- II. $\phi = - 27 + m + 32,4n - 1,3i + 79,2y$
- III. $\phi = + 30 + m + 32,4n + 0,2i + 25,6y$
- IV. $\phi = + 12 + m - 6,2n - 5,5i - 30,7y$
- V. $\phi = - 160 + m + 23,4n - 4,0i + 99,5y$
- VI. $\phi = - 102 + m - 16,3n + 4,1i + 98,2y$
- VII. $\phi = + 82 + m - 3,9n + 4,7i + 2,1y$
- VIII. $\phi = - 16 + m - 30,6n + 1,4i + 39,9y$
- IX. $\phi = + 67 + m + 30,4n + 1,6i - 49,2y$
- X. $\phi = - 47 + m - 22,2n + 3,4i - 98,1y$
- XI. $\phi = + 9 + m + 23,9n + 3,1i - 93,6y$
- XII. $\phi = + 143 + m + 17,1n + 4,0i - 96,0y$
- XIII. $\phi = - 7 + m - 4,4n - 5,5i - 22,6y$

nacti fu-
quibus
onis ab

§. 308. Hic statim commode euenit, ut errores calculi ab obseruationibus infra tria minuta prima subsistant, qui autem infra sesquiminutum primum deprimuntur, simul ac litterae y valor tribuitur unitati fere aequalis. Hincque ergo cognoscimus valorem ipsius y , quem quinario maiorem inueneramus, merito nobis fuisse suspectum, cum iam perspiciamus, eum unitatem superare non posse. Quamobrem ponamus $y = 1$, seu in formula nostra pro longitudine lunae scribamus terminum $100''$ sin $(2\phi - 2\pi - 2r)$. Quod autem ad litteras m , n et i attinet, tentanti mox patebit, quoscunque ipsis valores tribuamus, errores inde non admodum posse diminui; interim tamen decem circiter minutis secundis diminuentur, si ponatur $y = \frac{1}{2}$; $n = \frac{1}{2}$; $i = -3$ et $m = -4$; quo facto errores vix vnum minutum primum superabunt.

CAPUT XVIII.

CONSTITUTIO ELEMENTORUM
PRO TABULIS LUNARIBUS.

§. 309.

Tabulae autem, quibus in praecedenti calculo sum-
mus, praebent pro meridiano Parisiensi ad epo-
cham 1701 seu ad meridiem diei primi anni
1700 tempore medio

Longitudinem Lunae mediam $5^{\circ}, 20', 19'', 47''$
et Anomaliam Lunae mediam $6, 13, 26, 51$

Hinc accuratius habebimus haec elementa pro eodem
tempore eodemque loco scilicet

Longitudinem Lunae mediam $5^{\circ}, 20', 19'', 43''$
Anomaliam Lunae mediam $6, 13, 24, 0$
unde Longitudo Apogei $11, 6, 55, 43$

§. 310. Si haec elementa comparemus cum Tabulis
astronomicis Cel. Cassini et Monnierii, reperiemus pro
eodem tempore et loco

	Cassini	Monnier
Long. mediam Lunae	5, 20, 18, 19	5, 20, 19, 28
Anom. mediam Lunae	6, 13, 10, 48	6, 13, 13, 2
Long. Apogei	11, 7, 7, 27	11, 7, 6, 26

Hic quidem longitudo media satis convenit cum ea,
quam ex observationibus conclusimus; verum anomalia
media inuenta superat Cassinianam $13', 12''$, Monnierianam autem $11'$, quod discrimen satis est notabile.
Verum

RUM

culo, sum
ad epo-
imi anni

, 47''

, 51

) eodem

, 43''

, 0

, 43

Tabulis
nus pro

nier

19, 28

13, 2

6, 26

um ea,
nomalia
Monnie-
notabile.
Verum

Verum si perpendamus motum lunae a tam multis va-
riis inaequalitatibus perturbari, mirum sane non est,
anomaliam mediam per solas observationes accuratius
definiri non potuisse; praesertim cum error $15'$ in ano-
malia media commissus in loco lunae ad summum er-
rorem $1', 45''$ gignere valeat.

§. 311. Excentricitatem autem orbitae lunaris,
quam statueram $= 0,0545$ iam $\frac{766}{10000}$ vel $0,00005$
augeri oportet, ita ut nunc sit excentricitatis valor
 $k = 0,05455$; qui a supra assumpto tam parum discre-
pat, ut anomalia vera inde ex media collecta pro satis
exacta haberi possit: aequationes autem ab excentrici-
tate pendentes aliquod augmentum capient, quod nunc
quidem diligentius definiri oportet. Primum ergo for-
mulam pro longitudine lunae inuentam hinc corriga-
mus; deinde vero etiam formulas pro distantia lunae
a terra, pro eius motu momentaneo, et pro loco nodi
veraue inclinatione orbitae lunaris ad eclipticam hinc
evoluamus.

§. 312. Ante omnia autem oportebit formulam
exhibere, cuius ope ex data quavis anomalia lunae me-
dia p elicere liceat, convenientem anomaliam veram r .
Ac substituto quidem pro k vero eius valore nunc in-
vento, coefficientibusque in minuta secunda conuersis,
formula supra (§. 306) exhibita sequentem induet for-
mam:

$$r = p - 22495'' \sin p + 766'' \sin 2p - 36'' \sin 3p$$

$$4,352086 \quad 2,884229 \quad 1,55630$$

L 1 2

Huius

Huius ergo formulae ope haud difficulter tabula computabitur, quae ad singulos anomaliae mediae gradus exhibeat valores anomaliae verae.

§. 313. Inuenta autem anomalia vera r , si habeatur quoque anomalia vera solis s , una cum angulo η et longitudinibus Φ , θ , π saltem proxime, formula longitudinem veram Φ datae mediae ξ respondentem exhibens, sequenti modo habebitur expressa:

$\Phi = \xi$		log. coeff.	
—	22466'' fin r	4,351535	I
—	462 fin $2r$	2,66456	
—	11 fin $3r$	1,0518	
+	701 fin s	2,84572	II
+	4 fin $2s$	0,602	
+	141 fin $(r-s)$	2,1492	III
—	118 fin $(r+s)$	2,0719	IV
—	175 fin η	2,2430	V
+	2115 fin 2η	3,32531	
+	4 fin 3η	0,602	
—	8 fin 4η	0,903	VI
+	59 fin $(\eta-r)$	1,7708	
+	352 fin $(2\eta-2r)$	2,5465	VII
—	2729 fin $(2\eta-r)$	3,67477	
—	93 fin $(4\eta-2r)$	1,9685	VIII
+	56 fin $(2\eta+r)$	1,7482	
+	59 fin $(4\eta-r)$	1,7708	IX
—	49 fin $(\eta+s)$	1,6902	X
—	76 fin $(2\eta-s)$	1,8808	XI
—	57 fin $(2\eta+s)$	1,7559	XII
+	154 fin $(2\eta-r+s)$	2,1875	XIII

+

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I

II

V

V

VI

VII

VIII

X

I

II

III

+

+	45 fin $(2\eta-r-s)$	1,6532	XIV
—	411 fin $(2\Phi-2\pi)$	2,6138	XV
—	205 fin $(2\theta-2\pi)$	2,3117	XVI
—	6 fin $(4\theta-4\pi)$	0,778	
+	187 fin $(2\Phi-2\pi-r)$	2,2718	XVII
+	80 fin $(2\Phi-2\pi-2r)$	1,9031	XVIII
—	15 fin $(2\theta-2\pi-r)$	1,176	XIX
—	10 fin $(2\theta-2\pi+r)$	1,000	XX

§. 314. Inaequalitates has ita disposui, ut eas, quae una tabula comprehendere possunt, coniunctim exposuerim, quo facilius calculus expediri queat. Hinc igitur patet omissis iis inaequalitatibus, quae 10th non superant, locum lunae per viginti inaequalitates corrigi debere, antequam vera eius longitudo obineatur.

§. 315. Haec autem expressio adhuc isto defectu laborat, quod pleraeque inaequalitates ipsam lunae longitudinem veram Φ , quae tamen demum quaeritur, involuant, ideoque calculus, cum longitudo lunae etiam nunc est incognita, commode expediri non possit. Quoniam tamen sufficit longitudinem lunae proxime tantum nosse, cum longitudo media per quatuor priores inaequalitates fuerit correctae, ea pro sequentibus inaequalitatibus loco longitudinis verae usurpari poterit, sicque tandem longitudo lunae multo exactior reperietur. Quo facto si accuratior desideretur, omnes inaequalitates post 4 priores denuo ad calculum reuocari conveniet, iisque evolutis longitudo lunae vera prodibit, quae nulla amplius correctione indigebit. Interim

L 1 3

tamen

tamen ne calculum per se satis taediosum bis repetere opus sit, non difficulter hanc expressionem ita transformare licet, ut locus lunae per quatuor tantum priores inaequalitates correctus sine errore in sequentibus loco ϕ adhiberi possit.

§. 316. Cum autem longitudo lunae iam per observationes fuerit cognita, haec expressio sine vlla immutatione ad calculum accommodabitur, ut hoc modo consensus theoriae cum veritate exploretur. In inaequalitatibus enim determinandis pro littera ϕ ubique longitudo lunae observata introducerur, calculoque perfecto patebit, quantum locus lunae per calculum definitus etiamnunc discrepet ab eius loco vero observato. Atque si hoc modo plurimae observationes calculo subiciantur, ex aberrationibus a veritate non solum elementa, quibus haec formula innititur, accuratius definire licbit, sed etiam inaequalitates, quae nondum satis certae videntur, inde emendari poterunt. Quin etiam novae inaequalitates, quas per Theoriam determinare non licuerat, hoc modo forte certius colligi poterunt.

§. 317. Antequam autem huiusmodi calculi specimen exhiberi queat, necesse est ut aequationem pro loco nodi vero inveniundo ad calculum accommodemus. Formulae autem supra (219) exhibitae, si pro r substituamus valorem inuentum $r = p - 2k \sin r$ — $\frac{1}{2} k k \sin 2r$, pars: Const. — 0,004053 p indicabit longitudinem nodi mediam. Hincque longitudo nodi vera erit

$$\pi =$$

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servato.
ilo sub-
um ele-
us defi-
um sa-
Quin
deter-
colligi

uli spe-
ctionem
ommo-
si pro
sin r
dicabit
o nodi
 $\pi =$

		Log. coeff.
$\pi =$ Long. med. —	107" $\sin r$	2, 0294
—	6 $\sin 2r$	0, 778
+	551 $\sin s$	2, 7411
—	453 $\sin 2\eta$	2, 6561
—	129 $\sin (2\eta - r)$	2, 1106
—	33 $\sin (2\eta + r)$	1, 518
+	55 $\sin (2\eta - 2r)$	1, 740
+	420 $\sin (2\phi - 2\pi)$	2, 6232
+	98 $\sin (2\phi - 2\pi - r)$	1, 991
+	30 $\sin (2\phi - 2\pi + r)$	1, 477
+	235 $\sin (2\phi - 2\pi - 2r)$	2, 3711
+	5426 $\sin (2\theta - 2\pi)$	3, 73448
+	75 $\sin (4\theta - 4\pi)$	1, 875
—	53 $\sin (2\theta - 2\pi - r)$	1, 724
+	53 $\sin (2\theta - 2\pi + r)$	1, 724
—	90 $\sin (2\theta - 2\pi - s)$	1, 954
—	32 $\sin (2\theta - 2\pi + s)$	1, 505

§. 318. In hoc calculo plerasque inaequalitates omittere licet, siquidem tantum longitudinem lunae investigare sit propositum: manifestum enim est, etiam si in loco nodi error plurium minutorum primorum committatur, inde vix errorem aliquot minutorum secundorum in longitudinem lunae redundare. Quod si vero eclipsis cuiuspiam omnia phaenomena diligenter definire velimus, tum locum nodi exactissime cognitum esse oportet. Praeterea vero pro latitudine assignanda vera inclinatio orbitae lunaris ad eclipticam ex media accuratissime erit definienda ope huius formulae:

$$p = e$$

	Log coeff.
$\varepsilon = 0$ — $2'' \cos r$	0,30
— $48 \cos 2\eta$	1,681
+ $11 \cos (2\eta - r)$	1,041
+ $3 \cos (2\eta + r)$	0,48
+ $36 \cos (2\Phi - 2\pi)$	1,556
+ $9 \cos (2\Phi - 2\pi - r)$	0,95
+ $3 \cos (2\Phi - 2\pi + r)$	0,48
+ $23 \cos (2\Phi - 2\pi - 2r)$	1,362
+ $484 \cos (2\theta - 2\pi)$	2,6848
+ $9 \cos (4\theta - 4\pi)$	0,95
— $5 \cos (2\theta - 2\pi - r)$	0,70
+ $5 \cos (2\theta - 2\pi + r)$	0,70
— $7 \cos (2\theta - 2\pi - s)$	0,84
— $3 \cos (2\theta - 2\pi + s)$	0,48

Tabula autem pro distantia lunae a terra, vnde eius parallaxis et diameter apparens definiatur, ex formulis supra exhibitis facile construatur.

ADDI-

ide eius
formulis

DDI-

ADDITAMENTUM

CONTINENS ALIAS METHODOS

INVESTIGANDI MOTUS LUNAE

INAEQUALITATES.

Qui methodum ante descriptam accuratius evoluerit, eam quidem in se spectatam satis bonam acque plerisque lunae inaequalitatibus definiendis aptam deprehendet; interim tamen fateri cogor, eam non solum maxime esse operosam, sed etiam ita comparatam, ut plures inaequalitates, quae tamen motum lunae imprimis afficere videntur, non satis exacte exhibeat, et quasi in dubio relinquat. Causa huius incertitudinis manifesto in hoc est sita, quod omnes inaequalitates ita inter se sunt connexae, ut nullius valor verus accurate definiri possit, quin simul reliquae inaequalitates omnes fuerint cognitae. Cum igitur eiusmodi methodo approximandi sum usus, ut primo quasdam inaequalitates tanquam cognitae assumerem, ex quibus deinceps reliquas definiuerim, probe notandum est ab his inuentis iterum priores, quae erant assumptae, leuem quandam mutationem pati; quae si statim ab initio nota fuisset, etiam reliquarum valores aliquantillum mutati prodissent: at quaedam inaequalitates adeo sunt lubricae, ut facta vel minima mutatione in iis, a quibus pendent, inde non exiguum alterationem trahant. Huc imprimis pertinet motus apogei, cuius investigatio omnes omnino inaequalitates

M m

lites

litates implicat, ita ut sine harum cognitione nequaquam accurate definiri queat.

Cum igitur haec methodus istis tantis incommodis sit obnoxia, aliam maxime diuersam tentavi viam, quae ab iis esset libera, etiam si negare nequeam, etiam hanc suis non carere incommodis, quae tamen prorsus alius sunt generis. Ex quo confido his duabus diuersis methodis combinandis haud exiguum fructum in veram motuum lunarium cognitionem esse redundaturum. Praecipuum autem discrimen versatur in electione anomaliae, quae in superiore methodo non ita est assumpta, ut distantia lunae a terra fieret vel maxima vel minima, si anomalia vel $= 0$ vel $= 180^\circ$ statuatur: neque enim differentiale distantiae dx evanescit, quando sinus anomaliae in nihilum abit, sed praeterea etiam nunc ab elongatione solis a luna seu angulo η pendet. Ita secundum hanc methodum neque apogaeum lunae neque perigaeum ibi statuitur, ubi angulus, quem motus lunae directio cum radio vectore facit, est rectus; sed plerumque in alia puncta incidunt, quae ab iis locis, ubi luna terrae vel est proxima, vel ab ea maxime remota, notabiliter sint diuersa. Etsi autem in hoc calculo non verae lineae absidum positio consideratur, hinc tamen methodus minime vitiosa est reputanda; propterea quod non est quaestio, quo nomine quaequam orbitae lunaris puncta appellentur, dummodo cunctae inaequalitates recte exprimantur. Sed quoniam circa has ipsas inaequalitates nonnulla grauiora dubia sunt orta, haud abs re fore arbitror, et alteram methodum hic proponere.

I. Sit

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Sit

I.

Sit igitur ut ante: Massa solis $= \odot$; terrae $= \otimes$ et lunae $= \textcircled{D}$; atque vis attractiua terrae in distantia d ut $\frac{1}{dd} - \frac{1}{bb}$; manente vi solis quadrato distantiae exacte proportionali. Tum vero sit

Longitudo lunae $= \phi$; latitudo $= \psi$; et distantia curtata $= x$

Longitudo solis $= \theta$; eiusque a terra distantia $= y$

Longitudo nodi ascendentis lunae $= \pi$ et inclinatio ad eclipticam $= \varrho$

ac ponatur breuitatis ergo elongatio lunae a sole $\phi - \theta = \eta$ et distantia $\sqrt{(xx \sec \psi^2 - 2xy \cos \eta + yy)} = z$.

Quibus positis supra §. 20. vidimus motum lunae his quatuor aequationibus contineri:

$$\text{I. } 2xdx d\phi + xdd\phi = -\frac{1}{2} ds^2 \odot \left(\frac{y}{x^3} - \frac{1}{yy} \right) \sin \eta$$

$$\text{II. } ddx - x d\phi^2 = -\frac{1}{2} ds^2 (\otimes + \textcircled{D}) \cos \psi^2 \left(\frac{1}{xx} - \frac{1}{bb} \right) \\ - \frac{1}{2} ds^2 \odot \left(\frac{x-y \cos \eta}{z^3} + \frac{\cos \eta}{yy} \right)$$

$$\text{III. } d\pi = -\frac{1}{2} ds^2 \odot \left(\frac{y}{x^3} - \frac{1}{yy} \right) \frac{\sin(\phi - \pi) \sin(\theta - \pi)}{xd\phi}$$

$$\text{IV. } d \text{ tang } \varrho = \frac{d\pi}{\text{tang}(\phi - \pi)}, \text{ et tang } \psi = \text{tg } \varrho \cos(\phi - \pi)$$

vbi elementum temporis dt sumtum est pro constante.

M m 2

II. Qua-

II.

Quatenus hic motus solis ingreditur, is pro regulari atque regulis Kepleri conformi haberi poterit: habebimus ergo

$$2dyd\theta + ydd\theta = 0. \text{ et } ddy - yd\theta^2 = -\frac{1}{2}dt^2 \cdot \frac{\odot + \delta}{yy}$$

vnde si ponamus orbitae solaris:

semiparametrum = a ; excentricitatem = e et anomaliam veram = ω

$$\text{erit } y = \frac{a}{1-e \cos u}; du = d\theta = \frac{dt}{yy} \sqrt{\frac{1}{2}e(\odot + \delta)}$$

Sit a semiaxis transversus orbitae solis, ac tempore = t sol motu medio absoluat angulum = ω , quo pro mensura temporis t utamur: erit ergo $d\omega = \frac{dt}{aa} \sqrt{\frac{1}{2}e(\odot + \delta)}$

ideoque $\frac{1}{2}dt^2 = \frac{a^3 d\omega^2}{\odot + \delta}$. At est $a = \frac{c}{1-ee}$. Hinc ergo fit

$$du = d\theta = \frac{ad\omega}{yy} \sqrt{ec} = \frac{ad\omega}{yy} \sqrt{(1-ee)} = \frac{d\omega(1-ee \cos u)^2}{(1-ee)\sqrt{(1-ee)}}$$

ficque tam du quam $d\theta$ per elementum $d\omega$ loco temporis introductum expressimus. Quia autem massa solis $\odot$ massam terrae δ tam enormiter excedit, sine errore pro $\frac{1}{2}dt^2$ scribi poterit $\frac{a^3 d\omega^2}{\odot}$, eruntque nostrae aequationes pro luna:

$$\text{I. } 2dx d\phi + xdd\phi = -a^3 d\omega^2 \left(\frac{y}{x^3} - \frac{1}{yy} \right) \sin \omega$$

$$\text{II. } ddx - x d\phi^2 = -\frac{a^3 (\delta + \odot) d\omega}{\odot} \cos \psi \left(\frac{1}{xx} - \frac{1}{bk} \right) - a^3 d\omega^2 \left(\frac{x-y \cos \eta}{x^3} + \frac{\cos \eta}{yy} \right)$$

III. dx

ro regulari: ha-

$$\frac{\odot + \delta}{yy}$$

ano-
=

re = t
o men-
($\odot + \delta$)

ergo fit
of u)²
(1-ee)

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solis $\odot$
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ationes

$$\left(\frac{1}{x} - \frac{1}{bk} \right)$$

III. dx

$$\text{III. } d\pi = -\frac{a^3 d\omega^2}{xd\phi} \left(\frac{y}{x^3} - \frac{1}{yy} \right) \sin(\phi - \pi) \sin(\theta - \pi)$$

$$\text{IV. } d. l \tan \varphi = \frac{d\pi}{\tan(\phi - \pi)}; \text{ atque ob } \tan \psi = \tan \varphi \cos(\phi - \pi), \text{ habebitur proxime } \cos \psi^2 = 1 - \frac{1}{2} \tan \varphi^2 - \frac{1}{2} \tan \varphi^2 \cos 2(\phi - \pi).$$

III.

Incipiamus a duabus aequationibus prioribus, ac ponamus breuitatis gratia

$$a^3 \left(\frac{y}{x^3} - \frac{1}{yy} \right) \sin \eta = M. \text{ et}$$

$$\frac{a^3 (\delta + \odot)}{\odot} \cos \psi \left(\frac{1}{xx} - \frac{1}{bk} \right) + a^3 \left(\frac{x-y \cos \eta}{x^3} + \frac{\cos \eta}{yy} \right) = \frac{A}{xx} + N$$

quandoquidem haec posterior expressio terminum involuit formae $\frac{A}{xx}$ prae ceteris incomparabiliter maiorem; atque habebimus has duas aequationes:

$$2dx d\phi + xdd\phi = -M d\omega^2 \text{ et } ddx - x d\phi^2 = -\frac{A d\omega^2}{xx} - N d\omega^2$$

quarum prior per $2x^3 d\phi$ multiplicata ob $d\omega$ constans habebit integrale:

$$x^4 d\phi^2 = -2d\omega^2 \int M x^3 d\phi$$

Tum prior multiplicata per $2x d\phi$ addatur ad posteriorem per $2dx$ multiplicatam, eritque aggregatum:

$$2x dx d\phi^2 + 2xx d\phi d\phi + 2dx ddx = -2M x d\omega^2 d\phi - \frac{2A d\omega^2 dx}{xx} - 2N d\omega^2 dx$$

Cuius integrale erit:

$$dx^2 + xx d\phi^2 = + \frac{2A d\omega^2}{x} - 2d\omega^2 \int (M x d\phi + N dx)$$

Mm 3

IV.

IV.

Ponantur formulae integrales, quae in his expressionibus insunt:

$\int Mx^3 d\phi = P$ et $\int (Mxd\phi + Ndx) = Q$
vt habeamus has duas aequationes;

$$x^4 d\phi^2 = 2P d\omega^2 \text{ et } dx^2 + xx d\phi^2 = \frac{2A d\omega^2}{x} + 2Q d\omega^2$$

unde cum sit $xx d\phi^2 = \frac{2P d\omega^2}{xx}$ erit

$$dx^2 = 2d\omega^2 \left(Q + \frac{A}{x} - \frac{P}{xx} \right) \text{ et } dx = \pm d\omega \sqrt{2 \left(Q + \frac{A}{x} - \frac{P}{xx} \right)}$$

ficque differentiale dx per $d\omega$ exprimitur. Deinde vero habetur

$$d\phi = \frac{d\omega}{xx} \sqrt{2P}$$

estque per hypothesin:

$$dP = -Mxd\omega \sqrt{2P} \text{ et } dQ = -\frac{Mdx}{x} \sqrt{2P} + Ndx \sqrt{2 \left(Q + \frac{A}{x} - \frac{P}{xx} \right)}$$

vbi quidem signorum ambiguum inferius locum habere statuamus, quia motum ab apogeo numerare in animo est, ita vt hinc exeundo distantia x minuat.

V.

Cum igitur differentiale dx in apogeo et perigeo euanescat, necesse est vt his locis formula irrationalis $\sqrt{Q + \frac{A}{x} - \frac{P}{xx}}$ in nihilum abeat, in reliquis autem locis valorem fortitur realem. Commodissime ergo haec formula per sinum cuiuspiam anguli v exhibebitur, qui cum in apogeo euanescat, in perigeo autem duobus re-

ctis

ctis aequalis fiat, anomaliam lunae referet: idque sensu vero, ita vt distantia x in apogeo prodeat maxima, in perigeo vero minima. Sit igitur vt formam motus regularis sequamur:

$$\text{semilatus rectum orbitae lunaris} = p$$

$$\text{excentricitas orbitae} = q$$

$$\text{et anomalia vera lunae} = v$$

$$\text{eritque hinc per eandem legem distantia } x = \frac{p}{1 - q \cos v}.$$

Verum hic quantitates p et q , quae in motu regulari essent constantes, nunc pro variabilibus sunt habendae, earumque variabilitas per variabilitatem quantitarum P et Q , quae in motu regulari itidem sunt constantes, determinari debet.

VI.

$$\text{Substituamus ergo valorem assumptum } x = \frac{p}{1 - q \cos v}$$

in formula irrationali $\sqrt{Q + \frac{A}{x} - \frac{P}{xx}}$, quae abibit in

$$\frac{1}{p} \sqrt{Qpp + Ap(1 - q \cos v) - P(1 - q \cos v)^2}$$

et euoluta dabit

$$\frac{1}{p} \sqrt{Qpp + Ap - P - Apq \cos v + 2Pq \cos v - Pqq \cos^2 v}$$

quae vt reducat ad formam $V \sin v$, statuatur

$$\text{primo } 2P - Ap = 0$$

$$\text{tum vero } Qpp + Ap - P = Pqq$$

$$\text{ac nostra formula fiet } = \frac{1}{p} \sqrt{Pqq \sin^2 v} = \frac{q \sin v}{p} \sqrt{P},$$

habebimusque

dx

$$dx = -\frac{q d\omega \sin v}{p} \sqrt{2P} \text{ et}$$

$$dQ = -\frac{M d\omega}{x} \sqrt{2P} + \frac{N q d\omega \sin v}{p} \sqrt{2P}$$

VII.

Cum iam sit $2P - Ap = 0$; erit $P = \frac{1}{2} Ap$: quo valore in altera formula substituto orietur:

$$Qpp + \frac{1}{2} Ap = \frac{1}{2} Apqq \text{ seu } Q = -\frac{A}{2p} (1 - qq)$$

Sumantur nunc differentialia; eritque

$$dP = -Mx d\omega \sqrt{2P} = \frac{1}{2} A dp, \text{ quae ob } 2P = Ap \text{ abit in hanc}$$

$$-Mx d\omega \sqrt{Ap} = \frac{1}{2} A dp, \text{ siue } dp = -\frac{2Mx d\omega}{A} \sqrt{Ap}$$

$$\text{vel etiam } \sqrt{Ap} = -\int Mx d\omega$$

Simili modo erit

$$dQ = +\frac{A dp (1 - qq)}{2pp} + \frac{A q dq}{p} = -\frac{Mx d\omega (1 - qq)}{pp} \sqrt{Ap} + \frac{A q dq}{p}$$

ideoque

$$\frac{A q dq}{p} = Mx d\omega \left(\frac{x(1 - qq)}{pp} - \frac{1}{x} \right) \sqrt{Ap} + \frac{N q d\omega \sin v}{p} \sqrt{Ap}$$

$$\text{At est } \frac{x(1 - qq)}{pp} - \frac{1}{x} = \frac{x}{pp} \left(1 - qq - \frac{pp}{xx} \right) = \frac{x}{pp} (1 - qq + 2q \cos v - qq \cos^2 v)$$

$$\text{siue } \frac{x(1 - qq)}{pp} - \frac{1}{x} = \frac{qx}{pp} (2 \cos v - q - q \cos^2 v)$$

Hinc ergo colligitur:

$$dq = \frac{Mx d\omega}{Ap} (2 \cos v - q - q \cos^2 v) \sqrt{Ap} + \frac{N d\omega \sin v}{A} \sqrt{Ap} \text{ siue}$$

$$dq = d\omega \left(\frac{M}{A} (2 \cos v - \frac{q \sin v^2}{1 - q \cos v}) + \frac{N}{A} \sin v \right) \sqrt{Ap}$$

VIII.

VIII.

Inuenta iam relatione differentialium dx , dp et dq ad differentiale temporis $d\omega$ scilicet:

$$dx = -\frac{q d\omega \sin v}{q} \sqrt{Ap}; \quad dp = -\frac{2Mx d\omega}{A} \sqrt{Ap}$$

$$\text{et } dq = d\omega \left(\frac{M}{A} (2 \cos v - \frac{q \sin v^2}{1 - q \cos v}) + \frac{N}{A} \sin v \right) \sqrt{Ap}$$

superest, ut quoque relationem elementi anomaliae dv definiamus. Cum igitur sit

$$x = \frac{p}{1 - q \cos v}, \text{ erit } 1 - q \cos v = \frac{p}{x}; \text{ hincque differentiendo}$$

$$q dv \sin v = dq \cos v + \frac{dp}{x} - \frac{p dx}{xx};$$

substituuntur valores pro dq , dp et dx inuenti; ac diuisione facta per $q \sin v$ prodibit

$$dv = \frac{d\omega}{xx} \sqrt{Ap} - \frac{d\omega}{q} \left(\frac{M}{A} (2 \sin v + \frac{q \sin v \cos v}{1 - q \cos v}) - \frac{N}{A} \cos v \right) \sqrt{Ap}$$

Pro elemento autem longitudinis $d\phi$ ob $2P = Ap$, ex antecedentibus habemus:

$$d\phi = \frac{d\omega}{xx} \sqrt{Ap} = \frac{d\omega (1 - q \cos v)^2}{pp} \sqrt{Ap}$$

IX.

Ex his formulis statim se offert motus apogei; cum enim longitudo apogei sit $= \phi - v$, erit eius differentiale pro tempusculo $d\omega$:

$$d\phi - dv = \frac{d\omega}{q} \left(\frac{M}{A} (2 \sin v + \frac{q \sin v \cos v}{1 - q \cos v}) - \frac{N}{A} \cos v \right) \sqrt{Ap}$$

cuius ergo integrale praebebit verum motum apogei cum omnibus inaequalitatibus, quibus perturbatur. Vnde

N n

qui-

Ap: quo

(1-qq)

p abit in

v \sqrt{Ap}

1p + \frac{A q dq}{p}

v \sqrt{Ap}

(v-qqcfv^2)

\sqrt{Ap} siue

Ap

VIII.

quidem perspicitur, quod per se est manifestum, si quantitates M et N evanescerent, motum apogei fore nullum, seu apogeu perpetuo in loco fixo esse permanfurum. Deinde etiam iuuabit notasse has formulas:

$$d. q \cos v = -q d\phi \sin v + \frac{2M}{A} d\omega \sqrt{Ap}$$

$$d. q \sin v = +q d\phi \cos v + d\omega \left(\frac{N}{A} - \frac{M}{A} \cdot \frac{q \sin v}{1 - q \cos v} \right) \sqrt{Ap}$$

Tandem quoque habemus ex motu solis $d\omega = d\theta = \frac{d\omega (1 - e \cos u)^2}{(1 - e^2) \sqrt{1 - e^2}}$ ideoque

$$d\eta = d\phi - d\theta = d\omega \left(\frac{(1 - q \cos v)}{pp} \sqrt{Ap} - \frac{(1 - e \cos u)^2}{(1 - e^2) \sqrt{1 - e^2}} \right)$$

X.

Inuentis nunc omnium differentialium relationibus ad elementum temporis $d\omega$, euoluamus valores litterarum M et N, ac primo quidem cum fit $z = V(yy - 2xy \cos \eta + xx \sec^2 \psi)$; quoniam quantitas z nonnisi in terminis minimis occurrit, pro sec. ψ tuto unitas scribi poterit, et quia y tantopere excedit x , erit proxime

$$\frac{1}{z^3} = \frac{1}{y^3} + \frac{3x}{y^4} \cos \eta + \frac{3xx}{2y^5} (5 \cos^2 \eta - 1) \text{ siue}$$

$$\frac{1}{z^3} = \frac{1}{y^3} + \frac{3x}{y^4} \cos \eta + \frac{3xx}{4y^5} (3 + 5 \cos 2\eta)$$

Ideoque hinc habebitur:

$$\frac{y}{z^3} - \frac{1}{yy} = \frac{3x}{y^3} \cos \eta + \frac{3xx}{4y^4} (3 + 5 \cos 2\eta)$$

Vnde

Vnde obtinemus:

$$M = x^3 \left(\frac{3}{2y^3} \sin 2\eta + \frac{3xx}{8y^4} (\sin \eta + 5 \sin 3\eta) \right)$$

$$N = \frac{x^3(\delta + 2)}{\odot} \cos \psi^3 \left(\frac{1}{xx} - \frac{1}{bb} \right) - \frac{A}{xx} - x^3 \left(\frac{x}{2y^3} (1 + 3 \cos 2\eta) + \frac{3xx}{8y^4} (3 \cos \eta + 5 \cos 3\eta) \right)$$

XI.

Cum sit proxime $\cos \psi^3 = 1 - \frac{3}{2} \tan^2 \phi - \frac{3}{2} \tan^2 \phi \cos 2(\phi - \pi)$, eius valor unitate erit minor, atque ex parte constante, et parte variabili constabit, quae illa multo erit minor. Ponatur ergo

$\cos \psi^3 = \lambda + \Pi$; vt fit $\Pi = 1 - \lambda - \frac{3}{2} \tan^2 \phi - \frac{3}{2} \tan^2 \phi \cos 2(\phi - \pi)$ vbi λ denotat partem constantem unitate proxime aequalem, Π vero partem variabilem.

Erit ergo:

$$N = \frac{\lambda x^3(\delta + 2)}{\odot} \left(\frac{1}{xx} - \frac{1}{bb} \right) - \frac{A}{xx} + \frac{x^3(\delta + 2)}{\odot} \Pi \left(\frac{1}{xx} - \frac{1}{bb} \right) - x^3 \left(\frac{x}{2y^3} (1 + 3 \cos 2\eta) + \frac{3xx}{8y^4} (3 \cos \eta + 5 \cos 3\eta) \right)$$

Statuatur nunc $A = \frac{\lambda x^3(\delta + 2)}{\odot}$; vt fiat

$$N = -\frac{A}{bb} + A \Pi \left(\frac{1}{xx} - \frac{1}{bb} \right) - x^3 \left(\frac{x}{2y^3} (1 + 3 \cos 2\eta) + \frac{3xx}{8y^4} (3 \cos \eta + 5 \cos 3\eta) \right)$$

N n 2

ac

ac ponatur breuitatis gratia: $p = b(1 + \xi)$

$$\text{erit } \frac{\sqrt{Ap}}{pp} = \sqrt{\frac{A}{b^3(1+\xi)^3}} = (1 - \frac{3}{2}\xi + \frac{1}{8}\xi^2) \sqrt{\frac{A}{b^3}}$$

ob ξ prae 1 vehementer paruum, sitque porro:

$$\sqrt{\frac{A}{b^3}} = \sqrt{\frac{\lambda a^3(8+\gamma)}{b^3}} = m,$$

atque habebitur $d\phi = m d\omega (1 - \frac{3}{2}\xi + \frac{1}{8}\xi^2) (1 - q \cos v)^2$

XII.

Substituantur nunc pro x et y valores $\frac{p}{1 - q \cos v}$

et $\frac{c}{1 - e \cos u}$, eritque

$$M = a^3 \left(\frac{3p(1 - e \cos u)^3}{2c^3(1 - q \cos v)} \sin 2\eta + \frac{3pp(1 - e \cos u)^4}{8c^4(1 - q \cos v)^2} (\sin \eta + 5 \sin 3\eta) \right)$$

$$N = -\frac{A}{bb} + A\Pi \left(\frac{(1 - q \cos v)^2}{pp} - \frac{1}{bb} \right),$$

$$-a^3 \left(\frac{p(1 - e \cos u)^3}{2c^3(1 - q \cos v)} (1 + 3 \cos 2\eta) + \frac{3pp(1 - e \cos u)^4}{8c^4(1 - q \cos v)^2} (3 \cos \eta + 5 \cos 3\eta) \right)$$

vbi quidem quoque terminus $\frac{A\Pi}{bb}$ prae termino $\frac{A}{bb}$

omitti potest. Nunc ut hinc valores $\frac{M}{A}\sqrt{Ap}$ et $\frac{N}{A}\sqrt{Ap}$

commode exprimantur, erit

$$\frac{a^3 b}{A c^3} \sqrt{Ab} = \frac{a^3}{m c^3} = \frac{1}{m(1 - ee)} = \frac{1 + 3ee}{m}$$

quoniam in his terminis minimis pro $1 - ee$ scribere licet 1.

Tum vero sit $\frac{b}{c} = n$, eritque n fractio valde parua.

XIII.

XIII.

Factis ergo his substitutionibus, ob $p = b(1 + \xi)$ habebimus:

$$\begin{aligned} \frac{M}{A} \sqrt{Ap} &= \frac{3(1 + 3ee)}{2m} \frac{(1 - e \cos u)^3}{1 - q \cos v} (1 + \frac{1}{2}\xi) \sin 2\eta \\ &+ \frac{3n}{8m} \frac{(1 - e \cos u)^4}{(1 - q \cos v)^2} (1 + \frac{1}{2}\xi) (\sin \eta + 5 \sin 3\eta) \end{aligned}$$

Pro altera valore $\frac{N}{A}\sqrt{Ap}$ statuatur terminus minimus:

$$\frac{\sqrt{Ab}}{bb} = \sqrt{\frac{\lambda a^3 b(8+\gamma)}{b^4}} = i; \text{ eritque}$$

$$\begin{aligned} \frac{N}{A} \sqrt{Ap} &= -\frac{(1 + 3ee)}{2m} \frac{(1 - e \cos u)^3}{1 - q \cos v} (1 + \frac{1}{2}\xi) (1 + 3 \cos 2\eta) \\ &- \frac{3n}{8m} \frac{(1 - e \cos u)^4}{(1 - q \cos v)^2} (1 + \frac{1}{2}\xi) (3 \cos \eta + 5 \cos 3\eta) \\ &+ m (1 - q \cos v)^2 (1 - \frac{1}{2}\xi) \Pi - i \end{aligned}$$

vbi notari oportet, terminos per n multiplicatos ratione praecedentium esse minimos; tum vero quantitates ξ et Π atque multo magis i esse fractiones prae unitate fere evanescentes.

XIV.

Quoniam hi ipsi termini quantitates M et N involuentes sunt valde parui, in iis sine errore altiores potestates variusque excentricitatis q et e negligi possunt. In terminis ergo primis simpliciter per m diuisis excentricitates tantum ad duas dimensiones intro-

N n 3

ducan-

ducantur, in terminis autem per $\frac{n}{m}$ multiplicatis penitus omittantur, quia fractio n iam fere quadrato excentricitatis q aequivalet. In termino autem littera minima Π affecto, quia is per numerum m facis magnum, vixote 13 fere, est multiplicatus, excentricitas q vnius dimensionis retineatur.

His observatis habebimus:

$$\begin{aligned} \frac{M}{A} \vee A p = & + \frac{3}{2m} (1 + \frac{1}{2} ee + \frac{1}{2} qq) \sin 2\eta + \frac{3q}{4m} \sin (2\eta - v) \\ & + \frac{3q}{4m} \sin (2\eta + v) - \frac{9e}{4m} \sin (2\eta - u) - \frac{9e}{4m} \sin (2\eta + u) \\ & + \frac{3qq}{8m} \sin (2\eta - 2v) + \frac{3qq}{8m} \sin (2\eta + 2v) \\ & + \frac{9ee}{8m} \sin (2\eta - 2u) + \frac{9ee}{8m} \sin (2\eta + 2u) \\ & - \frac{9eq}{8m} \sin (2\eta - v + u) - \frac{9eq}{8m} \sin (2\eta + v - u) \\ & - \frac{9eq}{8m} \sin (2\eta - v - u) - \frac{9eq}{8m} \sin (2\eta + v + u) \\ & + \frac{9}{4m} \xi \sin 2\eta + \frac{9}{8m} q \xi \sin (2\eta - v) + \frac{9}{8m} q \xi \sin (2\eta + v) \\ & - \frac{27}{8m} e \xi \sin (2\eta - u) - \frac{27}{8m} e \xi \sin (2\eta + u) \\ & + \frac{3n}{8m} \sin \eta + \frac{15n}{8m} \sin 3\eta \end{aligned}$$

$$\frac{N}{A} \vee A p =$$

$$\begin{aligned} \frac{N}{A} \vee A p = & - \frac{1}{2m} (1 + \frac{1}{2} ee + \frac{1}{2} qq) - \frac{3}{2m} (1 + \frac{1}{2} ee + \frac{1}{2} qq) \cos 2\eta \\ & - \frac{q}{2m} \cos v + \frac{3e}{2m} \cos u \\ & - \frac{3q}{4m} \cos (2\eta - v) - \frac{3q}{4m} \cos (2\eta + v) \\ & + \frac{9e}{4m} \cos (2\eta - u) + \frac{9e}{4m} \cos (2\eta + u) \\ & - \frac{qq}{4m} \cos 2v + \frac{3eq}{4m} \cos (v - u) + \frac{3eq}{4m} \cos (v + u) \\ & - \frac{3ee}{4m} \cos 2u \\ & - \frac{3qq}{8m} \cos (2\eta - 2v) - \frac{3qq}{8m} \cos (2\eta + 2v) \\ & - \frac{9ee}{8m} \cos (2\eta - 2u) - \frac{9ee}{8m} \cos (2\eta + 2u) \\ & + \frac{9eq}{8m} \cos (2\eta - v + u) + \frac{9eq}{8m} \cos (2\eta + v - u) \\ & + \frac{9eq}{8m} \cos (2\eta - v - u) + \frac{9eq}{8m} \cos (2\eta + v + u) \\ & - \frac{3}{4m} \xi - \frac{9}{4m} \xi \cos 2\eta - \frac{3q}{8m} \xi \cos v \\ & - \frac{9q}{8m} \xi \cos (2\eta - v) - \frac{9q}{8m} \xi \cos (2\eta + v) \\ & + \frac{9e}{4m} \xi \cos u + \frac{27e}{8m} \xi \cos (2\eta - u) \\ & + \frac{27e}{8m} \xi \cos (2\eta + u) - \frac{9n}{8m} \cos \eta - \frac{15n}{8m} \cos 3\eta \\ & + n \Pi - 2m q \Pi \cos v - \frac{1}{2} m \xi \Pi - i \end{aligned}$$

XV.

Quaeramus igitur valores evolutos nostrorum differentialium ad elementum temporis applicatorum: ac primo quidem habebimus:

$$\frac{d\Phi}{d\omega} = m(1 + \frac{1}{2}qq) - 2mq \cos v + \frac{1}{2}mqg \cos 2v - \frac{1}{2}m(1 + \frac{1}{2}qq)\xi + 3mq\xi \cos v - \frac{1}{2}mqg\xi \cos 2v + \frac{1}{8}m\xi\xi$$

$$\frac{du}{d\omega} = \frac{d\theta}{d\omega} = 1 + 2ee - 2e \cos u + \frac{1}{2}ee \cos 2u; \text{ vnde concludimus}$$

$$\frac{d\eta}{d\omega} = m(1 + \frac{1}{2}qq) - 1 - 2ee - 2mq \cos v + 2e \cos u + \frac{1}{2}mqg \cos 2v - \frac{1}{2}ee \cos 2u - \frac{1}{2}m(1 + \frac{1}{2}qq)\xi + 3mq\xi \cos v - \frac{1}{2}mqg\xi \cos 2v + \frac{1}{8}m\xi\xi$$

$$\text{Deinde cum sit } \frac{dp}{d\omega} = -2x \cdot \frac{M}{A} \sqrt{Ap} = -\frac{2b(1+\xi)}{1-qcfv} \cdot \frac{M}{A} \sqrt{Ap},$$

$$\text{ob } p = b(1+\xi) \text{ erit } \frac{d\xi}{d\omega} =$$

$$\left(-2(1 + \frac{1}{2}qq) - 2q \cos v - qq \cos 2v - 2\xi - 2q\xi \cos v \right) \frac{M}{A} \sqrt{Ap}$$

ac valorem pro $\frac{M}{A} \sqrt{Ap}$ inuentum substituendo obtinebimus sequentes formulas:

$$\frac{d\xi}{d\omega} =$$

$$\begin{aligned} \frac{d\xi}{d\omega} = & \left[-\frac{3}{m} \left(1 + \frac{1}{2}ee + \frac{1}{2}qq \right) \sin 2\eta - \frac{3q}{m} \sin(2\eta - v) - \frac{3q}{m} \sin(2\eta + v) \right. \\ & + \frac{9e}{2m} \sin(2\eta - v) + \frac{9e}{2m} \sin(2\eta + v) \\ & - \frac{9qq}{4m} \sin(2\eta - 2v) - \frac{9qq}{4m} \sin(2\eta + 2v) \\ & - \frac{9ee}{4m} \sin(2\eta - 2v) - \frac{9ee}{4m} \sin(2\eta + 2v) \\ & + \frac{9eq}{2m} \sin(2\eta - v + u) + \frac{9eq}{2m} \sin(2\eta + v - u) \\ & + \frac{9eq}{2m} \sin(2\eta - v - u) + \frac{9eq}{2m} \sin(2\eta + v + u) \\ & - \frac{15}{2m} \xi \sin 2\eta - \frac{15q}{2m} \xi \sin(2\eta - v) - \frac{15q}{2m} \xi \sin(2\eta + v) \\ & + \frac{45e}{4m} \xi \sin(2\eta - v) + \frac{45e}{4m} \xi \sin(2\eta + v) \\ & \left. - \frac{3n}{4m} \sin \eta - \frac{15n}{4m} \sin 3\eta \right] \end{aligned}$$

XVI.

$$\text{Porro cum sit } \frac{q \sin v^2}{1-q \cos v} = \frac{q - q \cos 2v}{2(1-q \cos v)} =$$

$$\frac{1}{2}q - \frac{1}{2}q \cos 2v + \frac{1}{2}qq \cos v - \frac{1}{2}qq \cos 3v; \text{ erit}$$

$$\frac{dq}{d\omega} = (2cfv - \frac{1}{2}q + \frac{1}{2}q \cos 2v - \frac{1}{2}qq \cos v + \frac{1}{2}qq \cos 3v) \frac{M}{A} \sqrt{Ap} + \sin v \cdot \frac{N}{A} \sqrt{Ap}$$

Facta ergo substitutione valorum pro $\frac{M}{A} \sqrt{Ap}$ et $\frac{N}{A} \sqrt{Ap}$ inuentorum, habebitur:

Oo

$$\frac{dq}{d\omega} =$$

$$\begin{aligned}
& + \frac{9}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \sin(2\eta - v) + \frac{3}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \\
& \sin(2\eta + v) - \frac{1}{2m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \sin v \\
& + \frac{3q}{4m} \sin 2\eta + \frac{3q}{2m} \sin(2\eta - 2v) + \frac{3q}{4m} \sin(2\eta + 2v) - \frac{q}{4m} \sin 2v \\
& - \frac{27e}{8m} \sin(2\eta - v - u) - \frac{27e}{8m} \sin(2\eta - v + u) - \frac{9e}{8m} \sin(2\eta + v - u) \\
& - \frac{9e}{8m} \sin(2\eta + v + u) + \frac{3e}{4m} \sin(v - u) + \frac{3e}{4m} \sin(v + u) \\
& + \frac{15qq}{16m} \sin(2\eta - 3v) + \frac{9qq}{16m} \sin(2\eta + 3v) - \frac{qq}{8m} \sin 3v \\
& - \frac{9eq}{8m} \sin(2\eta - u) - \frac{9eq}{8m} \sin(2\eta + u) - \frac{9eq}{4m} \sin(2\eta - 2v + u) \\
& - \frac{9eq}{4m} \sin(2\eta - 2v - u) - \frac{9eq}{8m} \sin(2\eta + 2v - u) - \frac{9eq}{8m} \sin(2\eta + 2v + u) \\
& + \frac{27ee}{16m} \sin(2\eta - v - 2u) + \frac{27ee}{16m} \sin(2\eta - v + 2u) \\
& + \frac{9ee}{16m} \sin(2\eta + v - 2u) + \frac{9ee}{16m} \sin(2\eta + v + 2u) \\
& + \frac{3eq}{8m} \sin(2v - u) + \frac{3eq}{8m} \sin(2v + u) \\
& - \frac{3ee}{8m} \sin(v - 2u) - \frac{3ee}{8m} \sin(v + 2u) \\
& + \frac{27}{8m} \xi \sin(2\eta - v) + \frac{9}{8m} \xi \sin(2\eta + v) - \frac{3}{4m} \xi \sin v \\
& + \frac{9q}{8m}
\end{aligned}$$

$$\begin{aligned}
& + \frac{9q}{8m} \xi \sin 2\eta + \frac{9q}{4m} \xi \sin(2\eta - 2v) + \frac{9q}{8m} \xi \sin(2\eta + 2v) \\
& - \frac{3q}{8m} \xi \sin 2v + \frac{9e}{8m} \xi \sin(v - u) + \frac{9e}{8m} \xi \sin(v + u) \\
& - \frac{81e}{16m} \xi \sin(2\eta - v - u) - \frac{81e}{16m} \xi \sin(2\eta - v + u) \\
& - \frac{27e}{16} \xi \sin(2\eta + v - u) - \frac{27e}{16m} \xi \sin(2\eta + v + u) \\
& + \frac{15n}{16m} \sin(\eta - v) - \frac{3n}{16m} \sin(\eta + v) \\
& + \frac{45n}{16m} \sin(3\eta - v) + \frac{15n}{16m} \sin(3\eta + v) \\
& + m \Pi \sin v - mq \Pi \sin 2v - \frac{1}{2} m \xi \Pi \sin v - i \sin v
\end{aligned}$$

XVII.

Deinde cum sit $\frac{q \sin v \cos v}{1 - q \cos v} = \frac{q \sin 2v}{2(1 - q \cos v)} =$
 $= \frac{1}{2} q \sin 2v + \frac{1}{4} q q \sin v + \frac{1}{4} q q \sin 3v$; erit
 pro motu elementari apogei:
 $\frac{q(d\Phi - dv)}{d\omega} = (2 \sin v + \frac{1}{2} q \sin 2v + \frac{1}{4} q q \sin v$
 $+ \frac{1}{4} q q \sin 3v) \frac{M}{A} \sqrt{Ap} - \cos v. \frac{N}{A} \sqrt{Ap}$
 ac facta substitutione obtinebitur:

O o 2

$$\frac{q(d\Phi - dv)}{d\omega} =$$

$$\begin{aligned}
& + \frac{9}{4m} (1 + \frac{2}{3}ee + \frac{2}{3}qq) \cos(2\eta - v) - \frac{3}{4m} (1 + \frac{2}{3}ee + \frac{2}{3}qq) \\
& \quad \cos(2\eta + v) + \frac{1}{2m} (1 + \frac{2}{3}ee + \frac{2}{3}qq) \cos v \\
& + \frac{3q}{4m} \cos 2\eta + \frac{3q}{2m} \cos(2\eta - 2v) - \frac{3q}{4m} \cos(2\eta + 2v) \\
& + \frac{q}{4m} \cos 2v + \frac{q}{4m} - \frac{3e}{4m} \cos(v - u) - \frac{3e}{4m} \cos(v + u) \\
& - \frac{27e}{8m} \cos(2\eta - v - u) - \frac{27e}{8m} \cos(2\eta - v + u) \\
& + \frac{9e}{8m} \cos(2\eta + v - u) + \frac{9e}{8m} \cos(2\eta + v + u) \\
& + \frac{15qq}{16m} \cos(2\eta - 3v) - \frac{9qq}{16m} \cos(2\eta + 3v) + \frac{qq}{8m} \cos 3v \\
& - \frac{9eq}{8m} \cos(2\eta - u) - \frac{9eq}{8m} \cos(2\eta + u) - \frac{3eq}{4m} \cos u \\
& - \frac{3eq}{8m} \cos(2v - u) - \frac{3eq}{8m} \cos(2v + u) \\
& - \frac{9eq}{4m} \cos(2\eta - 2v + u) - \frac{9eq}{4m} \cos(2\eta - 2v - u) \\
& + \frac{9eq}{8m} \cos(2\eta + 2v - u) + \frac{9eq}{8m} \cos(2\eta + 2v + u) \\
& + \frac{27ee}{16m} \cos(2\eta - v - 2u) + \frac{27ee}{16m} \cos(2\eta - v + 2u) \\
& - \frac{9ee}{16m} \cos(2\eta + v - 2u) - \frac{9ee}{16m} \cos(2\eta + v + 2u) \\
& + \frac{3ee}{8m} \cos(v - 2u) + \frac{3ee}{8m} \cos(v + 2u)
\end{aligned}$$

+ $\frac{27}{8m}$

 $\frac{g(d\Phi - dv)}{d\omega} =$

Euolu
et cum f

 $dx = - \frac{1}{2}$

Substitu

 $VAp =$
 $dx = - \frac{du}{m}$
+ $\frac{1}{2}qq$ + $2v$ + u

u)

)

f $3v$

u

$$\begin{aligned}
& + \frac{27}{8m} \xi \cos(2\eta - v) - \frac{9}{8m} \xi \cos(2\eta + v) + \frac{3}{4m} \xi \cos v \\
& + \frac{9q}{8m} \xi \cos 2\eta + \frac{9q}{4m} \xi \cos(2\eta - 2v) - \frac{9q}{8m} \xi \cos(2\eta + 2v) \\
& + \frac{3q}{8m} \xi + \frac{3q}{8m} \xi \cos 2v \\
& - \frac{81e}{16m} \xi \cos(2\eta - v - u) + \frac{27e}{16m} \xi \cos(2\eta + v - u) \\
& - \frac{81e}{16m} \xi \cos(2\eta - v + u) + \frac{27e}{16m} \xi \cos(2\eta + v + u) \\
& - \frac{9e}{8m} \xi \cos(v - u) - \frac{9e}{8m} \xi \cos(v + u) \\
& + \frac{15n}{16m} \cos(\eta - v) + \frac{3n}{16m} \cos(\eta + v) \\
& + \frac{45n}{16m} \cos(3\eta - v) - \frac{15n}{16m} \cos(3\eta + v) \\
& - m \Pi \cos v + m q \Pi + m q \Pi \cos 2v + \frac{1}{2} m \xi \Pi \cos v + \xi \cos v
\end{aligned}$$

XVIII.

Euoluamus simili modo valorem differentialium dx et dy ,
et cum sit $\frac{y}{x^3} - \frac{1}{y} = \frac{3x}{y^3} \cos \eta + \frac{3xx}{4y^4} (3 + 5 \cos 2\eta)$ et $d\Phi = \frac{du}{xx} VAp$, erit
 $dx = - \frac{x^3 x d\omega}{VAp} \left(\frac{3x}{y^3} \cos \eta + \frac{3xx}{4y^4} (3 + 5 \cos 2\eta) \right) \sin(\theta - \pi) \sin(\Phi - \pi)$
 Substitutis autem valoribus $x = \frac{p}{1 - q \cos v}$, $y = \frac{e}{1 - e \cos u}$, $p = b(1 + \frac{1}{2} \xi)$,
 $VAp = m V b^3 p = m b b (1 + \frac{1}{2} \xi)$, $\frac{x^3}{e^3} = \frac{1}{(1 - ee)^3} = 1 + 3ee$ et $\frac{b}{e} = n$, erit
 $dx = - \frac{du \sin(\theta - \pi) \sin(\Phi - \pi)}{m} \left[\frac{3(1 + \frac{1}{2} \xi)(1 + 3ee)(1 - e \cos u)^3}{(1 - q \cos v)^2} \cos \eta + \frac{3}{2} (3 + 5 \cos 2\eta) \right]$
 O o 3 Negle-

 $\frac{17}{m}$

Neglectis igitur terminis, qui nullum valorem sensibilem continent, habebimus

$$\frac{d\pi}{d\omega} = \left[\begin{aligned} & -\frac{3}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) - \frac{3}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \cos 2\eta \\ & + \frac{3}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \cos 2(\Phi - \pi) + \frac{3}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \cos 2(\theta - \pi) \\ & - \frac{3q}{2m} \cos v - \frac{3q}{4m} \cos (2\eta - v) - \frac{3q}{4m} \cos (2\eta + v) \\ & + \frac{9e}{4m} \cos u + \frac{9e}{8m} \cos (2\eta - u) + \frac{9e}{8m} \cos (2\eta + u) \\ & + \frac{3q}{4m} \cos (2\Phi - 2\pi - v) + \frac{3q}{4m} \cos (2\Phi - 2\pi + v) \\ & + \frac{3q}{4m} \cos (2\theta - 2\pi - v) + \frac{3q}{4m} \cos (2\theta - 2\pi + v) \\ & - \frac{9e}{8m} \cos (2\Phi - 2\pi - u) - \frac{9e}{8m} \cos (2\Phi - 2\pi + u) \\ & - \frac{9e}{8m} \cos (2\theta - 2\pi - u) - \frac{9e}{8m} \cos (2\theta - 2\pi + u) \\ & - \frac{9}{8m} \xi - \frac{9}{8m} \xi \cos 2\eta \\ & + \frac{9}{8m} \xi \cos (2\Phi - 2\pi) + \frac{9}{8m} \xi \cos (2\theta - 2\pi) \\ & - \frac{11n}{16m} \cos \eta + \frac{3n}{8m} \cos (\Phi + \theta - 2\pi) - \frac{5n}{16m} \cos 3\eta \\ & + \frac{5n}{16m} \cos (3\Phi - \theta - 2\pi) + \frac{5n}{16m} \cos (3\theta - \Phi - 2\pi) \end{aligned} \right]$$

XIX.

Sim
tang (Φ
tangens

$\frac{d \text{ tang } \Phi}{d\omega}$

fibi.

[2\eta

+ \pi)

v)

u)

XIX.

Simili autem modo praecedentem valorem per tang ($\Phi - \pi$) dividendo prodibit differentiale logarithmi tangens inclinationis ϱ , erit enim

$$\frac{d \text{ tang } \varrho}{d\omega} = \left[\begin{aligned} & + \frac{3}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \sin 2\eta - \frac{3}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \\ & \sin 2(\Phi - \pi) - \frac{3}{4m} (1 + \frac{1}{2}ee + \frac{1}{2}qq) \sin 2(\theta - \pi) \\ & + \frac{3q}{4m} \sin (2\eta - v) + \frac{3q}{4m} \sin (2\eta + v) \\ & - \frac{9e}{8m} \sin (2\eta - u) - \frac{9e}{8m} \sin (2\eta + u) \\ & - \frac{3q}{4m} \sin (2\Phi - 2\pi - v) - \frac{3q}{4m} \sin (2\Phi - 2\pi + v) \\ & - \frac{3q}{4m} \sin (2\theta - 2\pi - v) - \frac{3q}{4m} \sin (2\theta - 2\pi + v) \\ & + \frac{9e}{8m} \sin (2\Phi - 2\pi - u) + \frac{9e}{8m} \sin (2\Phi - 2\pi + u) \\ & + \frac{9e}{8m} \sin (2\theta - 2\pi - u) + \frac{9e}{8m} \sin (2\theta - 2\pi + u) \\ & + \frac{9}{8m} \xi \sin 2\eta - \frac{9}{8m} \xi \sin (2\Phi - 2\pi) - \frac{9}{8m} \xi \sin (2\theta - 2\pi) \\ & + \frac{n}{16m} \sin \eta - \frac{3n}{8m} \sin (\Phi + \theta - \pi) + \frac{5n}{16m} \sin 3\eta \\ & - \frac{5n}{16m} \sin (3\Phi - \theta - 2\pi) - \frac{5n}{16m} \sin (3\theta - \Phi - 2\pi) \end{aligned} \right]$$

XX.

XX.

Quo iam facilius has formulas admodum complicatas euoluere queamus, quadruplicis generis terminos distingui conuenit. Primum scilicet genus eos complectitur terminos, qui tantum ab excentricitate orbitae lunaris pendent, neque excentricitatem solis, neque parallaxin solis seu litteram π , neque inclinationem orbitae lunaris seu litteram Π inuoluunt. Ad secundum genus refero terminos, qui ad primum genus insuper excentricitatem solis adiungunt. Ad tertium autem eos, qui praeterea parallaxin solis seu litteram π inducunt. In quarto autem eas inaequalitates, quae insuper ab obliquitate orbitae lunaris proueniunt, complexurus sum. Ab inaequalitatibus ergo primi generis exordiar, ideoque cum excentricitatem solis e , tum eius parallaxin, tum quoque obliquitatem orbitae lunaris reiciam

INVESTIGATIO INAEQUALITATUM

LUNAE PRIMI GENERIS.

XXI.

Neglectis ergo excentricitate solis cum eius parallaxi et obliquitate orbitae lunaris, has habebimus aequationes:

$$\frac{d\xi}{d\omega} =$$

$$\begin{aligned} \frac{d\xi}{d\omega} = & \left[-\frac{3}{m} (1 + \frac{1}{2} q q) \sin 2\eta - \frac{3q}{m} \sin (2\eta - v) - \frac{3q}{m} \sin (2\eta + v) \right. \\ & - \frac{9qq}{4m} \sin (2\eta - 2v) - \frac{9qq}{4m} \sin (2\eta + 2v) \\ & - \frac{15}{2m} \xi \sin 2\eta - \frac{15q}{2m} \xi \sin (2\eta - v) - \frac{15q}{2m} \xi \sin (2\eta + v) \\ & + \frac{9}{4m} (1 + \frac{1}{2} q q) \sin (2\eta - v) + \frac{3}{4m} (1 + \frac{1}{2} q q) \sin (2\eta + v) \\ & - \frac{1}{2m} (1 + \frac{1}{2} q q) \sin v - i \sin v \\ & + \frac{3q}{4m} \sin 2\eta + \frac{3q}{2m} \sin (2\eta - 2v) + \frac{3q}{4m} \sin (2\eta + 2v) \\ & - \frac{q}{4m} \sin 2v \\ & + \frac{15qq}{16m} \sin (2\eta - 3v) + \frac{9qq}{16m} \sin (2\eta + 3v) - \frac{qq}{8m} \sin 3v \\ & + \frac{27}{8m} \xi \sin (2\eta - v) + \frac{9}{8m} \xi \sin (2\eta + v) - \frac{3}{4m} \xi \sin v \\ & + \frac{9q}{8m} \xi \sin 2\eta + \frac{9q}{4m} \xi \sin (2\eta - 2v) + \frac{9q}{8m} \xi \sin (2\eta + 2v) \\ & \left. - \frac{3q}{8m} \xi \sin 2v \right] \end{aligned}$$

P p

$$\frac{q(d\phi dv)}{d\omega} =$$

$$\frac{q(d\Phi \cdot dv)}{d\omega} = \begin{aligned} & + \frac{9}{4m} (1 + \frac{1}{2} qg) \cos(2\eta - v) - \frac{3}{4m} (1 + \frac{1}{2} qg) \cos(2\eta + v) \\ & + \frac{1}{2m} (1 + \frac{1}{2} qg) \cos v + i \cos v \\ & + \frac{3g}{4m} \cos 2\eta + \frac{3g}{2m} \cos(2\eta - 2v) - \frac{3g}{4m} \cos(2\eta + 2v) \\ & + \frac{g}{4m} \cos 2v + \frac{g}{4m} \\ & + \frac{15gq}{16m} \cos(2\eta - 3v) - \frac{9gq}{16m} \cos(2\eta + 3v) + \frac{gq}{8m} \cos 3v \\ & + \frac{27}{8m} \xi \cos(2\eta - v) - \frac{9}{8m} \xi \cos(2\eta + v) + \frac{3}{4m} \xi \cos v \\ & + \frac{9g}{8m} \xi \cos 2\eta + \frac{9g}{4m} \xi \cos(2\eta - 2v) - \frac{9g}{8m} \xi \cos(2\eta + 2v) \\ & + \frac{3g}{8m} \xi + \frac{3g}{8m} \xi \cos 2v \end{aligned}$$

$$\frac{d\Phi}{d\omega} = m(1 + \frac{1}{2} qg) - 2mq \cos v + \frac{1}{2} mqq \cos 2v - \frac{1}{2} m(1 + \frac{1}{2} qg) \xi$$

$$\frac{d\eta}{d\omega} = m(1 + \frac{1}{2} qg) - 1 - 2mq \cos v + \frac{1}{2} mqq \cos 2v - \frac{1}{2} m(1 + \frac{1}{2} qg) \xi$$

XXII.

Hic autem primo patet valores litterarum ξ et q sine cognitis rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ definiri non posse, has autem vicissim ipsas quantitates ξ et q inuoluere. Cum autem ad valores ξ et q inueniendos non opus sit rationes

tionones $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ eo praecisionis gradu nosse, quo ipsi illi valores desiderantur; patet si valores ξ et q prope tantum veri consent, iis in rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ adhibitis, eosdem multo exactiores repertum iri. Cum igitur, si motus esset regularis, foret $\xi = 0$ et $q = \text{constanti}$, hinc primam hypothesein constituamus. Sic ergo

$$\xi = 0 \quad \text{et} \quad q = g$$

et neglectis terminis, qui ob harum litterarum errores affici possent, utpote valde paruis prae reliquis, habebimus proxime

$$\frac{d\Phi}{d\omega} = m(1 + \frac{1}{2} gg) - 2mg \cos v; \quad \frac{d\eta}{d\omega} = (1 + \frac{1}{2} gg) - 1 - 2mg \cos v$$

$$\text{et} \quad \frac{d\Phi - dv}{d\omega} = \frac{9}{4mg} (1 + \frac{1}{2} gg) \cos(2\eta - v) - \frac{3}{4mg} (1 + \frac{1}{2} gg) \cos(2\eta + v) \\ + \frac{1}{2mg} (1 + \frac{1}{2} gg) \cos v + \frac{i}{g} \cos v + \frac{1}{4m}$$

ideoque

$$\frac{dv}{d\omega} = m(1 + \frac{1}{2} gg) - \frac{1}{4m} - (2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g}) \cos v \\ - \frac{9}{4mg} (1 + \frac{1}{2} gg) \cos(2\eta - v) + \frac{3}{4mg} (1 + \frac{1}{2} gg) \cos(2\eta + v)$$

XXIII.

Ponamus ad has formulas abbreviandas:

$$m(1 + \frac{1}{2} gg) - 1 = \alpha; \quad 2mg = \gamma \\ m(1 + \frac{1}{2} gg) - \frac{1}{4m} = \epsilon; \quad 2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g} = \delta$$

P p 2

et

et neglectis quadratis gg in reliquis terminis, habebimus has formulas simpliciores :

$$\frac{d\eta}{d\omega} = \alpha - \gamma \cos v$$

$$\frac{dv}{d\omega} = \epsilon - \delta \cos v - \frac{9}{4mg} \cos(2\eta - v) + \frac{3}{4mg} \cos(2\eta + v)$$

Tum vero pro valoribus ξ et q propius inveniendis has aequationes :

$$\frac{d\xi}{d\omega} = -\frac{3}{m} (1 + \frac{1}{2}gg) \sin 2\eta - \frac{3g}{m} \sin(2\eta - v) - \frac{3g}{m} \sin(2\eta + v)$$

$$\frac{dq}{d\omega} = +\frac{9}{4m} (1 + \frac{1}{2}gg) \sin(2\eta - v) + \frac{3}{4m} (1 + \frac{1}{2}gg) \sin(2\eta + v)$$

$$- \frac{1}{2m} (1 + \frac{1}{2}gg) \sin v - i \sin v$$

$$+ \frac{3g}{4m} \sin 2\eta + \frac{3g}{2m} \sin(2\eta - 2v) + \frac{3g}{4m} \sin(2\eta + 2v) - \frac{g}{4m} \sin 2v$$

XXIV.

Fingamus ergo primo :

$$\xi = \mathcal{A} \cos 2\eta + \mathcal{B} \cos(2\eta - v) + \mathcal{C} \cos(2\eta + v)$$

vbi notandum est terminos binos posteriores, vti in differentiali, multo esse minores primo. Quare cum etiam in differentialibus $d\eta$ et dv duplicis generis termini occurrant, quorum posteriores prae primis sint valde parui, in differentiatione solius primi termini totum differentialis $d\xi$ valorem pono, in duobus vero reliquis tantum valorem principalem; sic prodibit

$$\frac{d\xi}{d\omega} = -2\alpha \mathcal{A} \sin 2\eta + \gamma \mathcal{A} \sin(2\eta - v) + \gamma \mathcal{A} \sin(2\eta + v) \\ - (2\alpha - \epsilon) \mathcal{B} - (2\alpha + \epsilon) \mathcal{C}$$

Collato ergo hoc differentiali cum forma proposita obtinetur :

$$\mathcal{A} =$$

bimus

$2\eta + v$

lendis

$\eta + v$

$\sin 2v$

v

dif-

etiam

i oc-

valde

rum

quibus

$+ v$

ξ

ob-

$\mathcal{A} =$

$$\mathcal{A} = \frac{3}{2ma} (1 + \frac{1}{2}gg)$$

$$(2\alpha - \epsilon) \mathcal{B} = \gamma \mathcal{A} + \frac{3g}{m} \text{ ergo } \mathcal{B} = \frac{3(\gamma + 2\alpha g)}{2ma(\alpha - \epsilon)}$$

$$(2\alpha + \epsilon) \mathcal{C} = \gamma \mathcal{A} + \frac{3g}{m} \text{ ergo } \mathcal{C} = \frac{3(\gamma + 2\alpha g)}{2ma(2\alpha + \epsilon)}$$

XXV.

Simili modo fingatur :

$$q = g + A \cos(2\eta - v) + B \cos(2\eta + v) + C \cos v \\ + D \cos 2\eta + E \cos(2\eta - 2v) + F \cos(2\eta + 2v) + G \cos 2v \\ + H \cos 4\eta + J \cos(4\eta - 2v) + K \cos(4\eta + 2v)$$

vbi linea prior continet terminos multo maiores, quam binae inferiores. Hinc ergo sit differenciando secundum regulam supra datam :

$$\frac{dq}{d\omega} = -(2\alpha - \epsilon) A \sin(2\eta - v) - (2\alpha + \epsilon) B \sin(2\eta + v) - \epsilon C \sin v$$

$$+ \left(\frac{1}{2} (2\gamma - \delta) A + \frac{1}{2} (2\gamma + \delta) B + \frac{3}{2mg} C - 2\alpha D \right) \sin 2\eta$$

$$+ \left(\frac{1}{2} (2\gamma - \delta) A - \frac{9}{8mg} C - 2(\alpha - \epsilon) E \right) \sin(2\eta - 2v)$$

$$+ \left(\frac{1}{2} (2\gamma + \delta) B - \frac{3}{8mg} C - 2(\alpha + \epsilon) F \right) \sin(2\eta + 2v)$$

$$+ \left(\frac{1}{2} \delta C - \frac{3A + 9B}{8mg} - 2\epsilon G \right) \sin 2v$$

$$+ \left(\frac{3A + 9B}{8mg} - 4\alpha H \right) \sin 4\eta$$

$$+ \left(-\frac{9A}{8mg} - 2(2\alpha - \epsilon) J \right) \sin(4\eta - 2v)$$

$$+ \left(-\frac{3B}{8mg} - 2(2\alpha + \epsilon) K \right) \sin(4\eta + 2v)$$

P p 3

Hinc

Hincque elicientur sequentes coefficientium valores:

$$(2\alpha-6) A = -\frac{9}{4m} (1 + \frac{1}{2}gg)$$

$$(2\alpha+6) B = -\frac{3}{4m} (1 + \frac{1}{2}gg)$$

$$6C = \frac{1}{2m} (1 + \frac{1}{2}gg) + \delta$$

$$2\alpha D = \frac{1}{2} (2\gamma-\delta) A + \frac{1}{2} (2\gamma+\delta) B + \frac{3}{2mg} C - \frac{3g}{4m}$$

$$2(\alpha-6) E = \frac{1}{2} (2\gamma-\delta) A - \frac{9}{8mg} C - \frac{3g}{2m}$$

$$2(\alpha+6) F = \frac{1}{2} (2\gamma+\delta) A - \frac{3}{8mg} C - \frac{3g}{4m}$$

$$26G = -\frac{3A+9B}{8mg} + \frac{1}{2}\delta C + \frac{g}{4m}$$

$$4\alpha H = \frac{3A+9B}{8mg}; 2(2\alpha-6)J = -\frac{9}{8mg} A$$

$$2(2\alpha+6)K = -\frac{3}{8mg} B$$

XXVI.

Cum igitur his inuentis valoribus fit multo verius:

$$z = X \cos 2\eta \text{ et } q = g + A \cos(2\eta-v) + B \cos(2\eta+v) + C \cos v$$

vbi terminos minores data opera adhuc omitto, quia fortasse correctione egent, praecedentes operationes multo accuratius instituere atque ad ordinem terminorum vltiorem progredi poterimus. Obtinebimus ergo:

$$\frac{d\Phi}{d\omega} = m (1 + \frac{1}{2}gg - C) - 2mg \cos v$$

$$-m(\frac{1}{2}X+A+B) \cos(2\eta-mA \cos(2\eta-2v) - mB \cos(2\eta+2v) + m(\frac{1}{2}gg-C) \cos 2v$$

$$\text{hincque } \frac{d\eta}{d\omega} = \frac{d\Phi}{d\omega} - 1.$$

Porro

Porro ob $\frac{1}{q} = \frac{1}{g} - \frac{A}{gg} \cos(2\eta-v) - \frac{B}{gg} \cos(2\eta+v) - \frac{C}{gg} \cos v$
erit

$$\begin{aligned} \frac{d\Phi-dv}{d\omega} &= \frac{9}{4mg} (1 + \frac{1}{2}gg) \cos(2\eta-v) - \frac{3}{4mg} (1 + \frac{1}{2}gg) \cos(2\eta+v) \\ &+ \frac{1}{2mg} (1 + \frac{1}{2}gg) \cos v + \frac{\delta}{g} \cos v + \frac{1}{4m} \left(1 - \frac{9A+3B-2C}{2gg} \right) \\ &+ \frac{1}{4m} \left(3 - \frac{A-B-3C}{gg} \right) \cos 2\eta + \frac{1}{4m} \left(6 - \frac{2A-9C}{2gg} \right) \cos(2\eta-2v) \\ &- \frac{1}{4m} \left(3 + \frac{2B-3C}{2gg} \right) \cos(2\eta+2v) + \frac{1}{4m} \left(1 + \frac{3A-9B-2C}{2gg} \right) \cos 2v \\ &+ \frac{3A-9B}{8mgg} \cos 4\eta - \frac{9A}{8mgg} \cos(4\eta-2v) + \frac{3B}{8mgg} \cos(4\eta+2v) \end{aligned}$$

XXVII.

Ponatur ad abbreviandum:

$$m (1 + \frac{1}{2}gg - C) - 1 = \alpha; 2mg = \gamma$$

$$m (\frac{1}{2}X + A + B) = \epsilon; \text{ vt fit}$$

$$\frac{d\eta}{d\omega} = \alpha - \gamma \cos v - \epsilon \cos 2\eta$$

$$-mA \cos(2\eta-2v) - mB \cos(2\eta+2v) + m(\frac{1}{2}gg-C) \cos 2v$$

Porro fit

$$m (1 + \frac{1}{2}gg - C) - \frac{1}{4m} \left(1 - \frac{9A+3B-2C}{2gg} \right) = \epsilon$$

$$2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{\delta}{g} = \delta$$

$$+ m (\frac{1}{2}X + A + B) + \frac{1}{4m} \left(3 - \frac{A-B-3C}{gg} \right) = \delta$$

mA

$$mA + \frac{1}{4m} \left(6 - \frac{2A-9C}{2gg} \right) = \eta$$

$$mB - \frac{1}{4m} \left(3 + \frac{2B-3C}{2gg} \right) = \theta$$

$$m(C - \frac{1}{2}gg) + \frac{1}{4m} \left(1 + \frac{3A-9B-2C}{2gg} \right) = \kappa$$

vt habeatur

$$\frac{dv}{d\omega} = \xi - \delta \cos v - \frac{9}{4mg} \cos(2\eta - v) + \frac{3}{4mg} \cos(2\eta + v)$$

$$- \frac{1}{2} \cos 2\eta - \eta \cos(2\eta - 2v) - \theta \cos(2\eta + 2v) - \kappa \cos 2v$$

$$- \frac{3A+9B}{8mg} \cos 4\eta + \frac{9A}{8mg} \cos(4\eta - 2v) - \frac{3B}{8mg} \cos(4\eta + 2v)$$

vbi caueatur, ne coefficientes η , θ , cum angulis cognominibus confundantur.

XXVIII.

Opus plane non est, vt valores litterarum ξ et q accuratius determinemus, arque ad plures terminos, quam ante inuenimus, expediamus; verum hos ipsos terminos, quos ante inuenimus, accuratius obtinebimus, si litteris α et β eos valores tribuamus, quos nunc eis conuenire collegimus. Pluribus autem terminis non indigebimus tam ad longitudinem lunae ϕ , quam ad eius anomaliam veram v satis exacte definiendam. Verum ad hoc ipsum negotium valores differentiales $\frac{d\phi}{d\omega}$ et $\frac{d\phi + dv}{d\omega}$, ac praecipue hunc posteriorem, quo motus apogei continetur, accuratius euolui oportet, quoniam imprimis in motu medio apogei minimae particulae ingentis momenti esse possunt.

XXIX.

XXIX.

Cum igitur accuratius quam adhuc assumimus sit

$$\xi = \mathfrak{A} \cos 2\eta + \mathfrak{B} \cos(2\eta - v) + \mathfrak{C} \cos(2\eta + v) \quad \text{et}$$

$$q = g + A \cos(2\eta - v) + B \cos(2\eta + v) + C \cos v \\ + D \cos 2\eta + E \cos(2\eta - 2v) + F \cos(2\eta + 2v) + G \cos 2v \\ + H \cos 4\eta + J \cos(4\eta - 2v) + K \cos(4\eta + 2v)$$

erit terminus ad quartum vsque ordinem extensis

I.

II.

$$\frac{d\phi}{d\omega} = m \left(1 + \frac{1}{2} gg - C \right) - (2mg - \frac{1}{2} mg C + m G) \cos v$$

III.

$$- m \left(\frac{1}{2} \mathfrak{A} + A + B \right) \cos 2\eta - m A \cos(2\eta - 2v) - m B \cos(2\eta + 2v) \\ - m \left(C - \frac{1}{2} gg \right) \cos 2v$$

IV.

$$+ m \left(\frac{1}{2} g \mathfrak{A} - \frac{1}{2} \mathfrak{B} + g A + \frac{1}{2} g B - D - E \right) \cos(2\eta - v) \\ + m \left(\frac{1}{2} g \mathfrak{A} - \frac{1}{2} \mathfrak{C} + g B + \frac{1}{2} g A - D - F \right) \cos(2\eta + v) \\ + m \left(\frac{1}{2} g A - E \right) \cos(2\eta - 3v) + m \left(\frac{1}{2} g B - F \right) \cos(2\eta + 3v) \\ + m \left(\frac{1}{2} g C - G \right) \cos 3v \\ - m (H + J) \cos(4\eta - v) - m (H + K) \cos(4\eta + v) - m J \cos(4\eta - 3v) \\ - m K \cos(4\eta + 3v)$$

vnde cum esset ante $\gamma = 2mg$, nunc accuratius erit

$$\gamma = 2mg - \frac{1}{2} mg C + m G$$

Qq

XXX.

IX.

XXX.

Deinde cum nunc quoque fit accuratius :

II.

$$\frac{1}{g} = \left(\frac{1}{g} + \frac{AA + BB + CC}{2g^3} \right)$$

III.

$$-\frac{A}{gg} \cos(2\eta - v) - \frac{B}{gg} \cos(2\eta + v) - \frac{C}{gg} \cos v$$

IV.

$$\begin{aligned} & + \left(\frac{(A+B)C}{g^3} - \frac{D}{gg} \right) \cos 2\eta + \left(\frac{AC}{g^3} - \frac{E}{gg} \right) \cos(2\eta - 2v) \\ & + \left(\frac{BC}{g^3} - \frac{F}{gg} \right) \cos(2\eta + 2v) + \left(\frac{2AB + CC}{2g^3} - \frac{G}{gg} \right) \cos 2v \\ & + \left(\frac{AB}{g^3} - \frac{H}{gg} \right) \cos 4\eta + \left(\frac{AA}{2g^3} - \frac{J}{gg} \right) \cos(4\eta - 2v) \\ & + \left(\frac{BB}{g^3} - \frac{K}{gg} \right) \cos(4\eta + 2v) \end{aligned}$$

Hinc quoque ad terminos quarti ordinis vsque valor formulae $\frac{d\Phi - dv}{d\omega}$ definiri posset, sed expressio prodiret tantopere complicata, ut eius euolutio summam requireret patientiam; neque tamen hic labor vilius foret vsus, nisi forte in motu apogei exactius eruendo: ipsae enim inaequalitates nullius forent momenti; propterea quod error in anomalia commissus multo minorem errorem in longitudine producit.

XXXI.

XXXI.

Ponatur ergo longitudo apogei :

$$\Phi - v = \text{Const.}$$

$$\begin{aligned} & + A' \sin(2\eta - v) + B' \sin(2\eta + v) + C' \sin 2v \\ & + \Delta\omega + D' \sin 2\eta + E' \sin(2\eta - 2v) + F' \sin(2\eta + 2v) + G' \sin 2v \\ & + H' \sin 4\eta + J' \sin(4\eta - 2v) + K' \sin(4\eta + 2v) \end{aligned}$$

et erit differentiando :

$$\frac{d\Phi - dv}{d\omega} =$$

$$\begin{aligned} & (2\alpha - \epsilon) A' \cos(2\eta - v) + (2\alpha + \epsilon) B' \cos(2\eta + v) + \epsilon C' \cos v \\ & + \Delta - \frac{1}{2} \delta C' + \frac{9A'}{8mg} + \frac{3B'}{8mg} - \epsilon D' - (mA - \eta) E' \\ & - (mB + \theta) F' - \alpha G' - \frac{9AJ'}{8mg} - \frac{3BK'}{3mg} \\ & \cos 2\eta \left(-\frac{1}{2} (2\gamma - \delta) A' - \frac{1}{2} (2\gamma + \delta) B' - \frac{3C'}{4mg} + 2\alpha D' \right. \\ & \cos(2\eta - 2v) \left(-\frac{1}{2} (2\gamma - \delta) A' - \frac{9C'}{8mg} + 2(\alpha - \epsilon) E' \right. \\ & \cos(2\eta + 2v) \left(-\frac{1}{2} (2\gamma + \delta) B' + \frac{3C'}{8mg} + 2(\alpha + \epsilon) F' \right. \\ & \cos 2v \left(-\frac{1}{2} \delta C' - \frac{3A'}{8mg} - \frac{9B'}{8mg} + 2\epsilon G' \right. \\ & \cos 4\eta \left(-\frac{3A'}{8mg} - \frac{9B'}{8mg} + 4\alpha H' \right. \\ & \cos(4\eta - 2v) \left(\frac{9A'}{8mg} + 2(2\alpha - \epsilon) J' \right. \\ & \cos(4\eta + 2v) \left(\frac{3B'}{8mg} + 2(2\alpha + \epsilon) K' \right. \end{aligned}$$

Qq 2

XXXII.

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XL

XXXIII.

Calculo autem praeprimae in primis terminis accuratius expedito est :

$$\begin{aligned}
 & \frac{d\phi - dv}{d\omega} = \\
 & + \frac{1}{4mg} \cos(2\eta - v) \left(9 + \frac{1}{4} gg + \frac{27AA + 18BB + 15CC}{4gg} \right. \\
 & \quad \left. - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g} \right) \\
 & + \frac{1}{4mg} \cos(2\eta + v) \left(-3 - \frac{1}{4} gg - \frac{6AA - 9BB + 15CC}{4gg} \right. \\
 & \quad \left. + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2F - 9G - 9H + 3K}{2g} \right) \\
 & + \frac{1}{2mg} \cos v \left(1 + \frac{1}{4} gg + \frac{2AA + 2BB + 3CC}{4gg} \right. \\
 & \quad \left. + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g} + 2mi \right) \\
 & + \frac{1}{4m} \left(1 - \frac{9A + 3B - 2C}{2gg} \right) \\
 & + \frac{1}{4m} \left(3 - \frac{A - B - 3C}{gg} \right) \cos 2\eta + \frac{1}{4m} \left(1 + \frac{3A - 9B - 2C}{2gg} \right) \cos 2v \\
 & + \frac{1}{4m} \left(6 - \frac{2A - 9C}{2gg} \right) \cos(2\eta - 2v) - \frac{1}{4m} \left(3 + \frac{2B - 3C}{2gg} \right) \cos(2\eta + 2v) \\
 & + \frac{3A - 9B}{8mgg} \cos 4\eta - \frac{9A}{8mgg} \cos(4\eta - 2v) + \frac{3B}{8mgg} \cos(4\eta + 2v)
 \end{aligned}$$

Simili

Simili autem modo ex valore ipsius $\frac{dq}{d\omega}$ accuratius erit

$$(2a - \epsilon) A = -\frac{1}{4m} \left(9 + \frac{1}{4} gg + \frac{1}{2} C \right)$$

$$(2a + \epsilon) B = -\frac{1}{4m} \left(3 + \frac{1}{4} gg + 3C \right)$$

$$\epsilon C = \frac{1}{2m} \left(1 + \frac{1}{4} gg + \frac{1}{4} A + 2mi \right)$$

XXXIII.

Comparatione autem instituta reperitur :

$$\begin{aligned}
 (2a - \epsilon) A' &= \frac{1}{4mg} \left(9 + \frac{1}{4} gg + \frac{27AA + 18BB + 15CC}{4gg} \right. \\
 & \quad \left. - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g} \right)
 \end{aligned}$$

feu

$$\begin{aligned}
 (2a - \epsilon) A' &= -\frac{1}{g} (2a - \epsilon) A + \frac{1}{4mg} \left(\frac{1}{4} gg - \frac{1}{2} C + \frac{27AA + 18BB + 15CC}{4gg} \right. \\
 & \quad \left. - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g} \right)
 \end{aligned}$$

$$\begin{aligned}
 (2a + \epsilon) B' &= +\frac{1}{g} (2a + \epsilon) B + \frac{1}{4mg} \left(\frac{1}{4} gg + 3C - \frac{6AA - 9BB + 15CC}{4gg} \right. \\
 & \quad \left. + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2F - 9G - 9H + 3K}{2g} \right)
 \end{aligned}$$

$$\begin{aligned}
 \epsilon C' &= +\frac{1}{g} \epsilon C + \frac{1}{2mg} \left(\frac{1}{4} gg - \frac{1}{4} A + \frac{2AA + 2BB + 3CC}{4gg} \right. \\
 & \quad \left. + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g} \right)
 \end{aligned}$$

Q q 3

Quibus

Quibus valoribus substitutis obtinebitur pro apogei motu medio, qui in termino $\Delta\omega$ continetur:

$$\begin{aligned} \Delta &= \frac{I}{4m} + \frac{2mg\delta - I}{4m\delta g} C \\ &+ \frac{\delta}{4\delta mg} \left(\frac{1}{2} \delta g - \frac{1}{2} A + \frac{2AA + 2BB + 3CC}{4\delta g} \right. \\ &\quad \left. + \frac{AB + 6AC + 6BC}{2\delta g} - \frac{3D - 3E - 3F - G}{2g} \right) \\ &\quad - \frac{9}{32(2\alpha - \delta)m\delta g} \left(\frac{1}{2} \delta g - \frac{1}{2} C + \frac{27AA + 18BB + 15CC}{4\delta g} \right. \\ &\quad \left. - \frac{3AB + 2AC + BC}{\delta g} - \frac{2D - 2E + 3G + 3H - 9J}{2g} \right) \\ &\quad - \frac{3}{32(2\alpha + \delta)m\delta g} \left(\frac{1}{2} \delta g + \frac{1}{2} C - \frac{6AA - 9BB + 15CC}{4\delta g} \right. \\ &\quad \left. + \frac{9AB + AC + 2BC}{\delta g} - \frac{2D - 2F - 9G - 9H + 3K}{2g} \right) \\ &+ \delta D' + (mA - \gamma)E' + (mB + \delta)F' + \alpha G' + \frac{9AJ'}{8m\delta g} + \frac{3BK'}{8m\delta g} \end{aligned}$$

Quae expressio, cum omnino sit similis illi, quae methodo praecedente est inuenta, nullum etiam dubium relinquit, quin et hinc motus apogei proditurus sit observationibus conformis; ideoque littera illa δ omitti poterit.

XXXIV.

Hinc igitur patet ad motum apogei definiendum valores litterarum A, B, C et A', B', C' summa accuratone inuestigari debere, qui cum consent partibus duplicis ordinis, etiam si partes posterioris ordinis prae primo

primo admodum videantur paruae, eas tamen omni cura euolui oportet, propterea quod pro motu apogei partes primi ordinis se destruunt. Quod cum in determinatione reliquorum coefficientium vsu non eueniat, in his quoque non erit opus, ut partes istae minores in computum ducantur, sed sufficiet partibus principalibus vi. Scilicet etsi determinatio litterae Δ maxime est lubrica, atque a reliquorum coefficientium exactissimis valoribus pendet, reliqui tamen coefficientes tantam solertiam minime requirunt, sed satis exacte sine tanta opera definiri possunt.

XXXV.

Valores ergo reliquorum coefficientium sequenti modo neglectis exiguis particulis ita se habebunt,

$$\begin{aligned} (2\alpha - \delta)A' &= \frac{9}{4mg}; \quad (2\alpha + \delta)B' = -\frac{3}{4mg}; \quad \delta C' = \frac{1}{2mg}. \\ 2\alpha D' &= \frac{1}{2}(2\gamma - \delta)A' + \frac{1}{2}(2\gamma + \delta)B' + \frac{3C'}{4mg} + \frac{1}{4m} \left(3 - \frac{A - B - 3C}{\delta g} \right) \\ 2(\alpha - \delta)E' &= \frac{1}{2}(2\gamma - \delta)A' + \frac{9C'}{8mg} + \frac{1}{4m} \left(6 - \frac{2A - 9C}{2\delta g} \right) \\ 2(\alpha + \delta)F' &= \frac{1}{2}(2\gamma + \delta)B' - \frac{3C'}{8mg} - \frac{1}{4m} \left(3 + \frac{2B - 3C}{2\delta g} \right) \\ 2\delta G' &= \frac{1}{2}\delta C' + \frac{3A' + 9B'}{8mg} + \frac{1}{4m} \left(1 + \frac{3A - 9B - 2C}{2\delta g} \right) \\ 4\alpha H' &= \frac{3A' + 9B'}{8mg} + \frac{8A - 9B}{8m\delta g} = 0 \\ 2(2\alpha - \delta)J' &= -\frac{9A'}{8mg} - \frac{9A}{8m\delta g} = 0 \quad \left| \quad 2(2\alpha + \delta)K' = -\frac{3H'}{8mg} + \frac{3B}{8m\delta g} = 0 \right. \\ \text{ob } A' &= -\frac{A}{g}; \quad B' = \frac{B}{g} \quad \text{et } C' = \frac{C}{g} \text{ proxime.} \end{aligned}$$

XXXVI.

XXXVI.

Cum autem sit proxime: $\gamma = 2mg$; $\delta = 2mg + \frac{1}{2mg}$;
his valoribus quoque substitutis fiet:

$$(2\alpha - \epsilon)A' = \frac{9}{4mg}; (2\alpha + \epsilon)B' = -\frac{3}{4mg}; \epsilon C' = \frac{1}{2mg}; \text{fiue:}$$

$$A' = -\frac{A}{g}; B' = \frac{B}{g}; C' = \frac{C}{g};$$

$$2\alpha D' = \frac{3}{4m} - mA + 3mB;$$

$$2(\alpha - \epsilon)E' = \frac{3}{2m} - mA$$

$$2(\alpha + \epsilon)F' = -\frac{3}{4m} + 3mB$$

$$2\epsilon G' = \frac{1}{4m} + mC$$

et reliqui coefficientes H' , J' , K' pro evanescentibus
sunt habendi. Valores autem litterarum A , B , C , etc.
§. 25. sunt exhibiti.

XXXVII.

Quaeramus nunc quoque longitudinem lunae ϕ ,
huncque in finem fingamus:

$$\phi = \text{Const.}$$

$$\begin{aligned} &+ M'\omega + N'\sin v + O'\sin 2\eta + P'\sin(2\eta - 2v) + Q'\sin(2\eta + 2v) + R'\sin 2v \\ &+ S'\sin(2\eta - v) + T'\sin(2\eta + v) + U'\sin(2\eta - 3v) + V'\sin(2\eta + 3v) + W'\sin 3v \\ &+ X'\sin(4\eta - v) + Y'\sin(4\eta + v) + Z'\sin(4\eta - 3v) + \Omega'\sin(4\eta + 3v) \end{aligned}$$

ac

ac differentiatione instituta obtinebimus:

$$\frac{d\phi}{d\omega} = + M' - \frac{1}{2} \delta N'$$

$$+ \cos v \left(\epsilon N' - \frac{1}{2} \kappa N' + \frac{9D'}{4mg} + \frac{3E'}{4mg} - \delta F' \right)$$

$$+ \cos 2\eta \left(-\frac{3N'}{4mg} + 2\alpha E' \right)$$

$$+ \cos(2\eta - 2v) \left(-\frac{9N'}{8mg} + 2(\alpha - \epsilon)D' \right)$$

$$+ \cos(2\eta + 2v) \left(+\frac{3N'}{8mg} + 2(\alpha + \epsilon)E' \right)$$

$$+ \cos 2v \left(-\frac{1}{2} \delta N' + 2\epsilon F' \right)$$

$$+ \cos(2\eta - v) \left(-\frac{1}{2} \epsilon N' - \frac{1}{2} \eta N' - \gamma E' - (\gamma - \delta)D' + \frac{3F'}{4mg} + (2\alpha - \epsilon)G' \right)$$

$$+ \cos(2\eta + v) \left(-\frac{1}{2} \epsilon N' - \frac{1}{2} \theta N' - \gamma E' - (\gamma + \delta)E' - \frac{9F'}{4mg} + (2\alpha + \epsilon)G' \right)$$

$$+ \cos(2\eta - 3v) \left(-\frac{1}{2} \eta N' - (\gamma - \delta)D' - \frac{9F'}{4mg} + (2\alpha - 3\epsilon)G' \right)$$

$$+ \cos(2\eta + 3v) \left(-\frac{1}{2} \theta N' - (\gamma + \delta)E' + \frac{3F'}{4mg} + (2\alpha + 3\epsilon)G' \right)$$

$$+ \cos 3v \left(-\frac{1}{2} \kappa N' - \frac{3D'}{4mg} - \frac{9E'}{4mg} - \delta F' + 3\epsilon G' \right)$$

$$+ \cos(4\eta - v) \left(-\frac{3A + 9B}{16mgg} N' + \frac{9A}{16mgg} N' - \frac{3D'}{4mg} + (4\alpha - \epsilon)M' \right)$$

$$+ \cos(4\eta + v) \left(-\frac{3A + 9B}{16mgg} N' - \frac{3B}{16mgg} N' - \frac{9E'}{4mg} + (4\alpha + \epsilon)M' \right)$$

$$+ \cos(4\eta - 3v) \left(+\frac{9A}{16mgg} N' + \frac{9D'}{4mg} + (4\alpha - 3\epsilon)M' \right)$$

$$+ \cos(4\eta + 3v) \left(-\frac{3B}{16mgg} N' + \frac{3E'}{4mg} + (4\alpha + 3\epsilon)M' \right)$$

R r

XXXVIII.

XXXVIII.

Comparata iam hac forma cum valore ipsius $\frac{d\phi}{d\omega}$ in §. 29. exhibito, obtinebitur

$$\mathfrak{A}' = \frac{1}{2} \delta \mathfrak{B}' + m(1 + \frac{1}{2} g g - C)$$

$$\mathfrak{E} \mathfrak{B}' = \frac{1}{2} \kappa \mathfrak{B}' - \frac{9 \mathfrak{D}' - 3 \mathfrak{E}'}{4 m g} + \delta \mathfrak{F}' - 2 m g + \frac{1}{2} m g C - m G$$

$$2 \alpha \mathfrak{E}' = \frac{3 \mathfrak{B}'}{4 m g} - m(\frac{1}{2} \mathfrak{A} + A + B)$$

$$2(\alpha - \mathfrak{E}) \mathfrak{D}' = \frac{9 \mathfrak{B}'}{8 m g} - m A$$

$$2(\alpha + \mathfrak{E}) \mathfrak{E}' = -\frac{3 \mathfrak{B}'}{8 m g} - m B$$

$$2 \mathfrak{E} \mathfrak{F}' = \frac{1}{2} \delta \mathfrak{B}' - m(C - \frac{1}{2} g g)$$

$$(2\alpha - \mathfrak{E}) \mathfrak{G}' = \frac{1}{2} (\zeta + \eta) \mathfrak{B}' + \gamma \mathfrak{E}' + (\gamma - \delta) \mathfrak{D}' - \frac{3 \mathfrak{F}'}{4 m g} + m(\frac{1}{2} g \mathfrak{A} - \frac{1}{2} \mathfrak{B} + g A + \frac{1}{2} g B - D - E)$$

$$(2\alpha + \mathfrak{E}) \mathfrak{H}' = \frac{1}{2} (\zeta + \theta) \mathfrak{B}' + \gamma \mathfrak{E}' + (\gamma + \delta) \mathfrak{E}' + \frac{9 \mathfrak{F}'}{4 m g} + m(\frac{1}{2} g \mathfrak{A} - \frac{1}{2} \mathfrak{E} + g B + \frac{1}{2} g A - D - F)$$

$$(2\alpha - 3\mathfrak{E}) \mathfrak{I}' = \frac{1}{2} \eta \mathfrak{B}' + (\gamma - \delta) \mathfrak{D}' + \frac{9 \mathfrak{F}'}{4 m g} + m(\frac{1}{2} g A - E)$$

$$(2\alpha + 3\mathfrak{E}) \mathfrak{K}' = \frac{1}{2} \theta \mathfrak{B}' + (\gamma + \delta) \mathfrak{E}' - \frac{3 \mathfrak{F}'}{4 m g} + m(\frac{1}{2} g B - F)$$

$$3 \mathfrak{E} \mathfrak{L}' = \frac{1}{2} \kappa \mathfrak{B}' + \frac{3 \mathfrak{D}' + 9 \mathfrak{E}'}{4 m g} + \delta \mathfrak{F}' + m(\frac{1}{2} g C - G)$$

$$(4\alpha - \mathfrak{E}) \mathfrak{M}' = -\frac{6A - 9B}{16 m g g} \mathfrak{B}' + \frac{3 \mathfrak{D}'}{4 m g} - m(H + J)$$

$$(4\alpha + \mathfrak{E}) \mathfrak{N}' = +\frac{3A - 6B}{16 m g g} \mathfrak{B}' + \frac{9 \mathfrak{E}'}{4 m g} - m(H + K)$$

$$(4\alpha - 3\mathfrak{E}) \mathfrak{P}' = -\frac{9A}{16 m g g} \mathfrak{B}' - \frac{9 \mathfrak{D}'}{4 m g} - m J$$

$$(4\alpha + 3\mathfrak{E}) \mathfrak{Q}' = +\frac{3B}{16 m g g} \mathfrak{B}' - \frac{3 \mathfrak{E}'}{4 m g} - m K$$

xxxix.

XXXIX.

Inuentis iam valoribus litterarum $p = b(1 + \xi)$ et q vna cum anomalia vera v , distantia curtata lunae a terra $x = \frac{p}{1 - q \cos v}$ cognoscetur: ac si deinceps latitudinis lunae ψ ratio habebitur, erit distantia vera $= \frac{p}{(1 - q \cos v) \cos \psi}$.

In Astronomia autem non tam distantia lunae, quam eius diameter apparens et parallaxis requiri solet; quarum vtraque cum sit distantiae lunae a terra reciproce proportionalis, erit tam diameter apparens quam parallaxis vt $\frac{(1 - q \cos v) \cos \psi}{p}$; vnde si vtriusque valor medius ex obseruationibus fuerit definitus, ad quoduis tempus valor verus assignari poterit. Sit igitur siue diametri apparentis siue parallaxis horizontalis valor medius $= \sigma$, eritque is pro tempore dato $= \frac{b\sigma}{p} (1 - q \cos v) \cos \psi$. Est autem proxime $\cos \psi = 1 - \frac{1}{2} \tan g^2 - \frac{1}{2} \tan g^2 \cos 2(\phi - \pi) = \frac{2}{3} + \frac{\lambda + \Pi}{3}$, et $\frac{b}{p} = 1 - \xi + \xi\xi$: vnde fit diameter seu parallaxis $= \frac{1}{3} \sigma (2 + \lambda + \Pi) (1 - \xi + \xi\xi) (1 - q \cos v)$, quae euoluitur in hanc expressionem: ob $\frac{2 + \lambda}{3} = 1$ proxime: $\frac{1}{3} (2 + \lambda) \sigma [1 - q \cos v - \xi + q \xi \cos v + \xi\xi + \frac{1}{3} \Pi - \frac{1}{3} q \Pi \cos v]$

XL.

Pro praesenti ergo casu, quo parallaxin solis, eiusque excentricitatem vna cum inclinatione orbitae luna-

R r 2

ris

ris negligimus, erit lunæ diameter apparens vel paral-
laxis horizontalis

$$= \frac{1}{2} (2 + \lambda) \sigma \text{ mult. per}$$

$$1 - \frac{1}{2} C - (g + \frac{1}{2} G) \cos v$$

$$- \frac{1}{2} (2\mathcal{A} + A + B) \cos 2\eta - \frac{1}{2} A \cos (2\eta - 2v) - \frac{1}{2} B \cos (2\eta + 2v) - \frac{1}{2} C \cos 2v$$

$$- \frac{1}{2} (2\mathcal{B} + D + E) \cos (2\eta - v) - \frac{1}{2} (2\mathcal{C} + D + F) \cos (2\eta + v)$$

$$- \frac{1}{2} E \cos (2\eta - 3v) - \frac{1}{2} F \cos (2\eta + 3v) - \frac{1}{2} G \cos 3v$$

$$- \frac{1}{2} (H + J) \cos (4\eta - v) - \frac{1}{2} (H + K) \cos (4\eta + v) - \frac{1}{2} J \cos (4\eta - 3v) - \frac{1}{2} K \cos (4\eta + 3v)$$

vbi quidem factor constans $\frac{1}{2} (2 + \lambda) \sigma$ omitti potest,
siquidem tantum proportio vel diametri apparentis vel
parallax horizontalis desideretur.

XLI.

Si nunc hos valores in numeris euoluere velimus,
ex observationibus primum colligimus has determina-
tiones:

$$\mathcal{A}' = 13,3682; \Delta = 0,1123 \text{ et proxime } g = 0,05445$$

ac postremo quidem valore ipsius g tantum in terminis
minimis utar, in maioribus ipsam litteram g relicturus,
ut deinceps ex collatione calculi cum observationibus ac-
curatius fortasse determinari possit. Habemus ergo

$$13,3682 = \frac{1}{2} \delta \mathcal{B}' + m (1 + \frac{1}{2} gg - C)$$

vnde ob $\mathcal{B}'$, gg et C numeros admodum paruos, statim
prope colligitur $m = 13,3682$. Tum vero est prope

$$\mathcal{B}' = -\frac{2mg}{\delta}; \mathcal{C} = m \text{ et } \delta = 2mg + \frac{1}{2mg} \text{ seu } \mathcal{C} = 13,3682;$$

$$\delta = 2,$$

$$\delta = 2,1419; \text{ hinc } \frac{1}{2} \delta \mathcal{B}' = -0,1165, \text{ ergo accuratius}$$

$$m (1 + \frac{1}{2} gg - C) = 13,4847 = a + 1 \text{ et } a = 12,4847$$

$$\text{Porro ob } C = \frac{1}{2mm} \text{ erit satis exacte } m = 13,5039$$

$$\text{vnde ex valoribus } \mathcal{A} \text{ et } \mathcal{B} \text{ proxime collectis fit } \mathcal{C} = 13,0644$$

$$g = 26,9524g.$$

At valor ipsius δ duabus constat partibus, altera per g
multiplicata altera diuisa, quibus separatim expressis erit

$$\delta = 27,0355g + 0,0370. \frac{1}{g} = 2,1521 \text{ proxime.}$$

XLII.

Hinc iam computo instituto sequentes supra assum-
torum coefficientium eruuntur valores numerici:

$$\mathcal{A} = 0,008931; \mathcal{B} = 0,03895g = 0,00209; \mathcal{C} = 0,01219g = 0,00064$$

ideoque

$$\mathcal{E} = 0,008931 \cos 2\eta + 0,03895g \cos (2\eta - v) + 0,01219g \cos (2\eta + v)$$

Deinde reperitur:

$$A = -0,013995; B = -0,001460; C = +0,002834$$

$$D = -0,001213g + 0,00002198. \frac{1}{g} = -0,000280 \text{ proxime}$$

$$E = +0,25834g - 0,00001889. \frac{1}{g} = +0,014012 \text{ proxime}$$

$$F = -0,00225g - 0,00000207. \frac{1}{g} = -0,000161 \text{ proxime}$$

$$G = +0,00218g + 0,00001221. \frac{1}{g} = +0,000340 \text{ proxime}$$

$$H = -0,00001022. \frac{1}{g} = -0,000184$$

$$J = +0,00004897. \frac{1}{g} = +0,000882$$

$$K = +0,00000053. \frac{1}{g} = +0,000009$$

XLIII.

XLIII.

Hinc porro pro motu apogei eiusque inaequalitatibus colligitur: $\Delta = 0,1123$; qui quidem valor ex observationibus est desumptus

$$\begin{aligned} A' &= -0,013995 \cdot \frac{1}{g} = -0,25703 \text{ proxime} \\ B' &= -0,001460 \cdot \frac{1}{g} = -0,02682 \text{ proxime} \\ C' &= +0,002834 \cdot \frac{1}{g} = +0,05205 \text{ proxime} \\ D' &= +0,007432 \quad F' = -0,002249 \\ E' &= -0,259170 \quad G' = -0,002176 \end{aligned}$$

in minutis secundis

$$\begin{aligned} A' &= -53018'' = -14^\circ, 43', 38'' \\ B' &= -5532 = -1^\circ, 32', 12'' \\ C' &= +10736 = +2^\circ, 58', 56'' \\ D' &= +1533 = +0^\circ, 25', 33'' \\ E' &= -53459 = -14^\circ, 50', 59'' \\ F' &= -464 = -0^\circ, 7', 44'' \\ G' &= +449 = +0^\circ, 7', 29'' \end{aligned}$$

Ergo longitudo apogei in minutis secundis

$$\phi - v = \text{Const.}$$

$$\begin{aligned} +0,1123w &= 53018'' \sin(2\eta - v) + 1533'' \sin 2\eta \\ &- 5532 \sin(2+v) - 53459 \sin(2\eta - 2v) \\ &+ 10736 \sin v - 464 \sin(2\eta + 2v) \\ &+ 449 \sin 2v \end{aligned}$$

XLIV.

XLIV.

Iam pro longitudine ipsa inuenienda habentur primo ex §. 27. valores: $\gamma = 1,46756$

$$\begin{aligned} \delta &= +2,15210; \epsilon = -0,027791; \zeta = +0,07138 \\ \eta &= -0,06974; \theta = -0,039809; \kappa = -0,070920 \end{aligned}$$

Deinde cum sit proxime

$$6\mathfrak{B}' = -2mg \text{ seu } \mathfrak{B}' = -0,11256$$

erit quoque proxime

$$\mathfrak{C}' = -0,003485; \mathfrak{D}' = -0,014456$$

$$\mathfrak{E}' = +0,001509; \mathfrak{F}' = -0,005334$$

Hinc ergo accuratius elicietur $\mathfrak{B}' = -0,11019$, ideoque hic et reliqui coefficients tam absolute quam in numeris secundis erunt:

absolute		in minutis secundis
$\mathfrak{B}' = -0,11019$	$\mathfrak{B}' = -22728''$	$= -6^\circ, 18', 4''$
$\mathfrak{C}' = -0,00339$	$\mathfrak{C}' = -700$	$= -0^\circ, 11', 40''$
$\mathfrak{D}' = -0,01742$	$\mathfrak{D}' = -3594$	$= -0^\circ, 59', 54''$
$\mathfrak{E}' = +0,00149$	$\mathfrak{E}' = +306$	$= +0^\circ, 5', 6''$
$\mathfrak{F}' = -0,00524$	$\mathfrak{F}' = -1081$	$= -0^\circ, 18', 1''$
$\mathfrak{G}' = -0,01824$	$\mathfrak{G}' = -3762$	$= -1^\circ, 2', 42''$
$\mathfrak{H}' = -0,00056$	$\mathfrak{H}' = -115$	$= -0^\circ, 1', 55''$
$\mathfrak{I}' = +0,01368$	$\mathfrak{I}' = +2823$	$= +0^\circ, 47', 3''$
$\mathfrak{K}' = +0,00023$	$\mathfrak{K}' = +47$	$= +0^\circ, 0', 47''$
$\mathfrak{L}' = -0,00062$	$\mathfrak{L}' = -128$	$= -0^\circ, 2', 8''$
$\mathfrak{M}' = -0,00119$	$\mathfrak{M}' = -246$	$= -0^\circ, 4', 6''$
$\mathfrak{N}' = +0,00020$	$\mathfrak{N}' = +41$	$= +0^\circ, 0', 41''$
$\mathfrak{O}' = +0,00084$	$\mathfrak{O}' = +379$	$= +0^\circ, 6', 19''$
$\mathfrak{Q}' = -0,00001$	$\mathfrak{Q}' = -2$	$= -0^\circ, 0', 2''$

XLV.

alita-
or ex

(-2v)

+2v)

LIV.

XLV.

Hinc ergo si ad datum tempus iam cognita sit anomalia lunae vera v cum angulo η , longitudo lunae per aequationes in minutis secundis expressas erit

$$\phi = \text{Const.}$$

$$\begin{aligned} +13,3682\omega & - 22728'' \sin v & - & 700'' \sin 2\eta \\ & - 1081 \sin 2v & - & 3594 \sin (2\eta - 2v) \\ & - 128 \sin 3v & + & 306 \sin (2\eta + 2v) \\ & - 3762'' \sin (2\eta - v) & - & 246'' \sin (4\eta - v) \\ & - 115 \sin (2\eta + v) & + & 41 \sin (4\eta + v) \\ & + 2823 \sin (2\eta - 3v) & + & 379 \sin (4\eta - 3v) \\ & + 47 \sin (2\eta + 3v) & - & 2 \sin (4\eta + 3v) \end{aligned}$$

vbi Const. $+13,3682\omega$ denotat longitudinem mediam; in reliquis autem terminis continentur inaequalitates periodicae pro hac hypothese.

XLVI.

Inde iam vicissim anomalia vera lunae v colligitur,

$$\begin{aligned} \text{vt sit } v & = \\ 13,2559\omega & - 33464'' \sin v & - & 2233'' \sin 2\eta \\ & - 1530 \sin 2v & + & 49864 \sin (2\eta - 2v) \\ & - 128 \sin 3v & + & 770 \sin (2\eta + 2v) \\ & + 49256 \sin (2\eta - v) & - & 246 \sin (4\eta - v) \\ & + 5417 \sin (2\eta + v) & + & 41 \sin (4\eta + v) \\ & + 2823 \sin (2\eta - 3v) & + & 379 \sin (4\eta - 3v) \\ & + 47 \sin (2\eta + 3v) & - & 2 \sin (4\eta + 3v) \end{aligned}$$

vbi primus terminus $13,2559\omega$ designat anomaliā mediam lunae, quae sit $= \zeta$: tum ex ea primum quaeratur

ano-

fit ano-
nae per

17
27-2v)
27+2v)
47-v)
47+v)
47-3v)
47+3v)
ediam;
res pe-

ligitur,

1
27-2v)
27+2v)
47-v)
47+v)
47-3v)
47+3v)
m me-
aeratur
ano-

anomalia Kepleriana, quae scilicet a sola excentricitate pendet, sitque ea $= s$, vt sit

$s = \zeta - 33464'' \sin s - 1530'' \sin 2s - 128'' \sin 3s$
vnde quidem facile tabulae construuntur. Tum statuatur $v = s + z$, et quia angulus z est modicus, inde is satis prope poterit definiri. Interim tamen expedire videtur aliquot operationibus iterandis istam anomaliā veram v determinari; dum scilicet primum valor non nimis a veritate abhorrens pro v aestimando assumitur, ex eoque deinceps exactior colligitur; qui si nimis ab assumto discrepare reperiatur, ex hoc denuo exactior quaeratur, donec nulla amplius correctione fuerit opus.

XLVII.

Formula denique, cui tam diameter lunae apparens geocentrica quam parallaxis horizontalis est proportionalis, ex §. 40. reperitur

$$\begin{aligned} 1 & - 0,05470 \cos v & - & 0,00120 \cos 2\eta \\ & - 0,00142 \cos 2v & + & 0,00700 \cos (2\eta - 2v) \\ & - 0,00017 \cos 3v & + & 0,00073 \cos (2\eta + 2v) \\ & - 0,00898 \cos (2\eta - v) & - & 0,00035 \cos (4\eta - v) \\ & - 0,00042 \cos (2\eta + v) & + & 0,00009 \cos (4\eta + v) \\ & - 0,00701 \cos (2\eta - 3v) & - & 0,00044 \cos (4\eta - 3v) \\ & + 0,00008 \cos (2\eta + 3v) & - & 0,00001 \cos (4\eta + 3v) \end{aligned}$$

quorum quidem terminorum plures, qui pro parallaxi infra aliquot minuta secunda subsistunt, tuto omitti poterunt. His igitur tribus formulis pro anomalia vera v , longitudine ϕ et parallaxi seu diametro apparente inuentis motus lunae contineretur, si quidem tam solis parallaxis

S s

laxis

laxis quam eius excentricitas et inclinatio orbitae lunaris ad eclipticam negligatur. Hae autem sunt inaequalitates praecipuae, quae etiam ad reliquas eruendas adhiberi debent; unde nunc ad inaequalitates ab excentricitate solis oriundas progrediamur.

INVESTIGATIO INAEQUALITATUM
LUNAE SECUNDI GENERIS SEU AB
EXCENTRICITATE SOLIS
PENDENTIUM.

XLVIII.

Formulae nostrae differentiales, quatenus ab excentricitate orbitae solaris pendent, omittis terminis, quos iam constat esse minimos, erunt

$$\frac{d\xi}{d\omega} = \text{Praec.} + \frac{9e}{2m} \sin(2\eta - u) + \frac{9e}{2m} \sin(2\eta + u)$$

$$\begin{aligned} \frac{dq}{d\omega} &= \text{Praec.} + \frac{3e}{4m} \sin(v - u) + \frac{3e}{4m} \sin(v + u) \\ &\quad - \frac{27e}{8m} \sin(2\eta - v - u) - \frac{27e}{8m} \sin(2\eta - v + u) \\ &\quad - \frac{9e}{8m} \sin(2\eta + v - u) - \frac{9e}{8m} \sin(2\eta + v + u) \end{aligned}$$

$$\begin{aligned} \frac{q(d\phi - d\psi)}{d\omega} &= \text{Pr.} - \frac{3e}{4m} \cos(v - u) - \frac{3e}{4m} \cos(v + u) \\ &\quad - \frac{27e}{8m} \cos(2\eta - v - u) - \frac{27e}{8m} \cos(2\eta - v + u) \\ &\quad + \frac{9e}{8m} \cos(2\eta + v - u) + \frac{9e}{8m} \cos(2\eta + v + u) \end{aligned}$$

Quam-

Quoniam enim nunc tam ξ quam q etiam ab excentricitate e pendeant, tamen in his formulis, in quas hae quantitates ingrediuntur, haec mutatio earum sine errore pro nihilo haberi potest; quoniam hi termini per se sunt minimi, et quia iam terminos ab e et q simul pendentes omittimus. Tum vero erit

$$\frac{d\phi}{d\omega} = m(1 + \frac{1}{2}gg) - 2mq \cos v + \frac{1}{2}mqg \cos 2v - \frac{1}{2}m\xi + 3mq\xi \cos v$$

$$\text{et } \frac{du}{d\omega} = \frac{d\theta}{d\omega} = 1 + 2ee - 2e \cos u$$

XLIX.

Ad formulas has integrandas seu tantum ad eas integralium partes inveniendas, quae ab excentricitate solis e pendent, opus est vt formularum $\frac{d\eta}{d\omega}$, $\frac{dv}{d\omega}$ et $\frac{du}{d\omega}$ primum habeamus partes principales, tum vero etiam eas quae a simplici solis excentricitate e pendent: habebimus ergo primo

$$\begin{aligned} \frac{d\eta}{d\omega} &= m(1 + \frac{1}{2}gg) - 1 - 2ee - 2mg \cos v + 2e \cos u \\ \frac{dv}{d\omega} &= m(1 + \frac{1}{2}gg) - 2mg \cos v + \frac{3e}{4mg} \cos(v - u) + \frac{3e}{4mg} \cos(v + u) \\ &\quad - \frac{1}{4m} \left(1 - \frac{9A + 3B - 2C}{2gg} \right) + \frac{27e}{8mg} \cos(2\eta - v - u) + \frac{27e}{8mg} \cos(2\eta - v + u) \\ &\quad - \frac{9e}{8mg} \cos(2\eta + v - u) - \frac{9e}{8mg} \cos(2\eta + v + u) \end{aligned}$$

seu introducendis, vt supra §. 27. breuitatis gratia, litteris

$$\begin{aligned} \alpha &= m(1 + \frac{1}{2}gg - C) - 2ee & \gamma &= 2mg \\ \delta &= m(1 + \frac{1}{2}gg - C) - \frac{1}{4m} \left(1 - \frac{9A + 3B - 2C}{2gg} \right); & \delta &= 2mg + \frac{1}{2mg} + \frac{3g}{8m} \end{aligned}$$

S s 2

erit:

$$\text{erit: } \frac{d\eta}{d\omega} u - \gamma \cos v + 2\epsilon \cos u$$

$$\begin{aligned} \frac{dv}{d\omega} = & \epsilon - \delta \cos v - \frac{9}{4mg} \cos(2\eta - v) + \frac{3}{4mg} \cos(2\eta + v) \\ & + \frac{3\epsilon}{4mg} \cos(v - u) + \frac{3\epsilon}{4mg} \cos(v + u) + \frac{27\epsilon}{8mg} \cos(2\eta - v - u) \\ & + \frac{27\epsilon}{8mg} \cos(2\eta - v + u) - \frac{9\epsilon}{8mg} \cos(2\eta + v - u) - \frac{9\epsilon}{8mg} \cos(2\eta + v + u) \end{aligned}$$

$$\text{et } \frac{du}{d\omega} = 1 - 2\epsilon \cos u.$$

L

Fingamus nunc primo:

$\xi = A \cos 2\eta + B \cos(2\eta - v) + C \cos(2\eta + v) + D \cos(2\eta - u) + E \cos(2\eta + u)$
ac differentiendo eos tantum sumamus terminos, qui formulae differentiali respondent, quandoquidem reliquos iam inuenimus: eritque

$$\begin{aligned} \frac{d\xi}{d\omega} = & -2\epsilon A \sin(2\eta - u) - 2\epsilon A \sin(2\eta + u) \\ & - (2\alpha - 1) D - (2\alpha + 1) E \end{aligned}$$

vnde colligitur:

$$(2\alpha - 1) D = -\frac{9\epsilon}{2m} - 2\epsilon A; \quad (2\alpha + 1) E = -\frac{9\epsilon}{2m} - 2\epsilon A$$

Cum igitur sit $\epsilon = 0,0168$, erit in numeris:

$$D = -0,000247 \quad \text{et} \quad E = -0,000227.$$

LI

Fingatur porro: $g = g$

$$\begin{aligned} & + A \cos(2\eta - v) + B \cos(2\eta + v) + C \cos v + M \cos(v - u) + N \cos(v + u) \\ & + P \cos(2\eta - v - u) + Q \cos(2\eta - v + u) + R \cos(2\eta + v - u) + S \cos(2\eta + v + u) \end{aligned}$$

ac

ac differentiendo obtinebitur: $\frac{dg}{d\omega} =$

$$\begin{aligned} & -2\epsilon A \sin(2\eta - v - u) - 2\epsilon A \sin(2\eta - v + u) - 2\epsilon B \sin(2\eta + v - u) - 2\epsilon B \sin(2\eta + v + u) \\ & - (2\alpha - \epsilon - 1) P - (2\alpha - \epsilon + 1) Q - (2\alpha + \epsilon - 1) R - (2\alpha + \epsilon + 1) S \\ & - (\epsilon - 1) M \sin(v - u) - (\epsilon + 1) H \sin(v + u) \end{aligned}$$

Comparatione ergo instituta reperietur:

$$(\epsilon - 1) M = -\frac{3\epsilon}{4m}; \quad (\epsilon + 1) N = -\frac{3\epsilon}{4m}$$

$$(2\alpha - \epsilon - 1) P = \frac{27\epsilon}{8m} - 2\epsilon A; \quad (2\alpha - \epsilon + 1) Q = \frac{27\epsilon}{8m} - 2\epsilon A$$

$$(2\alpha + \epsilon - 1) R = \frac{9\epsilon}{8m} - 2\epsilon B; \quad (2\alpha + \epsilon + 1) S = \frac{9\epsilon}{8m} - 2\epsilon B$$

et in numeris

$$M = -0,00008; \quad P = +0,00042; \quad R = +0,00004$$

$$N = -0,00006; \quad Q = +0,00036; \quad S = +0,00004$$

LII

Hic autem in differentiatione negleximus partes ipsius $\frac{dv}{d\omega}$ ab excentricitate ϵ pendentes, quarum tamen eodem iure ratio haberi debuisset, arque partis in differentiali $\frac{d\eta}{d\omega}$; inde autem multo plures termini accedent ad valorum ipsius g , ponatur ergo ob hos terminos:

$$g = g$$

$$\begin{aligned} & + A \cos(2\eta - v) + B \cos(2\eta + v) + C \cos v + M \cos(v - u) \\ & + N \cos(v + u) + D \cos(2\eta - u) + E \cos(2\eta + u) \\ & + P \cos(2\eta - v - u) + Q \cos(2\eta - v + u) + R \cos(2\eta + v - u) \\ & + S \cos(2\eta + v + u) + K \cos(2v - u) + L \cos(2v + u) \end{aligned}$$

S s 3

+

$$\begin{aligned}
 &+ F \cos(2\eta-2v-u) + G \cos(2\eta-2v+u) + H \cos(2\eta+2v-u) \\
 &\quad + J \cos(2\eta+2v+u) + T \cos(4\eta-u) + V \cos(4\eta+u) \\
 &+ W \cos(4\eta-2v-u) + X \cos(4\eta-2v+u) + Y \cos(4\eta+2v-u) \\
 &\quad + Z \cos(4\eta+2v+u)
 \end{aligned}$$

et sumto differentiali pleno reperitur:

$$\begin{aligned}
 \frac{dq}{dt} = &+ \sin(2\eta-v-u) [-2eA - (2\alpha-5-1)P] \\
 &+ \sin(2\eta-v+u) [-2eA - (2\alpha-5+1)Q] - (5-1)M \sin(v-u) \\
 &+ \sin(2\eta+v-u) [-2eB - (2\alpha+5-1)R] \\
 &+ \sin(2\eta+v+u) [-2eB - (2\alpha+5+1)S] - (5+1)N \sin(v+u) \\
 &+ \sin(2\eta-u) \left(+ \frac{3eA}{8mg} - \frac{3eB}{8mg} - \frac{27eC}{16mg} - \frac{9eC}{16mg} - (2\alpha-1)D \right) \\
 &+ \sin(2\eta+u) \left(+ \frac{3eA}{8mg} - \frac{3eB}{8mg} - \frac{27eC}{16mg} - \frac{9eC}{16mg} - (2\alpha+1)E \right) \\
 &+ \sin(2\eta-2v+u) \left(+ \frac{3eA}{8mg} + \frac{27eC}{16mg} - (2\alpha-25+1)G \right) \\
 &+ \sin(2\eta-2v-u) \left(+ \frac{3eA}{8mg} + \frac{27eC}{16mg} - (2\alpha-25-1)F \right) \\
 &+ \sin(2\eta+2v-u) \left(- \frac{3eB}{8mg} + \frac{9eC}{16mg} - (2\alpha+25-1)H \right) \\
 &+ \sin(2\eta+2v+u) \left(- \frac{3eB}{8mg} + \frac{9eC}{16mg} - (2\alpha+25+1)J \right) \\
 &+ \sin u \left(+ \frac{27eA}{16mg} - \frac{27eA}{16mg} + \frac{9eB}{16mg} - \frac{9eB}{16mg} - \frac{3eC}{8mg} + \frac{3eC}{8mg} \right) \\
 &+ \sin(4\eta-2v-u) \left(+ \frac{27eA}{16mg} - (4\alpha-25-1)W \right) \\
 &+ \sin(4\eta-2v+u) \left(+ \frac{27eA}{16mg} - (4\alpha-25+1)X \right)
 \end{aligned}$$

+

$$\begin{aligned}
 &2\eta+2v-u) \\
 &\cos(4\eta+u) \\
 &4\eta+2v-u) \\
 &4\eta+2v+u)
 \end{aligned}$$

$$[\sin(v-u)]$$

$$[\sin(v+u)]$$

$$(-1)D)$$

$$+1)E)$$

$$i)$$

$$)$$

$$i)$$

$$)$$

$$+ \frac{3eC}{8mg})$$

+

$$\begin{aligned}
 &+ \sin(4\eta+2v-u) \left(+ \frac{9eB}{16mg} - (4\alpha+25-1)Y \right) \\
 &+ \sin(4\eta+2v+u) \left(+ \frac{9eB}{16mg} - (4\alpha+25+1)Z \right) \\
 &+ \sin(2v-u) \left(+ \frac{9eA}{16mg} - \frac{27eB}{16mg} - \frac{3eC}{8mg} - (25-1)K \right) \\
 &+ \sin(2v+u) \left(+ \frac{9eA}{16mg} - \frac{27eB}{16mg} - \frac{3eC}{8mg} - (25+1)L \right) \\
 &+ \sin(4\eta-u) \left(- \frac{9eA}{16mg} - \frac{27eB}{16mg} - (4\alpha-1)T \right) \\
 &+ \sin(4\eta+u) \left(- \frac{9eA}{16mg} - \frac{27eB}{16mg} - (4\alpha+1)V \right)
 \end{aligned}$$

vnde reperitur:

$$\begin{aligned}
 D &= -0,000010 ; & H &= +0,000001 \\
 E &= -0,000010 ; & J &= +0,000001 \\
 F &= +0,000005 ; & K &= -0,000006 \\
 G &= +0,000069 ; & L &= -0,000006 \\
 T &= +0,000004 ; & X &= -0,000022 \\
 V &= +0,000004 ; & Y &= -0,000009 \\
 W &= -0,000023 ; & Z &= -0,000009
 \end{aligned}$$

L'II.

Ponamus nunc etiam pro motu apogei

$$\phi - v = \text{Const.} + \Delta\omega$$

$$\begin{aligned}
 &+ A' \sin(2\eta-v) + B' \sin(2\eta+v) + C' \sin v + M' \sin(v-u) + N' \sin(v+u) \\
 &+ P' \sin(2\eta-v-u) + Q' \sin(2\eta-v+u) + R' \sin(2\eta+v-u) + S' \sin(2\eta+v+u) \\
 &+ D' \sin(2\eta-u) + E' \sin(2\eta+u) + K' \sin(2v-u) + L' \sin(2v+u) + O' \sin u \\
 &+ F' \sin(2\eta-2v-u) + G' \sin(2\eta-2v+u) + H' \sin(2\eta+2v-u) + J' \sin(2\eta+2v+u) \\
 &+ W' \sin(4\eta-2v-u) + X' \sin(4\eta-2v+u) + Y' \sin(4\eta+2v-u) + Z' \sin(4\eta+2v+u) \\
 &+ T' \sin(4\eta-u) + V' \sin(4\eta+u)
 \end{aligned}$$

et

et sumto differentiali pleno reperietur :

$$\begin{aligned} \frac{d\phi-dv}{du} = & \Delta + \cos(2\eta-v-u) [2eA' + (2a-6-1)P'] \\ & + \cos(2\eta-v+u) [2eA' + (2a-6+1)Q'] + (6-1)M'\cos(v-u) \\ & + \cos(2\eta+v-u) [2eB' + (2a+6-1)R'] \\ & + \cos(2\eta+v+u) [2eB' + (2a+6+1)S'] + (6+1)N'\cos(v+u) \\ & + \cos(2\eta-u) \left(-\frac{3e}{8mg}A' + \frac{3e}{8mg}B' + \frac{27e}{16mg}C' - \frac{9e}{16mg}C' + (2a-1)D' \right) \\ & + \cos(2\eta+u) \left(-\frac{3e}{8mg}A' + \frac{3e}{8mg}B' + \frac{27e}{16mg}C' - \frac{9e}{16mg}C' + (2a+1)E' \right) \\ & + \cos(2\eta-2v-u) \left(-\frac{3e}{8mg}A' + \frac{27e}{16mg}C' + (2a-26-1)F' \right) \\ & + \cos(2\eta-2v+u) \left(-\frac{3e}{8mg}A' + \frac{27e}{16mg}C' + (2a-26+1)G' \right) \\ & + \cos(2\eta+2v-u) \left(+\frac{3e}{8mg}B' + \frac{9e}{16mg}C' + (2a+26-1)H' \right) \\ & + \cos(2\eta+2v+u) \left(+\frac{3e}{8mg}B' + \frac{9e}{16mg}C' + (2a+26+1)J' \right) \\ & + \cos u \left(-\frac{27e}{16mg}A' - \frac{27e}{16mg}A' - \frac{9e}{16mg}B' - \frac{9e}{16mg}B' \right. \\ & \quad \left. + \frac{3e}{8mg}C' + \frac{3e}{8mg}C' + O' \right) \\ & + \cos(4\eta-2v-u) \left(-\frac{27e}{16mg}A' + (4a-26-1)W' \right) \\ & + \cos(4\eta-2v+u) \left(-\frac{27e}{16mg}A' + (4a-26+1)X' \right) \\ & + \cos(4\eta+2v-u) \left(-\frac{9e}{16mg}B' + (4a+26-1)Y' \right) \\ & + \cos(4\eta+2v+u) \left(-\frac{9e}{16mg}B' + (4a+26+1)Z' \right) \end{aligned}$$

+

$$\begin{aligned} & + \cos(2v-u) \left(+\frac{9e}{16mg}A' + \frac{27e}{16mg}B' + \frac{3e}{8mg}C' + (26-1)K' \right) \\ & + \cos(2v+u) \left(+\frac{9e}{16mg}A' + \frac{27e}{16mg}B' + \frac{3e}{8mg}C' + (26+1)L' \right) \\ & + \cos(4\eta-u) \left(+\frac{9e}{16mg}A' + \frac{27e}{16mg}B' + (2a-1)T' \right) \\ & + \cos(4\eta+u) \left(+\frac{9e}{16mg}A' + \frac{27e}{16mg}B' + (2a+1)V' \right) \end{aligned}$$

LIV.

Singuli iam hi termini multiplicentur per q , cuius valor quidem erit $=g$, quoniam hi termini in suo genere iam sunt minimi : sed quoniam valor $\frac{d\phi-dv}{du}$ adhuc hos terminos praecipuos continet :

$(2a-6)A'\cos(2\eta-v) + (2a+6)B'\cos(2\eta+v) + 6C'\cos v$
 si et hi per q multiplicentur, inde nascentur quoque termini angulum u inuoluentem, erit autem pro his, sumtis partibus tantum praecipuis :

$$q = \text{Praec.} + P\cos(2\eta-v-u) + Q\cos(2\eta-v+u)$$

Ergo ad illos terminos per q multiplicatos insuper accedent isti :

$$\begin{aligned} \cos u [\frac{1}{2}(2a-6)PA' + \frac{1}{2}(2a-6)QA'] & + \frac{1}{2}6PC'\cos(2\eta-u) \\ \cos(4\eta-2v-u) [\frac{1}{2}(2a-6)PA' + \frac{1}{2}6PC'] & + \frac{1}{2}6QC'\cos(2\eta+u) \\ \cos(4\eta-2v+u) [\frac{1}{2}(2a-6)QA' + \frac{1}{2}6QC'] & \\ \cos(2v+u) [\frac{1}{2}(2a+6)PB'] & + \cos(4\eta-u) [\frac{1}{2}(2a+6)PB'] \\ \cos(2v-u) [\frac{1}{2}(2a+6)QB'] & + \cos(4\eta+u) [\frac{1}{2}(2a+6)QB'] \end{aligned}$$

T c

LV.

LV.

Hinc ergo obtinentur sequentes determinaciones:

$$\begin{aligned}
 2eg A' + (2\alpha - 6 - 1)g P' &= -\frac{27e}{8m} \\
 2eg A' + (2\alpha - 6 + 1)g Q' &= -\frac{27e}{8m} \\
 2eg B' + (2\alpha + 6 - 1)g R' &= +\frac{9e}{8m} \\
 2eg B' + (2\alpha + 6 + 1)g S' &= +\frac{9e}{8m} \\
 (6-1)g M' &= -\frac{3e}{4m} ; (6+1)g N' = -\frac{3e}{4m} \\
 -\frac{3e}{8m}(A'-B') + \frac{9e}{8m}C' + (2\alpha-1)g D' + \frac{1}{2}6PC' &= 0 \\
 -\frac{3e}{8m}(A'-B') + \frac{9e}{8m}C' + (2\alpha+1)g E' + \frac{1}{2}6QC' &= 0 \\
 -\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2\alpha-26-1)g F' &= 0 \\
 -\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2\alpha-26+1)g G' &= 0 \\
 +\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2\alpha+26-1)g H' &= 0 \\
 +\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2\alpha+26+1)g J' &= 0 \\
 -\frac{27e}{8m}A' - \frac{9e}{8m}B' + \frac{3e}{8m}C' + g O' + \frac{1}{2}(2\alpha-6)(P+Q)A' &= 0 \\
 -\frac{27e}{8m}A' + (4\alpha-26-1)g W' + \frac{1}{2}(2\alpha-6)PA' + \frac{1}{2}6PC' &= 0 \\
 -\frac{27e}{8m}A' + (4\alpha-26+1)g X' + \frac{1}{2}(2\alpha-6)QA' + \frac{1}{2}6QC' &= 0 \\
 -\frac{9e}{16m}B' + (4\alpha+26-1)g Y' &= 0 ; -\frac{9e}{16m}B' + (4\alpha+26+1)g Z' = 0
 \end{aligned}$$

$$\begin{aligned}
 +\frac{9e}{16m}A' + \frac{27e}{16m}B' + \frac{3e}{8m}C' + (26-1)g K' + \frac{1}{2}(2\alpha+6)QB' &= 0 \\
 +\frac{9e}{16m}A' + \frac{27e}{16m}B' + \frac{3e}{8m}C' + (26+1)g L' + \frac{1}{2}(2\alpha+6)PB' &= 0 \\
 +\frac{9e}{16m}A' + \frac{27e}{16m}B' + (4\alpha-1)g T' + \frac{1}{2}(2\alpha+6)PB' &= 0 \\
 +\frac{9e}{16m}A' + \frac{27e}{16m}B' + (4\alpha+1)g V' + \frac{1}{2}(2\alpha+6)QB' &= 0
 \end{aligned}$$

LVI.

Valores ergo horum coefficientium iam ad minuta secunda reductorum erunt: $O' = +310''$

$$P' = -1285'' ; M' = -293'' ; F' = +401''$$

$$Q' = -1087 ; N' = -251 ; G' = +5412$$

$$R' = +148 ; D' = -52 ; H' = -2$$

$$S' = +141 ; E' = -45 ; J' = -2$$

$$W' = -91'' ; K' = +61''$$

$$X' = -95 ; L' = +61$$

$$Y' = -1 ; T' = +35$$

$$Z' = -1 ; V' = +31$$

Vnde ob excentricitatem orbitae solaris erit:

$$\xi = +0,008931 \cos 2\eta - 0,000247 \cos(2\eta - \eta)$$

$$+ 0,002090 \cos(2\eta - v) - 0,000227 \cos(2\eta + \eta)$$

$$+ 0,000640 \cos(2\eta + v)$$

$$\eta - g = -0,013595 \cos(2\eta - v) - 0,000280 \cos 2\eta$$

$$- 0,001460 \cos(2\eta + v) + 0,014012 \cos(2\eta - 2v)$$

$$+ 0,002834 \cos v - 0,000162 \cos(2\eta + 2v)$$

$$+ 0,000340 \cos 2v$$

$$- 0,000184 \cos 4\eta + 0,000200 \cos(2\eta - v - \eta)$$

$$+ 0,000882 \cos(4\eta - 2v) + 0,000360 \cos(2\eta - v + \eta)$$

$$+ 0,000000 \cos(4\eta + 2v)$$

T r 2

C ..

ADDITAMENTUM.

$$\phi - v = \text{Const.} + 0,1123\omega$$

$$\begin{aligned} & -53018'' \sin(2\eta - v) + 1533'' \sin 2\eta - 1285'' \sin(2\eta - v - u) \\ & - 5532 \sin(2\eta + v) - 53549 \sin(2\eta - 2v) - 1087 \sin(2\eta - v - u) \\ & + 10736 \sin v - 464 \sin(2\eta + 2v) + 148 \sin(2\eta + v - u) \\ & + 449 \sin 2v + 141 \sin(2\eta + v + u) \\ & - 293'' \sin(v - u) + 401'' \sin(2\eta + 2v - u) + 61'' \sin(2v - u) \\ & - 251 \sin(v + u) + 5412 \sin(2\eta - 2v + u) + 61 \sin(2v + u) \\ & - 52 \sin(2\eta - u) - 91 \sin(4\eta - 2v - u) + 35 \sin(4\eta - u) \\ & - 45 \sin(2\eta + u) - 95 \sin(4\eta - 2v + u) + 31 \sin(4\eta + u) \\ & + 310 \sin u \end{aligned}$$

neglectis scilicet terminis minimis.

LVII.

Denique pro longitudine lunae vera ϕ invenienda,
cum sit $\frac{d\phi}{d\omega} = \text{Praec.}$

$$\begin{aligned} & + mg. 0,00005 \cos(2\eta - v - u) - m. 0,00005 \cos(2\eta - u) \\ & + mg. 0,00002 \cos(2\eta - v + u) - m. 0,00002 \cos(2\eta + u) \\ & + mg. 0,00005 \cos(2\eta + v - u) - m. 0,00042 \cos(2\eta - 2v - u) \\ & + mg. 0,00002 \cos(2\eta + v + u) - m. 0,00036 \cos(2\eta - 2v + u) \\ & + mg. 0,00042 \cos(2\eta - 3v - u) \\ & + mg. 0,00036 \cos(2\eta - 3v + u) \end{aligned}$$

ponatur $\phi = \text{Const.}$

$$+ 2\omega + 2\sin v + 2\sin(2\eta - 2v) + 2\sin(2\eta - v) + 2\sin(2\eta - 3v)$$

vna cum nouis terminis

$$\begin{aligned} & + a' \sin(2\eta - v - u) + e' \sin(2\eta - u) + g' \sin(2\eta - 2v - u) + l' \sin u \\ & + b' \sin(2\eta - v + u) + f' \sin(2\eta + u) + h' \sin(2\eta - 2v + u) + m' \sin(2v - u) \\ & + c' \sin(2\eta + v - u) + i' \sin(2\eta - 3v - u) + n' \sin(2v + u) \\ & + d' \sin(2\eta + v + u) + j' \sin(2\eta - 3v + u) \end{aligned}$$

+ g'

ADDITAMENTUM.

+ v' sin(2\eta + 2v - u) + q' sin(v - u) + s' sin(4v - u)
+ p' sin(2\eta + 2v + u) + r' sin(v + u) + t' sin(4v + u)
pro reliquorum terminorum, quos forma differentialis
requirit, coefficientibus ponamus litteram l.

LVIII.

Differentiatione iam per regulas praecedentes insti-
tura erit: $\frac{d\phi}{d\omega} = \text{Praec.}$

$$\begin{aligned} & + \cos u \left(\frac{3e}{8mg} \mathfrak{B}' + \frac{3e}{8mg} \mathfrak{B}' - \frac{27e}{16mg} \mathfrak{B}' - \frac{27e}{16mg} \mathfrak{B}' + l' \right) \\ & + (2\alpha + 5 - 1) e' \cos(2\eta + v - u) \\ & + \cos(2v - u) \left(\frac{3e}{8mg} \mathfrak{B}' + \frac{9e}{16mg} \mathfrak{B}' - \frac{27e}{16mg} \mathfrak{B}' + (-5 - 1) m' \right) \\ & + (2\alpha + 5 + 1) e' \cos(2\eta + v + u) \\ & + \cos(2v + u) \left(\frac{3e}{8mg} \mathfrak{B}' + \frac{9e}{16mg} \mathfrak{B}' - \frac{18e}{16mg} \mathfrak{B}' + (2\alpha + 1) n' \right) \\ & + \cos(2\eta - u) \left(+ \frac{27e}{16mg} \mathfrak{B}' - \frac{9e}{16mg} \mathfrak{B}' - \frac{3e}{8mg} \mathfrak{B}' + (2\alpha - 1) e' \right) \\ & + \cos(2\eta + u) \left(+ \frac{27e}{16mg} \mathfrak{B}' - \frac{9e}{16mg} \mathfrak{B}' - \frac{3e}{8mg} \mathfrak{B}' + (2\alpha + 1) f' \right) \\ & + \cos(2\eta - 2v - u) \left(+ \frac{27e}{16mg} \mathfrak{B}' + 2e \mathfrak{D}' - \frac{9e}{8mg} \mathfrak{B}' - \frac{9e}{8mg} \mathfrak{B}' \right. \\ & \quad \left. + (2\alpha - 2\delta - 1) g' \right) \\ & + \cos(2\eta - 2v + u) \left(+ \frac{27e}{16mg} \mathfrak{B}' + 2e \mathfrak{D}' - \frac{3e}{8mg} \mathfrak{B}' - \frac{9e}{8mg} \mathfrak{B}' \right. \\ & \quad \left. + (2\alpha - 2\delta + 1) h' \right) \end{aligned}$$

T t 3

+

$$\begin{aligned}
& + \cos(4v-u) \left(+ \frac{27e}{16mg} \mathfrak{Z}' + (4\mathfrak{E}-1) \mathfrak{S}' \right) \\
& + \cos(4v+u) \left(+ \frac{27e}{16mg} \mathfrak{Z}' + (4\mathfrak{E}+1) \mathfrak{S}' \right) \\
& + \cos(2\eta+2v-u) \left(- \frac{9e}{16mg} \mathfrak{Z}' + (2\alpha+2\mathfrak{E}-1) \mathfrak{S}' \right) \\
& + \cos(2\eta+2v+u) \left(- \frac{9e}{16mg} \mathfrak{Z}' + (2\alpha+2\mathfrak{E}+1) \mathfrak{S}' \right) \\
& + \cos(2\eta-v-u) \left(- \frac{3e}{4mg} \mathfrak{D}' + 2e \mathfrak{S}' + (2\alpha-\mathfrak{E}-1) \mathfrak{A}' \right) \\
& + \cos(2\eta-3v+u) \left(- \frac{3e}{8mg} \mathfrak{D}' + 2e \mathfrak{Z}' + (2\alpha-3\mathfrak{E}+1) \mathfrak{F}' \right) \\
& + \cos(2\eta-v+u) \left(- \frac{3e}{4mg} \mathfrak{D}' + 2e \mathfrak{S}' + (2\alpha-\mathfrak{E}+1) \mathfrak{S}' \right) \\
& + \cos(2\eta-3v-u) \left(- \frac{3e}{4mg} \mathfrak{D}' + 2e \mathfrak{Z}' + (2\alpha-3\mathfrak{E}-1) \mathfrak{J}' \right) \\
& + \cos(v-u) \left(- \frac{27e}{8mg} \mathfrak{D}' + (\mathfrak{E}-1) \mathfrak{A}' \right) \\
& + \cos(v+u) \left(- \frac{27e}{8mg} \mathfrak{D}' + (\mathfrak{E}+1) \mathfrak{F}' \right) \\
& + \cos(4\eta-3v-u) \left(- \frac{27e}{8mg} \mathfrak{D}' + (4\alpha-3\mathfrak{E}-1) \mathfrak{I} \right) \\
& + \cos(4\eta-3v+u) \left(- \frac{27e}{8mg} \mathfrak{D}' + (4\alpha-3\mathfrak{E}+1) \mathfrak{I} \right) \\
& + \cos(3v-u) \left(+ \frac{9e}{8mg} \mathfrak{D}' + (3\mathfrak{E}-1) \mathfrak{I} \right) \\
& + \cos(3v+u) \left(+ \frac{9e}{8mg} \mathfrak{D}' + (3\mathfrak{E}+1) \mathfrak{I} \right)
\end{aligned}$$

+

$$\begin{aligned}
& + \cos(4\eta-v-u) \left(+ \frac{9e}{8mg} \mathfrak{D}' + (4\alpha-\mathfrak{E}-1) \mathfrak{I} \right) \\
& + \cos(4\eta-v+u) \left(+ \frac{9e}{8mg} \mathfrak{D}' + (4\alpha-\mathfrak{E}+1) \mathfrak{I} \right) \\
& + \cos(4\eta-2v-u) \left(- \frac{27e}{16mg} \mathfrak{S}' + \frac{27e}{16mg} \mathfrak{Z}' + (4\alpha-2\mathfrak{E}-1) \mathfrak{I} \right) \\
& + \cos(4\eta-2v+u) \left(- \frac{27e}{16mg} \mathfrak{S}' + \frac{27e}{16mg} \mathfrak{Z}' + (4\alpha-2\mathfrak{E}+1) \mathfrak{I} \right) \\
& + \cos(4\eta-u) \left(+ \frac{9e}{16mg} \mathfrak{S}' + (4\alpha-1) \mathfrak{I} \right) \\
& + \cos(4\eta+u) \left(+ \frac{9e}{16mg} \mathfrak{S}' + (4\alpha+1) \mathfrak{I} \right) \\
& + \cos(2\eta-4v-u) \left(- \frac{9e}{8mg} \mathfrak{Z}' + (2\alpha-4\mathfrak{E}-1) \mathfrak{I} \right) \\
& + \cos(2\eta-4v+u) \left(- \frac{9e}{8mg} \mathfrak{Z}' + (2\alpha-4\mathfrak{E}+1) \mathfrak{I} \right) \\
& + \cos(4\eta-4v-u) \left(- \frac{81e}{3mg} \mathfrak{Z}' + (4\alpha-4\mathfrak{E}-1) \mathfrak{I} \right) \\
& + \cos(4\eta-4v+u) \left(- \frac{81e}{3mg} \mathfrak{Z}' + (4\alpha-4\mathfrak{E}+1) \mathfrak{I} \right)
\end{aligned}$$

LIX.

Collectis hinc valoribus coefficientium assumtorum,
obtinebitur longitudo lunae vt sequitur :

Φ =

$$\Phi = C + 13,3682 u$$

$$\begin{aligned} & -22728'' \sin v & +2823'' \sin(2\eta-3v) & +100'' \sin u \\ & -1081 \sin 2v & +47 \sin(2\eta+3v) & -23 \sin(v-u) \\ & -128 \sin 3v & -246 \sin(4\eta-v) & -20 \sin(v+u) \\ & -700 \sin 2\eta & +41 \sin(4\eta+v) & +22 \sin(2v-u) \\ & -3594 \sin(2\eta-2v) & +379 \sin(4\eta-3v) & +21 \sin(2v+u) \\ & +306 \sin(2\eta+2v) & -2 \sin(4\eta+3v) & +2 \sin(3v-u) \\ & -3762 \sin(2\eta-v) & & +2 \sin(3v+u) \\ & -115 \sin(2\eta+v) & & -2 \sin(4v-u) \\ & & & -2 \sin(4v+u) \\ \\ & +17'' \sin(2\eta-u) & -1' \sin(2\eta-4v-u) \\ & +19 \sin(2\eta+u) & -1 \sin(2\eta-4v+u) \\ & +1 \sin(4\eta-u) & -11 \sin(4\eta-2v-u) \\ & +1 \sin(4\eta+u) & -10 \sin(4\eta-2v+u) \\ & +6 \sin(2\eta-v-u) & -28 \sin(4\eta-3v-u) \\ & +5 \sin(2\eta-v+u) & -23 \sin(4\eta-3v+u) \\ & +60 \sin(2\eta-2v-u) & -98 \sin(4\eta-4v-u) \\ & -194 \sin(2\eta-2v+u) & -245 \sin(4\eta-4v+u) \\ & -6 \sin(2\eta+2v-u) & +2 \sin(4\eta-v-u) \\ & -6 \sin(2\eta+2v+u) & +2 \sin(4\eta-v+u) \\ & +6 \sin(2\eta-3v-u) \\ & +8 \sin(2\eta-3v+u) \end{aligned}$$

LX.

Plurimae igitur prodeunt inaequalitates ab excen-
tricitate solis pendentes, quarum nonnullae ita sunt ma-
gnae,

gnae, ut sine notabili errore omitti nequeant; cuiusmo-
di sunt imprimis, quae ab angulis $2\eta-2v+u$ et $4\eta-4v+u$
pendent. Sed in his fere idem incommodum usu venit,
quo methodus praecedens premebatur, quod magnitudo
harum inaequalitatum per Theoriam non satis accurate
definiri queat. Cum enim pro his terminis inuenien-
dis diuisores $2a-2b+1$ et $4a-4b+1$ fiant perquam exi-
gui, manifestum est in diuidendis terminos minimos ne-
glectos non exigui fore momenti: praecipue cum pro
litteris g' et h' termini maiores fere se mutuo destru-
xissent. Vnde cum ex valore Φ tantum termini maio-
res B' , D' , G' et J' essent adhibiti, perspicuum est si
etiam reliqui minores fuissent introducti, ex iis insignem
mutationem in valore coefficientium g' et h' oriuram
fuisse.

LXI.

De his autem inaequalitatibus tenendum est, eas
per satis notabile temporis spatium vix immutari; nam
inaequalitates ab angulo $2\eta-2v+u$, ob quantitatem
 $2a-2b+1 = -0,1594$ periodum habent annorum circi-
ter $6\frac{1}{2}$ annorum, et intervallo 19 annorum ter tantum
revolvuntur: et inaequalitas ab angulo $4\eta-4v+u$ pen-
dens spatio 29 annorum 19 periodos absolvit. Ex quo
cum istae inaequalitates per theoriam saltem propemo-
dum fuerint definitae, eas deinceps per observationes
accuratius definiri conveniet: nisi forte quis laborem in
se suscipere voluerit, calculum hic adumbratum multo
accuratius instituendi terminorumque hic omissorum ra-
tionem habendi; tum vero etiam valores ξ , η et $\Phi-v$

V v

multo

multo maiori studio, quam hic feci, euolui oporteret, quoniam in horum determinatione multa neglexi, quae in calculo tandem ad notabilem quantitatem excrefcere potuissent.

LXII.

Interim tamen hic notari conuenit, hac methodo eas tantum inaequalitates prodire incertas, quae satis longis periodis absoluuntur; quae incertitudo minus officit, cum per observationes facilius emendari possit: praecedente vero methodo etiam aliae inaequalitates minoribus periodis circumscriptae aliquantum incertae prodierunt, quod sane ingens erat incommodum. Vnde ex hac parte haec methodus posterior priori antefenda videtur: verum si ingentem inaequalitatum numerum spectemus, quibus non solum lunae longitudo afficitur, sed etiam longitudo apogei, calculus tantopere fit operosus, ut etiam si has formulas accuratissime euolverem, tamen in praxi difficillimi foret usus. Quin etiam plurimae inaequalitates in motum apogei ingredi videntur, quarum effectum deinceps per alias longitudinis inaequalitates iterum destrui oportet, ita ut satius fuisset illas penitus omittere.

LXIII.

Multitudo autem harum inaequalitatum, quibus tam apogei, quam ipsius lunae longitudo turbatur, inde potissimum originem trahit, quod inaequalitates excentricitatis prae eius quantitate media admodum sint notabiles, atque adeo quadrantem mediae quantitatis superent; ita ut prae ea negligi minime queant. Multo plures autem

oporteret, lexi, quae excrefcere

methodo quae satis do minus ari possit: litates mi-
ertae pro-
1. Vnde
ri antefeu-
tum nu-
longitudo
tantopere
ime euol-
Quin
i ingredi
gitudinis
uisset illas

ibus tam
inde po-
excentri-
t notabi-
uperent;
ures au-
tem

tem adhuc inaequalitates essent accessurae, si excentricitas lunae media adhuc esset minor, quo certe casu calculi difficultates insuperabiles euasissent: hoc vero ipso casu methodus prior multo tractabilior redderetur, cum enim pleraeque inaequalitates ibi multo minores prodirent. Atque ob hanc causam minus expedire videtur, anomaliam lunae ita constituere, ut eius sinus tam pro maximis quam pro minimis distantis lunae a terra plane euanescat, etiam si haec ratio naturae rei maxime consentanea videatur.

LXIV.

Cum igitur numerus inaequalitatum iam tantopere increuerit, facile perspicitur eum adhuc multo magis auctum iri, si eas inaequalitates, quae cum a parallaxi solis, tum ab eius inclinatione ad eclipticam essent euoluturus, quo labore propterea, cum eius usus fere nullus futurus esset, superfedebat. Interim tamen hinc tantum colligere licet, inaequalitates ab angulis $27-28^\circ$ et $47-48^\circ$ ortas, minime esse contemnendas; quae cum methodo praecedente sint vel omissae vel non satis accurate determinatae, sine dubio causam in se continent, quod etiam accuratissimae tabulae per observationes emendatae adhuc ultra $4'$ saepe a veritate aberrant.

LXV.

Sufficiat igitur methodum exposuisse, cuius ope inaequalitates lunae tam ratione apogei, quam longitudinis ac latitudinis verae ex anomalia hic adhibita determinari queant; neque propterea laborem calculi reliqua

quarum inaequalitatum, quae vel ex solis parallaxi vel ex inclinatione orbitae lunaris ad eclipticam oriuntur, suscipio; quippe quarum numerus, siquidem omnes, quae alicuius momenti essent futurae, persequi vellem, in immensum excreveret. Non solum autem multitudo inaequalitatum hanc methodum omni utilitate in praxi priuabit, sed etiam ingentes aequationes, quas determinatio apogei, atque anomaliae inde pendentis requirit, ita sunt comparatae, ut ipsae iam satis exactam tam longitudinis quam anomaliae cognitionem requirant; quae res etsi initio supponi possent, deinceps iterata eadem operatione accuratius definiendae, tamen quia correctio apogei ultra 30° gradus assurgere potest, calculus ob inaequalitatum multitudinem per se taediosus, nimis crebro repeti deberet, antequam de conclusione certi esse possemus.

APPLICATIO FORMULARUM INVENTARUM AD ALIOS CALCULOS LUNARES.

LXVI.

Cum igitur calculus inaequalitatum motus lunae haecenus duplici modo sit institutus, dum priori anomalia vera regulis Keplerianis conformis est assumpta, posteriori vero ita constituta, ut eius sinus tam pro maximis lunae a terra distantis quam pro minimis prorsus euanesce-
ret, quorum uterque vi vidimus incommodis non caret: ita etiam infinitis aliis modis lunae inaequalitates praesentari poterunt, quos breuiter exposuisse haud abs-
re

vel
tur,
quae
im-
inae-
riua-
apo-
com-
uam
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ura-
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eret,

1

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nalia
riori
unae
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s re-
abs
re

re fore arbitror. Nullum enim est dubium, quin inter hos infinitos modos quidam reperiantur, qui ipsi naturae rei magis sint consentanei, neque iis incommodis laborent, quibus utrumque expositum non mediocriter impediri comperimus; etiamsi adhuc difficillimum videatur, inter hanc infinitam multitudinem modum conuenientissimum eligere.

LXVII.

Postquam autem inuestigationem ab aequationibus differentio-differentialibus ad aequationes simpliciter differentiales produximus, etiamsi ad hoc anomalia vera v cuius sinus in maximis ac minimis lunae a terra distantis euanescat, sinus v si, tamen haec conditio iam iterum exui potest. Cum enim tam sinus quam cosinus ipsius v ubique per quantitatem q sit multiplicatus, loco harum duarum variabilium q et v iam alias duas variables in calculum introducere poterimus, quod commodissime fiet ponendo $q \cos v = r$ et $q \sin v = s$, ut sit $qq = rr + ss$ et $\tan v = \frac{s}{r}$, tum enim vi formularum §. IX. exhibitarum habebimus istas aequationes:

$$d\phi = -s d\varphi + \frac{2M}{A} ds \sqrt{A} p;$$

$$ds = r d\varphi + \left(\frac{N}{A} - \frac{M_r}{A(1-r)} \right) ds \sqrt{A} p.$$

V v 3

LXVIII.

LXX.

Quodsi iam pro r statueretur iste valor $k \cos v$, ita ut k esset quantitas constans, oriretur modus initio traditus inaequalitates lunae repraesentandi, foret enim tum v anomalia vera Kepleri et k denotaret excentricitatem orbitae lunaris. Vnde patet etiam inaequalitates lunae per methodum primam erutas, ex his formulis inueniri posse, neque ad hoc aequationibus secundi gradus esse opus. Reduceretur autem hoc casu indoles differentio-differentialium ad inuentionem quantitatis s , quem in finem pro s assumi deberet series quaedam sinuum angulorum η , v , u et $\Phi - \pi$ formatorum cum indefinitis coefficientibus, quos deinceps determinare liceret: hoc autem modo solutio primum tradita esset proditura.

LXXI.

Cum sit $p = b(1 + \xi)$ et $VA = m\sqrt{b^3}$, ob $dx = -\frac{sdw}{p}VAp$, fiat $\frac{dx}{dw} = -\frac{mbs}{V(1+\xi)} = -mbs(1 - \frac{1}{2}\xi + \frac{1}{8}\xi^2)$, vnde patet si eiusmodi anomalia v introducatur, ut sit $s = q \sin v$, siue q sit quantitas constans siue variabilis, tum hanc anomaliam tam in maximis quam minimis distantis sinum euanescentem esse habituram. Ac si pro q quantitas vel constans vel ex angulis cognitis composita assumatur, tum inde coefficientes assumi ac praeterea valor litterae r determinabitur. Sin autem pro q eiusmodi quantitas incognita assumatur, ut sit praeterea $r = q \cos v$, tum solutio ante exposita resultabit.

LXXII.

LXXII.

Semper autem vsus astronomicus exigit, ut anomalia vera quaedam angulo v contenta introducatur, id quod infinitis modis fieri potest. Quo autem quantitas r variabilitatem distantiarum lunae a terra accuratius exprimat, et valor ipsius ξ quam minimas mutationes subeat, necesse est, ut quantitas r huiusmodi contineat terminum $k \cos v$, vbi k excentricitatem designet, qui sit quasi eius pars praecipua; hocque etiam locum habet, si pro r sumatur $q \cos v$, denotante q quantitatem variabilem, quippe cuius pars potior excentricitatem k praebere debet. Verum praeterea quantitas r alios terminos continere potest, qui ab angulo v vel pendeant vel non pendeant: ita poni posset: $r = k \cos v + A \cos 2\eta + B \cos 4\eta + C \cos(2\eta - v)$ etc. quo valore assumto litterae quoque s , ξ cum reliquis suis valores debitos obpinerent.

LXXIII.

Hoc modo illud incommodum evitari potest, quo methodum in hoc additamento traditam laborare vidimus, si excentricitas orbitae lunaris esset nimis parua, vel adeo euanescebat; tum enim distantiae maximae et minimae non amplius ab anomalia penderent, sed potius ab angulo η , atque imprimis quidem a cosinu dupli anguli 2η . Casu ergo quo excentricitas plane euanescebat, pro variabili r , cuius loco utique noua variabilis introduci debet, non conueniet anomaliam v introducere, sed praestabit assumi seriem cosinuum ex solis angulis 2η , u et

X x

et

et $\phi - \pi$ constantem, quorum coefficientes etsi sunt constantes, tamen quia terminorum numerus in infinitum excurrit, vicem novae variabilis sustinebunt. Tum autem valor ipsius r ex simili serie sinuum eorundem angulorum constabit.

LXXIV.

Quodsi ergo rem generatim pro quacunque excentricitate expedire velimus, poterimus ad hos terminos, qui ex hypothese excentricitatis evanescentis prodeunt, adhuc adiungere terminos ex anomalia v formatos. Ita neglectis tam inaequalitatibus parallaeticis, quam iis quae tum ab excentricitate orbitae solaris, tum ab inclinatione orbitae lunaris ad eclipticam pendent, poni conveniet:

$$\begin{aligned} r &= k \cos v + A \cos 2\eta + B \cos (2\eta - v) + C \cos (2\eta + v) \\ &\quad + D \cos 4\eta + E \cos (4\eta - v) \text{ etc.} \\ s &= \Delta t \sin v + \mathcal{A} \sin 2\eta + \mathcal{B} \sin (2\eta - v) + \mathcal{C} \sin (2\eta + v) \\ &\quad + \mathcal{D} \sin 4\eta + \mathcal{E} \sin (4\eta - v) \text{ etc.} \\ \xi &= .. \cos v + .. \cos 2\eta + .. \cos (2\eta - v) + .. \cos (2\eta + v) \\ &\quad + .. \cos 4\eta + .. \cos (4\eta - v) \text{ etc.} \end{aligned}$$

Atque si hoc modo omnes angulorum 2η et v combinationes adhibeantur, hique valores in aequationibus supra datis substituantur, primo inde elicietur ratio $dv : d\omega$, ac deinceps coefficientes determinationes suas nanciscuntur.

LXXV.

Manebunt autem coefficientes unius seriei veluti ipsius r indeterminati, propterea quod ipsa series haec

ab

constan-
excurrit,
valor ipsi-
constabit.

e excen-
terminos,
prodeunt,
cos. Ita
iis quae
inclinatione
veniet:

$$\begin{aligned} &\cos (2\eta + v) \\ &\eta - v) \text{ etc.} \\ &n (2\eta + v) \\ &\eta - v) \text{ etc.} \\ &\cos (2\eta + v) \\ &\eta - v) \text{ etc.} \end{aligned}$$

mbinatio-
bus supra
: $d\omega$, ac
iscuntur.

iei veluti
ries haec
ab

ab arbitrio nostro pendet, dum pro r vel solum primum terminum $k \cos v$, vel quotquot lubuerit, assumere potuissimus. Hinc autem id commodi consequemur, ut istos coefficientes ad scopum quam convenientissime definire valeamus: scilicet eos ita definiri conveniet, ut primo nullius reliquorum coefficientium determinatio lubrica et incerta euadat, uti in veraque methodo exposita vsu venit: deinde vero ut nulli coefficientes fiant nimis magni praeter necessitatem, ita ut eorum effectus per alios terminos iterum destrui necesse sit. Fateri quidem cogor calculum hoc modo instituendum admodum futurum esse prolixum, verum fortasse in ipsa operatione non contemnenda se offerent compendia; vade confido hanc speculationem, etiam si mihi ipsi eam suscipere non vacet, vsu non esse carituram.

BEROLINI, EX OFFICINA MICHAELIS.

6. 7 (-9)