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Review of *The History of Mathematics: A Source-Based Approach* (Vol. 2), Part I

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Abstract

Review of Part I from *The History of Mathematics: A Source-Based Approach* (Vol. 2), by June Barrow-Green, Jeremy Gray, and Robin Wilson. MAA Press, 2022, 330 + xiv pages.

Introduction

The history of mathematics can often seem to be a vast, roiling sea, with so many ideas crashing together that it is difficult to discern the currents that lie underneath. Moreover, the depths required to carry out an historical analysis can make the whole endeavor feel especially daunting. It is a delight, then, to have a comprehensive accounting of many significant developments in mathematics that retains a connection to the original authors and their own ideas on their work. This review concerns the contents of the first half of Volume 2 [2] (covering the 17th and 18th centuries), which is most likely to interest readers of *Euleriana*. Book I (which may be more clearly thought of as Part I of Volume 2) occupies pages 7–330 of Volume 2.

As the authors indicate, this work is the second in a two-volume set used at Open University for its year-long course on the history of mathematics. Volume 1 [1], published by MAA Press in 2019, is an accounting of mathematical contributions before 1650. What makes this text unique is its *methodology*, which I perceive to have two significant dimensions. First, as the title suggests, considerable attention is given to original source materials and their presentation within the narrative. Second, this text is decidedly *historical* in its approach, reflecting the authors' own status as historians of mathematics, rather than mathematicians who study its history.[a]

The devotion to source material has benefits and drawbacks, though the benefits clearly win the day. Each section contains extensive excerpts from the

[a]This distinction is perhaps more significant for educators in the United States, where many history of mathematics courses use the latter approach.

historical record, with single passages often taking up multiple pages in the text. The benefit should be clear: readers can examine an historical text in its original form and on its own terms. In fact, the sheer number of original sources quoted in this book makes it a valuable resource for any historian of mathematics. One drawback is that this choice requires the use of each historical author's original notation, which can take the reader considerable time to fully parse. Fortunately, the authors include brief recapitulations after and between these excerpts, and provide boxed expositions in which problems are explained with more modern language and notation. In general, I recommend that readers not strain too much to understand an original source on its first reading, waiting until after reading these additional items before attempting a full examination.

The historical approach is seen most clearly in the student exercises presented in the final chapter. These exercises are sorted into three types: (A) writing responses to specific quotations by mathematicians and historians, (B) essay questions pertaining to specific chapters from the book, and (C) more general essay questions that require a synthesis of materials and ideas from across the book. None of these exercises ask the student to complete a mathematics problem, and, as the authors state in the introduction, this is a deliberate choice. They write:

> We are interested in who did the mathematics, and why? Were they teachers—and if so, who were their students and why were they there? Was there a cultural of philosophical dimension to their mathematical work? What does it mean to *discover* something in mathematics? How was mathematical knowledge disseminated? Surprisingly rich answers to questions such as these can be obtained without one having to master the accompanying mathematics. What was done is interesting, but why it was done is interesting too.

These questions provide something of a mission statement for the book—they lead the authors to a deep study of the historical source material, and while there are some limitations to the book as a specific work (more on that momentarily), the approach taken here is a valuable one.

The Emergence of the Calculus

In Volume 2 the story begins around 1650, following the death of René Descartes (1596-1650), and not long before Isaac Newton (1643-1727) and Gottfried Leibniz (1646-1716) began their work on the Calculus. After an introductory chapter, the authors focus the following four chapters on the contributions of these

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two men (and several of their contemporaries) toward a coherent view of tangents and areas. Of particular interest is the extent to which European mathematicians of this period drew on their geometric heritage, both ancient and modern, to apply their new methods. For example, the reader can see how Leibniz sought a solution to an inverse tangent problem posed some 40 years earlier by Florimond Debaunne (b. 1601). Following in the tradition of Descartes, Debaunne stated his problem in entirely geometric terms, essentially seeking a recipe with which to construct a desired curve. The curve in question needed to have the property that the slope of the tangent at a point X be inversely proportional to the difference between X's ordinate and abscissa. As the authors note, Gilles de Roberval (1602-1675) and Descartes himself were only able to describe this curve approximately, which is where the problem stood for a generation. When the narrative reaches Leibniz's own work in the 1670s, we see that, while he translated the problem into the language of the Calculus and made significant progress toward solving it, at the end he felt compelled to return the matter to the geometric language in which it was originally posed.[b]

The authors also take care to assess the character of Newton and Leibniz's work as it matured in the latter decades of the 17th century. In particular, Chapter 5 is devoted to Newton's *Principia Mathematica*, in which Newton propounded the concept of *forces*, in opposition to Descartes' theory of vortices. In the authors' own words, "Newton's concept of force is novel, vastly more general than anything discussed by Galileo or Huygens, and fundamental to the *Principia*; the work is about forces at least as much as it is about the motion of physical objects." We then see Newton link the inverse square law for gravitation to Kepler's laws of planetary motion, and demonstrate that his theory of forces can be made to account for the motions of all the planets in their orbits. Interestingly, we also see how the Calculus—now considered so central to Newton's work on physics—does not play as central a role in the *Principia* as one might expect. The full working-out of this connection was left to scholars of subsequent generations.

The Eulerian Century

Chapters 6 to 11 are devoted to the work of 18th century scientists and mathematicians, with Leonhard Euler (1707-1783) being the most prominent among them. Chapter 6 chronicles the spread of the Newtonian and Leibnizian flavors of the Calculus across Europe, with important contributions by Johann

[b]The authors eventually demonstrate, in an argument illustrating the Fundamental Theorem of Calculus, that this curve can be expressed via logarithms.

Bernoulli (1667-1748), Pierre-Louis Moreau de Maupertuis (1698-1759), Émilie du Châtelet (1706-1749), and others. Chapters 7, 8 and 9 examine the century's many advances in algebra, number theory, and geometry, along with some early work on the foundations of the Calculus. Finally, Chapters 10 and 11 consider a number of scientific applications, including the vibrating string problem, the motion of solid bodies and fluids, and orbital mechanics.

It is in these middle chapters that Euler's influence can be most keenly felt. The reader is treated to Euler's lucid writing style, including a classification of conic sections, the foundations of infinitesimal calculus, and trigonometric functions. In addition, some of Euler's specific contributions to infinite series, number theory, topology, and differential equations are described in detail. Overall, the authors paint a portrait of a mathematician aware of—though not particularly troubled by—the philosophical questions raised by algebra and infinitesimal calculus, who was willing to follow the rules and relations in these areas to identify the mathematical and physical laws at hand. This transition away from the Cartesian geometry of the previous century and toward the algebraic and analytical methods exemplified by Joseph-Louis Lagrange (1736-1813) can be seen most clearly in the work of Euler.

Of course, numerous other contemporaries are featured in this narrative, including not only Lagrange but also Brook Taylor (1685-1731), Jean le Rond d'Alembert (1717-1783), Pierre-Simon Laplace (1749-1827), and Mary Somerville (1780-1872), among others. Among the numerous applications considered in Chapters 10 and 11, the vibrating string problem and the stability of the solar system are worthy of particular note here. In the former case, one sees how the theory of partial differential equations evolved alongside the solution, ultimately leading to the consideration of trigonometric series. In the latter case, Laplace's analysis of the relative motion of the planets was a triumph of Newtonian mechanics, and serves as an epilogue to the story that begin in Chapter 5.

And there is much more! Together, both volumes comprise approximately 1100 pages, of which this review is only concerned with 330 pages. Any interested party is encouraged to begin at the start of Volume 1, though a reading of this volume is not necessary to appreciate and understand the work presented in Volume 2.

Final Thoughts

Before concluding, it is worth noting the limitations of this project as a whole. As is often the case, the history of mathematics is presented here as a largely

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European story dominated by men. Given that many societies are patriarchal in nature, and given the history of colonialism and domination by European societies, this is to some extent unavoidable (at least in narratives focused on the modern period). However, I would have appreciated more time devoted to the multiple contributions of those outside this narrative. To paraphrase one of the authors' original questions: Who did *not* do the mathematics, and why? The presence of illustrious women mathematicians in the narrative gives some hint at an answer to such questions, but it is not made particularly clear in this work. At minimum, some thought-provoking questions along this line could have been included in the exercises.

I view this limitation more as a missed opportunity than a deliberate choice to exclude, especially since Volume 1 does include some important mathematical contributions from India, China, and the Islamic world. Nevertheless, the methodology is one that deserves more of a presence on university campuses, where mathematics can so often be employed in a way that is divorced from broader historical and societal questions. In the end, the authors' tenacity and devotion to the original sources of the period make it well worth reading.

References

[1] Barrow-Green, J., Gray, J., Wilson, R. (2019). *The History of Mathematics: A Source-based Approach*, Vol. 1. Providence, RI: American Mathematical Society.

[2] Barrow-Green, J., Gray, J., Wilson, R. (2022). *The History of Mathematics: A Source-based Approach*, Vol. 2. Providence, RI: American Mathematical Society.