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Euler's Anticipations

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Euler's Anticipations

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Welcome to Volume 3 of *Euleriana*. This issue highlights occasions where Euler's work anticipated future results from other others, sometimes by decades or even centuries!

This issue's [Translation & Commentary](#) section includes an English translation of "Observationes analyticae" (E326, "Analytical Observations") by Cynthia Huffman. The titular analysis begins with an infinite series whose coefficients are central trinomial coefficients: 1, 1, 3, 7, 19, 51, etc. As the name suggests, the coefficients arise from the central terms obtained by expanding trinomials of the form $(1 + x + x^2)^n$. Euler successfully identifies the recursion relationship for these coefficients, and then generalizes his work to include trinomials of the form $(a + bx + cx^2)^n$. Interestingly, Euler also derives a sequence related to the Fibonacci sequence that appears to match the sequence above, but fails starting at the 10th entry.

The first contribution in the [Articles & Notes](#) section connects Euler's work on infinite series to the modern theory of *binary digit extraction formulas*: series that allow the user to calculate any digit in the binary expansion of π without needing to calculate any previous digit. Nick Craig-Wood demonstrates that two of the infinite series appearing in Euler's "De novo genere serierum rationalium et valde convergentium, quibus ratio peripheriae ad diametrum exprimi potest" (E706, "On a new type of rational and highly convergent series, by which the ratio of the periphery to the diameter is able to be expressed") can be written in such a way to align with other, more recent formulas of this type.

Next, Alexander Aycock (building on prior work in *Euleriana*) demonstrates that the duplication formula for the gamma function, though attributed to Gauss, can also be deduced from formulas that appeared in Euler's "Variae

considerationes circa series hypergeometricas” (E661, “Several considerations about hypergeometric series”).

The final article in this section highlights Euler’s participation in the dissemination of scientific articles to the broader community. In particular, Michael Saclolo writes about how Euler translated two memoirs concerning the purported discovery of a moon of Venus. These articles were written by Armand Henri Baudouin de Guémadeuc and based on observations by Jacques Montaigne. In addition to translating the works into German, Euler also provided expert commentary and context, all while remaining professionally skeptical about the supposed moon, perhaps anticipating the reality of its non-existence.

In our ongoing [Euler Archive Spotlight](#) series, Erik Tou surveys two recent translations in the Euler Archive that have yet to appear in *Euleriana*. First of these is Mark Snively and Phil Woodruff’s translation of “Solutio problematis de investigatione trium numerorum, quorum tam summa quam productum nec non summa productorum ex binis sint numeri quadrati” (E270, “The Solution of a Problem of Searching for Three Numbers, of Which the Sum, Product, and the Sum of Their Products Taken Two at a Time, Are Square Numbers”). Euler’s solutions to the title problem involve very large numbers, with the smallest solution consisting of 12- and 13-digit numbers.

The second translation featured in the Euler Archive Spotlight is Stella Li’s “Theoremata circa residua ex divisione potestatum relicta” (E262, “Theorems about the residues left by division by powers”), which supplements Euler’s argument with modern notation and exposition, showing how this paper provides the theoretical underpinning of unit groups modulo n .

If you have ideas or articles to submit for subsequent issues, please let us know; we are always [accepting submissions](#).