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Review of *Change and Variations*

Abstract

Review of *Change and Variations: A History of Differential Equations to 1900*, by Jeremy Gray, Springer Undergraduate Mathematics Series, 2021, 419 + xxii pages.

Jeremy Gray of the Open University has just published a wonderful history of differential equations. In 2018 he published *A History of Abstract Algebra* and this new history textbook, published in 2021, is something of a companion volume. The two books are very similar in most aspects except for the mathematical content. Each one is based on a second course on the history of mathematics, which seem to be offered as undergraduate courses at the Open University. Each one consists of 30 Chapters, which roughly correspond to the thirty lectures that make up each of these two courses. Both books include two types of homework assignments: there are Exercises, which are short, mathematical, and technical, and Questions, which are open-ended, generally more historical, and could be used as essay prompts. Both books naturally fall into three Parts, each one concluding with a Chapter called "Revision and Assessment." Finally, both books contain dozens of pages of translated passages from key papers, original source material that is not available elsewhere in English.

Change and Variations is a thorough and quite complete account of the history of differential equations, both ordinary and partial. Gray confesses to its incompleteness, but it's likely that only an expert researcher in the field would find anything to be lacking. It is particularly good to see such a complete and accessible history of PDEs in the 19th century. I don't believe there is anything quite like last two-thirds of this book available anywhere in English.

I will focus this book review on the first Part of the book, which covers the period up to roughly 1800 and is largely devoted to the work of Leonhard Euler (1707-1783) and his contemporaries. It's my hope to publish a different review of the latter two-thirds of Gray's book – the history of ODEs and PDEs in the 19th century – in another journal.

Arguably, the study of differential equations begins with the work of Leonhard Euler during his first Petersburg period, but Gray begins the narrative in 1638, with Florimond de Baune's (1601-1652) famous problem, asking for the curve whose tangents satisfy a given condition. This quickly leads to a survey of some of the work of Gottfried von Leibniz (1646-1716) and the Bernoulli brothers [Jacob (1655-1705) and Johann I (1667-1748)] during the early period of the history of calculus. Before long, though - on page 8 in fact - Euler makes the first of his many appearances in the book. Section 1.4 begins with the sentence "We turn now to study how Euler rewrote the calculus."

Gray begins with some general remarks about Euler's understanding of formal expressions and functions and how this led to an algebraic rather geometric conception of the calculus. He then moves on to examine the general solution of the linear ordinary differential equation, as presented in *De integratione* *aequationum differentialium altiorum graduum* (E62) and eventually in the *Institutionem Calculi Integralis*, volumes 1 and 2 (E342 and E366). He illustrates Euler's evolving ideas on the problem with excerpts from his correspondence with Johann I and Daniel Bernoulli (1700-1782).

There are two major threads running through the remainder of the first Part of *Change and Variations*, that is, running through Chapters 2-8. The larger one is partial differential equations, but there is is also a presentation of the calculus of variations.

The study of problems whose solution is a curve satisfying a given maximal or minimal condition dates back to the late 17th century, particularly with the Brachistochrone Problem, which Johann I Bernoulli proposed as follows:

> Given points A and B in a vertical plane, to find the path AMB down which a movable point M must, by virtue of its weight, proceed from A to B in the shortest possible time.

The story of this challenge problem, and particularly of Newton's (1643-1727) "claw of the lion" solution, can be found in most general textbooks in the history of mathematics. Here, Gray first discusses the work of Galileo (1564 - 1642) and Pierre de Fermat (1601 - 1665) that Johann I Bernoulli relied upon in his solution. Then, in Section 2.3, he presents Johann's solution to the problem. This is the clearest exposition of Bernoulli's solution that I have seen anywhere in print. However, he also mentions some details of Jakob Bernoulli's alternate solution, which seems to have been more influential on Euler's approach to variational problems.

The study of partial differential equations begins in the 1740s with Jean d'Alembert's (1717-1783) solution to the problem of the vibrating string. Gray devotes his third Chapter to this breakthrough, contextualizing it beforehand with some earlier work by Nicholas I Bernoulli (1687 - 1759), Euler and Alexis Clairaut (1713 - 1765). The problem is then described and Gray examines d'Alembert's original solution in detail. Of course, Euler rewrote the problem and recast the solution; the details are included in the same Chapter, along with the methods introduced by d'Alembert in his 1752 hydrodynamics book.

Euler's use of partial differential equations in rational dynamics is the next step in the evolution of PDE's. Gray analyzes the PDE's that appear in Euler's hydrodynamics as presented in three papers volume 12 of the Berlin *Mémoires* (E225-E227), and later in *Principia motus fluidorum* (E258); in his study of the propagation of sound (*De la propagation du son*, E305); and in *Découverte d'un nouveau principe de mécanique* (E177), his general approach to mechanics. These particular examples thereby understood, Gray devotes the next Chapter to Euler's general theory of PDE's, given in the third volume of *Institutionem Calculi Integralis* (E385). This sets the stage for a Chpater on Joseph-Louis Lagrange's (1736 - 1813) general theory of PDE's, given in three papers in the Berlin *Mémoires* in the 1770s.

Lagrange and Euler having now been placed in the history of PDE's, Gray devotes a Chapter to their calculus of variation, the d-δ formalism and the Euler-Lagrange Equations. The 18th century comes to a close in the next Chapter with a survey of Gaspard Monge's (1746 - 1818) writings on both ODE's and PDE's.

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At this point in teaching his course, Prof. Gray pauses to review the material that has been presented so far (about 110 pages) and assigns the following essay to the student/reader:

> Imagine you are a British professor of mathematics in about the year 1770 who is recommending a good student to spend a year studying with either Euler or Lagrange. Explain to him or her:
>
> - EITHER In what ways is Euler"s theory of mechanics an improvement on Newton's *Principia*.
> - OR What is involved in the study of partial differential equations.

This first Part of Jeremy Gray's *Change and Variations* is an excellent survey of the development of differential equations and the calculus of variations in the 18th and late 17th centuries. Because of Euler's central role in this evolution, it is essential reading for Eulerians. Many of the results presented here, and the stories recounted here, can be found elsewhere in the secondary literature, but here they constitute a coherent whole, with appropriate historical and mathematical context provided throughout.

References are extensive, offering researchers starting points for further investigation. There are 278 references provided in this book, with no fewer than 30 of Euler's articles and books cited, from E9 to E710.

As a postscript, I'd like to note that Gray's 2018 *A History of Abstract Algebra* is also a volume of interest for Eulerians. Because of algebra's roots in algebraic number theory, Euler similarly makes contributions to the early Chapters of this history, although not to the same extent as he does in *Change and Variations*. His number theory is to be found in the first two Chapters and four of his papers are cited.