2021

Euler Archive Spotlight

Cynthia J. Huffman Ph.D.
Pittsburg State University, cjhuffman@pittstate.edu

Spotlight on the Euler Archive

Cynthia Huffman
Pittsburg State University
cjhuffman@pittstate.edu

Of the 850+ works by Euler, the one which has been downloaded from the Euler Archive the most is *Solutio problematis ad geometriam situs pertinentis* (*The solution of a problem relating to the geometry of position*) [E53], Euler's paper on the famous Königsberg bridge problem. Downloaded over 2,200 times in the last two years, it also has been the focus of more than 30 tweets. Currently there is no English translation of E53 available on the Euler Archive, though one by Michael Behrend can be found on his web site. Other English translations appear in at least two books, namely *The World of Mathematics* by James R. Newman [2, pp. 573-580] and *Graph Theory 1736-1936* by Norman Biggs, E. Keith Lloyd, and Robin J. Wilson (pp. 3-8). This particular paper by Euler has been credited as the genesis of graph theory.
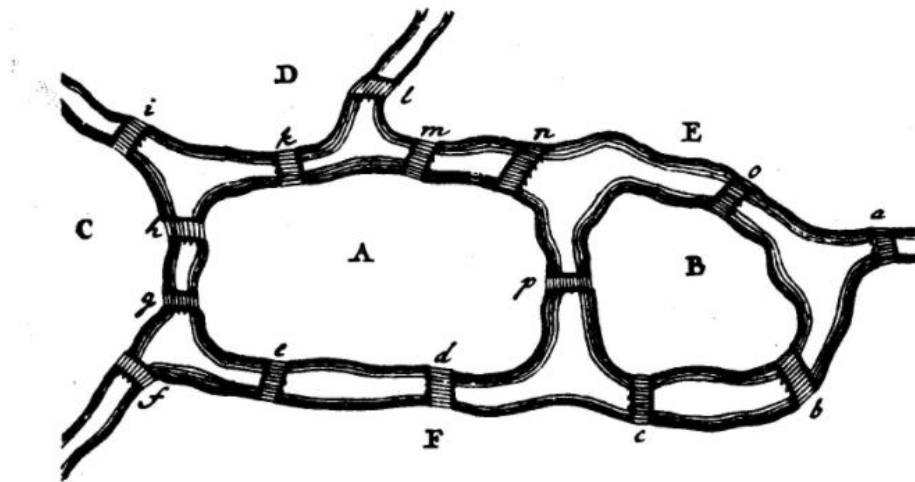

Figure 1: The illustration of the Königsberg bridge problem from E53.

One new addition to the Euler Archive is an English translation of *Problematis cuiusdam Pappi Alexandrini construction* (*The construction of a certain problem of Pappus of Alexandria*) [E543] by Cameron Friend, Research Fellow at Quest University, and Cynthia Huffman, University Professor at Pittsburg State University. The focus of E543 is Proposition 117 of Book 7 of Pappus' Collection. The problem of Pappus is, given a circle and 3 collinear points outside the circle, to inscribe a triangle on the circle, such that when the sides are extended, they pass through the three collinear points. In his paper, Euler

generalized the problem to the situation where the three points are not necessarily collinear. He provided a construction of the desired inscribed triangle, followed by two corollaries for when 2, and then all 3, of the points lie infinitely far from the circle. Euler concluded with a comment about the same problem on a sphere. In the paper immediately following E543 in Volume 4 of the *Acta Academiae Scientarum Imperialis Petropolitanae*, Euler's assistant Nikolai Fuss gave the context of the problem and a construction similar to Euler's except he used a second concentric circle instead of the cosine function. For an analysis of E543 and the accompanying article by Fuss, see Cameron Friend's Quest University Research Fellowship report.
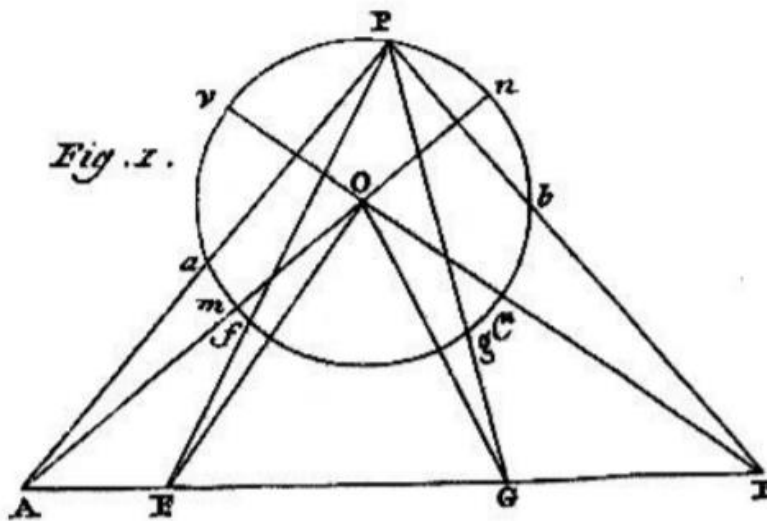

Figure 2: The first figure describing a problem of Pappus, from E543.

More recently, a translation of *Theorematum quorundam arithmeticorum demonstrationes* [E98] has been added to the Euler Archive. This English translation, *Proof of certain arithmetic theorems*, is by Paul R. Bialek of Trinity International University. German and Portuguese translations of E98 are also available on the Euler Archive. E98 is the first paper in which Euler began to investigate the list of Fermat's number theory claims. In subsequent works, Euler would go on to either prove or disprove the rest of Fermat's claims, except for the one now known as Fermat's Last Theorem, which wasn't settled until Andrew Wiles's 1995 proof. Among the many results contained in E98 is Euler's proof of the $n = 4$ case of Fermat's Last Theorem. Interestingly, E98 is the first of Euler's number theory papers written in the Theorem-Proof style, which eventually became the norm for mathematical papers. For more information on E98, see Chapter 37 of *The Early Mathematics of Leonhard Euler* by C. Edward Sandifer, published as part of the MAA Tercentenary Euler Celebration.
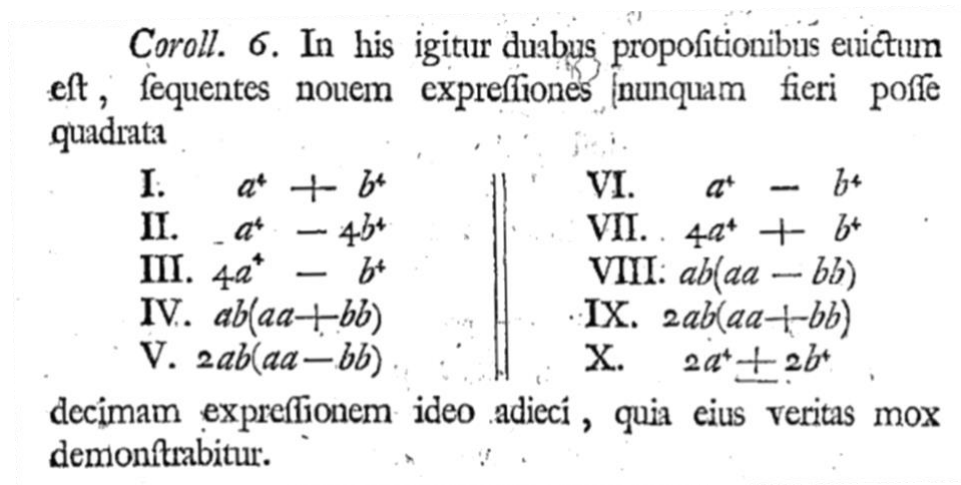
Figure 3: Corollary 6 from E98, showing several results Euler proved after tackling the list of Fermat's claims.

References

[1] Friend, Cameron. "Carrying Across: A Translation and Analysis of Leonhard Euler's Text 'Problematis Cuiusdam Pappi Alexandrini Constructio,' and the Accompanying Text by Nicolaus Fuss."

[2] Newman, James R. *The World of Mathematics*, *Vol 1.* Dover Publications, 1956.

[3] Biggs, Norman, Lloyd, E. Keith, and Robin J. Wilson. *Graph Theory 1736-1936*. Oxford University Press, 1976.

[E53] Euler, Leonhard. "Solutio problematis ad geometriam situs pertinentis," *Commentarii academiae scientiarum Petropolitanae*, Volume 8, pp. 128-140. Available online at the Euler Archive.

[E98] Euler, Leonhard. "Theorematum quorundam arithmeticorum demonstrationes," *Commentarii academiae scientiarum Petropolitanae*, Volume 10, pp. 125-146. Available online at the Euler Archive.

[E543] Euler, Leonhard. "Problematis cuiusdam Pappi Alexandrini constructio," *Acta Academiae Scientiarum Imperialis Petropolitanae*, Volume 1780: I, pp. 91-96. Available online at the Euler Archive.