2021

Spotlight on the Euler Archive

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To complement the tribute to C. Edward Sandifer included in this issue of *Euleriana*, this issue's Euler Archive Spotlight features works by Dr. Sandifer. His contributions to Eulerian scholarship are so numerous that it is impossible to include everything in this short note. Therefore, we have selected three samples to highlight: two translations and the *How Euler Did It* series.

Currently, the Euler Archive contains one translation by Ed Sandifer. It is an English translation, with additional notes, of Euler's article [E616] *De Transformatione Seriei Divergentis* $1 - mx + m(m+n)x^2 - m(m+n)(m+2n)x^3 + m(m+n)(m+2n)(m+3n)x^4$. *etc. in Fractionem Continuam* (*On the Transformation of the Divergent Series* $1 - mx + m(m+n)x^2 - m(m+n)(m+2n)x^3 + m(m+n)(m+2n)(m+3n)x^4$ *etc. into a Continued Fraction*).

In this article, Euler explains that transforming a specific hypergeometric series into a continued fraction led him to consider a more general case. Previously, in [E71], Euler had used the Riccati equation for the transformation into a continued fraction. In E616, which he presented to the Academy of Sciences in St. Petersburg on 11 January 1776 and which appeared in Volume 2 of *Nova Acta Academiae Scientiarum Imperialis Petropolitanae* in 1788, Euler gives a simpler method based on some strategic substitutions. As a result, Euler obtains a continued fraction representation of $\pi / 4$.

Another Euler translation by Dr. Sandifer, joint with Dr. Robert Bradley, appears in the MAA book by Ed Sandifer, titled *The Early Mathematics of Leonhard Euler*. This work is not a typical research article by Euler; rather it is an essay on the usefulness of higher mathematics. Penned in 1741, it did not appear in print until over a century later in 1847, when the essay was published in Volume 35 of *Journal fur die reine und angewandte Mathematik*. "Commentatio de matheseos sublimioris utilitate," which translates as "On the utility of higher mathematics," is available in the Euler Archive as E790. According to Sandifer,

> *Euler tries to convince the reader that, though many useful problems can be solved using just the "inferior parts of mathematics," arithmetic and algebra, some important problems require higher mathematics, especially calculus. He gives examples from mechanic, hydraulics, astronomy, artillery and navigation, without going into technical details. It is a simple and straightforward explanation of what Euler thought he was accomplishing with the way he did science and why he thought it was important. It is both reflective and forward-looking.*

An indispensable contribution to Eulerian scholarship by Ed Sandifer is the monthly column *How Euler Did It* he authored for the MAA from November 2003 to February 2010. Copies of these articles appear on the Euler Archive website at http://eulerarchive.maa.org/hedi/index.html. The columns also appear in print in two books from the Spectrum Series of MAA Press: *How Euler Did It* and *How Euler Did Even More*. The first book was published in 2007 as Volume 3 in the MAA special 5-volume set in celebration of the 300th anniversary of Euler's birth. It contains Dr. Sandifer's first 40 *How Euler Did It* articles. It also contains a helpful index that references Eneström numbers. So, a scholar interested in a particular work by Euler can easily find the related *How Euler Did It* column(s) to gain insight. The second book appeared in 2015 and contains the final 35 *How Euler Did It* columns, 32 of which were written by Ed Sandifer, 2 by Robert Bradley, and 1 by Dominic Klyve. While the pieces are organized by date on the Euler Archive, in the two books, they are organized by mathematical topic. It is interesting to note that a search for "Sandifer" in the Euler Archive reveals several translations in which the translator referenced one of Dr. Sandifer's columns.

Each self-contained *How Euler Did It* column focuses on a topic from one or more of Euler's works and the commentary is supplemented with images, equations, and quotes. In the introduction of the first book, Dr. Sandifer explained that he tried, "to follow the advice of the late John Fauvel and make sure that each column includes 'Content, Context and Significance.'" A sample of *How Euler Did It* columns are "Who Proved *e* is Irrational" (which references E71

from above), "The Euler-Pythagoras Theorem", "Gamma the Function", and "Euler and the Pirates".