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Number Theory and More

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Welcome to Issue 1 of Volume 4 of *Euleriana*.

This issue's Translation & Commentary section includes Erik Tou's translation of Lagrange's number-theoretical treatise "Recherches d'Arithmétique", or "Research on Arithmetic." In this work, Lagrange investigated the representation problem of binary quadratic forms, that is, determining which integers may be represented by a given form.

The second translation in this issue comes from Georg Ehlers, and covers Euler's "De casibus quibus hanc formulam $x^4+kxxyy+y^4$ ad quadratum reducere licet" (E696; "On the cases in which the form $x^4+kxxyy+y^4$ can be reduced to a square"). In this work, Euler solves the title problem by choosing particular forms for the coefficient k. He goes on to list tables of several possible solutions.

As in our previous issue, the Articles & Notes section contains a trio of papers by Alexander Aycock, with a common thread of mining the potential of Euler's works to see what his next steps could have been, and how he anticipated future advances in mathematics.

The first of these describes how a method from Euler's "De fractionibus continuis observationes" (E123; "Observations on continued fractions") can be used to solve homogeneous difference equations with linear coefficients, along with a connection to Legendre polynomials.

The second article shows that formulas found in Euler's "Plenior expositio serierum illarum memorabilium, quae ex unciis potestatum binomii formantur" (E663; "A more thorough exposition of those memorable series that are formed from the binomial coefficients") could have led directly to the Gaussian sum-

mation formula for the hypergeometric series.

The third article in this section lays out how Euler's work can be used to prove a functional equation equivalent to the Riemann zeta-function. This was a conjecture Euler made in "Remarques sur un beau rapport entre les series des puissances tant directes que reciproques" (E352; "Remarks on a beautiful relation between direct as well as reciprocal power series").

In our ongoing Euler Archive Spotlight series, Christopher Goff highlights the many search options available on the Euler Archive. In addition to the familiar Eneström index, the Euler Archive includes the ability to search by source publication name, date written, or decade of publication.

It's hard to believe that we launched *Euleriana* over three years ago, in the heart of the Covid-19 pandemic. Thank you for your continued interest and support for this work. As always, if you have ideas or articles to submit for subsequent issues, please let us know; we are always accepting submissions.

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